

445 **A Method Details**

446 **A.1 NeRF**

447 **Volume Rendering** A neural radiance field consists of two fields,  $\sigma_\phi(x), L_\psi(x, \omega)$  that encode  
 448 the density  $\sigma$  at every location  $x$  and the outgoing radiance  $L$  at that location in the direction  $\omega$ .  
 449 In NeRFs, both of these functions are represented by parameterized differentiable functions, such  
 450 as neural networks. Given a radiance field, we are able to march rays through an image plane  
 451 and reconstruct a camera image from a given camera pose and intrinsic matrix using the rendering  
 452 function:

$$I(x, \omega) = \int_0^T \sigma(t) \exp\left(\int_0^t \sigma(\hat{t})d\hat{t}\right)L(t, -\omega)dt \quad (\text{A.1})$$

453 Where  $L(t, \cdot)$  and  $\sigma(t)$  are shorthands for  $L(t\omega + x, \cdot)$  and  $\sigma(t\omega + x)$ , and  $I(x, \omega)$  is the intensity  
 454 at a location  $x$  given in world space in the direction  $\omega$ .

455 **Compositing** For our adversarial attacks to contain 3D semantics, it is crucial to insert the perturba-  
 456 tion in a 3D aware manner. For this we utilize another feature of neural radiance fields, which is  
 457 to output opacity values. Specifically, in Eqn. (A.1) we can extract the transmittance component,  
 458 which acts as a measure of the pixel transparency  $\alpha$ :

$$\alpha(x, \omega) = \exp\left(\int_0^t \sigma(\hat{t})d\hat{t}\right) \quad (\text{A.2})$$

459 Furthermore, we can replace the radiance term with distance in (A.1) to extract the expected termi-  
 460 nation depth of a ray  $z$ :

$$z(x, \omega) = \int_0^T t\sigma(t)\alpha(t)dt \quad (\text{A.3})$$

461 We consider the case of two radiance fields, the object radiance field  $\sigma_o, L_o$  and the background  
 462 radiance field  $\sigma_s, L_s$ . We use a transformation matrix to correspond ray coordinates between the  
 463 scene and the object radiance field.

464 By applying equations (A.1), (A.2), (A.3) to a single ray that corresponds to both the base scene  
 465 and the object radiance field, we obtain the values  $c_o, \alpha_o, z_o, c_s, \alpha_s, z_s$  respectively, where  $\alpha_*$   
 466 is the opacity and  $z_*$  is the depth along the ray. We denote the foreground and background values at a  
 467 pixel as

$$f = \underset{o,s}{\arg \min}(z_s, z_o) \quad (\text{A.4})$$

$$b = \underset{o,s}{\arg \max}(z_s, z_o) \quad (\text{A.5})$$

468 The final blended color is then given by:

$$c = \frac{\alpha_f c_f + (1 - \alpha_f)\alpha_b c_b}{\alpha_f + \alpha_b(1 - \alpha_f)} \quad (\text{A.6})$$

469 In the case of multiple object NeRFs, we simply repeat the alpha blending for each object to com-  
 470 posit them all into the same scene.

471 **A.2 Vehicle Dynamics**

472 The dynamics in equation (5) can take multiple forms, for the CARLA experiments, we choose the  
 473 simplest kinematic model of a car, a Dubin’s vehicle:

$$\dot{x} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ u \end{bmatrix} \quad (\text{A.7})$$

474 For the purposes of the CARLA deployment environment, we find that it is sufficient to consider the  
 475 kinematic model with fixed velocity, and only angular control. Thus, our imitation learning policy in

476 Eqn. (3) only outputs steering commands. We note that our approach is applicable to any dynamics  
 477 model, as long as it is differentiable.

478 For the real world experiments, we opted for a fixed velocity Ackerman steering model:

$$\dot{x} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \frac{v}{l} \tan(\theta) \end{bmatrix} \quad (\text{A.8})$$

479 where  $l$  is the robot wheelbase.

