

To identify the hidden car efficiently, I'll ask binary questions that maximize information gain, aiming to split the 94 cars into roughly equal groups to reduce entropy significantly each step. I'll calculate the belief distribution and entropy drop after each answer. Entropy is computed as $H = -\sum p_i \log_2 p_i$, where p_i is the probability of each car. Initially, with 94 cars, each has a probability of $\frac{1}{94} \approx 0.01064$, and the entropy is:

$$H_0 = -94 \cdot \left(\frac{1}{94} \log_2 \frac{1}{94}\right) = -\log_2 \frac{1}{94} = \log_2 94 \approx 6.555 \text{ bits}$$

Question 1: Is the hidden car a sedan?

Please answer "Yes" or "No," and I'll update the belief distribution and calculate the entropy drop.

Yes.

Thank you for answering that the hidden car is a sedan. Let's update the belief distribution and calculate the entropy drop based on this information.

Step 1: Update Belief Distribution

From the dataset, I identify the cars that are sedans (i.e., **sedan: True**). The cars with this attribute are:

1. Tesla Model S
2. Toyota Corolla
3. BMW 3 Series
4. Honda Civic
5. Chevrolet Bolt
6. Mercedes-Benz S-Class
7. Toyota Prius
8. Audi A4
9. Toyota Camry
10. Nissan Altima
11. Nissan Leaf
12. Hyundai Sonata
13. Tesla Model 3
14. Honda Accord
15. Hyundai Elantra
16. Hyundai Ioniq 6
17. Cadillac CTS
18. Rolls-Royce Phantom

19. Infiniti Q50
20. Genesis G70
21. Alfa Romeo Giulia
22. Lucid Air
23. Polestar 2

There are 23 sedans out of the original 94 cars. Thus, the new belief distribution assigns equal probability to each of these 23 cars:

- Probability per sedan: $\frac{1}{23} \approx 0.04348$
- Probability for non-sedan cars: 0 (since they are eliminated)

Entropy Calculation

The entropy of the new distribution is:

$$H_1 = -23 \cdot \left(\frac{1}{23} \log_2 \frac{1}{23} \right) = -\log_2 \frac{1}{23} = \log_2 23$$

To compute $\log_2 23$:

$$\log_2 23 = \frac{\log_{10} 23}{\log_{10} 2} \approx \frac{1.3617}{0.3010} \approx 4.524 \text{ bits}$$

Initial entropy (before the question):

$$H_0 = \log_2 94 \approx \frac{\log_{10} 94}{\log_{10} 2} \approx \frac{1.9731}{0.3010} \approx 6.555 \text{ bits}$$

Entropy Drop

The entropy drop is the reduction in uncertainty:

$$\Delta H = H_0 - H_1 \approx 6.555 - 4.524 \approx 2.031 \text{ bits}$$

Current Belief Distribution

Each of the 23 sedans has an equal probability of approximately 0.04348 (or $\frac{1}{23}$). The remaining 71 cars have a probability of 0.

Question 2

To further narrow down the possibilities, I aim to ask a question that splits the 23 sedans into roughly equal groups for maximum information gain. A good attribute to consider is **electric**, as it distinguishes electric vehicles from non-electric ones. Counting the sedans:

- Electric sedans (**electric: True**): Tesla Model S, Chevrolet Bolt, Nissan Leaf, Tesla Model 3, Hyundai Ioniq 6, Lucid Air, Polestar 2 (7 cars)
- Non-electric sedans (**electric: False**): Toyota Corolla, BMW 3 Series, Honda Civic, Mercedes-Benz S-Class, Toyota Prius, Audi A4, Toyota Camry, Nissan Altima, Hyundai Sonata, Honda Accord, Hyundai Elantra, Cadillac CTS, Rolls-Royce Phantom, Infiniti Q50, Genesis G70, Alfa Romeo Giulia (16 cars)

This split is not perfectly balanced (7 vs. 16), but it's a reasonable choice. Another option is `luxury` , which may provide a more balanced split:

- Luxury sedans (`luxury: True`): Tesla Model S, BMW 3 Series, Mercedes-Benz S-Class, Audi A4, Hyundai Sonata, Tesla Model 3, Honda Accord, Cadillac CTS, Rolls-Royce Phantom, Infiniti Q50, Genesis G70, Alfa Romeo Giulia, Lucid Air, Polestar 2 (14 cars)
- Non-luxury sedans (`luxury: False`): Toyota Corolla, Honda Civic, Chevrolet Bolt, Toyota Prius, Toyota Camry, Nissan Altima, Nissan Leaf, Hyundai Elantra (9 cars)

The `luxury` split (14 vs. 9) is closer to balanced, so I'll choose it.

Question 2: Is the hidden car a luxury vehicle?

