A Appendix (Supplementary Material)

A.1 Proofs Lower Bound Results

Theorem 3. *Let $q_1 \dots, q_d$ be a set collection of disjoint queries and let q_* be their sum. Let M be a randomized algorithm whose input is a dataset and whose output is a positively weighted dataset. Suppose M guarantees that for each query q_i and dataset $\mathfrak{D}$, $E[(q_i(\mathfrak{D}) - q_i(M(\mathfrak{D})))^2] \leq C^2$ and $E[(q_*(\mathfrak{D}) - q_*(M(\mathfrak{D})))^2] \leq D^2$ for some values C and D, where the expectation is **only** over the randomness in M.*

- *If M satisfies ϵ-differential privacy then for any $k > 0$, we have $e^{2\epsilon(2C+k)} \geq \frac{k(d-1)}{16C+8D+4k}$ which implies **(a)** if $D^2 \leq \lambda/\epsilon^2$ for some constant λ, then $C^2 \in \Omega(\frac{1}{\epsilon^2}\log^2(d))$, and **(b)** if $C \leq \lambda/\epsilon^2$ then $D \in \Omega(d^2/\epsilon^2)$.*
- *If M satisfies (ϵ, δ)-DP then for any $k > 0$, we have $\left(\frac{\delta}{\epsilon} + \frac{4C+2D+k}{k(d-1)}\right)e^{4\epsilon C + 2k\epsilon} \geq 1/4$, which implies **(a)** if $D^2 \leq \lambda/\epsilon^2$ for some constant λ, then $C^2 \in \Omega\left(\min(\frac{1}{\epsilon^2}\log^2(d),\ \frac{1}{\epsilon^2}\log^2\frac{\epsilon}{\delta})\right)$; **(b)** if $C \leq \lambda/\epsilon^2$ then either $\epsilon \in O(\delta)$ or $D^2 \in \Omega(d^2/\epsilon^2)$.*
- *If M satisfies ρ-zCDP, then the tradeoff function between C and D (which is more complex and omitted due to space constraints) implies: **(a)** if $D^2 \leq \lambda/\rho$ for some λ, then $C^2 \in \Omega(\log(d)/\rho)$, and **(b)** if $C^2 \leq \lambda/\rho$, then for any $\gamma \in (0,1)$, we must have $D^2 \in \Omega(d^{2\gamma}/\rho)$.*

Proof. The lower bounds for pure and approximate DP (but not zCDP) can be proved as consequences of the work of Balcer and Vadhan [3]. To make this material more self-contained, we write out a direct proof of the lower bounds by borrowing their proof technique.

For notational convenience, we will let $\mathbf{x}[i]$ denote $q_i(\mathfrak{D})$ (so $\sum_i \mathbf{x}[i] = \sum_i q_i(\mathfrak{D}) = q_*(\mathfrak{D})$). Similarly, we let $\widetilde{\mathbf{x}}[i]$ denote $q_i(\widetilde{\mathfrak{D}})$ (so $\sum_i \widetilde{\mathbf{x}}[i] = \sum_i q_i(\widetilde{\mathfrak{D}}) = q_*(\widetilde{\mathfrak{D}})$). Thus the vector $\mathbf{x}$ represents the true point query answers and $\widetilde{\mathbf{x}}$ represents the privacy protected point query answers. In particular, $\mathbf{x}$ is a vector of nonnegative integers and $\widetilde{\mathbf{x}}$ is a vector of nonnegative real numbers.

All probabilities are taken with respect to only the randomness in M.

In this proof, α, β, and k are constants that we will set later. For any fixed j, by Markov's inequality,

$$P(|\widetilde{\mathbf{x}}[j] - \mathbf{x}[j]| \geq \alpha C) \leq \frac{E\left[(\mathbf{x}[j] - \widetilde{\mathbf{x}}[j])^2\right]}{C^2\alpha^2} \leq \frac{1}{\alpha^2} \tag{2}$$

$$P\left(\Big|\sum_{i=1}^{d}\widetilde{\mathbf{x}}[i] - \sum_{i=1}^{d}\mathbf{x}[i]\Big| \geq \beta D\right) \leq \frac{1}{\beta^2} \tag{3}$$

For each positive integer n, positive number k, and $i = 2, \dots, d$, define the set

$$G_{i,n,k} = \left\{\widetilde{\mathbf{x}} \ :\ \substack{\widetilde{\mathbf{x}}[i] \in [k, k+2\alpha C], \\ \widetilde{\mathbf{x}}[1] \in [n-2\alpha C-k, n-k], \\ \sum_{j=1}^{d}\widetilde{x}[j] \in [n-\beta D, n+\beta D]}\right\}$$

The intuition behind the meaning of $G_{i,n,k}$ is that suppose we had a dataset $\mathfrak{D}_i$ with vector $\mathbf{x}_i$ of point query answers where the i^{th} point query satisfied $\mathbf{x}_i[i] = k + \alpha C$ and $\mathbf{x}_i[1] = n - k - \alpha C$ (all other entries are 0) then $G_{i,n,k}$ is the set of all possible outputs $M(\mathfrak{D}_i)$ where the 1^{st} and i^{th} entries are within αC of their true value and the sum is within βD of its true value.

For any fixed n and k, we next examine how many $G_{i,n,k}$ a vector $\widetilde{\mathbf{x}}$ can belong to (i.e., an overlap condition). A necessary condition for $\widetilde{\mathbf{x}}$ to belong to some $G_{i,n,k}$ is that $\widetilde{\mathbf{x}}[i] \geq k$ and $\widetilde{\mathbf{x}}[1] \geq n - 2\alpha C - k$. This means that after assigning the minimal necessary mass to the 1^{st} element, there is at most $k + 2\alpha C + \beta D$ mass to assign to the other elements (without exceeding the upper limit of $n + \beta D$ on the sum of all the cells). Since at least k units of this mass must be assigned to the i^{th} element in order for $\widetilde{\mathbf{x}}$ to belong to $G_{i,n,k}$, this means that $\widetilde{\mathbf{x}}$ can belong to $G_{i,n,k}$ for at most $\frac{2\alpha C+\beta D+k}{k}$ different choices of i.

Now define $\mathbf{x}_1, \dots, \mathbf{x}_d$ as follows. $\mathbf{x}_1[1] = n$ with all other entries being 0. Next for $i = 2, \dots, d$ we set $\mathbf{x}_i[1] = n - \alpha C - k$ and $\mathbf{x}_i[i] = \alpha C + k$ and all other entries of $\mathbf{x}_i$ are 0. For each i, Let $\mathfrak{D}_i$ be a database whose point query answers are $\mathbf{x}_i$, which is possible since the point queries are disjoint (and this means that $\mathfrak{D}_1$ differs from all of the others by the addition/removal of at least $2(\alpha C + k)$ records).

For pure differential privacy, we have:

$$
\begin{aligned}
1 &\geq P\left(M(\mathfrak{D}_1) \in \bigcup_{i=2}^{d} G_{i,n,k}\right) \geq \frac{k}{2\alpha C + \beta D + k} \sum_{i=2}^{d} P\left(M(\mathfrak{D}_1) \in G_{i,n,k}\right) \text{ by overlap condition} \\
&\geq e^{-\epsilon 2(\alpha C + k)} \frac{k}{2\alpha C + \beta D + k} \sum_{i=2}^{d} P\left(M(\mathfrak{D}_i) \in G_{i,n,k}\right) \text{ by group privacy property of } \epsilon\text{-DP [18]} \\
&\geq e^{-\epsilon 2(\alpha C + k)} \frac{k}{2\alpha C + \beta D + k} \sum_{i=2}^{d} \left(1 - \frac{2}{\alpha^2} - \frac{1}{\beta^2}\right) \text{ by the Markov inequality and union bound} \\
&= e^{-\epsilon 2(\alpha C + k)} \frac{k(d-1)}{2\alpha C + \beta D + k} \left(1 - \frac{2}{\alpha^2} - \frac{1}{\beta^2}\right)
\end{aligned}
$$

Now we set $\alpha = 2$ and $\beta = 2$ to get

$$
e^{2\epsilon(2C+k)} \geq \frac{k(d-1)}{16C + 8D + 4k}
$$

If D is allowed to be $\leq C$, then we set $k = C$ and get

$$
e^{6\epsilon C} \geq \frac{d-1}{28} \qquad \Rightarrow \qquad C \geq \frac{1}{6\epsilon} \log \frac{d-1}{28}
$$

In general, if $D \in O(C)$ (i.e., D is allowed to be at most some constant times C) then similar arguments show $C \in \Omega\left(\frac{1}{\epsilon}\log(d)\right)$.

If D is allowed to be $> C$ then we set $k = 1/\epsilon$ and get

$$
e^{4\epsilon C + 2} \geq \frac{(d-1)}{24\epsilon D + 4} \qquad \Rightarrow \qquad C \geq \frac{1}{4\epsilon}\left(\log\left(\frac{(d-1)}{24\epsilon D + 4}\right) - 2\right)
$$

In general, if $D \in \Omega(C)$ (i.e., D is required to be at least some constant times C) then similar arguments show that $C \in \Omega\left(\frac{1}{\epsilon}\log(\frac{d}{\epsilon D})\right)$.

Putting these facts together, we see that if $D \in O(1/\epsilon)$ then $C \in \Omega(\frac{1}{\epsilon}\log(d))$. Meanwhile, if $C \in O(1/\epsilon)$ then we must have $D \in \Omega(d/\epsilon)$.

For approximate, differential privacy, using the group privacy property of approximate differential privacy [3],

$$
\begin{aligned}
1 &\geq P\left(M(\mathfrak{D}_1) \in \bigcup_{i=2}^{d} G_{i,n,k}\right) \geq \frac{k}{2\alpha C + \beta D + k} \sum_{i=2}^{d} P\left(M(\mathfrak{D}_1) \in G_{i,n,k}\right) \\
&\geq \frac{k}{2\alpha C + \beta D + k} \sum_{i=2}^{d} \left(e^{-\epsilon 2(\alpha C + k)} P\left(M(\mathfrak{D}_i) \in G_{i,n,k}\right) - \delta/\epsilon\right) \text{ by group privacy} \\
&\geq \frac{k}{2\alpha C + \beta D + k} \sum_{i=2}^{d} \left(e^{-\epsilon 2(\alpha C + k)} \left(1 - \frac{2}{\alpha^2} - \frac{1}{\beta^2}\right) - \delta/\epsilon\right) \text{ Markov inequality, union bound} \\
&= \frac{k(d-1)}{2\alpha C + \beta D + k} \left(e^{-\epsilon 2(\alpha C + k)} \left(1 - \frac{2}{\alpha^2} - \frac{1}{\beta^2}\right) - \delta/\epsilon\right)
\end{aligned}
$$

Setting $\alpha = 2$ and $\beta = 2$ gives

$$
1 \geq \frac{k(d-1)}{4C + 2D + k}\left(\frac{1}{4}e^{-\epsilon 2(2C+k)} - \delta/\epsilon\right) \text{ and so } \left(1 + \frac{\delta}{\epsilon}\frac{k(d-1)}{4C + 2D + k}\right)e^{4\epsilon C + 2k\epsilon} \geq \frac{1}{4}\frac{k(d-1)}{4C + 2D + k}
$$

and this is the same as

$$\left(\frac{\delta}{\epsilon}+\frac{4C+2D+k}{k(d-1)}\right)e^{4\epsilon C+2k\epsilon}\geq 1/4$$

Noting that for any z, $1+z\leq 2\max(1,z)$ and so

$$e^{4\epsilon C+2k\epsilon}\geq\frac{1}{8}\min\left(\frac{k(d-1)}{4C+2D+k},\ \frac{\epsilon}{\delta}\right)$$

Proceeding as we did for pure differential privacy, if D is allowed to be $O(C)$, then $C\in\Omega\left(\min\left(\frac{1}{\epsilon}\log(d),\ \frac{1}{\epsilon}\log\frac{\epsilon}{\delta}\right)\right)$; if D is allowed to be $\Omega(C)$ then $C\in\Omega\left(\min(\frac{1}{\epsilon}\log\frac{d}{\epsilon D},\ \frac{1}{\epsilon}\log\frac{\epsilon}{\delta})\right)$.

Putting this together, if $D\in O(1/\epsilon)$ then $C\in\Omega\left(\min\left(\frac{1}{\epsilon}\log(d),\ \frac{1}{\epsilon}\log\frac{\epsilon}{\delta}\right)\right)$ and if $C\in O(1/\epsilon)$ then either $\epsilon\in O(\delta)$ or $D\in\Omega(d/\epsilon)$.

