Partially Linearized Update for Generative Inversion in Compressive Sensing

PLUGIn-CS: A simple algorithm for compressive sensing with generative prior

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Overview

We extend PLUGIn (a novel algorithm which can invert deep generative models) to compressive sensing and introduce the algorithm PLUGIn-CS, which can invert deep generative models from compressive observations.

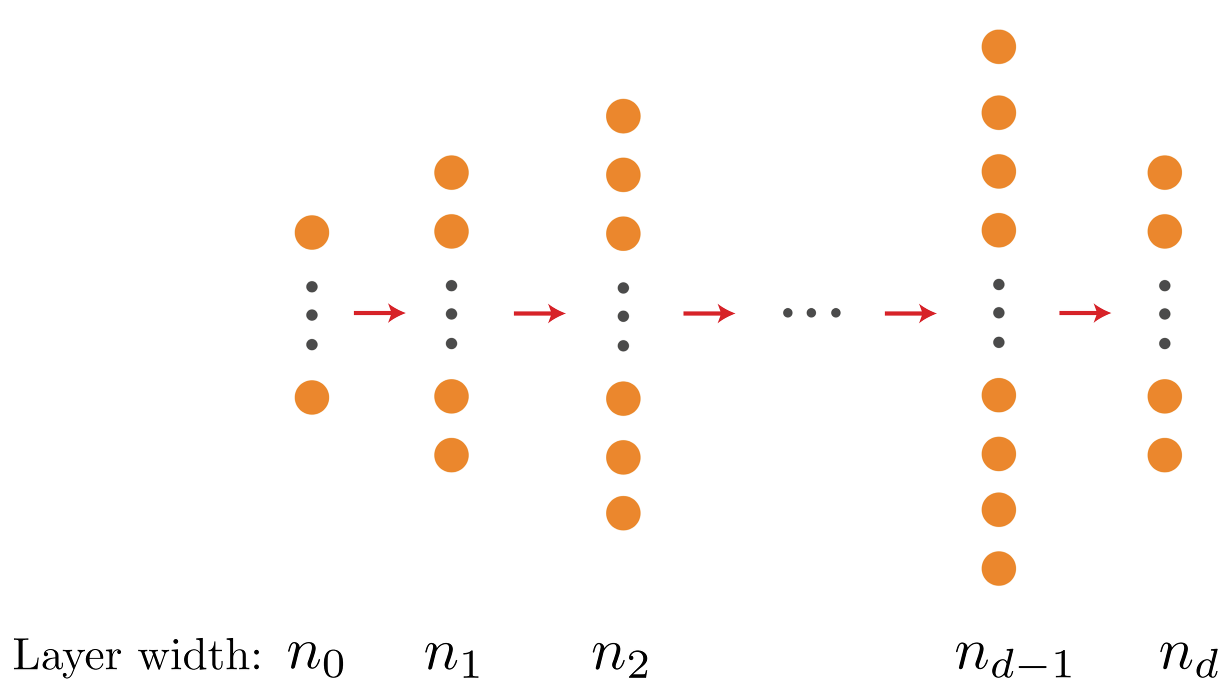
- PLUGIn-CS has a simple form and does not require gradient calculation.
- PLUGIn-CS has provable global geometric convergence[1].
- Our analysis for PLUGIn-CS allows some contractive layers in network.
- Numerical results show that PLUGIn-CS can effectively recover images from compressive measurements.

Problem formulation

- A deep generative model with ReLU activation functions:

$$\mathcal{G}(x) = \sigma(A_d\sigma(A_{d-1}\ldots\sigma(A_1 x)\ldots))$$

where $A_i \in \mathbb{R}^{n_i \times n_{i-1}}$.



- Given a noisy compressive observation:

$$y = \Phi\mathcal{G}(x^*) + \epsilon, \quad \Phi \in \mathbb{R}^{m \times n_d}.$$

Goal: Recover latent code x^* and/or signal $\mathcal{G}(x^*)$.

The algorithm

A natural way for recovery is to solve the following non-convex optimization problem using gradient descent

$$\min_{x\in\mathbb{R}^{n_0}} \|\Phi\mathcal{G}(x) - y\|^2.$$

The gradient descent update (with step size η) is given by

$$x^{k+1} = x^k - \eta(D_1A_1)^\intercal \cdots (D_dA_d)^\intercal \Phi^\intercal \left(\Phi\mathcal{G}(x^k) - y\right)$$

where each D_i is a diagonal 0 or 1 matrix *depending* on x^k and A_j $(j \le i)$.

