Mapping neural representations of topologically non-trivial spaces

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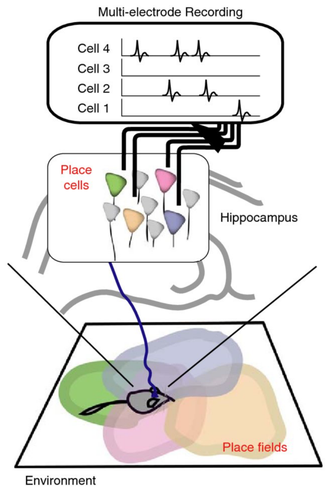
Intro: Place Cells

Place cells (PCs) are **neurons** found in the **hippocampus** (its CA1 region) of living brains of many species – that **map** the physical space[1] (also, spaces of other[2] stimuli) with **place (receptive) fields (PFs)**. **Nobel Prize** in Phys. & Med. of 2014 – John O'Keefe, May-Britt and Edvard I. Moser.



Inverse Problem: given signal of PCs –> **reconstruct**/learn the structure of the space of stimuli (e.g. physical space).

References

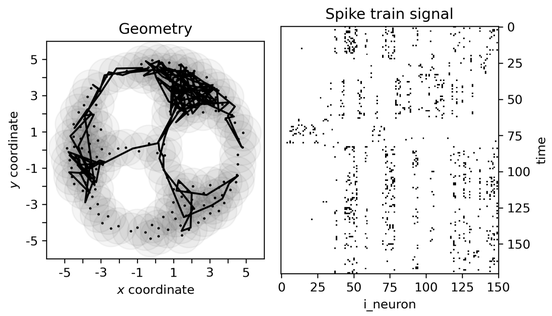
1. O'Keefe, J., & Dostrovsky, J. (1971). The hippocampus as a spatial map: preliminary evidence from unit activity in the freely-moving rat. *Brain research*.
2. Curto, C., & Itskov, V. (2008). Cell groups reveal structure of stimulus space. PLoS computational biology, 4(10), e1000205.
3. Image from: Wagatsuma, H., & Yamaguchi, Y. (2007). Neural dynamics of the cognitive map in the hippocampus. *Cognitive Neurodynamics*, *1*(2), 119-141.

Model

Following Dabaghian et al.[4], we model PCs' activity (**cognitive map**) as **Poisson processes**, so the **firing rates** of PCs depend only on the **position** of the cognitive **agent:**

$$\text{firing\_rate}_{\text{i\_neuron}}(\vec{r}(t)) \;\propto\; \sigma_{\text{RF}}^{-1}\,\exp(-||\vec{r}-\vec{c}_{\text{RF}}||^2/2\sigma_{\text{RF}}^2)$$

– when at time t, the agent is at position $\vec{r}$, at a certain distance from the center $\vec{c}_{\text{RF}}$ of receptive field (RF) of i-th place cell (neuron) – the **firing rate (FR)** of said neuron has **Gaussian** dependence on said distance.
This FR is the parameter of a Poisson process of i-th neuron's activity – modelled in discrete time as a Bernoulli trial – whether the neuron fires ("success") or not ("failure"), with $P(\text{success})$ proportional to the FR.



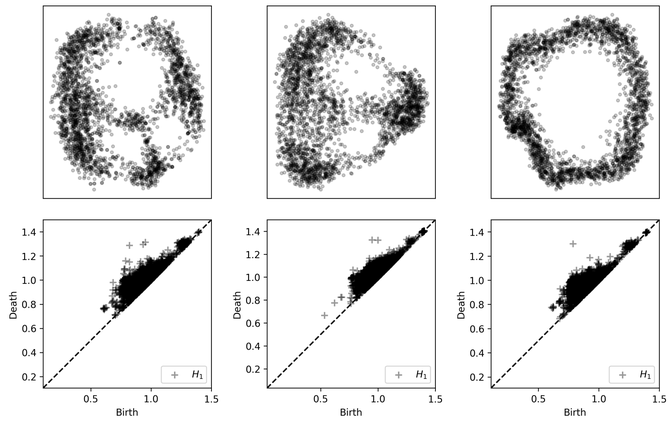
The agent is performing a **Levy flight random walk** over the arena: the lengths of its steps follow a **Pareto distr.** with decay exponent value = 1.5, angle chosen uniformly.

References

4. Dabaghian, Y., Mémoli, F., Frank, L., & Carlsson, G. (2012). A topological paradigm for hippocampal spatial map formation using persistent homology.

Analysis

We simulate such signals with 3 different arena **topologies**: **1, 2, 3 holes** (impenetrable obstacles).



We compute:

1) **Persistent homologies**[5] of PC signal only (spike train matrix after a proper normalization) – persistence diagrams (PDs) at the ***bottom*** of the above figure
2) **Isomap**[6] "projections" = reconstructions of the coordinates from signal only – shown at the ***top***

References

5. Bauer, U. (2021). Ripser: efficient computation of Vietoris–Rips persistence barcodes. Journal of Applied and Computational Topology, 5(3), 391-423.
6. Tenenbaum, J. B., Silva, V. D., & Langford, J. C. (2000). A global geometric framework for nonlinear dimensionality reduction. science, 290(5500), 2319-2323.

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"Topology of Cognitive Maps" arxiv.org/abs/2211.08718

🙏Thank you!🙏