For ρ-zCDP, consider a random variable X that is uniform over $\mathfrak{D}_2,\dots,\mathfrak{D}_d$ (i.e., with probability $1/(d-1)$, X is the dataset $\mathfrak{D}_i$). Note that the $\mathfrak{D}_i$ we have been using can be constructed so that $i\neq j$, $\mathfrak{D}_i$ and $\mathfrak{D}_j$ differ on the addition/removal of $2aC+2k$ people. Let $I(\cdot;\cdot)$ denote mutual information and $H(\cdot)$ denote entropy. By the group privacy property of zCDP [8] we have two facts relating group privacy to mutual information: (1) $\rho(2\alpha C+2k)^2\geq I(M(\mathfrak{D}_i);M(\mathfrak{D}_j))$ for all i and j (from Proposition 5.3 proof in [8]) and (2) the corollary that $\rho(2\alpha C+2k)^2\geq I(X,M(X))$ (from Proposition 6.1 proof in [8]). Then

$$\begin{aligned}\rho(2\alpha C+2k)^2&\geq I(X;M(X))=H(X)-H(X\mid M(X))\\&=\log_2(d-1)-H(X\mid M(X))\end{aligned}\tag{4}$$

and now we need to upper bound $H(X\mid M(X))$. Define G to be the event that $M(X)$ is in the $G_{i,n,k}$ that corresponds to the realized value of X (i.e., the event $X=\mathfrak{D}_j\Rightarrow M(X)\in G_{j,n,k}$ for $j=2,\dots,d$). Then we obtain a Fano-like inequality (following the proof structure in [12]) as follows:

$$\begin{aligned}H(X\mid M(X))&=H(X\mid M(X))+H(G\mid X,M(X))\\&\quad\text{(the last entropy is 0 since G is a deterministic function of X and $M(X)$)}\\&=H(G,X\mid M(X))\quad\text{by the chain rule for conditional entropy}\\&=H(G\mid M(X))+H(X\mid G,M(X))\quad\text{by chain rule for conditional entropy}\\&\leq 1+H(X\mid G,M(X))\quad\text{since G is binary, its entropy is ≤ 1}\\&=1+P(G=0)H(X\mid M(X),G=0)+P(G=1)H(X\mid M(X),G=1)\\&\leq 1+P(G=0)\log_2(d-1)+P(G=1)H(X\mid M(X),G=1)\\&\quad\text{(since the entropy of X is $\leq\log_2(d-1)$)}\\&\leq 1+P(G=0)\log_2(d-1)+P(G=1)\log_2\left(\frac{2\alpha C+\beta D+k}{k}\right)\\&\text{(This follows from $G=1$, by the overlap condition, since then $M(X)$ can}\\&\text{belong to at most $\frac{2\alpha C+\beta D+k}{k}$ of the $G_{i,n,k}$ so conditioned on knowing $M(X)$}\\&\text{there are at most $\frac{2\alpha C+\beta D+k}{k}$ possible choices for X}\\&\text{and hence $\log_2$ of this quantity upper bounds the conditional entropy)}\\&\leq 1+P(G=0)\log_2(d-1)+\log_2\left(\frac{2\alpha C+\beta D+k}{k}\right)\\&\leq 1+(\frac{2}{\alpha^2}+\frac{1}{\beta^2})\log_2(d-1)+\log_2\left(\frac{2\alpha C+\beta D+k}{k}\right)\end{aligned}\tag{5}$$

Where the last inequality follows from the Markov inequality and union bound on $P(G=0)$. Now, setting $\alpha=\beta=2$ and combining Equations 4 and 5, we have:

$$\rho(4C+2k)^2 \geq \frac{1}{4}\log_2(d-1) - \log_2\left(\frac{4C+2D+k}{k}\right) - 1$$

If D is allowed to be $\leq C$, we set $k = C$ to get

$$\begin{aligned}\rho(6C)^2 &\geq \frac{1}{4}\log_2(d-1) - \log_2\left(\frac{4C+2D+C}{C}\right) - 1 \geq \frac{1}{4}\log_2(d-1) - \log_2\left(\frac{7C}{C}\right) - 1\\ &\geq \frac{1}{4}\log_2(d-1) - 4 \Rightarrow C \geq \frac{1}{6\sqrt{\rho}}\sqrt{\frac{\log_2(d-1)}{4} - 4}\end{aligned}$$

so in general, if $D \in O(C)$ then similar arguments show that $C \in \Omega\left(\sqrt{\frac{1}{\rho}\log(d)}\right)$.

If D is allowed to be $> C$, then let γ be any number strictly between 0 and 1. Then we set $k = 1/\sqrt{\rho}$, and $\alpha = \beta = \sqrt{\frac{3}{(1-\gamma)}}$. Combining Equations 4 and 5

$$\begin{aligned}&(2\alpha\sqrt{\rho}C + 2)^2\\ &\geq \left(1 - \frac{2}{\alpha^2} - \frac{1}{\beta^2}\right)\log_2(d-1) - \log_2\left(2\alpha\sqrt{\rho}C + \beta\sqrt{\rho}D + 1\right) - 1\\ &= \gamma\log_2(d-1) - \log_2\left(2\alpha\sqrt{\rho}C + \beta\sqrt{\rho}D + 1\right) - 1\\ &\geq \gamma\log_2(d-1) - \log_2\left(3\sqrt{\frac{3}{1-\gamma}}\sqrt{\rho}D + 1\right) - 1\end{aligned}$$

which implies

$$C \geq \sqrt{\frac{1-\gamma}{12}}\sqrt{\frac{\gamma\log_2(d-1) - \log_2(3\sqrt{\frac{3}{1-\gamma}}\sqrt{\rho}D + 1) - 1}{\rho}} - \frac{1}{\sqrt{\rho}}\sqrt{\frac{1-\gamma}{3}}$$

In general, if D is allowed to be $\Omega(C)$ then similar arguments show that $C \in \Omega\left(\sqrt{1-\gamma}\sqrt{\frac{\gamma\log_2(d-1) - \log_2(\sqrt{\frac{3}{1-\gamma}}\sqrt{\rho}D+1) - 1}{\rho}} - \frac{1}{\sqrt{\rho}}\sqrt{\frac{1-\gamma}{3}}\right)$.

Putting all of this together, if $D \in O(1/\sqrt{\rho})$ then $C \in \Omega\left(\sqrt{\log(d)/\rho}\right)$. But in order to get $C = O(1/\sqrt{\rho})$, we must have $D \in \Omega(d^{\gamma}/\sqrt{\rho})$.

□

A.2 Proof of Upper Bound Results

We first need some facts about Gaussian and Laplace random variables.

Lemma 1. *Let $z_1, \ldots, z_d$ be i.i.d. random variables from a distribution F.*

- *If F is $N(0, \sigma^2)$ then*
  - $E[z_i^2] = \sigma^2$ *for all* i
  - $E[|z_i|] \leq \sigma$ *for all* i
  - $E[\max_i |z_i|] \in O(\sigma\sqrt{\log(d)})$
  - $E[\max_i z_i^2] \in O(\sigma^2 \log(d))$
- *If F is $Lap(1/\epsilon)$ then*
  - $E[z_i^2] = 2/\epsilon^2$ *for all* i
  - $E[|z_i|] = 1/\epsilon$ *for all* i
  - $E[\max_i |z_i|] \leq \frac{1}{\epsilon}(\ln(d) + 1)$
  - $E[\max_i z_i^2] \leq \frac{1}{\epsilon^2}(\ln^2(d) + 2\ln(d) + 2)$

Proof. The variance of a Gaussian is known to be σ^2 and that of the Laplace distribution is known to be $2/\epsilon^2$.

The absolute value of a Laplace is an Exponential random variable with rate ϵ and so the expectation is $1/\epsilon$. Next, by Jensen's inequality $(E[|z_i|])^2 \leq E[z_i^2]$ and so $E[|z_i|] \leq \sqrt{(E[z_i^2])}$. Thus, in the case of a Gaussian, this is upper bounded by σ.

To compute the expectation of the maxes, we note that if z_i' follows the Lap(1) distribution, then z'/ϵ follows the Lap$(1/\epsilon)$ and if z' follows $N(0, 1)$ then σz follows the $N(0, \sigma^2)$ distribution. Thus we compute the expectations under the assumption that the scale variables are 1 and then we multiply by $1/\epsilon$ or σ for the first moment, and $1/\epsilon^2$ or σ^2 for the second moment to get the results for z_i from the results for z_i'.

Next we let G be the cdf of a continuous nonnegative random variable and g the corresponding pdf. Then for any $p \geq 1$,

$$\begin{aligned} E_{X\sim G}[X^p] &= \int_0^\infty x^p g(x)\, dx = \int_0^\infty g(x) \left(\int_0^\infty pt^{p-1} 1_{\{t\leq x\}}\, dt \right) dx \\ &= \int_0^\infty pt^{p-1} \left(\int_0^\infty g(x) 1_{\{t\leq x\}}\, dx \right) dt = \int_0^\infty pt^{p-1}(1 - G(t))\, dt \end{aligned}$$

Now we let F_+ be the cdf of $|z_1'|$ (the random variables with location parameter 1), and let G be the distribution of $\max_i |z_i'|$. Then for all t, $G(t) = F_+(t)^d$ and also by the union bound, $1 - F_+(t)^d = 1 - G(t) \leq d(1 - F_+(t))$. For any $\gamma > 0$,

$$\begin{aligned} E\left[\max_i |z_i'|^p\right] &= \int_0^\infty pt^{p-1}(1 - F_+(t)^d)\, dt \\ &= \int_0^\gamma pt^{p-1}(1 - F_+(t)^d)\, dt + \int_\gamma^\infty pt^{p-1}(1 - F_+(t)^d)\, dt \\ &\leq \int_0^\gamma pt^{p-1}\, dt + \int_\gamma^\infty pt^{p-1} d(1 - F_+(t))\, dt \\ &= \gamma^p + \int_\gamma^\infty pt^{p-1} d(1 - F_+(t))\, dt \end{aligned}$$

For the Laplace distribution, $F_+(t) = 1 - e^{-t}$ thus, for any $\gamma > 0$

$$E\left[\max_i |z_i'|\right] \le \gamma + \int_\gamma^\infty de^{-t}\, dt = \gamma + de^{-\gamma}$$
$$E\left[\max_i |z_i'|^2\right] \le \gamma^2 + \int_\gamma^\infty 2tde^{-t}\, dt = \gamma^2 + 2\gamma de^{-\gamma} + 2de^{-\gamma}$$

Setting $\gamma = \ln(d)$ and converting from z_i' to z_i, we get $E[\max_i |z_i|] \le \frac{1}{\epsilon}(\ln(d) + 1)$ and $E[\max_i |z_i|^2] \le \frac{1}{\epsilon^2}(\ln^2(d) + 2\ln(d) + 2)$.

For the Gaussian distribution, a well-known tail bound on the Gaussian is that $1 - F_+(t) \le \frac{2}{t}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}$. Thus we get

$$\begin{aligned}
E\left[\max_i |z_i'|\right] &\le \gamma + \int_\gamma^\infty d\frac{2}{t}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\, dt \le \gamma + \frac{2d}{\gamma}\int_\gamma^\infty \frac{1}{\sqrt{2\pi}}e^{-t^2/2}\, dt \\
&\le \gamma + 2d\frac{2}{\gamma^2}\frac{1}{\sqrt{2\pi}}e^{-\gamma^2/2} \\
E\left[\max_i |z_i'|^2\right] &\le \gamma^2 + \int_\gamma^\infty 2td\frac{2}{t}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\, dt = \gamma^2 + 4d\int_\gamma^\infty \frac{1}{\sqrt{2\pi}}e^{-t^2/2}\, dt \\
&= \gamma^2 + 4d\frac{1}{\gamma}\frac{1}{\sqrt{2\pi}}e^{-\gamma^2/2}
\end{aligned}$$

Setting $\gamma = \sqrt{2\ln(d)}$ and converting from z_i' to z_i, we get $E[\max_i |z_i|] \le \sigma(\sqrt{2\ln(d)} + \frac{4}{\sqrt{2\pi}}\frac{1}{2\ln(d)})$ and $E[\max_i |z_i|^2] \le \sigma^2(2\ln(d) + \frac{4}{\sqrt{2\pi}}\frac{1}{\sqrt{2\ln(d)}})$. □

We next need a technical lemma about the solution to a constrained nonnegative least squares problem.

Lemma 2. *Let $a_1, \dots, a_d$ be real numbers and let $a_* \ge 0$. The solution to the optimization problem*

$$\begin{aligned}
\arg\min_{x_1,\dots,x_d} &\frac{1}{2}\sum_{i=1}^d (x_i - a_i)^2 \\
\textit{s.t.} &\sum_{i=1}^d x_i = a_* \\
&x_i \ge 0, \textit{ for } i = 1, \dots, d
\end{aligned}$$

is $x_i = \max\{a_i - \gamma, 0\}$ (for all i) where γ is chosen so that $\sum_{i=1}^d \max\{0, a_i - \gamma\} = a_*$.

Proof. Let us use the shorthand $(a - \gamma)_+$ to mean $\max\{0, a - \gamma\}$.

First, it is easy to see that by continuity, there exists a γ such that $\sum_i (a_i - \gamma)_+ = a_*$.

The gradient of the objective function with respect to the x_i is:

$$\frac{\partial \text{obj}}{\partial x_i} = (x_i - a_i) = (a_i - \gamma)_+ - a_i$$

and if this choice of x_i is not optimal, then any descent direction $(y_1, \dots, y_n)$ (i.e., for which $x_1 + y_1, \dots, x_1 + y_2$ is feasible and reduces the objective function) must satisfy (1) $\sum_{i=1}^d y_i = 0$ to maintain feasibility of the equality constraint, (2) $\sum_{i=1}^d y_i((a_i - \gamma)_+ - a_i) < 0$ to be a descent direction, (3) $y_i \ge 0$ when $a_i \le \gamma$ and $y_i \ge \gamma - a_i$ when $a_i > \gamma$ to maintain nonnegativity of $x_i + y_i \equiv (a_i - \gamma)_+ + y_i$.

Now,

$$\begin{aligned}
\sum_{i=1}^{d} y_i((a_i-\gamma)_+ - a_i) & \sum_{i:a_i>\gamma} y_i((a_i-\gamma)_+ - a_i) + \sum_{i:a_i\leq\gamma} y_i((a_i-\gamma)_+ - a_i) \\
&= -\gamma \sum_{i:a_i>\gamma} y_i - \sum_{i:a_i\leq\gamma} y_i a_i \\
&= -\gamma \sum_{i:a_i>\gamma} y_i - \sum_{i:a_i\leq\gamma} y_i \gamma \\
&\quad \text{since feasibility of } x_i + y_i \text{ requires } y_i \geq 0 \text{ when } a_i \leq \gamma \\
&= -\gamma \sum_{i=1}^{d} y_i = 0 \quad \text{since feasibility requires } \sum_i y_i = 0
\end{aligned}$$

contradicting that $y_1, \ldots, y_d$ is a descent direction.

