A Closer Look at Reference Learning for Fourier Phase Retrieval

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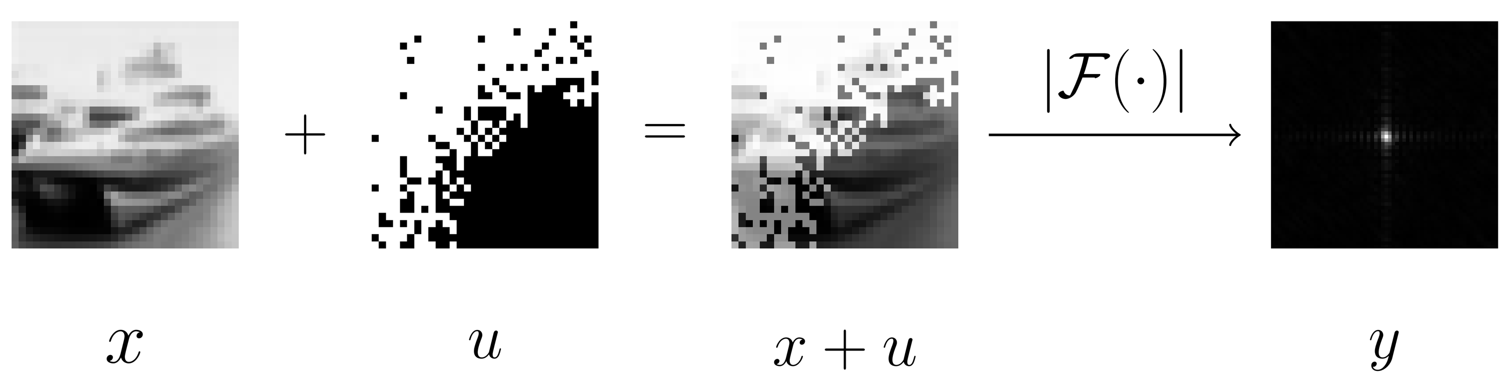
Problem Definition

- We consider a modified phase retrieval problem [3], which allows for a reference image u to be added onto the image x before the Fourier magnitudes y are measured, i.e.,

$$y = |\mathcal{F}(x + u)|.$$

- We consider the oversampled as well as the more difficult non-oversampled problem.
- The measurement process can be visualized as follows:



Our Contribution

- We modify the Gerchberg-Saxton (GS) algorithm [1] to utilize a reference image for Fourier phase retrieval.
- By unrolling the GS algorithm we can learn good reference images using gradient descent.
- We compare the performance of learned and simple sampled reference images and show in which cases learned references are better than sampled ones.

Unrolling the Gerchberg-Saxton Algorithm

Algorithm 1: GS algorithm with reference image

Input: Fourier magnitude $y \in \mathbb{R}^{d \times d}$, reference image $u \in \mathbb{R}^{d \times d}$, initialization $x_0 \in \mathbb{R}^{d \times d}$, number of iterations n

Output: Reconstruction $x_n \in \mathbb{R}^{d \times d}$

for $k = 1, \ldots, n-1$ **do**
- $p_{k+1} \leftarrow \mathcal{F}(x_k + u) / |\mathcal{F}(x_k + u)|$
- $\bar{x}_{k+1} \leftarrow \mathcal{F}^{-1}(p_{k+1} \odot y) - u$
- $x_{k+1} \leftarrow \max(0, \bar{x}_{k+1})$

end

return x_n

- We calculate the mean squared error between the original image x and the reconstruction x_n.
- By back-propagating through the unrolled GS iterations, we can calculate the gradient w.r.t. the reference image and update it.

Constructing a Simple Reference

- To sample our simple reference, we mimic the appearance of the learned references, e.g., the asymmetric, large flat areas.
- Our simple reference can be created by the following steps:

1. Start with a black image with a white square in the bottom right corner
2. Blur the image using a Gaussian filter
3. Re-normalize the image such that the pixel values are between $[0, 1]$
4. Add Poission noise onto the image
5. Finally, binarize the image by thresholding the pixel intensities

References

[1] Ralph W Gerchberg. A practical algorithm for the determination of phase from image and diffraction plane pictures. *Optik*, 35:237--246, 1972.

