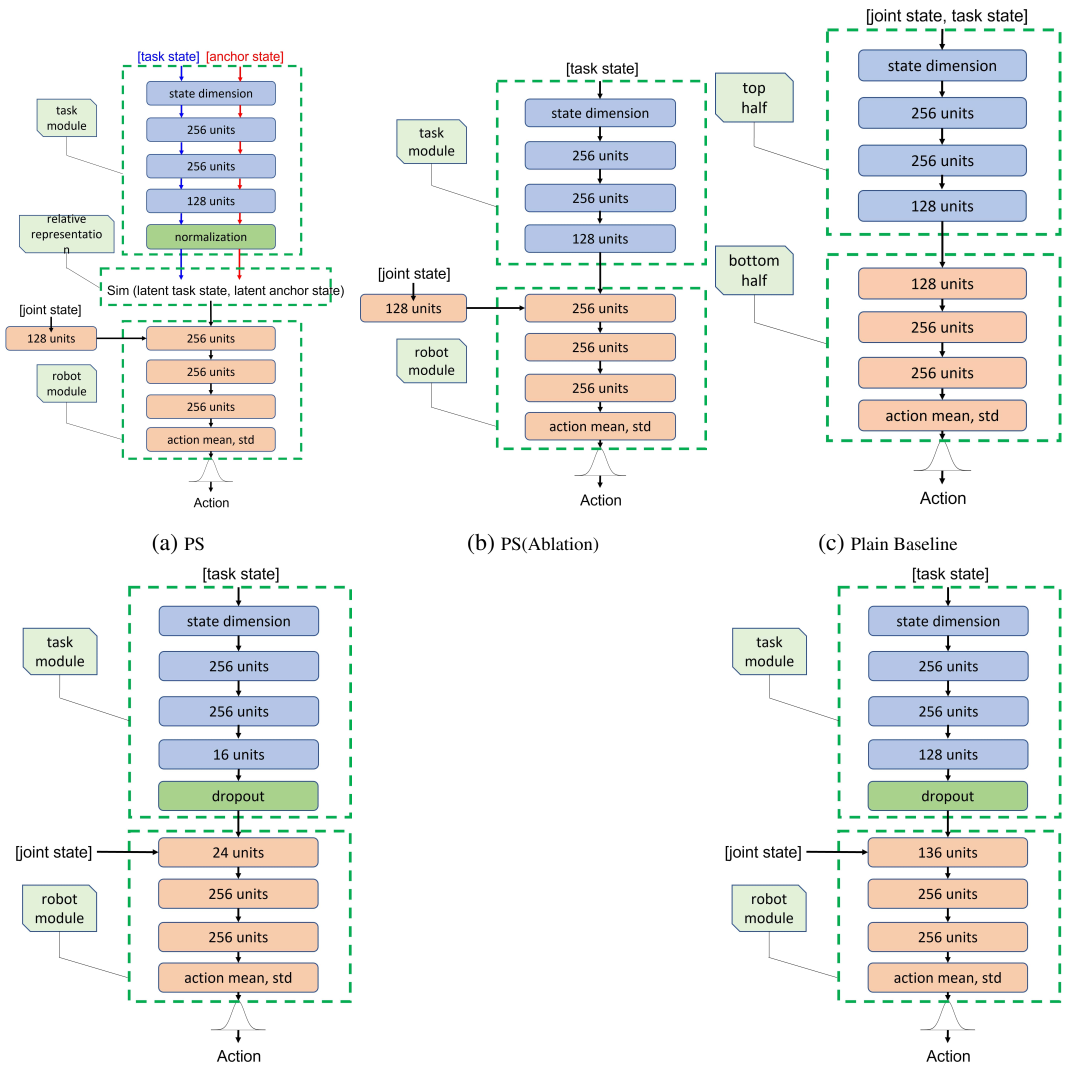
Appendix

A Network Structure



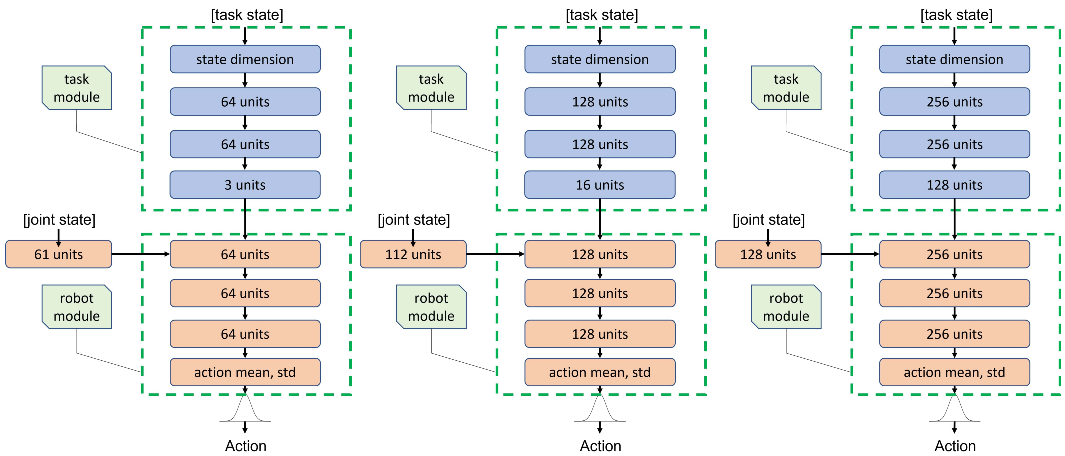
(a) PS (b) PS(Ablation) (c) Plain Baseline

(d) Devin et al. [27] (e) Devin et al. [27] (Large)

Fig. 8: Detailed network structure of PS, PS(Ablation), Devin et al., Devin et al.(Large) and Plain Baseline methods.

B Analysis of the Module Interface

We carry out additional analysis of the latent representations across different sizes of modular networks. We build three different sizes of modular networks, each interface dimension being 3D, 16D, and 128D as shown in Fig.9. The transferable representation is added to these networks as shown in Fig.8a. We construct six networks (3 different sizes, with and without relative representation) to perform the reaching task as described in Fig.1.



(a) Small modular network (b) Medium modular network (c) Large modular network

Fig. 9: The detailed architectures of the modular networks with three different interface dimensions.

B.1 Visualization of the latent representations at the modules interface

