FILTRA: RETHINKING STEERABLE CNN BY FILTER TRANSFORM APPENDIX

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A VERIFICATION OF LEMMA 1 ON (22)

(22) can be verified to follow Lemma 1 as:

$$\begin{aligned} \mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N}(\phi+\theta_{i_1}) &= \operatorname{diag}\big(P(i_1)\mathsf{K}\big)\beta_k, \quad \text{c.f. (17b)} \\ &= P(i_1)\operatorname{diag}(\mathsf{K})P(i_1)^{-1}\beta_k = \rho_{\text{reg}}^{\mathrm{C}_N}(g)\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N}\psi_{0,k}(g)^{-1}, \quad \text{c.f. (11a).} \end{aligned} \tag{27}$$

We can also verify this for

$$\overline{\mathsf{K}}_{k\to\text{reg}}^{\mathrm{C}_N} = \operatorname{diag}(\overline{\mathsf{K}})\beta_k. \tag{28}$$

B VERIFICATION OF LEMMA 1 ON (23)

First note it is easy to verify that for $i_0=0$, i.e. $g=(0,i_1)$, the Lemma 1 holds in the same way as (27),

$$\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}(\phi+\theta) = \rho_{\text{reg}}^{\mathrm{D}_N}(g)\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}\psi_{j,k}(g)^{-1}. \tag{29}$$

We then generalize (20) on $\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N}$ and $\overline{\mathsf{K}}_{k\to\text{reg}}^{\mathrm{C}_N}$ given a reflected action $g=(1,i_1)$:

$$\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N}(-\phi+\theta_{i_1}) = \operatorname{diag}\big(\mathsf{K}(-\phi+\theta_{i_1})\big)\beta_k = B(i_1)\operatorname{diag}(\overline{\mathsf{K}})B(i_1)^{-1}\beta_k, \quad \text{c.f. (11b)} \tag{30a}$$

$$= B(i_1)\operatorname{diag}(\overline{\mathsf{K}})\beta_k\psi_{0,k}(g)^{-1} = -B(i_1)\operatorname{diag}(\overline{\mathsf{K}}^{\mathrm{C}_N})\beta_k\psi_{1,k}(g)^{-1} \tag{30b}$$

$$= B(i_1)\operatorname{diag}(\mathsf{K}^{\mathrm{C}_N})\beta_k\psi_{0,k}(g)^{-1} = -B(i_1)\operatorname{diag}(\mathsf{K}^{\mathrm{C}_N})\beta_k\psi_{1,k}(g)^{-1}. \tag{30c}$$

Note that (30b) and (30c) both have two equivalent forms denoted with $\psi_{0,k}(g)$ and $\psi_{1,k}(g)$ respectively. Now we can show $\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}$ follows Lemma 1 for $j=0$, $i_0=1$, i.e. $g=(1,i_1)$ as:

$$\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}(-\phi+\theta_{i_1}) = \left[\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N}(-\phi+\theta_{i_1})^\top \quad \overline{\mathsf{K}}_{k\to\text{reg}}^{\mathrm{C}_N}(-\phi+\theta_{i_1})^\top\right]^\top \tag{31a}$$

$$= \left[B(i_1)\operatorname{diag}(\overline{\mathsf{K}}^{\mathrm{C}_N})\beta_k\psi_{0,k}(g)^{-1} \quad B(i_1)\operatorname{diag}(\mathsf{K}^{\mathrm{C}_N})\beta_k\psi_{0,k}(g)^{-1}\right]^\top \quad \text{c.f. (30b)} \tag{31b}$$

$$= \rho_{\text{reg}}^{\mathrm{D}_N}(g)\left[\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N\,\top} \quad \overline{\mathsf{K}}_{k\to\text{reg}}^{\mathrm{C}_N\,\top}\right]^\top \psi_{0,k}(g)^{-1} \quad \text{c.f. (22), (28)} \tag{31c}$$

$$= \rho_{\text{reg}}^{\mathrm{D}_N}(g)\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}\psi_{0,k}(g)^{-1}. \tag{31d}$$

The verification is similar for $j=1$, $i_0=1$, i.e. $g=(1,i_1)$:

$$\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}(-\phi+\theta_{i_1}) = \left[\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N}(-\phi+\theta_{i_1})^\top \quad -\overline{\mathsf{K}}_{k\to\text{reg}}^{\mathrm{C}_N}(-\phi+\theta_{i_1})^\top\right]^\top \tag{32a}$$

$$= \left[-B(i_1)\operatorname{diag}(\overline{\mathsf{K}}^{\mathrm{C}_N})\beta_k\psi_{1,k}(g)^{-1} \quad B(i_1)\operatorname{diag}(\mathsf{K}^{\mathrm{C}_N})\beta_k\psi_{1,k}(g)^{-1}\right]^\top \quad \text{c.f. (30b)} \tag{32b}$$

$$= \rho_{\text{reg}}^{\mathrm{D}_N}(g)\left[\mathsf{K}_{k\to\text{reg}}^{\mathrm{C}_N\,\top} \quad -\overline{\mathsf{K}}_{k\to\text{reg}}^{\mathrm{C}_N\,\top}\right]^\top \psi_{0,k}(g)^{-1} \quad \text{c.f. (22), (28)} \tag{32c}$$

$$= \rho_{\text{reg}}^{\mathrm{D}_N}(g)\mathsf{K}_{j,k\to\text{reg}}^{\mathrm{D}_N}\psi_{0,k}(g)^{-1}. \tag{32d}$$

C VERIFICATION OF LEMMA 1 ON (25)

This kernel can be verified as follows for $g = (0, i_1)$:

$$\mathsf{K}^{\mathrm{C}_N}_{\mathrm{reg}\to\mathrm{reg}}(\phi + \theta_{i_1}) = \left[\rho^{\mathrm{C}_N}_{\mathrm{reg}}(g)\mathsf{K}^{\mathrm{C}_N}_{0\to\mathrm{reg}}\psi_{0,0}(g)^{-1}, \cdots, \rho^{\mathrm{C}_N}_{\mathrm{reg}}(g)\mathsf{K}^{\mathrm{C}_N}_{\lfloor\frac{N}{2}\rfloor\to\mathrm{reg}}\psi_{0,\lfloor\frac{N}{2}\rfloor}(g)^{-1}\right] V^{-1} \tag{33a}$$

$$= \rho^{\mathrm{C}_N}_{\mathrm{reg}}(g)\left[\mathsf{K}^{\mathrm{C}_N}_{0\to\mathrm{reg}} \cdots \mathsf{K}^{\mathrm{C}_N}_{\lfloor\frac{N}{2}\rfloor\to\mathrm{reg}}\right] D^{\mathrm{C}_N} V^{-1} \tag{33b}$$

$$= \rho^{\mathrm{C}_N}_{\mathrm{reg}}(g)\left[\mathsf{K}^{\mathrm{C}_N}_{0\to\mathrm{reg}} \cdots \mathsf{K}^{\mathrm{C}_N}_{\lfloor\frac{N}{2}\rfloor\to\mathrm{reg}}\right] V^{-1} V D^{\mathrm{C}_N} V^{-1} = \rho^{\mathrm{C}_N}_{\mathrm{reg}}(g)\mathsf{K}^{\mathrm{C}_N}_{\mathrm{reg}\to\mathrm{reg}}{\rho^{\mathrm{C}_N}_{\mathrm{reg}}}^{-1}. \tag{33c}$$