□

Theorem 4 (Upper bound for pure DP and zCDP)**.** *Let $q_1, \ldots, q_d$ be a set of disjoint queries and let q_* be their sum. Given privacy parameters $\epsilon > 0$ and $\rho > 0$, there exist algorithms $M_\epsilon, M_\rho, M'_\epsilon, M'_\rho$, $M'_{\epsilon,\delta}$ that output a positively weighted dataset and have the following properties:*

1. *M_ϵ satisfies ϵ-DP, and for all $\mathfrak{D}$ and i, $E\left[(q_i(M_\epsilon(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right] \leq 2/\epsilon^2$ and $E\left[(q_*(M_\epsilon(\mathfrak{D})) - q_*(\mathfrak{D}))^2\right] \leq 2d^2/\epsilon^2$.*
2. *M_ρ satisfies ρ-zCDP, and for all $\mathfrak{D}$ and i, $E\left[(q_i(M_\rho(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right] \leq 1/(2\rho)$ and $E\left[(q_*(M_\rho(\mathfrak{D})) - q_*(\mathfrak{D}))^2\right] \leq d^2/(2\rho)$.*
3. *M'_ϵ satisfies ϵ-DP, and for all $\mathfrak{D}$ and i, $E\left[(q_i(M'_\epsilon(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right] \in O(\log^2(d)/\epsilon^2)$ and $E\left[(q_*(M'_\epsilon(\mathfrak{D})) - q_*(\mathfrak{D}))^2\right] \in O(1/\epsilon^2)$*
4. *M'_ρ satisfies ρ-zCDP, and for all $\mathfrak{D}$ and i, $E\left[(q_i(M'_\rho(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right] \in O(\log(d)/\rho)$ and $E\left[(q_*(M'_\rho(\mathfrak{D})) - q_*(\mathfrak{D}))^2\right] \in O(1/\rho)$*
5. *$M'_{\epsilon,\delta}$ satisfies (ϵ, δ)-DP and for all $\mathfrak{D}$ and i, $E\left[(q_i(M'_{\epsilon,\delta}(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right] \in O(\log^2(1/\delta)/\epsilon^2 + 1)$ and $E\left[(q_*(M'_{\epsilon,\delta}(\mathfrak{D})) - q_*(\mathfrak{D}))^2\right] \in O(1/\epsilon^2)$. Also note M_ϵ and M'_ϵ satisfy ϵ, δ-DP.*

Proof. The double-sided geometric mechanism $DGeo(\epsilon)$ is a discrete version of the Laplace distribution, supported over integers, with probability mass function $p(k) = \frac{1-e^{-\epsilon}}{1+e^{-\epsilon}} e^{-\epsilon|k|}$ [22]. It has several useful properties: (a) its mean is 0, (b) its variance is $2\frac{e^{-\epsilon}}{(1-e^{-\epsilon})^2} \leq 2/\epsilon^2$, (c) given an integer-valued query q, adding $DGeo(\epsilon/\Delta_1(q))$ to its answer satisfies ϵ-differential privacy.

Similarly, the discrete Gaussian $DGauss(0, 1/(2\rho))$ is a discrete version of the Gaussian distribution [9] with several useful properties: (a) its mean is 0, (b) its variance is less than that of $N(0, 1/(2\rho))$, (c) given an integer-valued query q, adding $DGauss(0, \Delta_2(q)^2/(2\rho))$ to its answer satisfies ρ-zcdp.

To prove Item 1, let $r_1, \ldots, r_d$ be records satisfying the predicates for point queries $q_1, \ldots, q_d$, respectively. Let M_ϵ be the algorithm that first computes nonnegative noisy query answers $a_i = \max\{0, q_i(\mathfrak{D}) + DGeo(1/\epsilon)\}$ for $i = 1, \ldots, d$ and then outputs the synthetic dataset $\widetilde{\mathfrak{D}}$ that has a_i copies of record r_i for each i. Note that M_ϵ does not obtain a noisy answer to q_*, and so it satisfies ϵ-differential privacy since $\Delta_1(q_1, \ldots, q_d) = 1$ Since $q_i(\mathfrak{D}) \geq 0$ for all i, we have:

$$\begin{aligned}
E\left[(q_i(\mathfrak{D}) - q_i(M_\epsilon(\mathfrak{D})))^2\right] &= E\left[(q_i(\mathfrak{D}) - \max\{0, q_i(\mathfrak{D}) + DGeo(\epsilon)\})^2\right] \\
&\leq E\left[(q_i(\mathfrak{D}) - (q_i(\mathfrak{D}) + DGeo(\epsilon)))^2\right] \leq 2/\epsilon^2
\end{aligned}$$

Furthermore

$$E\left[(q_*(\mathfrak{D}) - q_*(M_\epsilon(\mathfrak{D})))^2\right] = E\left[(\sum_i q_i(\mathfrak{D}) - \sum_i q_i(M_\epsilon(\mathfrak{D})))^2\right]$$
$$= \sum_i E\left[(q_i(\mathfrak{D}) - \max\{0, q_i(\mathfrak{D}) + DGeo(\epsilon)\})^2\right]$$
$$+ 2\sum_{i,j:i<j} E\left[\Big(q_i(\mathfrak{D}) - \max\{0, q_i(\mathfrak{D}) + DGeo(\epsilon)\}\Big)\right] E\left[\Big(q_j(\mathfrak{D}) - \max\{0, q_j(\mathfrak{D}) + DGeo(\epsilon)\}\Big)\right]$$
$$\leq d\frac{2}{\epsilon^2} + d(d-1)\frac{2}{\epsilon^2} = d^2\frac{2}{\epsilon^2}$$

To prove Item 2, we use the same proofs as before, except that M_ρ synthesizes $\widetilde{\mathfrak{D}}$ using the noisy answers $a_i = q_i(\mathfrak{D}) + \max\{0, DGauss(0, 1/(2\rho))\}$. Following essentially the same calculations, we see that the expected squared error of each point query q_i is at most $1/(2\rho)$ and for the sum query q_* it is at most $d^2/(2\rho)$.

To prove Item 3, let $r_1, \dots, r_d$ be records satisfying the predicates for point queries $q_1, \dots, q_d$, respectively. Let M'_ϵ be the algorithm that does the following. First, it obtains noisy answers for each query: $a_i = q_i(\mathfrak{D}) + Lap(2/\epsilon)$ for $i = 1, \dots, d$ and $a_* = q_*(\mathfrak{D}) + Lap(2/\epsilon)$. (Since $\Delta_1(q_1, \dots, q_d, q_*) = 2$, this clear satisfies ϵ-differential privacy). Next, M solves the following optimization problem:

$$\begin{aligned} \arg\min_{x_1,\dots,x_d} \frac{1}{2}\sum_{i=1}^d (x_i - a_i)^2 \\ \text{s.t. } \sum_{i=1}^d x_i = \max\{0, a_*\} \\ x_i \geq 0, \text{ for } i = 1, \dots, d \end{aligned}$$

and creates a privacy protected microdata $\widetilde{\mathfrak{D}}$ that consists of the records $r_1, \dots, r_d$ with respective weights $x_1, \dots, x_d$.

Since the sum query is nonnegative and the problem is constrained so that $\sum_i x_i$ is equal to $\max\{0, a_*\}$, clearly $E\left[(q_*(M'_\epsilon(\mathfrak{D})) - q_*(\mathfrak{D}))^2\right] \leq 2/\epsilon^2$.

Now let us derive an upper bound on $E\left[(q_i(M'_\epsilon(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right]$ for a point query q_i.

For each i, let $z_i = a_i - q_i(\mathfrak{D})$ and $z_* = a_* - q_*(\mathfrak{D})$ be the actual noises that are added (they are all i.i.d. Laplace$(2/\epsilon)$.

We know from Lemma 2 that the solution x_i have the form $\max\{a_i - \gamma, 0\}$ (which is $\max\{0, q_i(\mathfrak{D}) + z_i - \gamma\}$) for some γ such that $\sum_i \max\{a_i - \gamma, 0\} = \max\{0, a*\}$ and note that the left hand side is monotonic in γ.

We first find a suitable upper and lower bound on γ. Define $L = -|z_*| + \min_i z_i$ and $U = |z_*| + \max_i z_i$. Then we have:

$$\begin{aligned} \sum_i \max\{0, a_i - U\} = \sum_i \max\{0, q_i(D) + z_i - U\} \leq \sum_i \max\{0, q_i(\mathfrak{D}) - |z_*|\} \\ \leq \max\{0, \left(\sum_i q_i(\mathfrak{D})\right) - |z_*|\} \\ \text{since the } q_i(\mathfrak{D}) \text{ are nonnegative} \\ = \max\{0, q_*(\mathfrak{D}) - |z_*|\} \\ \leq \max\{0, a_*\} \end{aligned}$$

and so $\gamma \leq U$.

Next,

$$\begin{aligned}\sum_i \max\{0, a_i - L\} &= \sum_i \max\{0, q_i(D) + z_i - L\} \geq \sum_i \max\{0, q_i(\mathfrak{D}) + |z_*|\} \\ &= \sum_i (q_i(\mathfrak{D}) + |z_*|) \\ &\text{since the } q_i(\mathfrak{D}) \text{ are nonnegative} \\ &\geq \left(\sum_i q_i(\mathfrak{D})\right) + |z_*| \\ &= q_*(\mathfrak{D}) + |z_*| \geq \max\{0, a_*\}\end{aligned}$$

and so $\gamma \leq L$.

We next find a bound on $E\left[(q_i(M'_\epsilon(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right]$ in terms of γ.

$$\begin{aligned}E\left[(q_i(M'_\epsilon(\mathfrak{D})) - q_i(\mathfrak{D}))^2\right] &= E\left[(\max\{0, q_i(\mathfrak{D}) + z_i - \gamma\} - q_i(\mathfrak{D}))^2\right] \\ &\text{note the random variable here are } z_i \text{ and } \gamma \\ &\leq E\left[((q_i(\mathfrak{D}) + z_i - \gamma) - q_i(\mathfrak{D}))^2\right] \\ &\text{since } q_i(\mathfrak{D}) \text{ is nonnegative and removing the max moves the} \\ &\text{left part further away from } q_i(\mathfrak{D}) \\ &= E\left[(z_i - \gamma)^2\right] \leq E\left[(|z_i| + \max\{|L|, |U|\})^2\right] \\ &\leq E\left[\left(|z_i| + |z|_* + \max_j |z_j|\right)^2\right] \\ &\text{since the noises } z_j \text{ are symmetric around 0} \\ &\leq E\left[\left(|z|_* + 2\max_j |z_j|\right)^2\right] \\ &= E\left[z_*^2\right] + 4E\left[|z_*|\right] E\left[\max_j |z_j|\right] + 4E\left[(\max_j |z_j|)^2\right] \qquad (6) \\ &\in O(\frac{1}{\epsilon^2}\log^2(d)) \quad \text{by Lemma 1 for Laplace noise}\end{aligned}$$

To prove Item 4 we follow the same steps as before, but using $N(0, 1/(\rho))$ noise instead of $\text{Lap}(2/\epsilon)$ (noting that $\Delta_2(q_1, \ldots, q_d, q_*) = \sqrt{2}$) and again see that the variance of the sum query is at most $1/\rho$, while for the point queries, the only thing that changes are the calculations after Equation 6, where we use the Lemma 1 results for Gaussian noise, to conclude that $E[(q_i(\mathfrak{D}) - q_i(M(\mathfrak{D})))^2] \in O(\frac{1}{\rho}\log(d))$ for each i.

To prove Item 5, we again follow the same steps as before but with a different noise distribution. Recall that the double geometric distribution $DGeo(\epsilon)$ is supported over the integers. If $z \sim DGeo(\epsilon)$ then $P(z = k) = \frac{1-e^{-\epsilon}}{1+e^{-\epsilon}}e^{-\epsilon|k|}$. Furthermore, if $k \geq 0$, $P(z \geq k) = P(z \leq -k) = \frac{1}{1+e^{-\epsilon}}e^{-\epsilon k}$.

For any integer $B > 0$, the truncated double geometric distribution $TDGeo(\epsilon, B)$ is obtained by clipping a $\text{DGeo}(\epsilon)$ at B and $-B$. Specifically, if $z' \sim TDGeo(\epsilon, B)$ then

$$P(z' = k) = \begin{cases} \frac{1}{1+e^{-\epsilon}}e^{-\epsilon B} & \text{if } k = B \\ \frac{1-e^{-\epsilon}}{1+e^{-\epsilon}}e^{-\epsilon|k|} & \text{for } k = -B+1, \ldots, B-1 \\ \frac{1}{1+e^{-\epsilon}}e^{-\epsilon B} & \text{if } k = -B \end{cases}$$

So, we follow the same approach as in the proof of Item 3 but we use $TDGeo(\epsilon/2, B)$ noise to answer each query (detail queries and sum query). We first determine the value of B needed to satisfy $(\epsilon/2, \delta/2)$-DP.

First note that for any integer v, and integer $k \in [v - B + 1, v + B - 2]$

$$e^{-\epsilon/2} \leq \frac{P(v + TDGeo(\epsilon/2, B) = k)}{P(v - 1 + TDGeo(\epsilon/2, B) = k)} \leq e^{\epsilon/2}$$

(the significance of these points are that they are not in the boundary of $v+TDGeo$ or $v-1+TDGeo$).