For the small networks with 3D interfaces, we plot the 3D latent representations directly. For the medium and large networks with 16D and 128D interfaces, we use PCA [66] to reduce the dimension to 2D. Fig.10 shows the visualization of the interface of the six networks trained with different random seeds. The isometric transformation relationship is shown for the PS(Ablation) method across all sizes of modular networks, and with the help of transferable presentation, PS achieves near invariance. Similarly, Fig.11 shows the interface of the networks trained with different robot types. The transferable representation achieves near invariance to isometric transformations across all types of robots.

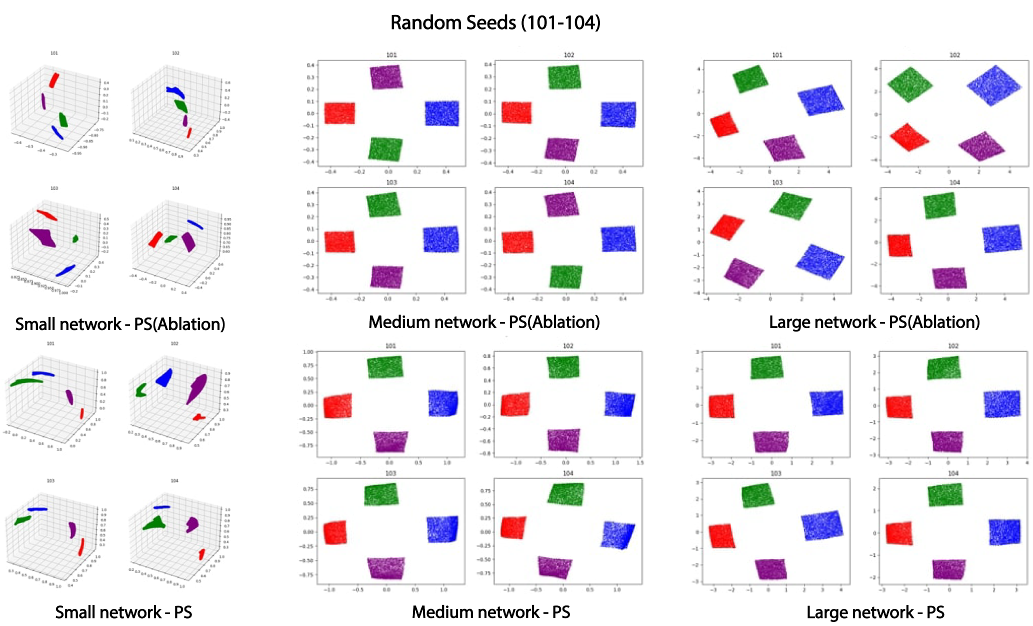


Fig. 10: **Latent Space Visualization** Train each policy network four times with four different random seeds (101-104). Without transferable representation, the latent representations at the interfaces have an approximate isometric transformation relationship. With relative representation, these latent representations are isometrically similar.

