Solving Inverse Problems in Medical Imaging with Score-based Generative Model

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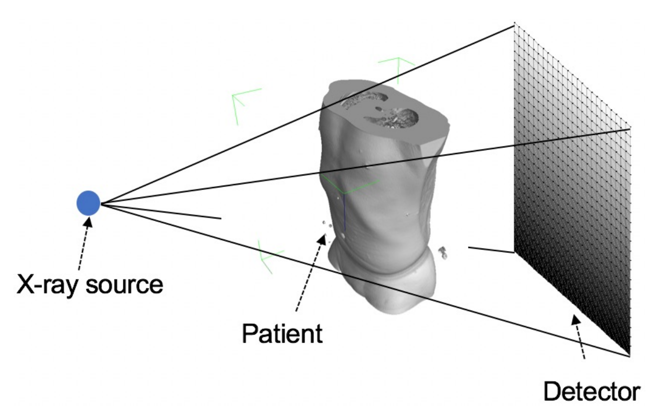
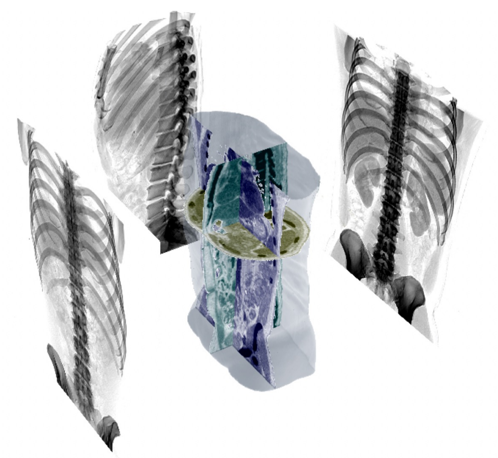
Background and Motivation

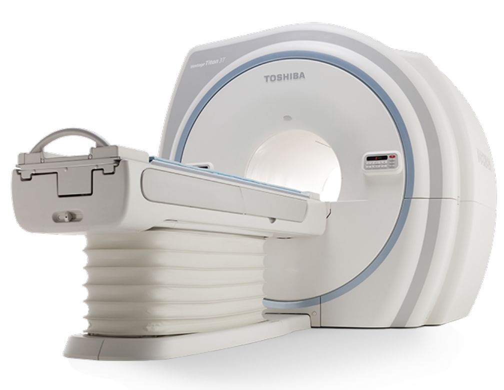
Why care sparse-sampling medical imaging?

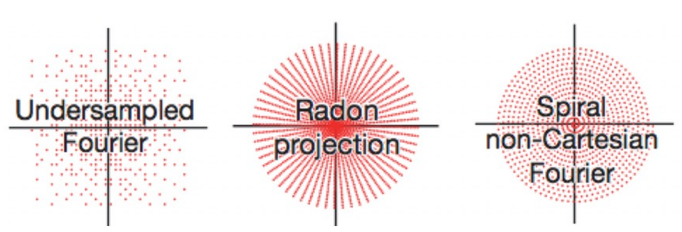
- X-ray Computed Tomography (CT) imaging
- Reduce radiation injury in CT: sample sparse projection views





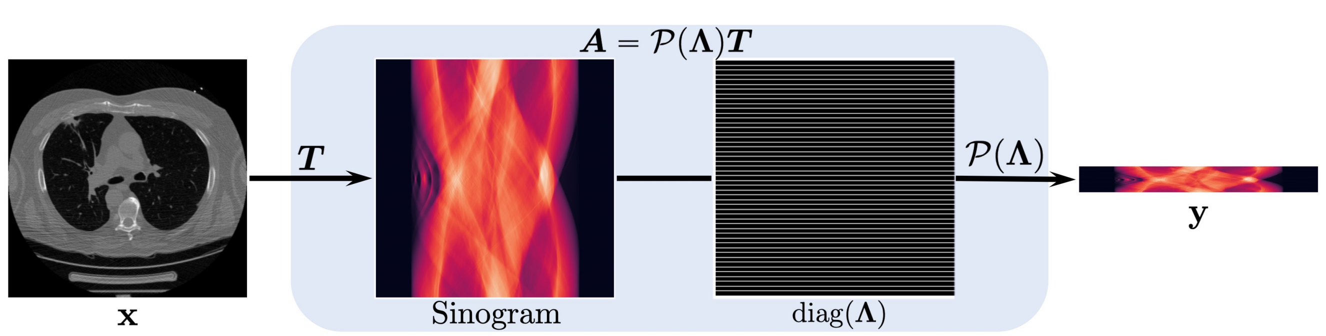
- Magnetic Resonance Imaging (MRI)
- Accelerate MRI scanning: under-sample k-space data



