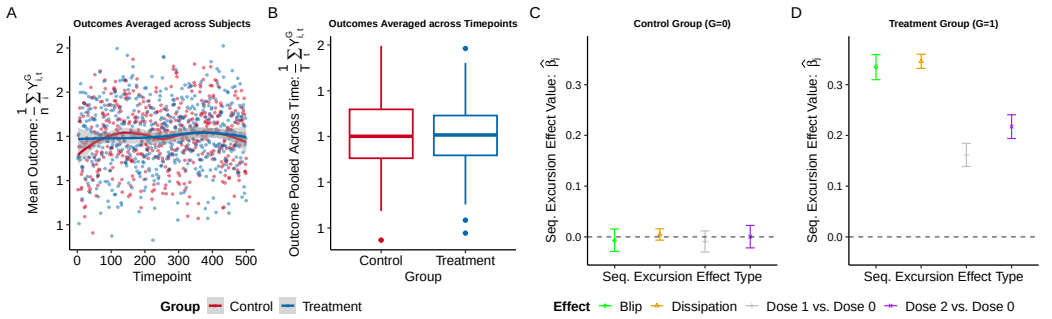

Figure 1: **Treatment–Confounder Feedback Example** Sequential excursion effects reveal causal effects obscured in "macro" summaries. Results from new illustrative example, following argument in Appendix 2, taking $n = 100$, $T = 500$, $\gamma_1 = \gamma_3 = 1$, $\gamma_2 = \gamma_4 = 0.5$. [A] Each dot is an outcome value, $Y_{i,t}^G$, for subject i at timepoint t from "control" ($G = 0$), or from "treatment" ($G = 1$) groups. Lines are timepoint-specific means (averaged across subjects), estimated using a linear smoother (`loess`). (B) Same data as (A), but each point in boxplot is a subject's mean outcome value (averaged across timepoints). In (A)-(B), "macro" summaries show no differences due to treatment–confounder feedback: mean outcome values (averaged across subjects or timepoints) are nearly identical in both groups. (C)-(D) Point estimates and 95% CIs (error bars) of sequential excursion effects reveal "local" causal effects (in Treatment group only), obscured in "macro" summaries (shown in (A)-(B)).

	$T = 50$				$T = 500$			
Effect	$n = 6$	$n = 10$	$n = 30$	$n = 100$	$n = 6$	$n = 10$	$n = 30$	$n = 100$
Blip	1.91 ± 0.09	1.10 ± 0.05	0.30 ± 0.02	0.03 ± 0.01	0.11 ± 0.01	0.03 ± 0.01	0.00 ± 0.00	0.00 ± 0.00
Dissip	2.11 ± 0.09	1.23 ± 0.06	0.33 ± 0.02	0.04 ± 0.01	0.11 ± 0.01	0.03 ± 0.01	0.00 ± 0.00	0.00 ± 0.00
Dose0	0.73 ± 0.04	0.40 ± 0.02	0.08 ± 0.01	0.00 ± 0.00	0.02 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
Dose1	0.90 ± 0.04	0.51 ± 0.03	0.10 ± 0.01	0.00 ± 0.00	0.02 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
Dose2	3.52 ± 0.15	2.05 ± 0.09	0.58 ± 0.03	0.12 ± 0.01	0.25 ± 0.02	0.11 ± 0.01	0.01 ± 0.00	0.00 ± 0.00

Table 1: **Simulation Results: MSE** Our estimator's MSE decreases to 0 as T or n grows. Denoting the estimated effect j (e.g., $j =$ "Blip") for replicate r as $\hat{\beta}_{j,r}$, we show $\text{MSE}_j := \frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}_{j,r} - \beta_j)^2$ for $R = 1000$ simulation replicates ($\pm$ SE) for a sample size, n, and timepoints, T. Values are scaled by 100 for readability (e.g., 0.01 is shown in the table as 1.0). Thus 0 indicates a value $< 1e-4$.