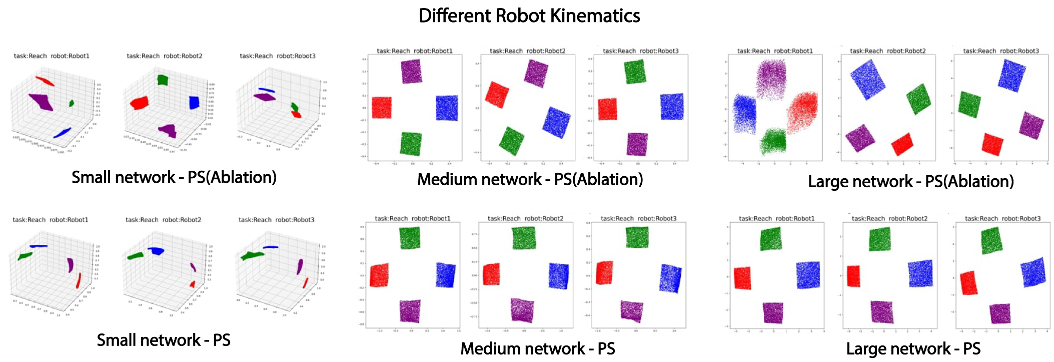


Fig. 11: **Latent Space Visualization** Train each policy network for the reaching task with three different types of robots as shown in Figure 3. With relative representation, the latent representations at the module interfaces are nearly the same across different environments. Without it, they have an approximately isometric transformation relationship.

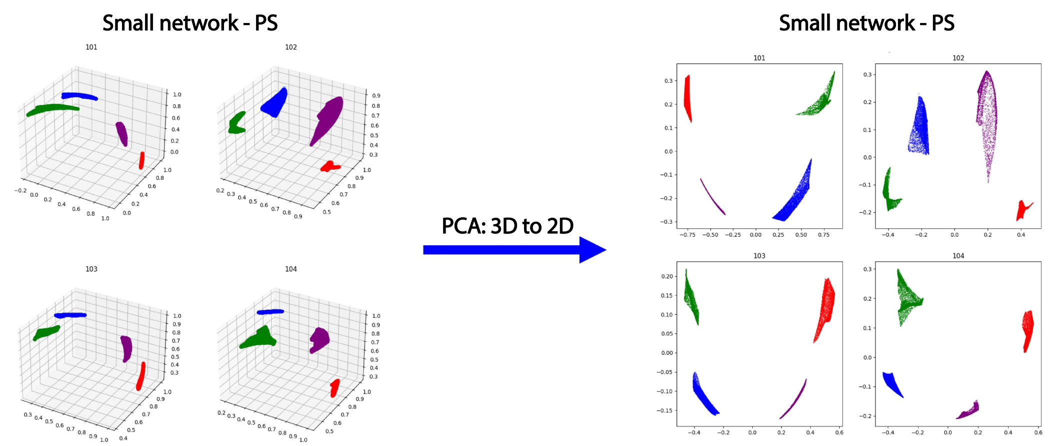
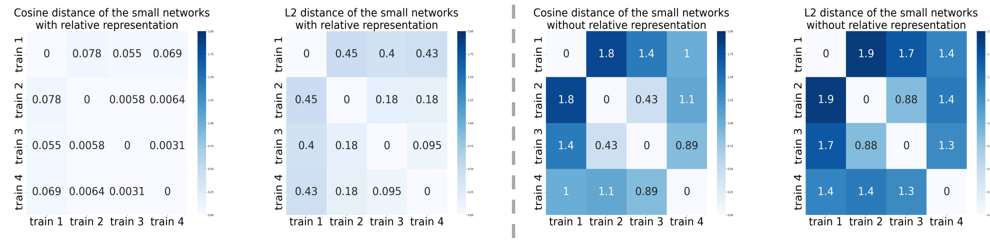


Fig. 12: **Limitation of visualization with PCA.** Raw latent distributions are very similar to each other in the 3D space, but after compression to 2D with PCA, the visualization results are quite different.