### 480 A.3 Implicit Differentiation

481 To carry out the adjoint method for obtaining gradients of the trajectory optimization problem stated  
 482 in Equation (1), we need to perform two passes over the trajectory.

483 Explicitly, the method performs a forward simulation to compute the variables  $x_t$  and then subse-  
 484 quently a backward pass to compute adjoint variables  $\lambda_t$  by solving the equations:

$$\frac{\partial G(x_{t-1}, x_t)}{\partial x_t}^\top \lambda_t = -\frac{\partial C(x_t)}{\partial x_t}^\top - \frac{\partial G(x_t, x_{t+1})}{\partial x_t}^\top \lambda_{t+1} \quad (\text{A.9})$$

485 with the boundary condition:

$$\frac{\partial G(x_{T-1}, x_T)}{\partial x_T}^\top \lambda_T = -\frac{\partial C(x_T)}{\partial x_T}^\top \quad (\text{A.10})$$

486 Finally, the gradient of the loss can be calculated as:

$$\nabla_{\theta} J = \lambda_1^\top \frac{\partial G(x_0, x_1, \theta)}{\partial x_0} \frac{\partial x_0}{\partial \theta} + \sum_{t=1}^T \lambda_t^\top \frac{\partial G(x_{t-1}, x_t, \theta)}{\partial \theta} \quad (\text{A.11})$$

487 Throughout both passes we do not need to store large intermediate variables and only need to accu-  
 488 mulate the gradient at each step.

### 489 A.4 Optimization Details

490 As described in Section 4.1, following prior work, we do not propagate gradients of camera param-  
 491 eters through the sensor model function. Specifically, we set,

$$o_t = h_{\gamma, \theta}(\text{stop\_gradient}(x_t)) \quad (\text{A.12})$$

492 Thus gradients of the observation will only be taken with respect to the adversarial object parameters  
 493  $\theta$  and not the state of the car. The gradient with respect to  $x_t$  corresponds to exploiting higher order  
 494 effects of how the observation would change if the car was looking in a slightly different direction  
 495 due to previous steps of the attacks, and leads to a very non-smooth loss objective that is not useful  
 496 for finding practical attacks.

497 For experiments in the real world, we found the attacks were sometimes very sensitive to the robot’s  
 498 pose. To alleviate this issue, we chose to optimize multiple randomly sampled initial poses simul-  
 499 taneously. The samples were normally distributed around the nominal car starting location, with a  
 500 standard deviation of 0.1.

#### 501 A.4.1 Optimization parameters

502 In all our experiments, our optimization parameters  $\theta$  correspond to values on the NGP voxel grid.  
 503 Since we have removed the decoder, the grid values directly correspond to the color for a given  
 504 position in the volume. Due to this, the parametrization even for small models can get quite large,  
 505 in the order of a 5 million for the hydrant.

## 506 **B Experimental Details**

### 507 **B.1 NeRF Models**

508 When training the surrogate NeRF models of the background scene and objects, we use the default  
509 Instant-NGP hyperparameters and optimize over 50 epochs using the Adam optimizer.

510 The source 3D assets for our objects were obtained from the Objaverse dataset [45] and posed  
511 images produced by rendering with Blender[46]. For our object models, we choose to use Instant-  
512 NGP without a decoder, instead directly encoding the colour values in the feature grid. Furthermore,  
513 we remove view dependence for better multi-view consistency. Finally, we use lower resolutions for  
514 the object feature grids as compared to the scene feature grids. The object feature grids contain  
515 resolutions up to  $128^3$  and  $64^3$  features for the car and hydrant, respectively. Since our adversarial  
516 objective does not have any smoothness constraint, we found it critical to use lower resolution grids  
517 and remove the positionally encoded feature decoders to avoid aliasing effects.

### 518 **B.2 Driving Policy.**

519 We train our own policy on which the attack will be performed. Our policy is an end-to-end RGB  
520 image neural network policy and the architecture is taken from [47]. We make a slight addition to  
521 goal condition the policy by adding a goal input to the linear component and increasing the capacity  
522 of the linear layers. The policy is trained via imitation learning, specifically DAGger [48], [49].