Please answer "Yes" or "No," and I'll update the belief distribution and calculate the entropy drop.

Yes.

Thank you for answering that the hidden car is a luxury vehicle. Let's update the belief distribution and calculate the entropy drop based on this information.

Step 2: Update Belief Distribution

From the previous step, we narrowed down to 23 sedans. Now, we filter those sedans to include only those that are luxury vehicles (`luxury: True`). The sedans that are both `sedan: True` and `luxury: True` are:

1. Tesla Model S
2. BMW 3 Series
3. Mercedes-Benz S-Class
4. Audi A4
5. Hyundai Sonata
6. Tesla Model 3
7. Honda Accord
8. Cadillac CTS
9. Rolls-Royce Phantom
10. Infiniti Q50
11. Genesis G70
12. Alfa Romeo Giulia
13. Lucid Air
14. Polestar 2

There are 14 luxury sedans out of the 23 sedans from the previous step (and 94 total cars). Thus, the new belief distribution assigns equal probability to each of these 14 cars:

- Probability per luxury sedan: $\frac{1}{14} \approx 0.07143$
- Probability for all other cars (including non-luxury sedans and non-sedans): 0

Entropy Calculation

The entropy of the new distribution is:

$$H_2 = -14 \cdot \left(\frac{1}{14} \log_2 \frac{1}{14} \right) = -\log_2 \frac{1}{14} = \log_2 14$$

To compute $\log_2 14$:

$$\log_2 14 = \frac{\log_{10} 14}{\log_{10} 2} \approx \frac{1.1461}{0.3010} \approx 3.807 \text{ bits}$$

Previous entropy (after Question 1):

$$H_1 = \log_2 23 \approx 4.524 \text{ bits}$$

Initial entropy (before any questions):

$$H_0 = \log_2 94 \approx 6.555 \text{ bits}$$

Entropy Drop

The entropy drop from the previous step is:

$$\Delta H_1 = H_1 - H_2 \approx 4.524 - 3.807 \approx 0.717 \text{ bits}$$

The cumulative entropy drop from the initial state is:

$$\Delta H_{\text{total}} = H_0 - H_2 \approx 6.555 - 3.807 \approx 2.748 \text{ bits}$$

Current Belief Distribution

Each of the 14 luxury sedans has an equal probability of approximately 0.07143 (or $\frac{1}{14}$). The remaining 80 cars (94 - 14) have a probability of 0.

Question 3

To further narrow down the 14 luxury sedans, I aim to ask a question that splits them into roughly equal groups. Let's consider the **electric** attribute:

- Electric luxury sedans (**electric: True**): Tesla Model S, Tesla Model 3, Lucid Air, Polestar 2 (4 cars)
- Non-electric luxury sedans (**electric: False**): BMW 3 Series, Mercedes-Benz S-Class, Audi A4, Hyundai Sonata, Honda Accord, Cadillac CTS, Rolls-Royce Phantom, Infiniti Q50, Genesis G70, Alfa Romeo Giulia (10 cars)

This split (4 vs. 10) is not very balanced. Another option is **all_wheel_drive** :

- Luxury sedans with all-wheel drive (`all_wheel_drive: True`): Tesla Model S, Audi A4, Tesla Model 3, Cadillac CTS, Infiniti Q50, Genesis G70, Lucid Air, Polestar 2 (8 cars)
- Luxury sedans without all-wheel drive (`all_wheel_drive: False`): BMW 3 Series, Mercedes-Benz S-Class, Hyundai Sonata, Honda Accord, Rolls-Royce Phantom, Alfa Romeo Giulia (6 cars)

The `all_wheel_drive` split (8 vs. 6) is more balanced and likely to maximize information gain.

Question 3: Does the hidden car have all-wheel drive?

Please answer "Yes" or "No," and I'll update the belief distribution and calculate the entropy drop.

Yes.

Thank you for answering that the hidden car has all-wheel drive. Let's update the belief distribution and calculate the entropy drop based on this information.

Step 3: Update Belief Distribution

From the previous step, we narrowed down to 14 luxury sedans. Now, we filter those to include only those with all-wheel drive (`all_wheel_drive: True`). The luxury sedans that are `sedan: True` , `luxury: True` , and `all_wheel_drive: True` are:

1. Tesla Model S
2. Audi A4
3. Tesla Model 3
4. Cadillac CTS
5. Infiniti Q50
6. Genesis G70
7. Lucid Air
8. Polestar 2

There are 8 cars that satisfy these criteria out of the 14 luxury sedans from the previous step (and 94 total cars). Thus, the new belief distribution assigns equal probability to each of these 8 cars:

- Probability per car: $\frac{1}{8} = 0.125$
- Probability for all other cars: 0

Entropy Calculation

The entropy of the new distribution is:

$$H_3 = -8 \cdot \left(\frac{1}{8} \log_2 \frac{1}{8}\right) = -\log_2 \frac{1}{8} = \log_2 8 = 3 \text{ bits}$$

Previous entropy (after Question 2):

$$H_2 = \log_2 14 \approx 3.807 \text{ bits}$$

Initial entropy (before any questions):

$$H_0 = \log_2 94 \approx 6.555 \text{ bits}$$

Entropy Drop

The entropy drop from the previous step is:

$$\Delta H_2 = H_2 - H_3 \approx 3.807 - 3.000 \approx 0.807 \text{ bits}$$

The cumulative entropy drop from the initial state is:

$$\Delta H_{\text{total}} = H_0 - H_3 \approx 6.555 - 3.000 \approx 3.555 \text{ bits}$$

Current Belief Distribution

Each of the 8 cars (Tesla Model S, Audi A4, Tesla Model 3, Cadillac CTS, Infiniti Q50, Genesis G70, Lucid Air, Polestar 2) has an equal probability of 0.125 (or $\frac{1}{8}$). The remaining 86 cars (94 - 8) have a probability of 0.

Question 4

To further narrow down the 8 cars, I aim to ask a question that splits them into roughly equal groups. Let's consider the **electric** attribute:

- Electric cars (**electric: True**): Tesla Model S, Tesla Model 3, Lucid Air, Polestar 2 (4 cars)
- Non-electric cars (**electric: False**): Audi A4, Cadillac CTS, Infiniti Q50, Genesis G70 (4 cars)

This split (4 vs. 4) is perfectly balanced, maximizing information gain.

Question 4: Is the hidden car electric?

Please answer "Yes" or "No," and I'll update the belief distribution and calculate the entropy drop.

Yes.

Thank you for answering that the hidden car is electric. Let's update the belief distribution and calculate the entropy drop based on this information.

Step 4: Update Belief Distribution

From the previous step, we narrowed down to 8 luxury sedans with all-wheel drive. Now, we filter those to include only those that are electric (`electric: True`). The cars that are `sedan: True` , `luxury: True` , `all_wheel_drive: True` , and `electric: True` are:

1. Tesla Model S
2. Tesla Model 3
3. Lucid Air
4. Polestar 2

There are 4 cars that satisfy these criteria out of the 8 cars from the previous step (and 94 total cars). Thus, the new belief distribution assigns equal probability to each of these 4 cars:

- Probability per car: $\frac{1}{4} = 0.25$
- Probability for all other cars: 0

Entropy Calculation

The entropy of the new distribution is:

$$H_4 = -4 \cdot \left(\frac{1}{4} \log_2 \frac{1}{4}\right) = -4 \cdot \left(\frac{1}{4} \cdot (-2)\right) = 4 \cdot \frac{2}{4} = 2 \text{ bits}$$

Previous entropy (after Question 3):

$$H_3 = \log_2 8 = 3 \text{ bits}$$

Initial entropy (before any questions):

$$H_0 = \log_2 94 \approx 6.555 \text{ bits}$$

Entropy Drop

The entropy drop from the previous step is:

$$\Delta H_3 = H_3 - H_4 = 3.000 - 2.000 = 1.000 \text{ bit}$$

The cumulative entropy drop from the initial state is:

$$\Delta H_{\text{total}} = H_0 - H_4 \approx 6.555 - 2.000 \approx 4.555 \text{ bits}$$

Current Belief Distribution

Each of the 4 cars (Tesla Model S, Tesla Model 3, Lucid Air, Polestar 2) has an equal probability of 0.25 (or $\frac{1}{4}$).

The remaining 90 cars (94 - 4) have a probability of 0.

Question 5

To further narrow down the 4 cars, I aim to ask a question that splits them into roughly equal groups. Let's consider the `domestic` attribute:

- Domestic cars (`domestic: True`): Tesla Model S, Tesla Model 3, Lucid Air (3 cars)
- Non-domestic cars (`domestic: False`): Polestar 2 (1 car)

This split (3 vs. 1) is not balanced. Another option is **expensive** :

- Expensive cars (**expensive: True**): Tesla Model S, Lucid Air (2 cars)
- Non-expensive cars (**expensive: False**): Tesla Model 3, Polestar 2 (2 cars)

The **expensive** split (2 vs. 2) is perfectly balanced, maximizing information gain.

Question 5: Is the hidden car expensive?

Please answer "Yes" or "No," and I'll update the belief distribution and calculate the entropy drop.

No.

Thank you for answering that the hidden car is not expensive. Let's update the belief distribution and calculate the entropy drop based on this information.