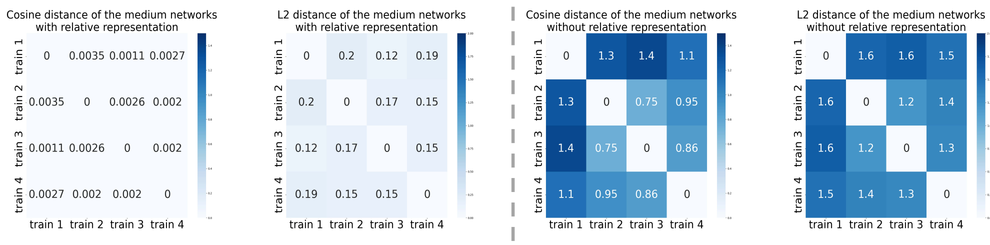
PCA is an information lossy compression process. The PCA method only guarantees identical output results when the input data sets are identical. When the input data sets are similar but not identical, the output results may vary considerably. In our experiments using PCA for visualization, we have observed that the PCA results of most interfaces with transferable representations are similar. However, in rare cases, we have noticed significant differences in the PCA results. As shown in Fig.12, the original 3D latent states have very similar distributions across the four different runs, but after the dimension reduction to 2D with PCA, the visualization results show isometric transformations. Moreover, in the case of small modular networks with 3D interfaces, achieving a high success rate of approximately 100% often requires a considerable amount of training time. Occasionally, the network may converge to a local minimum with a success rate of around 90%. When it converges to a local minimum, the latent representation at its interface typically differs from those that converge to the global minimum. PCA only provides an intuitive idea of the behavior at module interface, thus, we accompany these visualizations with quantitative analysis.

B.2 Quantitative analysis of the latent representations at the modules interface

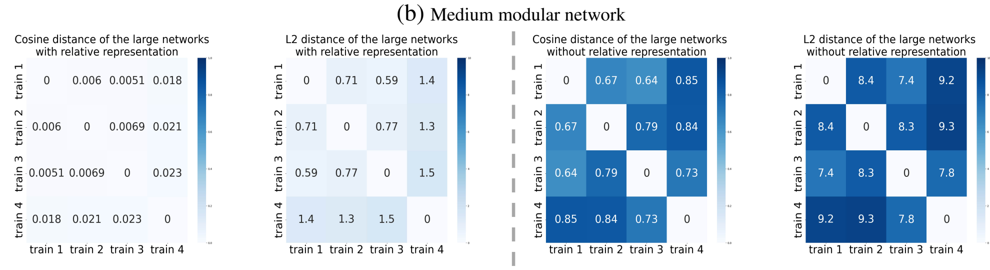
To measure the similarity between two different latent representations, we use cosine and L2 pairwise distances. We compute the pairwise distance between two latent task states derived from the same input state. By considering a dataset of input states, we calculate the mean of the pairwise distances across all input states, obtaining the average pairwise distance between two modular networks.



(a) Small modular network



(b) Medium modular network



(c) Large modular network

Fig. 13: Cosine and L2 distances of different networks with and without transferable representation for different random seeds.

Given an input task state set $\mathbb{S}_{E,\mathcal{T}}$, the average pairwise cosine distance and L2 distance are defined as

$$\bar{d}_{cos} = \sum_{i=1}^{|\mathbb{S}_{E,\mathcal{T}}|} \left(1 - S_C\left(g_k^1\left(s_{E,\mathcal{T}}^i\right), g_k^2\left(s_{E,\mathcal{T}}^i\right)\right)\right) / \left|\mathbb{S}_{E,\mathcal{T}}\right|, \tag{4}$$

$$\bar{d}_{L2} = \sum_{i=1}^{|\mathbb{S}_{E,\mathcal{T}}|} d_{L2}\left(g_k^1\left(s_{E,\mathcal{T}}^i\right), g_k^2\left(s_{E,\mathcal{T}}^i\right)\right) / \left|\mathbb{S}_{E,\mathcal{T}}\right|, \tag{5}$$

where $S_C(\boldsymbol{a}, \boldsymbol{b}) = \frac{\boldsymbol{a}\boldsymbol{b}}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|}$ is the cosine similarity and $d_{L2}(p, q) = \|p - q\|$ is the L2 distance. Fig.13 shows the average pairwise distances of modular networks trained with four different random seeds. We calculate the distances for different sizes of networks shown in Figure 9 . We also calculate the mean and standard deviations of the data in Fig.9 and present them in Tab. 2. The results show that the transferable representation largely reduces the average pairwise distances of the latent spaces between different training runs.

