Dear Authors of Submission #1293,

Thank you for your submission to ICCV 2025. The review process has now concluded. Below you will find the meta-review and final reviews for your submission, which will be available on OpenReview shortly.

Congratulations, your submission #1293, titled "Axis-level Symmetry Detection with Group-Equivariant Representation" has been accepted to ICCV 2025.
You will receive additional information for submitting the camera-ready version shortly. Please note that acceptance is contingent on passing a plagiarism check. Papers that do not comply with plagiarism and dual-submission rules will be rejected.

This year, we received 11,239 valid submissions that underwent the review process. The program committee recommended 2,698 papers for acceptance, resulting in an acceptance rate of 24%. All papers were initially evaluated by at least three independent reviewers. Following the author rebuttal and reviewer discussions, moderated by an Area Chair (AC), a triplet of three ACs reviewed each paper holistically, considering the reviews, author rebuttal, and reviewer discussions. For challenging cases, additional ACs and/or Program Chairs (PCs) were consulted.

We would like to thank the reviewers and Area Chairs for their contributions to the review process.

Best Regards,
ICCV Program Chairs

Axis-level Symmetry Detection with Group-Equivariant Representation

Supplementary Material

A. Regular representation and group convolution

A.1. Discrete group representation

Regular group representation. The regular representation of a finite group $G = \{g_1, \dots, g_N\}$ acts on a vector space $\mathbb{R}^{|G|}$. For any element $g \in G$, the regular representation $\sigma^G_{\text{reg}}(g)$ is defined as:

$$\sigma^G_{\text{reg}}(g) = [\mathbf{e}_{g\cdot g_1}, \dots, \mathbf{e}_{g\cdot g_N}], \tag{1}$$

where each group element $g_i \in G$ is associated with a basis vector $\mathbf{e}_{g_i} \in \mathbb{R}^{|G|}$. In regular representation, $\sigma^G_{\text{reg}}(g) \in \mathbb{R}^{|G|\times|G|}$ is a permutation matrix that maps each basis vector $\mathbf{e}_{g_i}$ to $\mathbf{e}_{g\cdot g_i}$ for all $g_i \in G$.

Cyclic group representation. The cyclic group C_N, consisting of N discrete planar rotations, is defined as $\{r^0, r^1, \dots, r^{(N-1)}\}$ with rotation generator r. With the group law $r^a \cdot r^b = r^{(a+b) \bmod N}$, the regular representation of r^n is given by:

$$\sigma^{\mathrm{C}_N}_{\text{reg}}(r^n) = [\mathbf{e}_{r^n}, \mathbf{e}_{r^{(n+1) \bmod N}}, \dots, \mathbf{e}_{r^{(n+N-1) \bmod N}}], \tag{2}$$

where the basis vectors are defined from:

$$\sigma^{\mathrm{C}_N}_{\text{reg}}(r^0) = \mathrm{I}_N, \tag{3}$$

where I_N being the $N\times N$ identity matrix. Here, the regular representation of the cyclic group corresponds to a cyclic permutation matrix.

Dihedral group representation. The dihedral group $\mathrm{D}_N = \{r^0, \dots, r^{N-1}, b, rb, \dots, r^{N-1}b\}$, consisting of $2N$ elements, is an extension of the cyclic group that includes an additional reflection generator b. The regular representation of the element $r^n b$ is given by:

$$\begin{aligned}\sigma^{\mathrm{D}_N}_{\text{reg}}(r^n b) = [&\mathbf{e}_{r^n b}, \mathbf{e}_{r^n b\cdot r}, \dots, \mathbf{e}_{r^n b\cdot r^{N-1}},\\ &\mathbf{e}_{r^n b\cdot b}, \mathbf{e}_{r^n b\cdot rb}, \dots, \mathbf{e}_{r^n b\cdot r^{N-1} b}]\end{aligned} \tag{4}$$

using the group laws $b^2 = e$ and $r^n b = br^{-n}$. By changing the order of cyclic rotation and reflection, the equation can be transformed as:

$$\begin{aligned}\sigma^{\mathrm{D}_N}_{\text{reg}}(br^n) = [&\mathbf{e}_{br^n}, \mathbf{e}_{br^n\cdot r}, \dots, \mathbf{e}_{br^n\cdot r^{N-1}},\\ &\mathbf{e}_{br^n\cdot b}, \mathbf{e}_{br^n\cdot rb}, \dots, \mathbf{e}_{br^n\cdot r^{N-1} b}]\end{aligned} \tag{5}$$