Similar to PLUGIn, we linearize $(D_iA_i)^\intercal$ to $A_i^\intercal$ and obtain

$$\textbf{PLUGIn-CS:} \quad x^{k+1} = x^k - \eta A_1^\intercal A_2^\intercal \cdots A_d^\intercal \Phi^\intercal \left(\Phi\mathcal{G}(x^k) - y\right)$$

- PLUGIn-CS becomes PLUGIn if Φ is identity matrix.
- $A_1^\intercal A_2^\intercal \cdots A_d^\intercal$ is *static* (does not depend on x^k). No gradient calculation required.
- If Φ and A_i are i.i.d. Gaussian (properly normalized), first iterate of PLUGIn-CS can provide an unbiased estimate of x^* with step size $\eta = 2^d$.

Convergence results

We assume:

- Each A_i has i.i.d. $\mathcal{N}(0, 1/n_i)$ entries and are independent.
- Layer width satisfy

$$n_i \gtrsim 5^i n_0 \log\left(\prod_{j=0}^{i-1} \frac{en_j}{n_0}\right), \quad 1 \le i \le d.$$

- Φ has i.i.d. $\mathcal{N}(0, 1/m)$ entries (independent from weight matrices) with

$$m \gtrsim 2^d n_0 \log\left(\prod_{j=0}^{d} \frac{en_j}{n_0}\right).$$

- The noise ϵ does not depend on $\{A_i\}_{i\le d}$ or Φ.

Also let R be a positive number such that $\|x^0 - x^*\| \le R$.

We prove that, the k-th estimate given by PLUGIn-CS algorithm with constant step size $\eta = 2^d$ satisfies

$$\|x^k - x^*\| \le 2^{-k}R + 30 \cdot 2^d\sqrt{n_0/m}\|\epsilon\|, \text{ and}$$
$$\|\mathcal{G}(x^k) - \mathcal{G}(x^*)\| \le 2^{-k}(3R) + 90 \cdot 2^d\sqrt{n_0/m}\|\epsilon\|$$

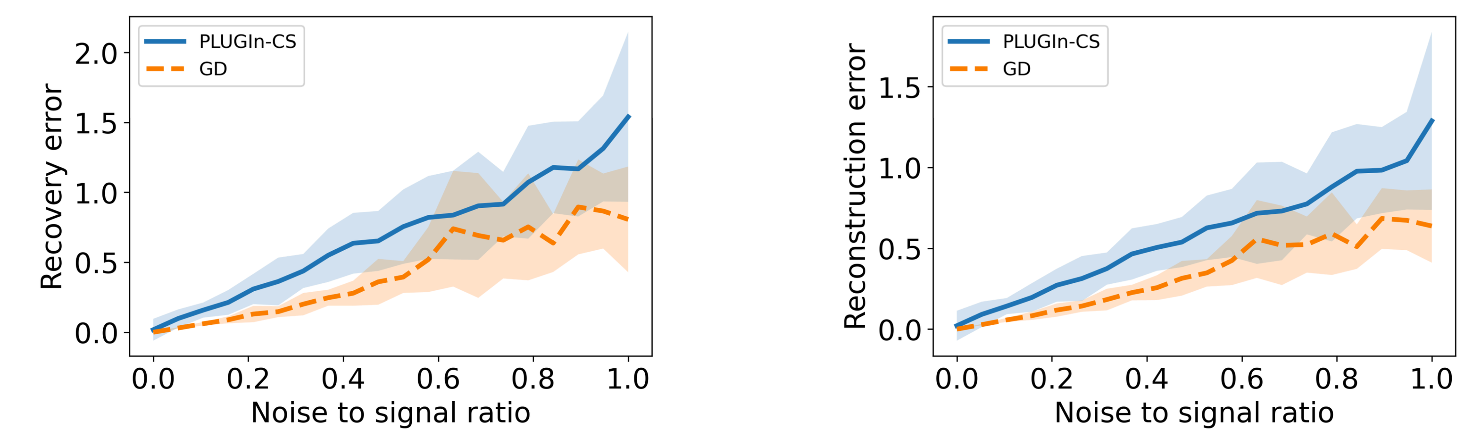
with probability at least $1 - 2(k+4)e^{-10n_0}$.