Meanwhile $P(v + TDGeo(\epsilon/2) \in \{v - B, v + B - 1, v + B\}) = P(DGeo(\epsilon/2) \geq B - 1) + P(DGeo(\epsilon/2) \leq -B) = \frac{1}{1+e^{-\epsilon/2}} e^{-\epsilon B/2} + \frac{1}{1+e^{-\epsilon/2}} e^{-\epsilon(B-1)/2} \leq 2e^{-\epsilon(B-1)/2}$. These are the boundary points where the probability ratios may be large.

Setting this equal to $\delta/2$ (and then performing similar calculations when the $v - 1$ term is in the numerator), we see that adding $TDGeo(\epsilon/2, B)$ noise satisfies ϵ, δ-DP if $B \geq \frac{2}{\epsilon} \log(4/\delta) + 1$.

Thus using a naive composition result of approximate differential privacy [18], we can add TGeo$(\epsilon/2, B)$ noise to each point query and the sum query to satisfy (ϵ, δ)-DP.

Using the same postprocessing as in the proof of Item 3, we see that the expected squared error of the sum query (when computed from the postprocessed privacy-protected data) is at most variance$(TDGeo(\epsilon/2, B)) \leq$ variance$(DGeo(\epsilon/2)) \leq 8/\epsilon^2$.

For the point queries, the only thing that changes are the calculations after Equation 6. Since the absolute value of the noises is bounded by B, we get that the expected squared error of the point queries is $\in O(B^2) = O(\frac{1}{\epsilon^2} \log^2(1/\delta) + 1)$.

□

B Full Data Benchmark Description

Our benchmarks contain 15 real datasets and 16 synthetic datasets. The datasets are designed to be small enough to enable thousands of runs (in order to compute expected squared errors) but large enough to clearly illustrate postprocessing errors and present a challenge to many open-source optimizers.

B.1 Real Datasets

The real datasets are drawn from the 2016 American Community Survey Public Use Microdata Sample (PUMS) [39], which provides records for geographies known as Public Use Microdata Areas (PUMA).

To create a benchmark data set that adequately captured the diversity of real world demographic data, we drew from outlier geographies in the 2016 ACS PUMS. We chose 15 Public Use Microdata Areas whose data distributions had been identified as conflicting significantly with the majority distributions in their states, according to the k-marginal metric used by NIST in their Differential Privacy challenge [36]. The data spanned historically redlined areas, a variety of immigrant communities, wealthy and diverse urban neighborhoods, rural agricultural communities, and included every major region in the United States.

For each of the 15 regions, we created a 9×24 Race by Hispanic Origin histogram. These were two separate questions in the ACS questionnaire. Although the questionnaire allowed respondents to select multiple races (from a list of 15 categories and 3 fill-in text boxes), most individuals belong to three or fewer races, and the 2016 ACS PUMS did not include detailed racial breakdowns for individuals with more than 3 races. To mimic the extreme sparsity and geographically diverse correlation patterns in the multi-racial checkbox variable, we selected two variables (called RAC1P and HISP; full definitions below): a smaller race variable with 9 possible values which primarily records single races, and a detailed Hispanic origin variable with 24 possible values. Any of the 216 possible combinations of race and Hispanic origin is valid; individuals of all races have origins from all across Latin America. However, in any given community the vast majority of these counts will be zero, resulting in sparse distributions. At the same time, communities with different immigration histories will differ significantly with respect to which counts are nonzero and in the size of the other counts. Algorithms which performed well across all cases in the PUMS benchmark data set should be expected perform well on the edge case complexities of national data.

```
RAC1P
    Recoded detailed race code
1. White alone
2. Black or African American alone
3. American Indian alone
4. Alaska Native alone
5. American Indian and Alaska Native tribes specified; or American
 Indian or Alaska Native, not specified and no other races
6. Asian alone
7. Native Hawaiian and Other Pacific Islander alone
8. Some Other Race alone
9. Two or More Races

HISP
Detailed Hispanic origin
01. Not Spanish/Hispanic/Latino
02. Mexican
03. Puerto Rican
04. Cuban
05. Dominican
06. Costa Rican
07. Guatemalan
08. Honduran
09. Nicaraguan
10. Panamanian
11. Salvadoran
12. Other Central American
13. Argentinean
14. Bolivian
15. Chilean
16. Colombian
17. Ecuadorian
18. Paraguayan
19. Peruvian
20. Uruguayan
21. Venezuelan
22. Other South American
23. Spaniard
24. All Other Spanish/Hispanic/Latino
```

B.2 Synthetic Data

The synthetic data are modeled after the proofs of our lower bound results. The main idea is that suppose noise from a distribution F is added to a histogram, and that there are k zero cells and one cell with a count of C in that histogram. Based on the noisy cell values, it is difficult to guess which cell had value C when C is smaller than the median of the distribution of $\max\{X_1, \dots, X_k\}$ (whose CDF is $F^k(t)$), where each $X_i \sim F$. Thus we created datasets with sparsity patterns.

Each histogram had 100 elements, from which we created a 1-dimensional version (a 100-element vector) and a 2-dimensional version (reshaping it to a 10×10 histogram). In all of the datasets, the first histogram cell is relatively large (10,000) and should be easy to distinguish from 0 based on the noisy counts (although ordinary nonnegative least squares fails to do so).

The synthetic histograms come from 4 categories, defined as follows:

- **Level**. In the Level **k** histograms, all cells have the same value k (except the first, which has value 10,000). The benchmarks include 1- and 2-dimensional versions of Level0 (i.e., only the first element is nonzero), Level1, Level16, and Level32. The Level1 dataset presents a tricky case where each cell (other than the first), based on its noisy value, may look similar to 0, but the overall sum of these small cells is clearly distinguishable from 0. The Level16 and Level32 datasets are designed to force algorithms to try to estimate the number of cells that are likely to have true value of 0. Note that 16 is roughly the 40th percentile of the distribution of $\max X_1, \dots, X_{100}$ when each X_i has the Laplace$(1/\epsilon)$ distribution with $\epsilon = 0.25$. So having a few cell noisy cell counts near 16 is possible when a histogram is mostly 0, but having many noisy counts near 16 is a sign that the histogram is not sparse.

- **Stair**. The Stair data is a histogram that looks like this: $[10000, 1, 2, 3, 4, \dots]$ in one dimension (and is reshaped into a 10×10 matrix in 2 dimensions. It is designed to simulate a dataset with small, medium, and large values.
- **Step**. The Step k dataset is a step function. The first element is 10000, the next 49 are 0 and the last 50 are k. This is an interpolation between the sparse dataset synthetic dataset Level0 and Level k. For our benchmark, we use Step16 (i.e., $k = 16$) as a dataset of medium difficulty and Step50 as an easy dataset.
- **SplitStairs**. The SplitStairs dataset is an interpolation between Stair and a very sparse dataset. The first half looks like the Stair dataset but cells 50 until the end all have value 0. This ensures that all true cell counts that can be dominated by 50 random zero-mean Laplace random variables are represented in the dataset.

Combined, these synthetic datasets give 8 1-dimensional histograms (4 Level, 1 Stair, 2 Step, 1 SplitStairs) and 8 2-dimensional histograms.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes] **Our claims are that we present an uncertainty principle (for which we prove upper and lower bounds in Section 3 with credit to [3] for important heavy lifting), some algorithms that try to mitigate its effects (Section 4) and creation of benchmark datasets (briefly described in Section 5 and more fully described in the supplementary material. The supplementary material contains 3 items: our source code (which includes the benchmark data), a version of this paper with appendix containing full proofs and data description, and a third document containing all of the experimental results we generated)**
   (b) Did you describe the limitations of your work? [Yes] **We discussed the limitations of the uncertainty principle in Remark 2 in Section 3 which explains that the principle only occurs in some (worst-case) datasets, although real-world datasets seem to be closer to the worst-case. We also explain in the introduction that we do not have theoretical guarantees for our mitigation algorithms and experiments show that they are good but not perfect.**
   (c) Did you discuss any potential negative societal impacts of your work? [Yes] **We believe that privacy research has positive impacts. However, we note that the uncertainty principle shows that the requirement that privacy-preserving algorithms should produce microdata is a requirement that can lead to further accuracy loss. In the conclusions we recommend that algorithms should be based on postprocessing noisy query answers so that the noisy query answers can also be released. We believe that this is good theoretically-motivated guidance to data publishers.**
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes] **We also note that the datasets we use are public and our code can be fully open-sourced.**
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] **Our Theorems 3 and 4 are self-contained and include their assumptions**
   (b) Did you include complete proofs of all theoretical results? [Yes] **In the supplementary materials.**
3. If you ran experiments...

(a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] **In the supplementary materials.**

(b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] **We explained our experimental settings, and more details are added in the separate supplementary document that contains all of our experiments.**

(c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] **Due to space constraints, we report a range on the standard errors of our experiments. The actual standard errors appear in the supplementary files in the document that has the full experiments**

(d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] **We made the benchmarks small enough to avoid the need for clusters and specialized hardware. All of the experiments were run on a desktop computer**

4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...

   (a) If your work uses existing assets, did you cite the creators? [Yes] **We cited the Julia language, the COSMO optimizer and the 2016 ACS Public Use Microdata Samples used to create our dataset histograms.**

   (b) Did you mention the license of the assets? [N/A] **ACS is public domain, and we do not distribute COSMO or Julia (users need to install it themselves).**

   (c) Did you include any new assets either in the supplemental material or as a URL? [Yes] **Our code is in the supplementary material**

   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A] **We only use public data (2016 ACS Public Microdata Samples (PUMS))**

   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [No] **In general, the 2016 ACS PUMS may contain PII despite the U.S. Census Bureau's efforts to provide confidentiality protections. However, we only use the Hispanic origins and race variables of people in Public Use Microata Areas which contain at least 100,000 people.**

5. If you used crowdsourcing or conducted research with human subjects...

   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]

   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]

   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

Complete experimental results.

In this document, we present our full experimental results. The datasets used are the PUMS datasets (2-dimensional), the 1-dimensional synthetic data, and the 2-dimensional synthetic data. These datasets are described in the appendix of the full version of the paper, which appears in the supplementary material file.

For the one-dimensional datasets, we use either the Laplace mechanism (for pure differential privacy) or the Gaussian mechanism (for zCDP) to obtain noisy answers to:

- The sum query (the sum of the histogram cells)
- The identity queries (the count in each cell).

For the two-dimensional datasets, we use either the Laplace or Gaussian mechanisms to obtain noisy answers to:

- The sum query (the sum of the histogram cells)
- The identity queries (the count in each cell).
- The marginal on the first dimension.
- The marginal on the second dimension.

We use the NNLS (referred to as nnlsalg in the tables), Max fitting, Sequential Fitting, and Weighted Fitting (with confidence parameter 0.99) postprocessing methods to obtain the privacy preserving positively weighted data $\widetilde{D}$. Sequential Fitting prioritizes queries in the order listed above. We also use OLS fitting (NNLS fitting without the nonnegativity constraints), which is referred to as olsalg in the experiments. The OLS fitting method is known to improve the squared error of the queries compared to the original noisy answers (this is a consequence of the Gauss-Markov theorem) but does not result in a positively weighted dataset. Hence the goal of the methods is not to do much worse than the OLS fitting method.

The code was written in Julia. In order to make the code fully open source, we experimented with several open source solvers compatible with Julia's JuMP framework. Out of these, the COSMO solver performed the best. However, the relatively complex multi-stage optimizations in Max fitting and Sequential fitting caused problems. In some cases the solver claimed infeasibility for problems in latter stages of the optimization (likely due to poor quality solutions in earlier stages), numerical errors, or slow convergence (hitting the iteration limit). To reduce the chance of poor solutions in earlier stages of an optimization, we set the absolute and relative tolerances to 1e-7 and an iteration limit of 20,000, which is 4 times the default. We also converted equality constraints of the form $x = \mathit{constant}$ to $x \leq \mathit{constant} + 0.001$ and $x \geq \mathit{constant} - 0.001$. For the Max Fitting solve, after it gets an L_∞ distance estimate in the first stage of the solve, we added a slack of 0.01 to this distance to prevent it from failing in the second stage.

Despite tuning parameter and setting slack tolerances to equalities and inequalities, not all runs were successful, so we only kept the ones where all stages of the optimization were optimal. This likely optimistically biased the results of Max fitting and Sequential fitting and increased their estimated standard errors.

These optimization problems did not affect OLS, NNLS, or the Weighted Fitting approaches.

Each experiment is an average over 1000 runs (thus the expected error of a query is estimated the average of its errors across 1000 runs). However, for more complex constrained methods, the average was among fewer runs if some stage of the multi-stage optimization failed to find an optimal solution.

In each table, we evaluate the error of different queries.

- For the Sum query (as in Table 2), we display its expected error along with estimated standard deviation.
- For the Identity queries (as in Table 1), each cell i in the histogram corresponds to a query q_i (the count in that cell). For each cell i, we estimate its expected squared error $e_i = E[((q_i\,\mathfrak{D}) - q_i(\widetilde{\mathfrak{D}}))^2]$ by averaging the error across trials. Then we report $\max_i e_i$ and $\sum_i e_i$ along with standard errors. Again, we emphasize that our Max metric is $\max_i E[((q_i\,\mathfrak{D}) - q_i(\widetilde{\mathfrak{D}}))^2]$ and not outlier error $E[\max_i(((q_i\,\mathfrak{D}) - q_i(\widetilde{\mathfrak{D}})))^2]$.
- For the two dimensional datasets, we also have tables for each marginal and report the max and total squared errors as for the identity queries.