The basis vectors for the dihedral group are defined from:

$$\sigma^{\mathrm{D}_N}_{\text{reg}}(r^0 b^0) = \mathrm{I}_{2N}. \tag{6}$$

A.2. Discrete group convolution

Conventional convolutional neural networks (CNNs) are inherently equivariant to translations, meaning that a translation of the input results in a corresponding translation of the output. The standard 2D convolution operation can be expressed as:

$$(f * \psi)(\mathbf{x}) = \sum_{\mathbf{y}\in\mathbb{Z}^2} f(\mathbf{y})\,\psi(\mathbf{x}-\mathbf{y}), \tag{7}$$

where $f : \mathbb{Z}^2 \to \mathbb{R}^{\mathcal{C}_{\text{in}}}$ is the input function with $\mathcal{C}_{\text{in}}$ channels, $\psi : \mathbb{Z}^2 \to \mathbb{R}^{\mathcal{C}_{\text{in}}\times\mathcal{C}_{\text{out}}}$ is the filter, and $\mathbf{x}, \mathbf{y} \in \mathbb{Z}^2$ are spatial coordinates. Here, plane feature map is defined only along the spatial dimension $\mathbb{Z}^2$. To associate discrete group within the feature map, an additional dimension corresponding to the group G should be constructed, resulting in the mapping $f_G : G \times \mathbb{Z}^2 \to \mathbb{R}^{\mathcal{C}}$. In the discrete group convolution, this additional dimension is constructed through the lifting operation:

$$f_G = \bigoplus_{g\in G} (f * g\psi). \tag{8}$$

The order of the stack corresponds to the order of group elements in the initial state. Since the lifted feature map contains features corresponding to each group element, transformations must account for both spatial changes and the group structure. Applying a specific group element $g' \in G$ to the lifted feature map thus requires both spatial transformation and permutation of the group dimension:

$$(g' \cdot f_G)(\mathbf{x}) = \sigma^G_{\text{reg}}(g') \cdot f_G(g'^{-1}\mathbf{x}), \tag{9}$$

where $\sigma^G_{\text{reg}}(g')$ is the block diagonal form of the regular representation of g' repeated $\mathcal{C}$ times, permuting along the group dimension, while $g'^{-1}\cdot\mathbf{x}$ applies the spatial transformation. Following the lifting operation, group convolution for the lifted feature map is defined as:

$$\begin{aligned}&[f_G *_G \psi](g, \mathbf{x})\\ &\quad= \sum_{g'\in G}\sum_{\mathbf{y}\in\mathbb{Z}^2} f_G(g', \mathbf{y})\left[\sigma^G_{\text{reg}}(g)\psi(g^{-1}(\mathbf{x}-\mathbf{y}))\right](g').\end{aligned} \tag{10}$$

Here, $\psi : G\times\mathbb{Z}^2 \to \mathbb{R}^{\mathcal{C}_{\text{in}}\times\mathcal{C}_{\text{out}}}$ represents the group convolution filter, where $f_G : G \times \mathbb{Z}^2 \to \mathbb{R}^{\mathcal{C}_{\text{in}}}$ is the lifted feature map, and $g, g' \in G$ are group elements of G. The key property of group convolution is its equivariance to group elements, expressed as:

$$\begin{aligned}[(g' \cdot f_G) *_G \psi](\mathbf{x}) &= [g' \cdot (f_G *_G \psi)](\mathbf{x}) \\ &= \sigma_{\text{reg}}^{G}(g') \cdot (f_G *_G \psi)(g'^{-1} \cdot \mathbf{x}),\end{aligned} \tag{11}$$

for any $g' \in G$. Here, $(g' \cdot f_G) *_G \psi$ represents the group convolution applied to the transformed input, while $g' \cdot (f_G *_G \psi)$ is the action of g' on the result of the group convolution. This equality demonstrates that the order of applying group transformations and group convolutions is interchangeable, preserving the group structure throughout the network layers.