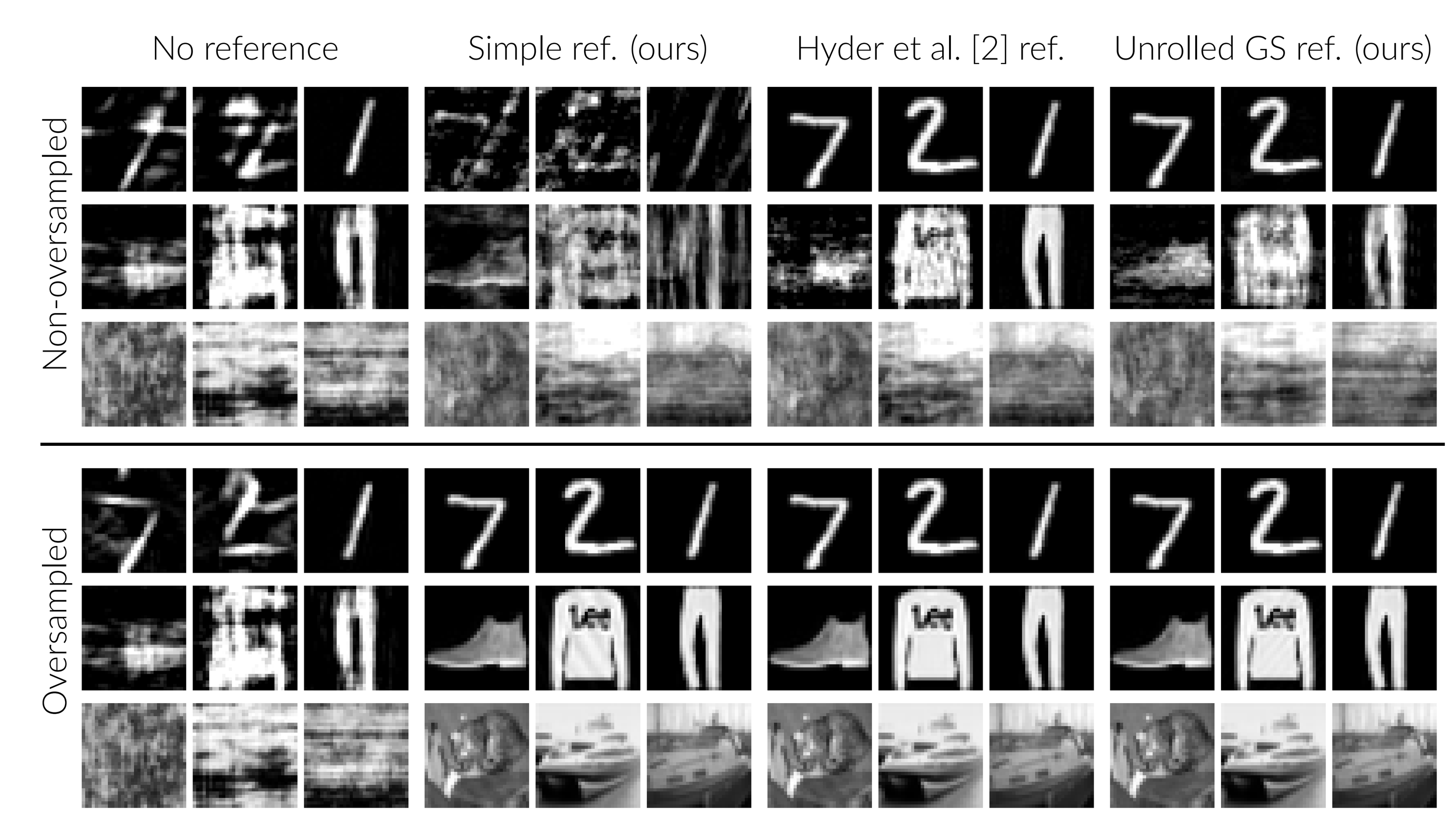
[2] Rakib Hyder, Zikui Cai, and M Salman Asif. Solving phase retrieval with a learned reference. In *European Conference on Computer Vision*, pages 425--441. Springer, 2020.

[3] Wooshik Kim and Monson H Hayes. Phase retrieval using two fourier-transform intensities. *JOSA A*, 7(3):441--449, 1990.

Experimental Results

- Our unrolled GS algorithm is able to learn references of similar quality as the method proposed by Hyder et al. [2].
- We see that the reconstruction errors of our simple reference are quite close to the errors of the learned methods in the oversampled case.
- The following table shows the mean squared error for reconstructions from oversampled and non-oversampled Fourier magnitude measurements using different reference images:

	Method	MNIST	EMNIST	FMNIST	SVHN	CIFAR-10
Non-oversampled	No reference	0.035615	0.063414	0.042417	0.013985	0.035134
	Random ref.	0.052724	0.079784	0.046670	0.012393	0.030222
	Random ref. (binary)	0.055324	0.081130	0.049436	0.012447	0.029299
	Simple ref. (ours)	0.060027	0.089347	0.067549	0.011270	0.024128
	Hyder et al. [2]	0.002607	**0.014687**	**0.013649**	0.010131	0.024141
	Unrolled GS (ours)	**0.002181**	0.015427	0.019863	**0.008775**	**0.020020**
Oversampled	No reference	0.020566	0.032907	0.021068	0.007516	0.020518
	Random ref.	0.005350	0.025308	0.011484	0.003670	0.009848
	Random ref. (binary)	0.001170	0.010994	0.006842	0.002462	0.007310
	Simple ref. (ours)	0.000761	0.003681	0.000848	0.000187	0.000495
	Hyder et al. [2] ref.	0.000132	**0.000023**	0.000073	0.000126	0.001415
	Unrolled GS ref. (ours)	**0.000071**	0.000257	**0.000055**	**0.000055**	**0.000125**



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