523 Expert actions are given by a lane following controller tuned for the simulator that gets access to  
524 the ground truth state, unlike the policy. The expert queried from various states random distances  
525 from the center of the road to recover from. Furthermore, random noise augmentation is used on the  
526 images during training to make the policy more robust to noisy observations.

### 527 **B.3 CARLA**

528 We fit the background scene model using a dataset of 1800 images and their corresponding camera  
529 poses, which provide a dense covering of the CARLA scene.

530 When transferring our attacks back to the deployment scene, opacity values are usually not available.  
531 In order to evaluate our attacks, we assume that objects are opaque ( $\alpha = 1$ ), and thus our method of  
532 blending in Equation A.3 can be calculated using just the depth and color values. We observe from  
533 experiments on the CARLA simulator that this type of composition is sufficient for the evaluation  
534 in the deployment environment.

535 **Driving Policy.** For our driving policy the initial training dataset of images is collected from the  
536 intersection in CARLA. We further fine-tuned the policy with some additional data collected from  
537 our surrogate simulator to ensure that our policy is not trivially failing due to slight visual differ-  
538 ences. We use a total dataset of 120000 images in CARLA and 60000 images in the surrogate  
539 simulator in order to train the policy. We validated our policy on a hold out validation set consisting  
540 of 12000 images captured purely from the surrogate simulator. All data were collected by running  
541 the expert on the 3 reference trajectories. The policy was trained using behaviour cloning, where we  
542 gave examples of recovery from deviation by collecting data from random start locations around the  
543 nominal trajectory.

### 544 **B.4 Real World**

545 We fit the background to a room in the real world using a dataset of 2161 images captured from an  
546 iPhone camera at 4K resolution. We collect data covering the room by walking around, then attach  
547 the iPhone to the robot to collect further data from the driving view points. The captured videos are  
548 processed using COLMAP [50, 51] for both camera intrinsic and the poses.

549 **Driving Policy.** We train a driving policy to track a square track in the room marked by green  
550 tape, this policy was trained using an expert PID controller with global positioning supplied by the



Figure B.1: Picture of driving area for the real world scenario experiments.

551 VICON system providing 9584 images. We further augment this again with 12000 images from  
552 driving data in the NeRF scene. An overview of our working area is given in Figure B.1.

553 For all real world attacks we optimize the color of a cube in the surrogate NeRF scene, placed at one  
554 of the corners such that the camera will encounter this cube as the car takes the turn.

#### 555 **B.4.1 Robot**

556 We carry out experiments using the RACECAR/J<sup>2</sup> platform. The robot is equipped with a ZED  
557 stereo camera, of which we only utilize the RGB data from the left sensor, which has been configured  
558 to a resolution of 366x188 at 10 frames per second. We operate the robot inside a VICON system  
559 that positions the robot at a rate of 50Hz streaming through a remotely connected computer that runs  
560 policy as well as the image processing for some of the attacks.

#### 561 **B.4.2 Green Screen Attack**

562 For the green screen attack, we utilized a VICON system to accurately position both our robot and  
563 the green screen target. Using the green screen target position, as well as the camera parameters,  
564 we project one face of the cube on the input image to the policy. We opt to overlay the cube in  
565 such a manner to keep the policy driving in real time and to ensure that there is no penalty on  
566 control frequency. The image compositions is done at the remote computer where the controls are  
567 computed, which are then sent wirelessly to the robot to execute.

#### 568 **B.4.3 Monitor Attack**

569 To replace the green screen with a physical object, we place a monitor and display the same attack  
570 as above on the monitor. We place the monitor in a location such that it is visually consistent with  
571 the NeRF and green screen attacks. For the monitor attack, we utilize a 27-inch monitor with a 16:9  
572 aspect ratio. Since the adversarial objects optimized in earlier examples are cubes we only use the  
573 center of the monitor to display the attack.