B. Cyclic group-equivariance of the reflectional matching

B.1. Cyclic group-equivariance of the single fiber reflectional matching

Given a D_N-equivariant feature map $\mathbf{F} \in \mathbb{R}^{\mathcal{C}|\mathrm{D}_N| \times H \times W}$ under the regular representation σ_{reg}, we need to prove that $\mathbf{H}$ from Reflectional Matching without spatial expansion is equivariant to the cyclic group C_N with its element r^k:

$$\begin{aligned}&\bigoplus_{n=0}^{N-1} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^n)\mathbf{F}_{\mathbf{x}}^{(0,k)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^n)\mathbf{F}_{\mathbf{x}}^{(0,k)}\right) \\ &= \sigma_{\text{reg}}^{\mathrm{D}_N}(r^k) \bigoplus_{n=0}^{N-1} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^n)\mathbf{F}_{\mathbf{x}}^{(0,0)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^n)\mathbf{F}_{\mathbf{x}}^{(0,0)}\right),\end{aligned} \tag{12}$$

where $\mathbf{F}_{\mathbf{x}}^{(l,n)}$ is the fiber at position $\mathbf{x}$, with the regular representation corresponding to l reflections and n rotations added. Using the property $\sigma(g)\sigma(h) = \sigma(gh)$, the equation can be rewritten as:

$$\begin{aligned}&\bigoplus_{n=0}^{N-1} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^n)\mathbf{F}_{\mathbf{x}}^{(0,k)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^n)\mathbf{F}_{\mathbf{x}}^{(0,k)}\right) \\ &= \bigoplus_{n=0}^{N-1} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k+n})\mathbf{F}_{\mathbf{x}}^{(0,0)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^{k+n})\mathbf{F}_{\mathbf{x}}^{(0,0)}\right).\end{aligned} \tag{13}$$

Here, h is the similarity function defined as:

$$h(\mathbf{f}^1, \mathbf{f}^2) = \bigoplus_{c=1}^{\mathcal{C}} \frac{\mathbf{f}_c^1 \cdot \mathbf{f}_c^2}{\|\mathbf{f}_c^1\|\|\mathbf{f}_c^2\|} \in \mathbb{R}^{\mathcal{C}}, \tag{14}$$

Since permutation matrices preserve the norm of a vector, and using the rule $r^{a+b} = r^{(a+b) \bmod N}$, the equation can be reformulated as:

$$\begin{aligned}&\bigoplus_{n=0}^{N-1}\bigoplus_{c=1}^{\mathcal{C}} \frac{1}{\|\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\|^2}\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k+n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)} \cdot \sigma_{\text{reg}}^{\mathrm{D}_N}(br^{k+n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\right) \\ &= \bigoplus_{c=1}^{\mathcal{C}} \frac{1}{\|\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\|^2} \bigoplus_{n=0}^{N-1}\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k+n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)} \cdot \sigma_{\text{reg}}^{\mathrm{D}_N}(br^{k+n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\right)\end{aligned} \tag{15}$$

$$= \bigoplus_{c=1}^{\mathcal{C}} \frac{1}{\|\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\|^2} \bigoplus_{n=k}^{k+N-1}\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)} \cdot \sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\right) \tag{16}$$

$$\begin{aligned}&= \bigoplus_{c=1}^{\mathcal{C}} \frac{1}{\|\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\|^2} \sigma_{\text{reg}}^{\mathrm{D}_N}(r^k) \\ &\qquad \bigoplus_{n=0}^{N-1}\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)} \cdot \sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\right)\end{aligned} \tag{17}$$

$$\begin{aligned}&= \sigma_{\text{reg}}^{\mathrm{D}_N}(r^k) \bigoplus_{c=1}^{\mathcal{C}} \frac{1}{\|\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\|^2} \\ &\qquad \bigoplus_{n=0}^{N-1}\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)} \cdot \sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}_{c,\mathbf{x}}^{(0,0)}\right)\end{aligned} \tag{18}$$

$$= \sigma_{\text{reg}}^{\mathrm{D}_N}(r^k) \bigoplus_{n=0}^{N-1} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^n)\mathbf{F}_{\mathbf{x}}^{(0,0)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^n)\mathbf{F}_{\mathbf{x}}^{(0,0)}\right), \tag{19}$$

where $\mathbf{F}_{c,\mathbf{x}}$ denotes the feature at position $\mathbf{x}$ in channel c.

