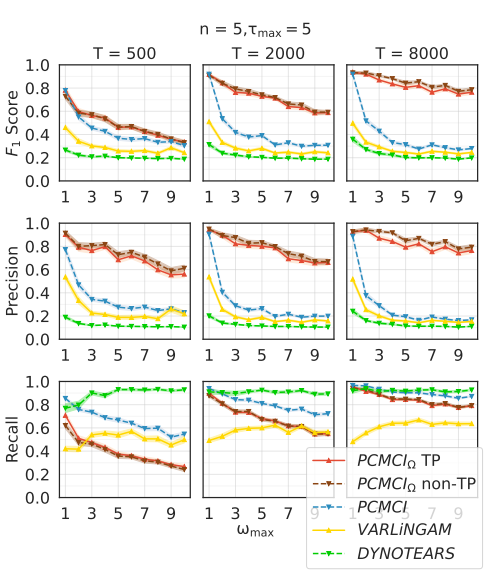
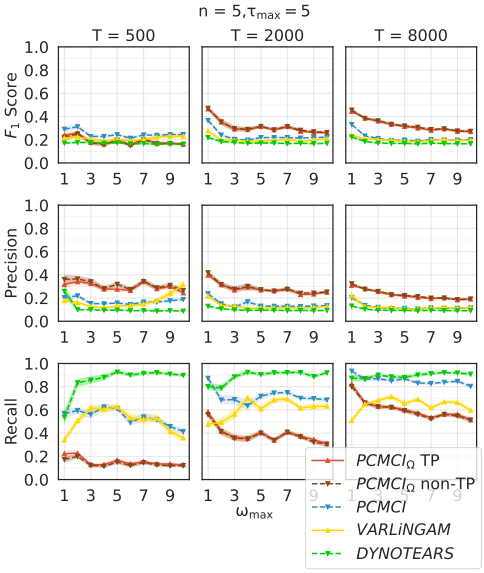
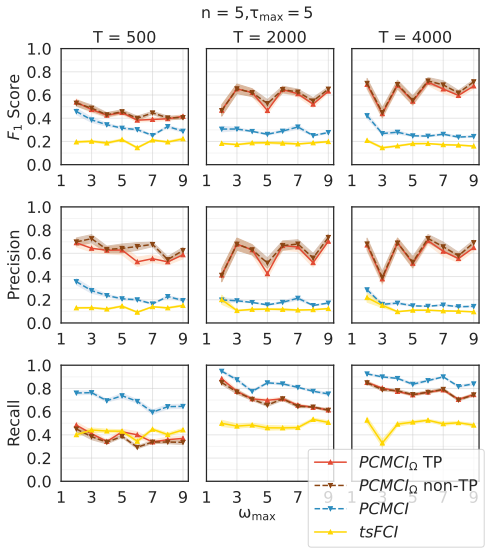


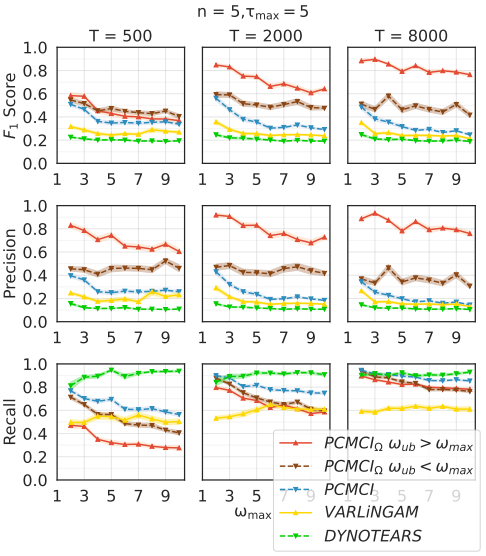
(a) Performance with and without turning point



(b) Performance in non-stationary setting without periodicity



(c) Performance in non-linear SCM



(d) Performance when $\omega_{ub} > \omega_{max}$ and $\omega_{ub} < \omega_{max}$

Figure 1: Multiple algorithms are tested on 5-multivariate time series with different time length T. PCMCI$_\Omega$ TP means that the turning point rules are utilized when choosing ω. See lines 13-16 in Algorithm C1 and section F in the supplementary material. PCMCI$_\Omega$ non-TP means that turning point rule is not applied and ω is chosen based on Lemma 3.4. Every line corresponds to a different algorithm. Every marker corresponds to the average performance over 50 trials. In (a), the consistent performance of PCMCI under different chosen rules of ω supports our theoretical result, that is, the correct periodicity ω leads to the most sparse causal graph. In (b), data sets are in a non-stationary setting without periodicity. In (c), the structural causal model (SCM) is non-linear. In (d), algorithm PCMCI$_\Omega$ are tested under conditions that $\omega_{ub} > \omega_{max}$ and $\omega_{ub} < \omega_{max}$ respectively.