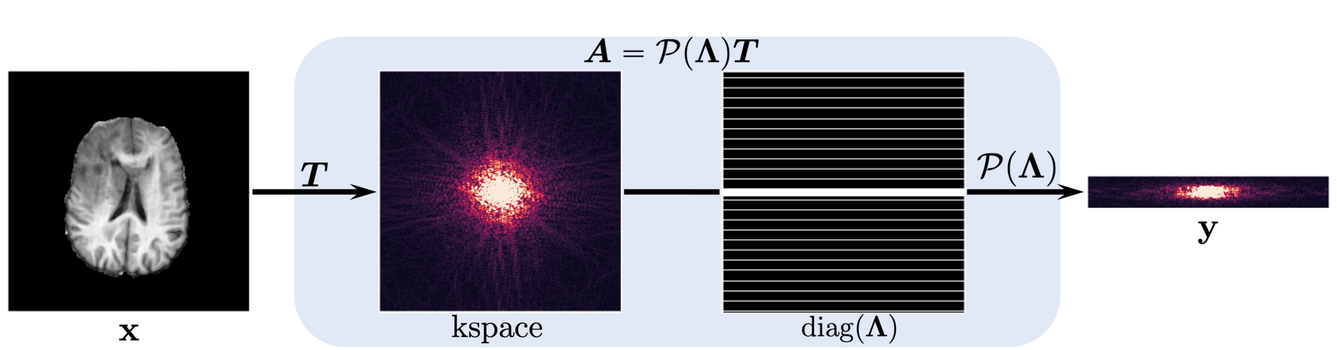


Inverse problem in medical imaging

- Measurement process for sparse-view CT
- Radon Transform + masking



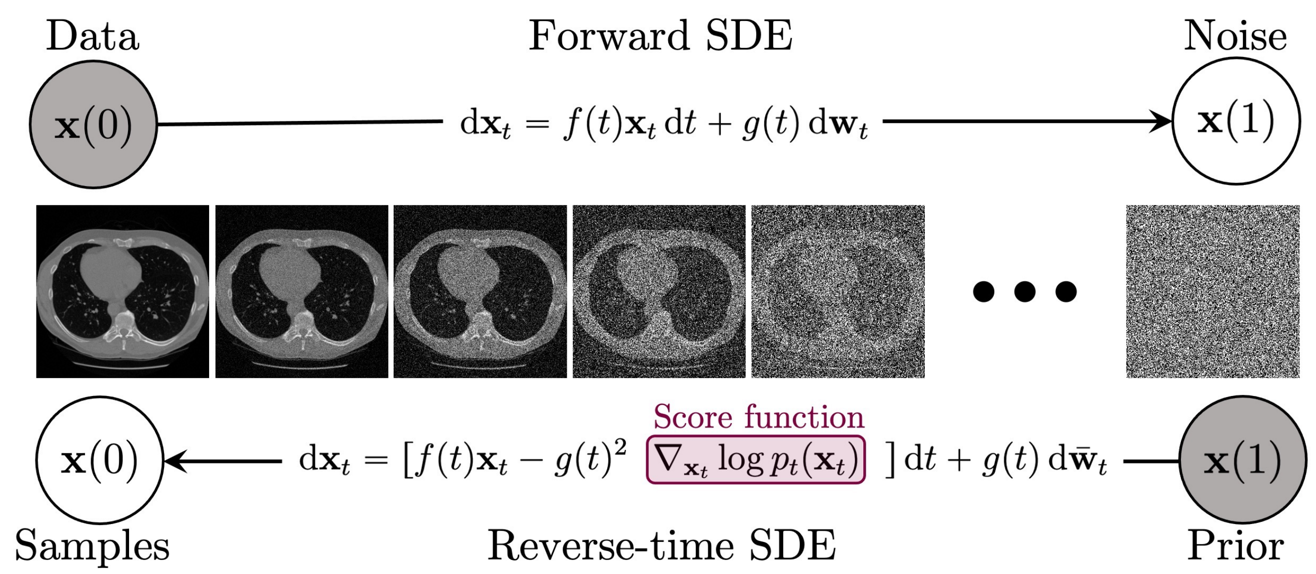
- Measurement process for under-sampled MRI
- Fourier Transform + masking