B.2. Cyclic group equivariance of spatially expanded reflectional matching

We now have to to prove the spatial expansion of single fiber Reflectional Matching is also equivariant to the cyclic group C_N:

$$\begin{aligned}&\bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^n)\mathbf{F}_{\mathbf{x}+r^{k+n}(\mathbf{q})}^{(0,k)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^n)\mathbf{F}_{\mathbf{x}+br^{k+n}(\mathbf{q})}^{(0,k)}\right) \\ &= \sigma_{\text{reg}}^{\mathrm{D}_N}(r^k) \bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} h\left(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^n)\mathbf{F}_{\mathbf{x}+r^n(\mathbf{q})}^{(0,0)}, \sigma_{\text{reg}}^{\mathrm{D}_N}(br^n)\mathbf{F}_{\mathbf{x}+br^n(\mathbf{q})}^{(0,0)}\right),\end{aligned} \tag{20}$$

where $\mathbf{q} \in \mathcal{Q}$ is the offset, $r^n(\mathbf{q})$ represents the spatially rotated offset, and $br^n(\mathbf{q})$ denotes the offset that is first rotated and then reflected. Same as single fiber, the equation can be written as:

$$\bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} h\Big(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k+n})\mathbf{F}^{(0,0)}_{\mathbf{x}+r^{k+n}(\mathbf{q})},$$

$$\sigma_{\text{reg}}^{\mathrm{D}_N}(br^{k+n})\mathbf{F}^{(0,0)}_{\mathbf{x}+br^{k+n}(\mathbf{q})}\Big)$$

$$= \bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} \bigoplus_{c=1}^{\mathcal{C}} \frac{\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k+n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{k+n}(\mathbf{q})}\cdot}{\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{k+n}(\mathbf{q})}\|\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{k+n}(\mathbf{q})}\|}$$

$$\cdot\,\sigma_{\text{reg}}^{\mathrm{D}_N}(br^{k+n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{k+n}(\mathbf{q})} \tag{21}$$

$$= \bigoplus_{c=1}^{\mathcal{C}} \bigoplus_{n=k}^{k+N-1} \sum_{\mathbf{q}\in\mathcal{Q}} \frac{\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{n}(\mathbf{q})}\cdot}{\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{n}(\mathbf{q})}\|\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{n}(\mathbf{q})}\|}$$

$$\cdot\,\sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{n}(\mathbf{q})} \tag{22}$$

$$= \bigoplus_{c=1}^{\mathcal{C}} \sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k}) \bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} \frac{\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{n}(\mathbf{q})}\cdot}{\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{n}(\mathbf{q})}\|\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{n}(\mathbf{q})}\|}$$

$$\cdot\,\sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{n}(\mathbf{q})} \tag{23}$$

$$= \sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k}) \bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} \bigoplus_{c=1}^{\mathcal{C}} \frac{\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{n}(\mathbf{q})}\cdot}{\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+r^{n}(\mathbf{q})}\|\|\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{n}(\mathbf{q})}\|}$$

$$\cdot\,\sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}^{(0,0)}_{c,\mathbf{x}+br^{n}(\mathbf{q})} \tag{24}$$

$$= \sigma_{\text{reg}}^{\mathrm{D}_N}(r^{k}) \bigoplus_{n=0}^{N-1} \sum_{\mathbf{q}\in\mathcal{Q}} h\Big(\sigma_{\text{reg}}^{\mathrm{D}_N}(r^{n})\mathbf{F}^{(0,0)}_{\mathbf{x}+r^{n}(\mathbf{q})},$$

$$\sigma_{\text{reg}}^{\mathrm{D}_N}(br^{n})\mathbf{F}^{(0,0)}_{\mathbf{x}+br^{n}(\mathbf{q})}\Big). \tag{25}$$