Step 5: Update Belief Distribution

From the previous step, we narrowed down to 4 electric, luxury, all-wheel-drive sedans: Tesla Model S, Tesla Model 3, Lucid Air, and Polestar 2. Now, we filter those to include only those that are not expensive (**expensive: False**). The cars that are **sedan: True** , **luxury: True** , **all_wheel_drive: True** , **electric: True** , and **expensive: False** are:

1. Tesla Model 3
2. Polestar 2

There are 2 cars that satisfy these criteria out of the 4 cars from the previous step (and 94 total cars). Thus, the new belief distribution assigns equal probability to each of these 2 cars:

- Probability per car: $\frac{1}{2} = 0.5$
- Probability for all other cars: 0

Entropy Calculation

The entropy of the new distribution is:

$$H_5 = -2 \cdot \left(\frac{1}{2} \log_2 \frac{1}{2} \right) = -2 \cdot \left(\frac{1}{2} \cdot (-1) \right) = 2 \cdot \frac{1}{2} = 1 \text{ bit}$$

Previous entropy (after Question 4):

$$H_4 = \log_2 4 = 2 \text{ bits}$$

Initial entropy (before any questions):

$$H_0 = \log_2 94 \approx 6.555 \text{ bits}$$

Entropy Drop

The entropy drop from the previous step is:

$$\Delta H_4 = H_4 - H_5 = 2.000 - 1.000 = 1.000 \text{ bit}$$

The cumulative entropy drop from the initial state is:

$$\Delta H_{\text{total}} = H_0 - H_5 \approx 6.555 - 1.000 \approx 5.555 \text{ bits}$$

Current Belief Distribution

Each of the 2 cars (Tesla Model 3, Polestar 2) has an equal probability of 0.5 (or $\frac{1}{2}$). The remaining 92 cars (94 - 2) have a probability of 0.

Question 6

To distinguish between the Tesla Model 3 and Polestar 2, I need an attribute that differs between them. Comparing their attributes:

- **Tesla Model 3:** sedan: True , luxury: True , all_wheel_drive: True , electric: True , expensive: False , domestic: True , sports: False , hybrid: False , gas: False , off_road: False , rear_wheel_drive: False , front_wheel_drive: False , seats_5: True
- **Polestar 2:** sedan: True , luxury: True , all_wheel_drive: True , electric: True , expensive: False , domestic: False , sports: False , hybrid: False , gas: False , off_road: False , rear_wheel_drive: False , front_wheel_drive: False , seats_5: True

The key difference is domestic :

- Tesla Model 3: domestic: True
- Polestar 2: domestic: False

This attribute perfectly distinguishes the two cars.

Question 6: Is the hidden car domestic?

Please answer "Yes" or "No," and I'll update the belief distribution and calculate the entropy drop.

Yes.

Thank you for answering that the hidden car is domestic. Let's update the belief distribution and calculate the entropy drop based on this information.

Step 6: Update Belief Distribution

From the previous step, we narrowed down to 2 cars: Tesla Model 3 and Polestar 2. Now, we filter those to include only the car that is domestic (`domestic: True`). The cars that are `sedan: True` , `luxury: True` , `all_wheel_drive: True` , `electric: True` , `expensive: False` , and `domestic: True` are:

1. Tesla Model 3

Only 1 car satisfies these criteria out of the 2 cars from the previous step (and 94 total cars). Thus, the new belief distribution assigns:

- Probability for Tesla Model 3: 1.0
- Probability for all other cars (including Polestar 2): 0

Entropy Calculation

The entropy of the new distribution is:

$$H_6 = -1 \cdot (1.0 \log_2 1.0) - 0 \cdot \log_2 0 = 0 \text{ bits}$$

(Since $\log_2 1.0 = 0$, and terms with zero probability contribute nothing to entropy.)

Previous entropy (after Question 5):

$$H_5 = \log_2 2 = 1 \text{ bit}$$

Initial entropy (before any questions):

$$H_0 = \log_2 94 \approx 6.555 \text{ bits}$$

Entropy Drop

The entropy drop from the previous step is:

$$\Delta H_5 = H_5 - H_6 = 1.000 - 0.000 = 1.000 \text{ bit}$$

The cumulative entropy drop from the initial state is:

$$\Delta H_{\text{total}} = H_0 - H_6 \approx 6.555 - 0.000 \approx 6.555 \text{ bits}$$

Current Belief Distribution

- Tesla Model 3: 1.0 (100% probability)
- All other cars: 0.0

Conclusion

The hidden car is the **Tesla Model 3**, as it is the only car that satisfies all the provided answers:

- Sedan: Yes
- Luxury: Yes
- All-wheel drive: Yes

- Electric: Yes
- Expensive: No
- Domestic: Yes

Since we have identified a single car and the entropy is 0 (no uncertainty remains), there are no more distinguishing attributes to consider, and we can stop here.

Final Answer: The hidden car is the Tesla Model 3.