Score-based Generative Model

Train score-based generative model to capture prior data distribution

- Perturbation process: Forward SDE
- Sampling process: Reverse-time SDE

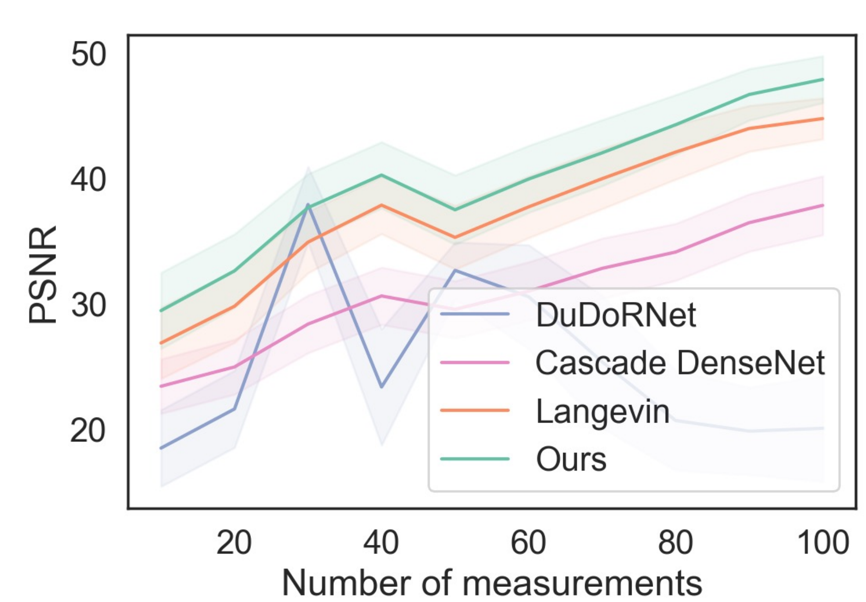


Results

- Comparable or better performance to supervised learning methods for sparse-sampling MRI/CT reconstruction

Method	Measurements	PSNR	SSIM
Undersampled MRI on BraTS 240 × 240			
Cascade DenseNet	30	$28.35_{\pm 2.30}$	$0.845_{\pm 0.038}$
DuDoRNet	30	$\mathbf{37.88}_{\pm \mathbf{3.03}}$	$\mathbf{0.985}_{\pm \mathbf{0.007}}$
Langevin	30	$36.44_{\pm 2.28}$	$0.952_{\pm 0.016}$
Ours	30	$37.63_{\pm 2.70}$	$0.958_{\pm 0.015}$
Sparse-view CT on LIDC 320 × 320			
FISTA-TV	23	$20.08_{\pm 4.89}$	$0.799_{\pm 0.061}$
cGAN	23	$19.83_{\pm 3.07}$	$0.479_{\pm 0.103}$
Neumann	23	$17.18_{\pm 3.79}$	$0.454_{\pm 0.128}$
SIN-4c-PRN	23	$30.48_{\pm 3.99}$	$0.895_{\pm 0.047}$
Ours	23	$\mathbf{35.24}_{\pm \mathbf{2.71}}$	$\mathbf{0.905}_{\pm \mathbf{0.046}}$

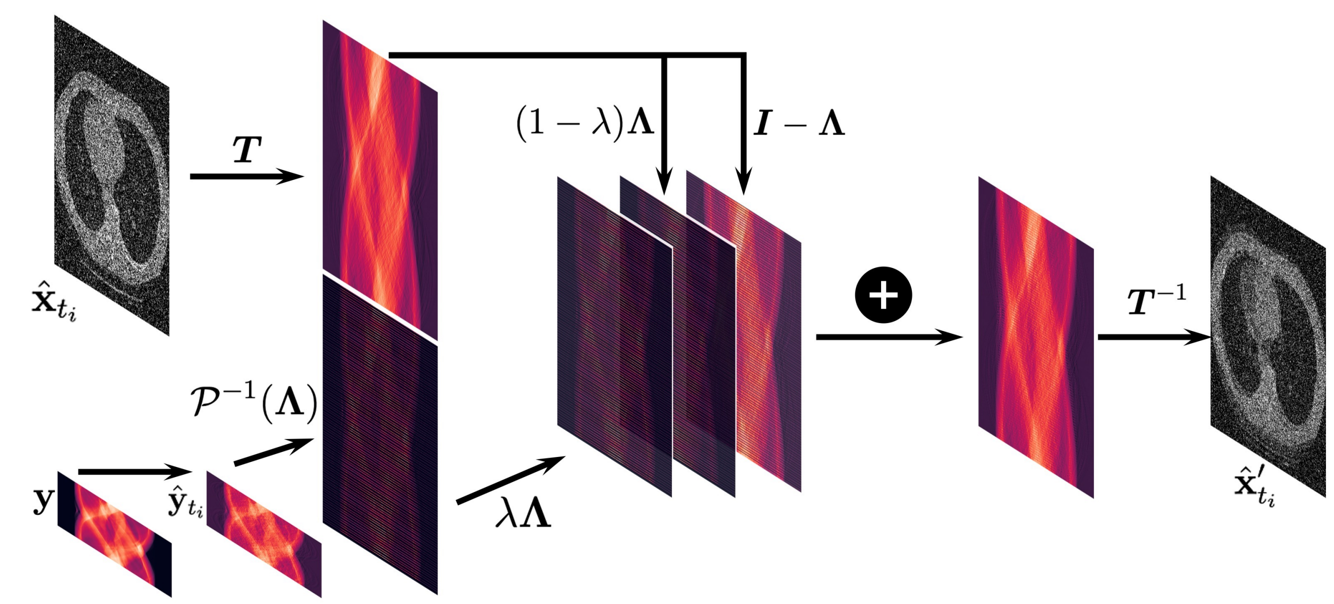
- Better generalization to unknown measurement processes