Note that the goal is to avoid extreme errors that are much larger than the OLS error.

The experiments are organized first by privacy definition (pure DP and zCDP). Within each privacy definition, we first present results for the 1-dimensional synthetic data (for 3 privacy parameters) followed by the 2-dimensional synthetic data (for 3 privacy parameters) followed by the PUMS data (for 3 privacy parameters).

1. Pure Differential Privacy

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-1d	789.4	9.2	61.4	28.8	64.5	30.0	57.8	39.5	11.4	6.2
	±5.5	±0.6	±1.4	±0.9	±1.9	±1.2	±1.3	±1.1	±0.7	±0.5
Level01-1d	789.4	9.2	300.9	8.8	303.5	9.0	298.8	8.9	296.4	8.0
	±5.5	±0.6	±3.2	±0.6	±4.2	±0.8	±3.2	±0.6	±3.2	±0.6
Level16-1d	789.4	9.2	788.0	9.2	796.9	10.4	788.0	9.2	783.2	9.2
	±5.5	±0.6	±5.5	±0.6	±10.1	±1.5	±5.5	±0.6	±5.4	±0.6
Level32-1d	789.4	9.2	789.4	9.2	791.9	11.7	789.4	9.2	789.4	9.2
	±5.5	±0.6	±5.5	±0.6	±11.9	±2.7	±5.5	±0.6	±5.5	±0.6
SplitStairs-1d	789.4	9.2	535.1	9.5	528.9	9.5	535.0	9.5	519.8	13.1
	±5.5	±0.6	±4.5	±0.7	±5.8	±0.8	±4.5	±0.7	±4.4	±0.6
Stair-1d	789.4	9.2	779.2	9.1	778.7	11.0	779.2	9.1	781.3	9.1
	±5.5	±0.6	±5.5	±0.7	±10.2	±1.8	±5.5	±0.7	±5.5	±0.7
Step16-1d	789.4	9.2	560.2	9.6	555.4	10.1	560.2	9.6	644.4	11.8
	±5.5	±0.6	±4.6	±0.7	±6.2	±0.9	±4.6	±0.7	±4.9	±0.7
Step50-1d	789.4	9.2	561.5	9.6	563.9	10.1	561.5	9.6	427.2	9.1
	±5.5	±0.6	±4.7	±0.7	±6.4	±1.0	±4.7	±0.7	±4.2	±0.7

Table 1. Squared Errors (with standard deviations). Id Query. 1-d datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-1d	7.7	29.7	31.7	7.8	6.8
	±0.5	±0.9	±1.3	±0.5	±0.5
Level01-1d	7.7	8.8	9.4	7.8	7.7
	±0.5	±0.5	±0.7	±0.5	±0.5
Level16-1d	7.7	7.7	6.9	7.8	7.7
	±0.5	±0.5	±0.8	±0.5	±0.5
Level32-1d	7.7	7.7	8.3	7.8	7.7
	±0.5	±0.5	±1.0	±0.5	±0.5
SplitStairs-1d	7.7	8.4	8.4	7.8	7.7
	±0.5	±0.5	±0.7	±0.5	±0.5
Stair-1d	7.7	7.7	7.6	7.8	7.7
	±0.5	±0.5	±0.8	±0.5	±0.5
Step16-1d	7.7	8.3	8.4	7.8	7.7
	±0.5	±0.5	±0.7	±0.5	±0.5
Step50-1d	7.7	8.3	8.9	7.8	7.8
	±0.5	±0.5	±0.7	±0.5	±0.5

Table 2. Squared Error (with standard deviations). Sum Query. 1-d datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-1d	3157.5	37.0	245.7	115.2	250.2	121.2	231.2	158.1	45.0	24.8
	±22.2	±2.3	±5.5	±3.7	±8.4	±5.8	±5.1	±4.3	±2.9	±1.8
Level01-1d	3157.5	37.0	744.2	43.6	733.0	42.6	723.9	44.6	691.9	31.9
	±22.2	±2.3	±10.5	±2.5	±13.2	±2.9	±10.3	±2.5	±10.4	±2.2
Level16-1d	3157.5	37.0	3016.8	35.6	3113.9	38.8	3016.8	35.6	3021.0	35.6
	±22.2	±2.3	±19.3	±2.1	±31.7	±3.8	±19.3	±2.1	±19.4	±2.1
Level32-1d	3157.5	37.0	3151.8	37.0	3145.6	40.3	3151.8	37.0	3131.8	36.7
	±22.2	±2.3	±21.8	±2.3	±37.3	±5.0	±21.8	±2.3	±21.7	±2.3
SplitStairs-1d	3157.5	37.0	2053.5	37.5	2053.1	39.1	2053.0	37.6	2126.2	45.9
	±22.2	±2.3	±17.0	±2.5	±21.6	±3.1	±17.0	±2.5	±17.7	±2.7
Stair-1d	3157.5	37.0	3057.2	36.3	3063.7	40.2	3057.2	36.3	3074.2	36.5
	±22.2	±2.3	±21.4	±2.7	±31.8	±4.8	±21.4	±2.7	±21.5	±2.5
Step16-1d	3157.5	37.0	2142.9	35.8	2153.2	36.5	2142.5	35.8	2161.2	36.6
	±22.2	±2.3	±16.4	±2.3	±20.9	±2.7	±16.4	±2.3	±17.0	±2.5
Step50-1d	3157.5	37.0	2245.9	38.4	2258.7	38.8	2245.9	38.5	1814.2	41.3
	±22.2	±2.3	±19.0	±2.7	±24.8	±3.3	±18.9	±2.7	±22.1	±4.1

TABLE 3. Squared Errors (with standard deviations). Id Query. 1-d datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-1d	30.9	118.9	116.6	31.3	26.9
	±1.9	±3.7	±5.7	±1.9	±1.8
Level01-1d	30.9	44.2	43.8	31.2	31.0
	±1.9	±2.4	±3.1	±1.9	±1.9
Level16-1d	30.9	30.9	34.7	31.2	31.0
	±1.9	±1.9	±3.3	±1.9	±1.9
Level32-1d	30.9	30.9	33.3	31.2	30.9
	±1.9	±1.9	±3.7	±1.9	±1.9
SplitStairs-1d	30.9	33.8	33.7	31.2	31.0
	±1.9	±2.1	±2.6	±1.9	±1.9
Stair-1d	30.9	30.9	33.9	31.2	30.9
	±1.9	±1.9	±3.2	±1.9	±1.9
Step16-1d	30.9	33.5	30.5	31.2	31.0
	±1.9	±2.1	±2.3	±1.9	±1.9
Step50-1d	30.9	33.4	31.9	31.2	31.1
	±1.9	±2.1	±2.5	±1.9	±1.9

TABLE 4. Squared Error (with standard deviations). Sum Query. 1-d datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-1d	78938.2	924.2	6142.8	2879.9	5628.8	2631.4	5787.0	3953.1	1121.6	617.5
	±554.5	±58.3	±138.0	±93.5	±328.5	±232.4	±127.3	±107.5	±72.4	±46.0
Level01-1d	78938.2	924.2	8021.2	2029.4	8061.8	2028.1	7109.2	2344.4	4107.4	787.6
	±554.5	±58.3	±167.0	±80.8	±404.4	±175.3	±151.5	±86.2	±127.9	±54.3
Level16-1d	78938.2	924.2	39776.1	826.1	39237.6	839.7	39689.7	827.1	39637.2	797.0
	±554.5	±58.3	±348.5	±56.2	±609.1	±116.8	±347.6	±56.2	±351.9	±55.8
Level32-1d	78938.2	924.2	56529.7	798.5	57415.6	781.4	56511.8	798.6	56541.6	797.0
	±554.5	±58.3	±385.1	±55.7	±669.3	±94.1	±384.9	±55.7	±387.8	±55.8
SplitStairs-1d	78938.2	924.2	34261.9	929.3	33432.3	1284.1	34129.4	935.2	34009.3	797.0
	±554.5	±58.3	±310.5	±58.8	±627.5	±178.6	±308.6	±58.9	±317.7	±55.8
Stair-1d	78938.2	924.2	62862.6	884.3	62820.4	966.5	62856.9	885.2	63033.7	892.8
	±554.5	±58.3	±429.5	±62.7	±726.0	±126.3	±429.6	±62.7	±435.1	±64.1
Step16-1d	78938.2	924.2	27709.8	962.5	28196.6	1199.7	27460.6	971.2	27113.5	797.0
	±554.5	±58.3	±291.7	±59.5	±773.9	±207.2	±289.0	±59.7	±294.4	±55.8
Step50-1d	78938.2	924.2	47908.3	848.4	48145.1	889.8	47873.6	850.2	47937.5	797.0
	±554.5	±58.3	±360.7	±57.0	±617.2	±85.8	±359.9	±57.1	±367.2	±55.8

Table 5. Squared Errors (with standard deviations). Id Query. 1-d datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-1d	771.8	2972.2	2857.8	782.8	670.3
	±47.6	±93.3	±246.6	±48.7	±46.2
Level01-1d	771.8	2090.0	1846.9	780.9	780.0
	±47.6	±80.3	±149.2	±48.5	±47.8
Level16-1d	771.8	817.0	861.1	780.9	774.3
	±47.6	±50.4	±89.6	±48.5	±48.0
Level32-1d	771.8	780.5	779.3	780.9	774.3
	±47.6	±48.5	±74.1	±48.5	±48.0
SplitStairs-1d	771.8	927.5	819.0	780.9	774.3
	±47.6	±54.5	±107.5	±48.5	±48.0
Stair-1d	771.8	780.0	732.1	781.5	774.3
	±47.6	±48.5	±81.3	±48.5	±48.0
Step16-1d	771.8	970.1	932.6	780.9	774.3
	±47.6	±56.0	±138.7	±48.5	±48.0
Step50-1d	771.8	847.9	822.8	780.9	774.3
	±47.6	±52.0	±82.5	±48.5	±48.0

Table 6. Squared Error (with standard deviations). Sum Query. 1-d datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	264.7	31.2	75.5	18.9	87.2	23.9	88.1	51.4	29.7	12.7
	±5.2	±1.9	±1.9	±0.9	±10.2	±5.1	±6.2	±4.8	±1.7	±0.7
Level01-2d	264.7	31.2	190.6	22.0	180.3	20.5	253.9	30.4	245.7	28.1
	±5.2	±1.9	±3.5	±1.3	±7.3	±2.8	±5.8	±2.3	±5.3	±1.8
Level16-2d	264.7	31.2	264.1	31.2	276.9	32.1	291.3	34.5	290.8	34.4
	±5.2	±1.9	±5.2	±1.9	±11.6	±4.6	±6.0	±2.2	±6.0	±2.2
Level32-2d	264.7	31.2	264.7	31.2	278.1	31.4	291.5	34.6	266.8	31.6
	±5.2	±1.9	±5.2	±1.9	±10.3	±3.6	±6.0	±2.2	±5.3	±1.9
SplitStairs-2d	264.7	31.2	241.9	28.3	245.0	28.9	290.7	33.9	289.4	34.3
	±5.2	±1.9	±4.6	±1.7	±7.3	±2.5	±6.2	±2.2	±6.0	±2.2
Stair-2d	264.7	31.2	264.0	31.1	273.6	31.9	291.7	34.6	287.8	34.3
	±5.2	±1.9	±5.2	±1.9	±14.0	±4.7	±6.0	±2.2	±6.0	±2.2
Step16-2d	264.7	31.2	246.6	28.5	247.1	27.7	293.1	33.8	290.8	34.5
	±5.2	±1.9	±4.8	±1.7	±11.2	±4.1	±6.3	±2.2	±6.0	±2.2
Step50-2d	264.7	31.2	247.4	28.6	240.2	30.3	291.6	34.9	246.5	28.3
	±5.2	±1.9	±4.8	±1.7	±10.2	±4.6	±6.0	±2.2	±4.9	±1.7

TABLE 7. Squared Errors (with standard deviations). Marg1 Query. 2-d datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	253.7	26.7	73.6	18.3	88.1	26.3	76.0	45.4	31.0	12.9
	±5.1	±1.5	±1.8	±0.9	±10.2	±4.6	±5.6	±4.7	±1.9	±0.7
Level01-2d	253.7	26.7	184.4	20.9	182.4	21.6	245.5	28.9	235.8	26.1
	±5.1	±1.5	±3.5	±1.2	±7.9	±2.9	±5.7	±2.1	±5.2	±1.6
Level16-2d	253.7	26.7	253.2	26.6	250.8	33.4	279.2	29.5	278.6	29.4
	±5.1	±1.5	±5.1	±1.5	±10.4	±4.3	±5.9	±1.8	±6.0	±1.8
Level32-2d	253.7	26.7	253.7	26.7	251.5	29.5	279.3	29.5	256.0	27.0
	±5.1	±1.5	±5.1	±1.5	±9.8	±3.4	±5.9	±1.8	±5.2	±1.6
SplitStairs-2d	253.7	26.7	279.9	35.5	291.7	37.0	188.1	30.1	160.6	28.0
	±5.1	±1.5	±4.9	±1.7	±8.1	±3.5	±4.8	±1.7	±4.8	±1.6
Stair-2d	253.7	26.7	253.0	26.7	280.3	41.9	279.3	29.5	262.3	28.4
	±5.1	±1.5	±5.1	±1.5	±16.9	±7.7	±5.9	±1.8	±5.4	±1.8
Step16-2d	253.7	26.7	290.1	33.9	301.3	36.6	219.0	29.8	185.4	28.3
	±5.1	±1.5	±5.1	±1.9	±12.1	±4.0	±5.4	±2.0	±5.0	±1.9
Step50-2d	253.7	26.7	291.2	34.0	300.8	37.4	215.5	29.6	207.2	27.6
	±5.1	±1.5	±5.1	±1.9	±11.5	±5.5	±5.2	±1.9	±5.0	±1.8