- Global and geometrical convergence for estimation errors.
- Network is expansive on "average", thus can have some contractive layers.
- PLUGIn-CS can recover latent code and signal when m/n_0 is large.
- Exact step size can be relaxed to a constant step size $\eta \le 2^d$.

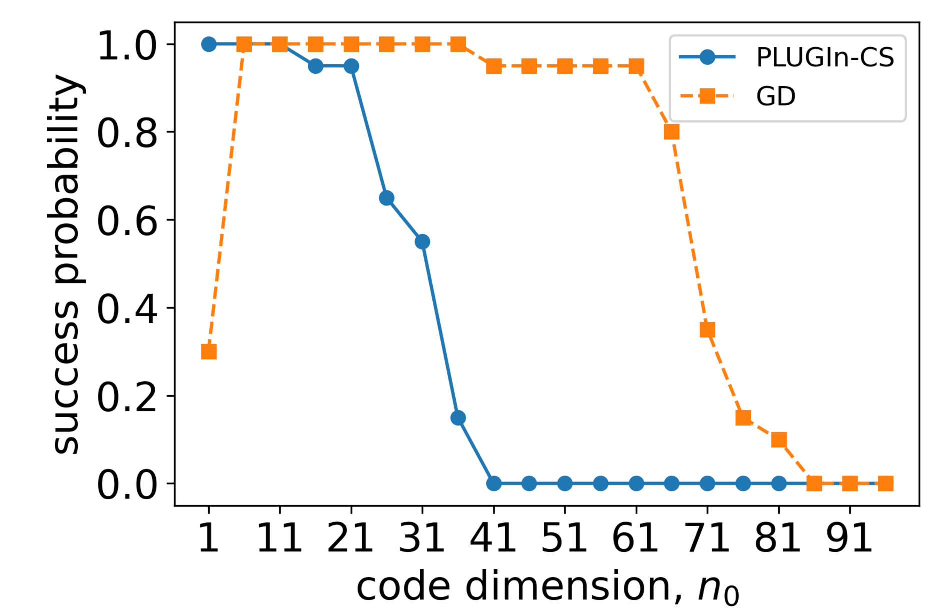
Numerical results

1) Performance of PLUGIn-CS and gradient descent (GD) on a 2-layer synthetic networks with random Gaussian weights.

(a) We compare their relative recovery error $\|\hat{x} - x^*\|/\|x^*\|$ and relative reconstruction error $\|\mathcal{G}(\hat{x}) - \mathcal{G}(x^*)\|/\|\mathcal{G}(x^*)\|$ w.r.t. noise level.

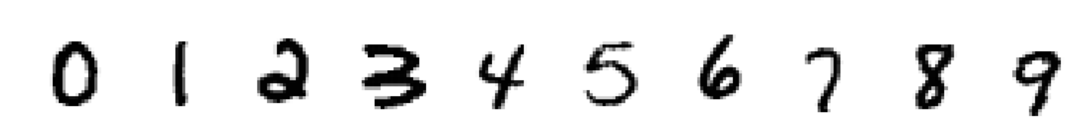


(b) We compare their success probability from 20 independent trials w.r.t. different code dimension n_0 (with $n_1 = 250$, $n_2 = 700$, $m = 150$ fixed).



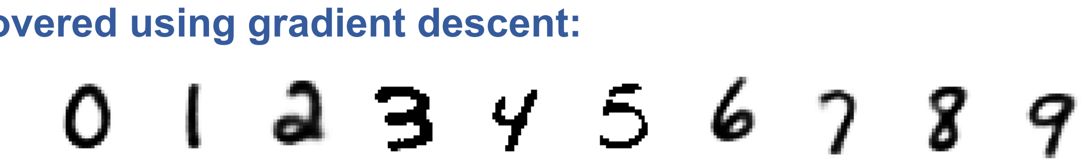
2) We trained a generative model on MNIST dataset, then took compressive measurements (with $m = 150$ and $n_d = 784$) of some images and tried PLUGIn-CS as well as gradient descent for recovery.

Original images:



Recovered using PLUGIn-CS:

Recovered using gradient descent:



1. Under assumptions listed in convergence results section.