We also train the modular networks in different environments and calculate the pairwise distances at the interfaces. Specifically, we train the policy networks on the reaching task with different robots shown in Figure 3. The average pairwise distances are shown in Figure 14 and we calculate the mean values and standard deviations in Tab.3. These quantitative results show that the relative representation makes the module interfaces much more similar to each other when trained in different environments.

	cosine distance	L2 distance
PS	$\mathbf{0.0363 \pm 0.0319}$	$\mathbf{0.289 \pm 0.141}$
PS(Ablation)	1.106 ± 0.434	1.426 ± 0.322

(a) Small modular network

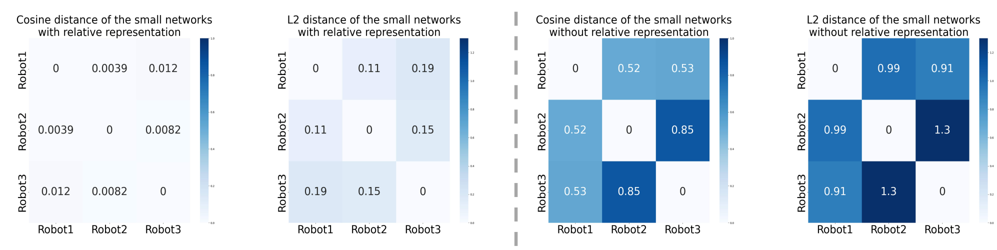
	cosine distance	L2 distance
PS	$\mathbf{0.00231 \pm 0.00073}$	$\mathbf{0.1633 \pm 0.0294}$
PS(Ablation)	1.054 ± 0.218	1.442 ± 0.151

(b) Meidum modular network

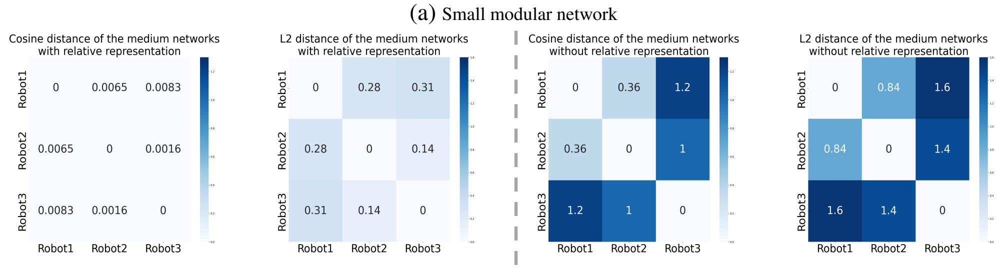
	cosine distance	L2 distance
PS	$\mathbf{0.013 \pm 0.007}$	$\mathbf{1.051 \pm 0.368}$
PS(Ablation)	0.753 ± 0.081	8.386 ± 0.687

(c) Large modular network

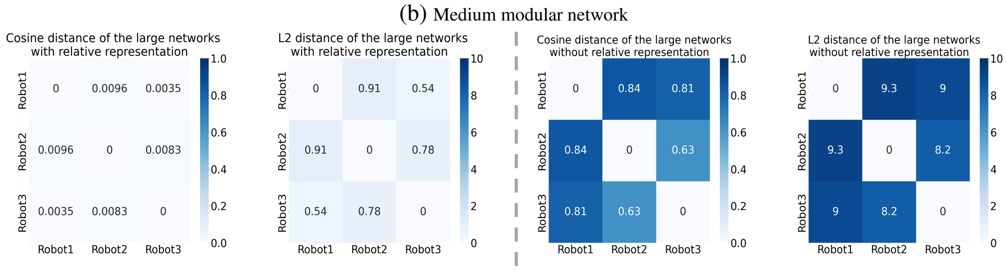
Tab. 2: Mean and standard deviation values of the average pairwise distances between trainings with four different random seeds (101-104)



(a) Small modular network



(b) Medium modular network



(c) Large modular network

Fig. 14: Cosine and L2 distances of different networks with and without transferable representation for different robot setup.

	cosine distance	L2 distance
PS	$\mathbf{0.0082 \pm 0.0035}$	$\mathbf{0.149 \pm 0.032}$
PS(Ablation)	0.633 ± 0.153	1.066 ± 0.165

(a) Small modular network

	cosine distance	L2 distance
PS	$\mathbf{0.0055 \pm 0.0029}$	$\mathbf{0.240 \pm 0.075}$
PS(Ablation)	0.865 ± 0.367	1.275 ± 0.311

(b) Medium modular network

	cosine distance	L2 distance
PS	$\mathbf{0.0071 \pm 0.0026}$	$\mathbf{0.743 \pm 0.152}$
PS(Ablation)	0.760 ± 0.094	8.822 ± 0.448

(c) Large modular network

Tab. 3: Mean and standard deviation values of the average pairwise distances between trainings with three different types of robot kinematics