## 574 **C Additional Experimental Results**

### 575 **C.1 Incorporating Discovered Adversarial Scenarios in the Training Set**

576 Our primary focus in this paper was to discover adversarial attacks for the evaluation of pretrained  
577 self-driving policy. Here we perform some preliminary investigations on fine-tuning our self-driving  
578 policies, on the old data and the adversarial attacks we found. Specifically, we take the attacks  
579 discovered by the gradient-based optimization and use them to collect additional imitation learning  
580 data. The collection is performed in the CARLA simulator using the depth compositing approach

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<sup>2</sup><https://racecarj.com/>

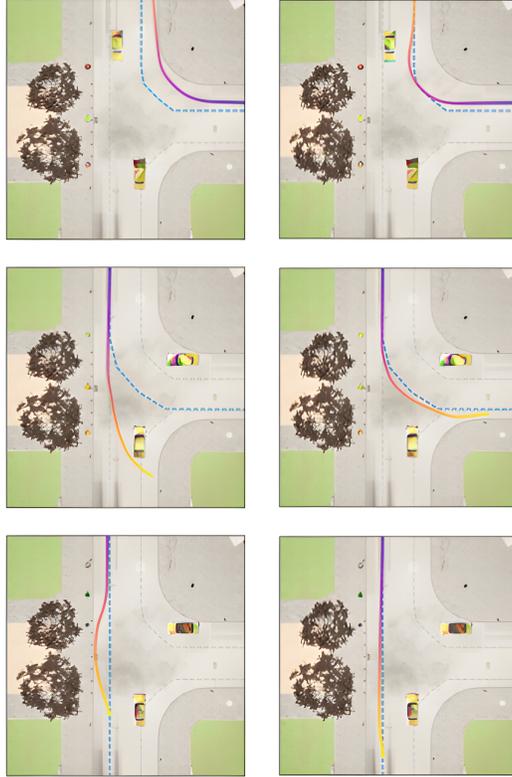


Figure C.1: The performance of the driving policy before (left) and after (right) retraining on the discovered adversarial scenarios.

Scenario	CARLA	Attack Transfer in CARLA		CARLA Attack After Retraining
	Unperturbed	Random	Gradient	Gradient
Straight	1166	$1193 \pm 19$	$1702. \pm 160$	1250
Right	1315	$1476 \pm 12$	$2101. \pm 75$	1307
Left	1448	$1158 \pm 163$	$2240. \pm 574$	1419

Table 2: Comparison of the total cross-track error for the retraining experiment over the 3 different trajectories. Results are extending the results from the main paper Table 1 shown for the following cases: (1) no attack in CARLA (unperturbed), (2) an attack in the CARLA scene, (3) an attack in the CARLA scene after the driving policy is retrained using adversarial data.