TABLE 8. Squared Errors (with standard deviations). Marg2 Query. 2-d datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg		nnlsalg		maxalg		seqalg		weightalg	
	Total	Max	Total	Max	Total	Max	Total	Max	Total	Max
Level00-2d	2629.1	31.0	86.1	36.9	106.7	47.5	107.7	75.8	39.9	19.6
	±16.7	±2.1	±1.9	±1.5	±11.4	±8.1	±6.3	±5.8	±1.9	±1.3
Level01-2d	2629.1	31.0	442.6	26.3	442.8	30.9	381.7	26.9	395.2	29.5
	±16.7	±2.1	±4.6	±1.6	±10.5	±4.8	±4.2	±1.8	±5.4	±2.3
Level16-2d	2629.1	31.0	2556.7	29.4	2665.4	42.8	2565.1	29.6	2690.1	35.1
	±16.7	±2.1	±15.1	±1.7	±33.4	±7.6	±14.9	±1.7	±17.4	±2.6
Level32-2d	2629.1	31.0	2626.4	31.0	2711.8	34.6	2631.7	31.1	2999.9	36.8
	±16.7	±2.1	±16.5	±2.1	±33.2	±4.8	±16.2	±2.0	±24.5	±4.1
SplitStairs-2d	2629.1	31.0	1195.5	27.0	1206.8	27.6	1141.5	27.0	1710.2	62.2
	±16.7	±2.1	±9.6	±1.6	±14.6	±2.6	±9.3	±1.6	±16.0	±3.8
Stair-2d	2629.1	31.0	2536.8	29.8	2504.3	40.7	2538.8	29.8	3621.0	94.1
	±16.7	±2.1	±16.0	±2.0	±42.0	±14.5	±15.7	±1.9	±22.6	±4.0
Step16-2d	2629.1	31.0	1334.2	37.9	1353.0	36.1	1296.8	54.0	1344.1	29.1
	±16.7	±2.1	±10.2	±2.0	±23.8	±3.9	±10.1	±2.5	±12.1	±2.2
Step50-2d	2629.1	31.0	1368.3	37.9	1370.9	40.2	1330.9	53.6	1242.2	29.2
	±16.7	±2.1	±11.3	±2.0	±26.4	±4.2	±10.8	±2.4	±10.0	±2.1

TABLE 9. Squared Errors (with standard deviations). Id Query. 2-d datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-2d	25.3	115.5	109.6	39.5	27.3
	±1.4	±3.1	±15.4	±4.2	±1.7
Level01-2d	25.3	35.3	32.9	32.4	26.4
	±1.4	±1.8	±3.8	±2.3	±1.6
Level16-2d	25.3	25.3	23.5	31.3	25.4
	±1.4	±1.4	±2.5	±1.9	±1.4
Level32-2d	25.3	25.3	21.4	31.2	25.3
	±1.4	±1.4	±2.3	±1.9	±1.4
SplitStairs-2d	25.3	35.9	37.4	30.5	25.3
	±1.4	±1.8	±3.0	±1.9	±1.5
Stair-2d	25.3	25.3	29.0	31.3	25.4
	±1.4	±1.4	±4.1	±1.9	±1.4
Step16-2d	25.3	33.6	27.8	31.6	25.4
	±1.4	±1.8	±3.1	±2.0	±1.5
Step50-2d	25.3	33.6	36.6	31.5	27.7
	±1.4	±1.8	±3.5	±2.0	±1.6

TABLE 10. Squared Error (with standard deviations). Sum Query. 2-d datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	1058.9	124.7	301.9	75.8	376.1	105.5	351.1	170.4	118.5	50.8
	±20.9	±7.6	±7.5	±3.6	±67.4	±42.0	±18.6	±12.8	±6.9	±3.0
Level01-2d	1058.9	124.7	585.6	75.9	595.1	80.5	784.9	112.8	727.4	91.6
	±20.9	±7.6	±11.3	±4.2	±35.3	±15.6	±19.9	±8.7	±19.8	±6.1
Level16-2d	1058.9	124.7	1041.2	123.2	1049.6	117.3	1166.1	138.3	1163.7	137.8
	±20.9	±7.6	±20.6	±7.5	±30.8	±10.6	±23.9	±8.7	±24.1	±8.7
Level32-2d	1058.9	124.7	1056.4	124.7	1020.9	123.4	1164.3	136.8	1163.7	137.8
	±20.9	±7.6	±20.9	±7.6	±35.7	±13.6	±23.9	±8.6	±24.1	±8.7
SplitStairs-2d	1058.9	124.7	953.0	111.1	998.2	115.7	1167.9	135.9	1163.1	137.7
	±20.9	±7.6	±18.1	±6.6	±33.3	±11.6	±26.0	±9.3	±24.1	±8.7
Stair-2d	1058.9	124.7	1051.8	123.6	987.3	116.5	1166.7	138.4	1163.4	137.8
	±20.9	±7.6	±20.8	±7.6	±35.1	±11.4	±23.9	±8.7	±24.1	±8.7
Step16-2d	1058.9	124.7	962.8	111.8	963.3	110.9	1178.1	137.5	1161.8	137.9
	±20.9	±7.6	±18.6	±6.8	±39.4	±12.9	±26.0	±9.4	±24.1	±8.7
Step50-2d	1058.9	124.7	989.2	114.3	960.6	108.0	1166.2	139.7	1163.4	138.0
	±20.9	±7.6	±19.1	±6.9	±40.7	±14.0	±24.2	±8.8	±24.1	±8.7

TABLE 11. Squared Errors (with standard deviations). Marg1 Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	1014.9	106.8	294.2	73.4	435.8	120.0	320.8	158.0	123.4	51.4
	±20.4	±6.2	±7.3	±3.4	±72.5	±30.6	±17.6	±12.2	±7.5	±2.8
Level01-2d	1014.9	106.8	568.8	75.4	562.0	69.0	753.1	99.2	705.9	94.3
	±20.4	±6.2	±11.3	±4.2	±34.2	±16.0	±18.1	±7.3	±19.7	±5.8
Level16-2d	1014.9	106.8	997.1	105.0	1033.3	117.5	1117.0	117.8	1114.6	117.7
	±20.4	±6.2	±20.1	±6.1	±30.7	±11.3	±23.6	±7.0	±23.9	±7.1
Level32-2d	1014.9	106.8	1012.8	106.6	1058.9	119.1	1116.8	117.0	1114.6	117.7
	±20.4	±6.2	±20.4	±6.2	±41.5	±13.3	±23.7	±7.0	±23.9	±7.1
SplitStairs-2d	1014.9	106.8	1093.6	144.5	1096.2	141.7	770.2	122.2	654.3	112.5
	±20.4	±6.2	±19.3	±6.8	±32.3	±12.2	±20.3	±7.0	±19.5	±6.5
Stair-2d	1014.9	106.8	1009.5	106.7	967.8	115.2	1117.5	117.8	1095.4	117.5
	±20.4	±6.2	±20.3	±6.1	±33.5	±14.8	±23.7	±7.0	±23.3	±7.0
Step16-2d	1014.9	106.8	1127.2	130.6	1195.6	145.6	884.6	119.8	740.6	113.1
	±20.4	±6.2	±20.1	±7.3	±43.1	±16.2	±22.5	±8.3	±20.1	±7.5
Step50-2d	1014.9	106.8	1164.1	136.0	1211.6	157.2	868.0	117.9	740.5	113.1
	±20.4	±6.2	±20.5	±7.6	±47.3	±24.7	±20.8	±7.6	±20.1	±7.5

TABLE 12. Squared Errors (with standard deviations). Marg2 Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	10516.5	124.0	344.2	147.4	443.7	173.1	437.3	283.6	159.2	78.4
	±66.8	±8.3	±7.7	±6.0	±77.1	±53.5	±17.3	±15.1	±7.8	±5.3
Level01-2d	10516.5	124.0	960.2	110.7	1049.6	149.7	775.0	118.4	790.1	105.6
	±66.8	±8.3	±12.8	±6.1	±44.7	±21.6	±12.1	±7.8	±19.0	±9.0
Level16-2d	10516.5	124.0	8551.1	109.9	8627.8	108.6	8579.5	109.2	8741.9	140.3
	±66.8	±8.3	±48.1	±6.9	±71.0	±9.9	±47.0	±6.6	±53.3	±10.2
Level32-2d	10516.5	124.0	10227.0	117.5	10545.2	143.4	10255.8	118.2	10761.4	140.3
	±66.8	±8.3	±60.3	±6.8	±113.8	±24.0	±59.6	±6.8	±69.6	±10.2
SplitStairs-2d	10516.5	124.0	4296.4	106.6	4274.2	113.4	4087.0	106.9	4501.3	140.3
	±66.8	±8.3	±34.1	±6.4	±59.4	±12.7	±34.0	±6.1	±42.9	±10.2
Stair-2d	10516.5	124.0	9650.6	118.7	9689.6	127.9	9654.7	118.3	12121.6	207.2
	±66.8	±8.3	±60.9	±7.9	±111.7	±13.3	±59.6	±7.7	±89.5	±13.3
Step16-2d	10516.5	124.0	4518.6	150.2	4593.0	184.9	4408.7	205.6	4313.0	106.8
	±66.8	±8.3	±32.0	±7.8	±69.6	±21.2	±32.5	±9.9	±35.2	±9.1
Step50-2d	10516.5	124.0	5451.1	151.7	5576.9	178.4	5301.0	215.9	6483.3	142.2
	±66.8	±8.3	±43.9	±7.8	±106.7	±26.7	±42.2	±9.7	±56.2	±10.2

TABLE 13. Squared Errors (with standard deviations). Id Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-2d	101.3	461.9	533.9	149.2	108.5
	±5.7	±12.5	±76.7	±14.8	±7.0
Level01-2d	101.3	189.5	215.3	126.4	105.7
	±5.7	±8.6	±27.2	±8.9	±6.3
Level16-2d	101.3	101.8	107.0	125.0	101.6
	±5.7	±5.7	±8.6	±7.8	±5.7
Level32-2d	101.3	101.3	100.0	125.1	101.6
	±5.7	±5.7	±9.9	±7.8	±5.7
SplitStairs-2d	101.3	146.6	141.7	124.5	101.1
	±5.7	±7.4	±11.3	±8.5	±5.8
Stair-2d	101.3	101.5	100.0	125.0	101.5
	±5.7	±5.7	±9.7	±7.8	±5.7
Step16-2d	101.3	136.7	153.6	123.1	101.7
	±5.7	±7.2	±17.7	±8.1	±5.8
Step50-2d	101.3	134.3	153.3	124.9	101.7
	±5.7	±7.2	±20.2	±7.9	±5.8

TABLE 14. Squared Error (with standard deviations). Sum Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	26471.7	3118.5	7548.3	1894.1	5694.6	2431.7	8088.6	3727.6	2958.3	1269.9
	±523.4	±190.8	±186.9	±89.1	±1376.7	±1126.3	±396.5	±293.5	±173.8	±74.6
Level01-2d	26471.7	3118.5	8866.5	1805.8	6245.0	1635.9	9712.2	3237.9	6134.4	1757.0
	±523.4	±190.8	±211.7	±89.7	±1262.2	±398.3	±322.7	±199.7	±255.9	±105.1
Level16-2d	26471.7	3118.5	21759.9	2580.4	20142.9	3174.1	28382.5	3365.1	28246.4	3309.6
	±523.4	±190.8	±404.8	±150.6	±1602.1	±705.0	±556.6	±203.4	±570.3	±198.6
Level32-2d	26471.7	3118.5	24466.9	2921.1	24229.8	3687.0	28991.1	3449.8	28907.3	3427.4
	±523.4	±190.8	±475.8	±175.8	±1377.6	±602.9	±591.7	±216.9	±598.3	±216.0
SplitStairs-2d	26471.7	3118.5	20081.4	2275.5	28035.9	4753.6	27437.2	3119.0	26088.0	2901.0
	±523.4	±190.8	±364.6	±133.2	±4283.3	±2378.7	±554.6	±197.6	±544.4	±180.8
Stair-2d	26471.7	3118.5	25170.6	2959.2	24453.9	2980.0	29157.7	3457.4	29081.5	3445.1
	±523.4	±190.8	±494.3	±179.8	±1384.4	±678.3	±598.2	±216.8	±602.2	±216.9
Step16-2d	26471.7	3118.5	17895.7	2105.1	21071.0	2841.1	24033.8	2835.4	22665.5	2550.1
	±523.4	±190.8	±329.4	±120.1	±2083.2	±751.8	±472.4	±189.9	±521.5	±160.1
Step50-2d	26471.7	3118.5	23086.7	2695.9	23437.1	3793.6	29141.9	3431.7	28085.5	3320.2
	±523.4	±190.8	±439.7	±160.4	±2507.8	±954.8	±595.8	±216.0	±580.2	±208.3