	$T = 50$				$T = 500$			
Effect	$n = 6$	$n = 10$	$n = 30$	$n = 100$	$n = 6$	$n = 10$	$n = 30$	$n = 100$
Blip	0.23 ± 0.55	0.42 ± 0.43	0.42 ± 0.24	0.10 ± 0.13	0.07 ± 0.17	0.05 ± 0.13	0.08 ± 0.08	0.03 ± 0.04
Dissip	0.82 ± 0.93	1.04 ± 0.71	0.45 ± 0.39	0.26 ± 0.22	0.17 ± 0.27	0.22 ± 0.21	0.10 ± 0.13	0.06 ± 0.07
Dose0	0.28 ± 1.48	0.15 ± 1.17	0.12 ± 0.68	0.42 ± 0.36	0.28 ± 0.48	0.23 ± 0.35	0.34 ± 0.20	0.13 ± 0.11
Dose1	0.12 ± 0.42	0.05 ± 0.32	0.28 ± 0.18	0.06 ± 0.10	0.10 ± 0.13	0.08 ± 0.10	0.04 ± 0.06	0.01 ± 0.03
Dose2	0.41 ± 0.35	0.09 ± 0.27	0.12 ± 0.15	0.10 ± 0.08	0.03 ± 0.11	0.05 ± 0.08	0.03 ± 0.05	0.01 ± 0.03

Table 2: **Simulation Results: Bias** Our estimator is unbiased. Moreover, the absolute relative bias decreases to 0 as T and/or n grows. Denoting the estimated effect j (e.g., $j =$ "Blip") for replicate r as $\hat{\beta}_{j,r}$, we show Absolute Relative $\text{Bias}_j := |\frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}_{j,r} - \beta_j)/\beta_j|$ for $R = 1000$ replicates ($\pm$ standard error). Values are scaled by 100 for readability (e.g., 0.01 is shown in the table as 1.0).

		$T = 50$				$T = 500$			
Effect	*CI*	$n = 6$	$n = 10$	$n = 30$	$n = 100$	$n = 6$	$n = 10$	$n = 30$	$n = 100$
Blip	*HC*	0.94 ± 0.01	0.94 ± 0.01	0.95 ± 0.01	0.95 ± 0.01	0.99 ± 0.00	0.97 ± 0.01	0.95 ± 0.01	0.95 ± 0.01
	LS	0.86 ± 0.01	0.88 ± 0.01	0.93 ± 0.01	0.95 ± 0.01	0.86 ± 0.01	0.89 ± 0.01	0.93 ± 0.01	0.94 ± 0.01
Dissip	*HC*	0.93 ± 0.01	0.94 ± 0.01	0.95 ± 0.01	0.95 ± 0.01	0.97 ± 0.01	0.96 ± 0.01	0.94 ± 0.01	0.94 ± 0.01
	LS	0.85 ± 0.01	0.89 ± 0.01	0.94 ± 0.01	0.95 ± 0.01	0.88 ± 0.01	0.91 ± 0.01	0.93 ± 0.01	0.94 ± 0.01
Dose0	*HC*	0.92 ± 0.01	0.92 ± 0.01	0.94 ± 0.01	0.94 ± 0.01	0.94 ± 0.01	0.94 ± 0.01	0.96 ± 0.01	0.95 ± 0.01
	LS	0.86 ± 0.01	0.88 ± 0.01	0.92 ± 0.01	0.94 ± 0.01	0.86 ± 0.01	0.91 ± 0.01	0.95 ± 0.01	0.95 ± 0.01
Dose1	*HC*	0.95 ± 0.01	0.95 ± 0.01	0.96 ± 0.01	0.95 ± 0.01	1.00 ± 0.00	0.97 ± 0.01	0.94 ± 0.01	0.96 ± 0.01
	LS	0.88 ± 0.01	0.91 ± 0.01	0.95 ± 0.01	0.95 ± 0.01	0.86 ± 0.01	0.90 ± 0.01	0.92 ± 0.01	0.96 ± 0.01
Dose2	*HC*	0.95 ± 0.01	0.94 ± 0.01	0.96 ± 0.01	0.96 ± 0.01	1.00 ± 0.00	0.98 ± 0.00	0.96 ± 0.01	0.96 ± 0.01
	LS	0.86 ± 0.01	0.89 ± 0.01	0.95 ± 0.01	0.95 ± 0.01	0.87 ± 0.01	0.90 ± 0.01	0.94 ± 0.01	0.96 ± 0.01

Table 3: **Simulation Results: CI Coverage** We achieve 95% confidence interval (CI) coverage using either small sample size-adjusted *HC3* (shown as *HC*), or our *Large Sample* (shown as *LS*) sandwich variance estimators. Mean of $R = 1000$ replicates is shown ($\pm$ standard error). We recommend *HC3* when n is low. When n is high, *LS* achieves nominal coverage, confirming our asymptotic theory.