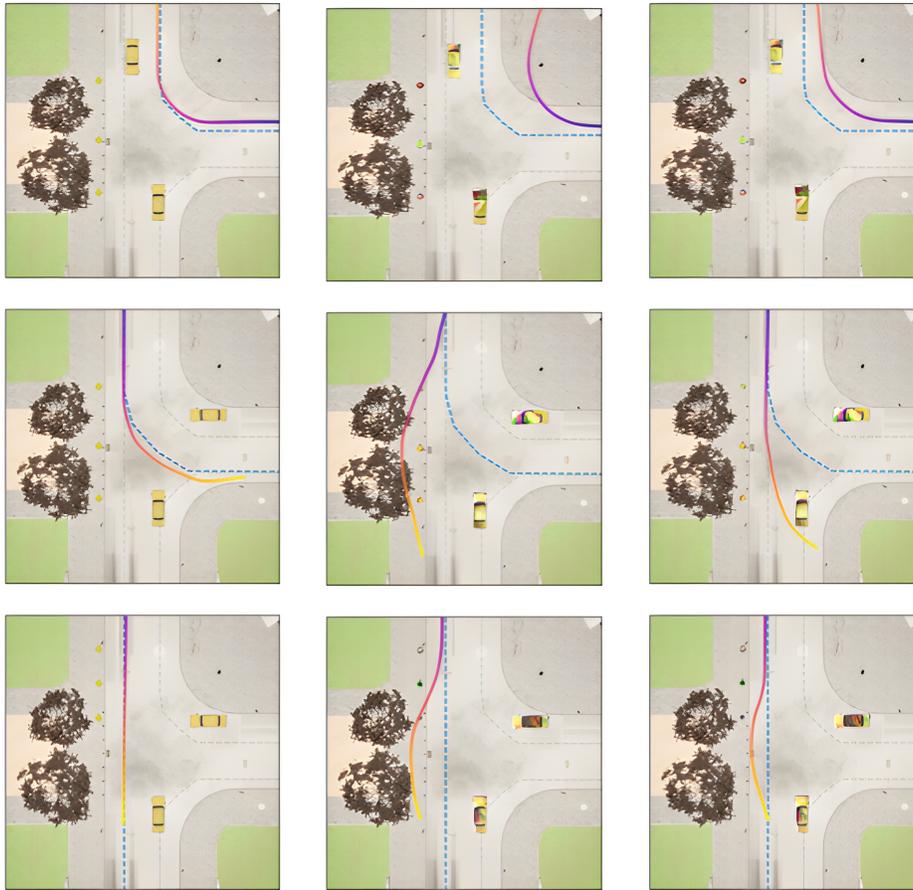
581 to insert the adversarial objects, as was done for the evaluation in the main paper. Apart from  
 582 the object compositing, the data is collected in the same way as the original CARLA data used  
 583 to train the base policy. We collect 24000 total frames over three trajectories with two different  
 584 starting points. After fine-tuning our policy on the combination of the original dataset and the new  
 585 adversarially augmented dataset, we evaluate the fine-tuned agent in the same scenario. We visualize  
 586 the trajectories of the fine-tuned policy in Figure C.1 and report on the total deviation compared to  
 587 before fine-tuning in Table 2. We find that the policy is no longer susceptible to the adversarial  
 588 attacks, even though the initial starting position for evaluation was unseen during training.

## 589 C.2 CARLA Visualizations

590 We show first person visualizations of our discovered adversarial attacks inserted back into the  
 591 CARLA deployment simulator in Figure C.2. We note the smoothness of the texture discovered  
 592 by our method. Purely perceptual single-frame attacks typically exhibit a much higher frequency  
 593 texture.



Figure C.2: Sample renderings of the left turn trajectory with the adversarial perturbations in CARLA from the ego vehicle's point of view. Four different snapshots from the evolution of the trajectory are shown.

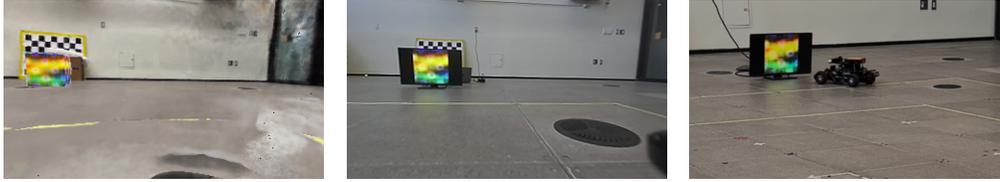


(a) Unperturbed

(b) Attacks in NERF

(c) Transferred

Figure C.3: Overhead views of three distinct trajectories driven by the policy. (a) shows the policy driving behavior in CARLA when no adversarial perturbation is introduced. (b) shows the policy driving behavior in the surrogate simulator with the discovered adversarial perturbation. (c) shows the same perturbation transferred to the deployment scene.



(a) Surrogate Simulator

(b) First person view

(c) Third person view

Figure C.4: Real-world adversarial monitor attack visualizations.

594 We show additional overhead trajectory views of adversarially attacked trajectories from one  
595 CARLA scene in Figure C.3.

### 596 C.3 Real-world Visualizations

597 We show aligned visualizations of the same adversarial real-world monitor attack in Figure C.4.