TABLE 15. Squared Errors (with standard deviations). Marg1 Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	25372.5	2668.9	7355.9	1833.8	14218.1	9473.1	7665.7	3512.6	3080.0	1285.3
	±510.4	±154.1	±182.7	±85.1	±1243.4	±105.1	±365.1	±262.1	±187.2	±69.1
Level01-2d	25372.5	2668.9	8646.9	1759.3	18406.0	5264.1	9151.9	3042.7	6193.8	1788.8
	±510.4	±154.1	±209.6	±87.2	±5237.9	±3718.8	±301.6	±184.7	±272.2	±98.7
Level16-2d	25372.5	2668.9	20984.9	2250.9	23854.3	3431.9	27062.7	2939.7	26963.2	2873.5
	±510.4	±154.1	±398.4	±132.8	±2344.5	±1162.7	±535.8	±175.0	±555.5	±180.1
Level32-2d	25372.5	2668.9	23472.4	2481.9	24557.9	3312.0	27908.4	2954.4	27757.4	2924.1
	±510.4	±154.1	±464.9	±141.4	±1452.2	±534.7	±585.4	±176.2	±588.5	±174.8
SplitStairs-2d	25372.5	2668.9	22466.1	3354.0	23583.4	4580.8	18798.5	3011.1	21400.8	5680.5
	±510.4	±154.1	±401.0	±153.1	±3017.1	±1448.8	±473.0	±171.4	±556.3	±228.8
Stair-2d	25372.5	2668.9	24634.2	2607.9	24122.0	3112.3	27780.8	2943.5	27156.0	2924.4
	±510.4	±154.1	±480.4	±167.7	±1303.0	±637.1	±576.0	±175.7	±567.2	±174.2
Step16-2d	25372.5	2668.9	20455.3	2297.4	22047.5	4131.7	20990.4	2825.8	22687.6	3290.4
	±510.4	±154.1	±368.2	±117.6	±2756.9	±1537.4	±475.7	±177.9	±558.5	±210.0
Step50-2d	25372.5	2668.9	26829.3	3076.4	29360.4	4341.9	21590.8	2950.8	18446.5	2816.4
	±510.4	±154.1	±481.0	±163.5	±2504.2	±1149.5	±516.3	±189.3	±495.3	±185.0

TABLE 16. Squared Errors (with standard deviations). Marg2 Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	262911.4	3100.0	8605.7	3684.9	11473.9	4702.8	10107.9	6442.2	3969.7	1957.5
	±1671.2	±207.7	±192.5	±150.6	±2457.8	±1883.9	±389.6	±352.0	±195.8	±132.6
Level01-2d	262911.4	3100.0	10798.5	3274.3	12020.4	4404.7	10372.8	4925.4	6766.7	2206.0
	±1671.2	±207.7	±214.0	±149.5	±2257.9	±1463.9	±270.5	±229.7	±260.6	±159.1
Level16-2d	262911.4	3100.0	68991.2	2595.8	67044.4	1549.9	63441.2	2564.4	65066.2	3271.1
	±1671.2	±207.7	±615.4	±160.2	±2491.7	±425.1	±511.1	±150.1	±673.4	±240.5
Level32-2d	262911.4	3100.0	124733.6	2616.5	123979.9	2573.2	122210.1	2588.6	123469.4	3496.1
	±1671.2	±207.7	±882.3	±166.1	±2800.4	±862.2	±817.7	±158.4	±906.2	±254.5
SplitStairs-2d	262911.4	3100.0	50234.5	2996.2	52915.5	1999.9	45193.8	3302.0	44590.7	2844.4
	±1671.2	±207.7	±464.6	±156.8	±3467.3	±912.6	±403.3	±166.0	±550.9	±228.3
Stair-2d	262911.4	3100.0	155160.3	3097.7	155361.3	2718.6	153483.3	3581.2	152550.7	2916.9
	±1671.2	±207.7	±997.9	±178.7	±2725.5	±370.6	±950.4	±187.6	±1064.5	±229.7
Step16-2d	262911.4	3100.0	37999.0	3529.7	39280.2	3791.0	34028.3	4686.3	31919.8	2665.9
	±1671.2	±207.7	±408.0	±178.7	±3077.8	±1182.3	±353.0	±219.2	±511.3	±227.3
Step50-2d	262911.4	3100.0	89586.0	3702.8	94050.9	3428.8	86582.5	5201.5	82313.0	2673.2
	±1671.2	±207.7	±674.3	±191.3	±3765.8	±693.1	±623.4	±235.0	±665.9	±227.0

Table 17. Squared Errors (with standard deviations). Id Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-2d	2532.8	11547.9	21245.2	3869.8	2706.6
	±141.5	±313.5	±204.6	±318.7	±174.9
Level01-2d	2532.8	8668.1	9288.8	2672.6	2609.7
	±141.5	±281.0	±2043.4	±187.5	±153.9
Level16-2d	2532.8	3021.9	2354.3	3096.8	2643.9
	±141.5	±164.5	±426.9	±193.2	±156.6
Level32-2d	2532.8	2656.6	2517.2	3131.0	2541.3
	±141.5	±149.7	±336.6	±194.7	±142.8
SplitStairs-2d	2532.8	4363.3	3599.9	3038.5	2637.0
	±141.5	±203.0	±809.9	±193.4	±154.5
Stair-2d	2532.8	2635.1	3014.5	3126.8	2537.6
	±141.5	±148.5	±483.3	±194.2	±142.5
Step16-2d	2532.8	4413.9	4285.1	3133.3	2644.7
	±141.5	±208.2	±1574.4	±197.5	±157.4
Step50-2d	2532.8	3540.9	3890.6	3114.6	2536.0
	±141.5	±184.7	±1362.8	±196.2	±146.5

Table 18. Squared Error (with standard deviations). Sum Query. 2-d datasets. Lap Mechanism ($\epsilon = 0.1$).

	olsalg		nnlsalg		maxalg		seqalg		weightalg	
Dataset	Total	Max	Total	Max	Total	Max	Total	Max	Total	Max
PUMA0101301	249.0	32.2	138.3	19.3	149.2	19.4	157.0	33.5	147.0	44.9
	±5.3	±2.0	±2.8	±1.0	±4.2	±1.3	±4.7	±2.4	±4.1	±2.1
PUMA0800803	249.0	32.2	185.7	41.8	193.3	38.8	181.6	30.2	169.0	35.3
	±5.3	±2.0	±3.6	±2.0	±6.2	±2.7	±6.3	±2.8	±4.5	±1.9
PUMA1304600	249.0	32.2	194.6	25.3	199.5	26.7	196.9	33.2	196.4	46.6
	±5.3	±2.0	±3.7	±1.3	±6.2	±2.4	±7.2	±2.6	±5.1	±2.5
PUMA1703529	249.0	32.2	184.9	29.9	195.3	31.2	207.8	36.4	185.1	46.2
	±5.3	±2.0	±3.6	±1.5	±6.0	±2.4	±7.8	±3.4	±4.9	±2.5
PUMA1703531	249.0	32.2	127.2	19.2	138.7	21.1	148.2	32.4	136.1	26.5
	±5.3	±2.0	±2.6	±1.0	±4.1	±1.6	±4.3	±2.6	±4.0	±1.4
PUMA1901700	249.0	32.2	205.6	31.2	211.8	31.3	198.4	34.9	163.2	28.3
	±5.3	±2.0	±3.9	±1.6	±6.8	±2.9	±7.3	±3.0	±4.5	±1.7
PUMA2401004	249.0	32.2	228.5	57.1	245.7	59.2	191.5	36.3	178.5	41.5
	±5.3	±2.0	±4.3	±2.4	±7.5	±4.2	±12.7	±5.9	±4.6	±2.0
PUMA2602702	249.0	32.2	175.8	32.9	191.6	34.4	189.6	34.7	167.4	33.8
	±5.3	±2.0	±3.4	±1.6	±5.8	±2.7	±6.1	±2.7	±4.3	±1.6
PUMA2801100	249.0	32.2	145.0	23.9	150.5	23.5	156.5	32.0	133.4	24.4
	±5.3	±2.0	±2.9	±1.2	±4.4	±1.6	±5.3	±2.6	±3.8	±1.3
PUMA2901901	249.0	32.2	165.3	29.4	174.4	32.0	188.9	35.3	149.3	30.1
	±5.3	±2.0	±3.3	±1.4	±5.3	±2.2	±6.5	±3.1	±4.2	±1.8
PUMA3200405	249.0	32.2	225.0	30.5	237.2	32.4	225.5	35.8	216.7	33.6
	±5.3	±2.0	±4.2	±1.7	±8.1	±3.1	±7.5	±3.2	±5.1	±2.1
PUMA3603710	249.0	32.2	240.7	34.2	245.8	32.0	216.4	33.0	227.1	34.3
	±5.3	±2.0	±4.5	±1.8	±7.6	±2.8	±7.2	±2.7	±5.4	±2.2
PUMA3604010	249.0	32.2	204.0	33.7	210.2	32.5	192.3	32.4	207.4	49.9
	±5.3	±2.0	±3.8	±1.8	±6.6	±2.9	±7.5	±2.9	±5.6	±2.6
PUMA5101301	249.0	32.2	209.2	45.1	227.7	45.0	198.2	34.4	172.2	41.2
	±5.3	±2.0	±4.0	±2.1	±7.3	±3.4	±6.8	±3.1	±4.5	±2.3
PUMA5151255	249.0	32.2	239.3	41.4	249.3	39.7	192.8	30.8	160.5	28.0
	±5.3	±2.0	±4.4	±2.0	±7.6	±2.9	±8.2	±3.4	±4.4	±1.6

Table 19. Squared Errors (with standard deviations). Marg1 Query. PUMS datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	656.6	30.8	199.1	62.3	223.5	67.5	185.2	45.9	152.4	58.3
	±8.6	±2.1	±3.4	±2.1	±5.6	±3.1	±5.1	±2.5	±4.4	±2.2
PUMA0800803	656.6	30.8	292.9	62.7	312.7	60.6	319.5	38.5	239.6	26.2
	±8.6	±2.1	±4.2	±2.3	±7.7	±3.6	±8.8	±3.4	±5.8	±1.7
PUMA1304600	656.6	30.8	315.8	53.3	325.7	52.2	293.2	32.5	288.6	46.8
	±8.6	±2.1	±4.4	±2.1	±7.5	±3.3	±8.7	±2.7	±6.0	±2.0
PUMA1703529	656.6	30.8	291.3	58.6	308.7	62.3	277.9	40.2	217.0	30.3
	±8.6	±2.1	±4.3	±2.2	±7.5	±3.6	±8.3	±3.3	±5.6	±1.9
PUMA1703531	656.6	30.8	186.0	78.1	205.7	81.9	134.4	58.5	77.0	26.2
	±8.6	±2.1	±3.3	±2.4	±5.5	±3.5	±4.2	±2.9	±3.2	±1.5
PUMA1901700	656.6	30.8	337.3	49.5	359.7	53.7	350.1	38.1	301.8	31.3
	±8.6	±2.1	±4.6	±2.1	±8.4	±3.4	±9.5	±3.4	±6.5	±1.7
PUMA2401004	656.6	30.8	387.1	49.0	400.6	47.9	534.6	40.7	447.1	37.4
	±8.6	±2.1	±4.9	±2.1	±8.5	±3.1	±23.8	±7.3	±8.1	±2.2
PUMA2602702	656.6	30.8	257.1	61.9	271.7	64.9	257.1	35.2	208.0	28.8
	±8.6	±2.1	±3.9	±2.3	±6.9	±3.7	±6.9	±2.4	±5.2	±1.5
PUMA2801100	656.6	30.8	212.2	68.6	228.6	70.6	174.2	41.6	136.5	26.5
	±8.6	±2.1	±3.5	±2.3	±6.0	±3.3	±4.8	±2.5	±3.9	±1.5
PUMA2901901	656.6	30.8	256.4	64.8	278.7	68.3	235.9	38.9	197.8	42.7
	±8.6	±2.1	±3.9	±2.3	±7.1	±3.6	±6.5	±2.6	±5.0	±1.8
PUMA3200405	656.6	30.8	406.8	49.7	422.1	53.9	403.9	35.1	393.9	37.9
	±8.6	±2.1	±5.3	±2.1	±10.1	±3.8	±10.6	±3.5	±7.6	±2.1
PUMA3603710	656.6	30.8	445.5	39.0	475.8	44.0	466.8	34.4	414.6	43.3
	±8.6	±2.1	±5.7	±1.9	±10.8	±3.4	±11.4	±3.3	±8.0	±2.3
PUMA3604010	656.6	30.8	330.4	47.7	357.8	47.6	353.5	37.2	328.3	38.9
	±8.6	±2.1	±4.5	±2.0	±8.3	±3.1	±9.6	±3.1	±6.9	±2.0
PUMA5101301	656.6	30.8	330.6	54.9	348.9	60.2	390.5	34.2	326.9	29.6
	±8.6	±2.1	±4.5	±2.2	±8.3	±4.0	±9.8	±3.3	±6.7	±2.1
PUMA5151255	656.6	30.8	398.5	41.6	412.3	46.3	465.7	34.7	424.5	33.0
	±8.6	±2.1	±5.1	±1.9	±9.2	±3.4	±12.8	±3.9	±7.6	±2.0