Method

Unsupervised technique for inverse problem solving

- Incorporate data consistency constraints into the sampling process



- Optimization problem with data prior and data consistency

$$\hat{\mathbf{x}}'_t = \arg\min_{\boldsymbol{z} \in \mathbb{R}^n} \{(1-\lambda)\,\|\boldsymbol{z} - \hat{\mathbf{x}}_t\|_{\boldsymbol{T}}^2 + \min_{\boldsymbol{u} \in \mathbb{R}^n} \lambda\,\|\boldsymbol{z} - \boldsymbol{u}\|_{\boldsymbol{T}}^2\}$$
$$s.t. \quad \boldsymbol{A}\boldsymbol{u} = \hat{\mathbf{y}}_t,$$

- Closed-form solution

$$\hat{\mathbf{x}}'_t = \lambda \boldsymbol{T}^{-1}\boldsymbol{\Lambda}\mathfrak{C}_{\boldsymbol{\Lambda}}^{-1}\hat{\mathbf{y}}_t + (1-\lambda)\boldsymbol{T}^{-1}\boldsymbol{\Lambda}\boldsymbol{T}\hat{\mathbf{x}}_t + \boldsymbol{T}^{-1}(\boldsymbol{I}-\boldsymbol{\Lambda})\boldsymbol{T}\hat{\mathbf{x}}_t.$$

- Convert sampler to an inverse problem solver

Algorithm 1 Unconditional sampling

Require: N
$\hat{\mathbf{x}}_1 \sim \pi(\mathbf{x}), \Delta t \leftarrow \frac{1}{N}$
for $i = N-1$ **to** 0 **do**
$t \leftarrow \frac{i+1}{N}$
$\hat{\mathbf{x}}_{t-\Delta t} \leftarrow \hat{\mathbf{x}}_t + g(t)^2 \boldsymbol{s}_{\boldsymbol{\theta}*}(\hat{\mathbf{x}}_t, t)\Delta t$
$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$
$\hat{\mathbf{x}}_{t-\Delta t} \leftarrow \hat{\mathbf{x}}_{t-\Delta t} + g(t)\sqrt{\Delta t}\,\mathbf{z}$
return $\hat{\mathbf{x}}_0$

Algorithm 2 Inverse problem solving

Require: $N, \mathbf{y}, \lambda$
$\hat{\mathbf{x}}_1 \sim \pi(\mathbf{x}), \Delta t \leftarrow \frac{1}{N}$
for $i = N-1$ **to** 0 **do**
$t \leftarrow \frac{i+1}{N}$
$\hat{\mathbf{y}}_t \sim p_{0t}(\mathbf{y}_t \mid \mathbf{y})$
$\hat{\mathbf{x}}_t \leftarrow \lambda \boldsymbol{T}^{-1}\boldsymbol{\Lambda}\mathfrak{C}_{\boldsymbol{\Lambda}}^{-1}\hat{\mathbf{y}}_t + (1-\lambda)\boldsymbol{T}^{-1}\boldsymbol{\Lambda}\boldsymbol{T}\hat{\mathbf{x}}_t + \boldsymbol{T}^{-1}(\boldsymbol{I}-\boldsymbol{\Lambda})\boldsymbol{T}\hat{\mathbf{x}}_t$
$\hat{\mathbf{x}}_{t-\Delta t} \leftarrow \hat{\mathbf{x}}_t + g(t)^2 \boldsymbol{s}_{\boldsymbol{\theta}*}(\hat{\mathbf{x}}_t, t)\Delta t$
$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$
$\hat{\mathbf{x}}_{t-\Delta t} \leftarrow \hat{\mathbf{x}}_{t-\Delta t} + g(t)\sqrt{\Delta t}\,\mathbf{z}$
return $\hat{\mathbf{x}}_0$

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