Table 20. Squared Errors (with standard deviations). Marg2 Query. PUMS datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	5976.5	35.7	235.4	41.0	260.1	43.8	236.5	60.8	302.3	93.0
	±26.4	±2.6	±3.2	±1.5	±5.5	±2.3	±4.3	±2.4	±5.8	±2.9
PUMA0800803	5976.5	35.7	385.8	26.6	412.8	30.3	401.9	64.5	352.6	31.1
	±26.4	±2.6	±4.2	±1.6	±8.0	±2.6	±6.8	±3.4	±6.1	±1.4
PUMA1304600	5976.5	35.7	477.2	30.0	491.1	31.0	416.1	40.1	473.3	44.7
	±26.4	±2.6	±4.6	±1.7	±8.1	±3.6	±6.7	±3.9	±6.0	±2.2
PUMA1703529	5976.5	35.7	396.6	28.8	412.5	28.9	344.7	30.8	442.2	56.8
	±26.4	±2.6	±4.5	±1.3	±8.0	±2.2	±6.2	±2.8	±7.1	±2.5
PUMA1703531	5976.5	35.7	180.4	23.2	200.0	24.9	148.1	28.9	153.0	26.6
	±26.4	±2.6	±2.5	±1.0	±4.4	±1.5	±3.0	±1.5	±3.7	±1.2
PUMA1901700	5976.5	35.7	515.9	32.0	527.4	31.0	451.5	40.8	554.9	49.7
	±26.4	±2.6	±5.0	±1.7	±8.6	±2.0	±6.8	±3.4	±8.0	±3.1
PUMA2401004	5976.5	35.7	628.8	35.6	659.9	37.0	672.5	78.3	606.3	38.3
	±26.4	±2.6	±5.8	±1.7	±10.3	±3.0	±16.6	±6.7	±7.6	±2.0
PUMA2602702	5976.5	35.7	326.3	26.9	349.1	27.6	305.2	40.2	296.3	37.4
	±26.4	±2.6	±3.9	±1.6	±7.1	±2.9	±5.3	±2.1	±5.3	±2.2
PUMA2801100	5976.5	35.7	233.5	25.2	253.1	26.3	203.3	30.8	210.1	27.6
	±26.4	±2.6	±2.8	±0.6	±5.1	±1.2	±3.5	±0.8	±3.9	±0.8
PUMA2901901	5976.5	35.7	307.1	26.9	333.1	26.8	279.8	33.4	276.7	38.0
	±26.4	±2.6	±3.8	±1.6	±6.9	±2.8	±4.9	±1.9	±5.0	±2.2
PUMA3200405	5976.5	35.7	759.7	31.0	787.4	30.3	685.8	41.0	803.3	63.2
	±26.4	±2.6	±6.3	±1.8	±12.0	±2.5	±8.4	±3.2	±8.7	±3.4
PUMA3603710	5976.5	35.7	916.9	35.5	960.2	35.0	815.7	45.6	992.8	52.9
	±26.4	±2.6	±7.3	±1.4	±13.3	±2.3	±9.1	±2.7	±10.2	±2.0
PUMA3604010	5976.5	35.7	502.6	31.6	523.4	28.8	464.5	37.8	534.7	62.4
	±26.4	±2.6	±4.7	±1.4	±8.6	±2.5	±6.7	±2.3	±7.8	±4.0
PUMA5101301	5976.5	35.7	510.9	29.3	529.3	28.7	506.6	63.4	472.3	54.7
	±26.4	±2.6	±4.9	±1.5	±8.7	±2.2	±7.2	±3.4	±6.4	±2.1
PUMA5151255	5976.5	35.7	741.7	34.7	760.6	34.9	688.8	46.2	762.4	92.8
	±26.4	±2.6	±6.4	±1.9	±10.9	±2.3	±10.1	±4.0	±9.9	±6.2

Table 21. Squared Errors (with standard deviations). Id Query. PUMS datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
PUMA0101301	26.8	130.2	139.6	31.9	28.4
	±1.5	±3.6	±5.3	±2.5	±1.6
PUMA0800803	26.8	100.5	104.4	29.5	26.8
	±1.5	±3.2	±5.5	±2.5	±1.6
PUMA1304600	26.8	89.9	90.6	30.7	27.7
	±1.5	±3.0	±5.0	±2.7	±1.6
PUMA1703529	26.8	96.9	97.1	32.3	27.7
	±1.5	±3.2	±4.9	±3.0	±1.6
PUMA1703531	26.8	135.7	137.0	28.6	27.2
	±1.5	±3.6	±5.0	±2.4	±1.6
PUMA1901700	26.8	88.4	94.5	33.7	27.0
	±1.5	±3.0	±5.6	±3.3	±1.6
PUMA2401004	26.8	83.2	82.2	25.4	27.6
	±1.5	±3.0	±4.8	±4.3	±1.6
PUMA2602702	26.8	102.8	104.1	29.5	27.1
	±1.5	±3.2	±5.3	±2.4	±1.6
PUMA2801100	26.8	123.5	120.5	28.4	27.4
	±1.5	±3.5	±4.8	±2.4	±1.6
PUMA2901901	26.8	109.4	106.7	29.1	27.3
	±1.5	±3.3	±4.7	±2.4	±1.6
PUMA3200405	26.8	72.8	73.3	31.4	27.4
	±1.5	±2.8	±4.6	±2.8	±1.6
PUMA3603710	26.8	66.8	65.1	32.2	27.3
	±1.5	±2.7	±4.3	±2.8	±1.6
PUMA3604010	26.8	90.3	88.0	31.7	27.8
	±1.5	±3.1	±4.8	±3.0	±1.6
PUMA5101301	26.8	87.3	86.2	27.6	27.6
	±1.5	±3.0	±4.8	±2.6	±1.6
PUMA5151255	26.8	73.2	73.9	34.9	26.8
	±1.5	±2.8	±4.7	±3.7	±1.6

Table 22. Squared Error (with standard deviations). Sum Query. PUMS datasets. Lap Mechanism ($\epsilon = 1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	995.9	128.6	505.6	70.7	535.3	86.8	598.3	141.0	513.0	142.6
	±21.1	±7.9	±10.4	±3.9	±46.0	±21.2	±24.2	±12.6	±14.5	±5.9
PUMA0800803	995.9	128.6	662.4	143.4	699.6	142.0	680.0	136.9	642.3	134.8
	±21.1	±7.9	±13.2	±6.7	±45.3	±17.9	±34.7	±20.0	±17.8	±7.2
PUMA1304600	995.9	128.6	682.1	87.5	718.7	96.8	782.3	151.1	790.2	158.6
	±21.1	±7.9	±13.1	±4.5	±41.1	±13.3	±44.8	±21.6	±20.2	±7.4
PUMA1703529	995.9	128.6	669.6	101.9	710.6	100.6	730.8	142.4	729.9	158.1
	±21.1	±7.9	±13.0	±5.1	±44.5	±14.8	±48.0	±26.0	±18.6	±7.5
PUMA1703531	995.9	128.6	444.9	72.3	499.9	77.2	464.5	141.8	387.0	86.2
	±21.1	±7.9	±9.6	±3.8	±36.4	±10.5	±17.8	±11.5	±12.7	±5.2
PUMA1901700	995.9	128.6	736.9	107.9	729.2	98.7	731.7	126.6	869.0	185.2
	±21.1	±7.9	±14.1	±5.5	±45.0	±16.0	±37.1	±15.4	±21.8	±9.2
PUMA2401004	995.9	128.6	843.7	224.8	854.5	205.0	823.1	145.8	636.1	122.7
	±21.1	±7.9	±16.0	±9.6	±49.3	±26.6	±49.1	±19.3	±17.2	±6.0
PUMA2602702	995.9	128.6	613.1	99.9	594.9	95.9	695.3	130.5	686.7	155.5
	±21.1	±7.9	±12.1	±4.8	±46.8	±18.0	±34.6	±16.3	±18.2	±7.0
PUMA2801100	995.9	128.6	502.2	78.8	522.8	74.0	527.2	125.3	390.3	85.6
	±21.1	±7.9	±10.6	±3.9	±32.8	±18.4	±26.2	±13.7	±12.5	±5.3
PUMA2901901	995.9	128.6	595.2	94.1	606.8	105.3	676.0	125.4	660.7	140.8
	±21.1	±7.9	±11.9	±4.6	±35.7	±14.4	±36.0	±12.5	±18.2	±7.7
PUMA3200405	995.9	128.6	821.3	118.0	892.0	150.7	886.9	143.4	795.7	137.1
	±21.1	±7.9	±15.7	±6.5	±67.3	±39.1	±37.8	±15.9	±20.1	±8.3
PUMA3603710	995.9	128.6	904.6	130.5	868.5	166.7	773.4	127.3	811.8	136.9
	±21.1	±7.9	±16.9	±6.8	±63.0	±26.5	±31.1	±13.4	±20.6	±8.6
PUMA3604010	995.9	128.6	726.3	115.1	715.9	145.3	760.1	129.0	825.6	155.7
	±21.1	±7.9	±13.7	±6.0	±42.4	±26.8	±33.0	±15.9	±21.0	±8.7
PUMA5101301	995.9	128.6	747.0	157.6	895.4	205.8	810.9	138.7	797.2	173.8
	±21.1	±7.9	±14.3	±7.4	±72.0	±38.5	±39.0	±16.7	±20.4	±9.4
PUMA5151255	995.9	128.6	874.0	153.5	976.9	213.0	795.4	141.3	607.1	115.6
	±21.1	±7.9	±16.3	±7.5	±63.5	±30.3	±38.1	±16.7	±17.5	±6.9

Table 23. Squared Errors (with standard deviations). Marg1 Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	2626.6	123.0	725.6	236.9	794.1	270.8	655.0	221.3	463.8	159.6
	±34.3	±8.3	±12.7	±8.1	±50.0	±34.6	±23.6	±14.7	±14.2	±4.8
PUMA0800803	2626.6	123.0	982.6	254.0	1070.7	262.4	959.4	179.6	795.2	174.2
	±34.3	±8.3	±15.3	±9.0	±59.8	±26.7	±37.0	±19.1	±20.8	±8.2
PUMA1304600	2626.6	123.0	1056.6	233.0	1141.7	261.0	989.6	157.0	866.7	154.0
	±34.3	±8.3	±15.8	±8.7	±61.9	±31.5	±44.2	±18.7	±21.5	±7.0
PUMA1703529	2626.6	123.0	1017.0	255.2	1053.7	236.1	936.6	180.5	784.3	238.7
	±34.3	±8.3	±15.9	±9.1	±44.8	±20.5	±55.5	±21.8	±20.6	±9.2
PUMA1703531	2626.6	123.0	644.0	275.4	679.4	255.9	466.5	264.2	232.5	98.0
	±34.3	±8.3	±12.0	±8.4	±40.8	±21.9	±18.5	±14.6	±11.8	±5.6
PUMA1901700	2626.6	123.0	1137.1	218.9	1270.6	248.5	1069.0	161.2	860.8	113.5
	±34.3	±8.3	±16.6	±8.5	±58.9	±24.7	±42.4	±18.7	±20.8	±7.1
PUMA2401004	2626.6	123.0	1348.6	208.8	1301.5	201.4	1532.4	123.3	1368.2	110.3
	±34.3	±8.3	±17.9	±8.4	±56.5	±23.4	±57.8	±15.5	±27.8	±7.4
PUMA2602702	2626.6	123.0	855.8	273.2	823.0	229.4	731.5	192.3	546.5	106.4
	±34.3	±8.3	±14.4	±9.4	±54.6	±25.4	±33.8	±18.5	±16.9	±6.4
PUMA2801100	2626.6	123.0	712.4	265.2	728.7	260.3	600.2	244.6	345.3	96.9
	±34.3	±8.3	±12.8	±8.8	±49.0	±34.5	±26.4	±18.3	±12.4	±5.4
PUMA2901901	2626.6	123.0	867.5	278.3	821.8	244.4	778.2	200.6	549.8	107.5
	±34.3	±8.3	±14.3	±9.4	±43.2	±27.2	±36.0	±18.3	±16.0	±5.0
PUMA3200405	2626.6	123.0	1407.8	210.4	1417.3	232.2	1403.1	154.6	1246.5	136.0
	±34.3	±8.3	±19.1	±8.6	±67.8	±33.0	±44.3	±15.5	±26.1	±7.7
PUMA3603710	2626.6	123.0	1617.8	156.4	1788.8	208.4	1572.7	129.6	1506.5	151.0
	±34.3	±8.3	±21.5	±6.9	±86.9	±28.0	±42.6	±12.1	±31.1	±8.7
PUMA3604010	2626.6	123.0	1092.7	213.7	1106.5	223.9	1136.2	154.5	969.6	122.3
	±34.3	±8.3	±15.7	±8.2	±43.7	±21.9	±38.1	±15.6	±23.9	±6.1
PUMA5101301	2626.6	123.0	1093.7	241.6	1079.9	235.9	1049.1	124.3	941.8	110.5
	±34.3	±8.3	±16.2	±9.0	±59.5	±35.6	±35.5	±12.8	±23.2	±7.1
PUMA5151255	2626.6	123.0	1388.6	183.6	1378.6	200.2	1523.9	133.0	1430.8	134.5
	±34.3	±8.3	±18.7	±7.9	±72.5	±28.8	±46.6	±15.4	±28.3	±8.7

Table 24. Squared Errors (with standard deviations). Marg2 Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	23906.2	142.9	809.0	135.6	910.7	144.6	782.9	179.7	731.3	209.8
	±105.7	±10.3	±11.2	±3.3	±48.1	±12.0	±19.3	±5.1	±15.4	±3.7
PUMA0800803	23906.2	142.9	1179.8	107.5	1235.3	125.4	1171.7	189.8	1123.8	141.9
	±105.7	±10.3	±14.7	±6.4	±52.9	±17.4	±33.2	±14.1	±22.3	±8.7
PUMA1304600	23906.2	142.9	1313.0	111.9	1385.5	142.3	1049.1	126.3	1264.4	136.0
	±105.7	±10.3	±14.3	±5.4	±50.1	±20.8	±25.4	±13.8	±20.8	±6.9
PUMA1703529	23906.2	142.9	1243.8	105.3	1257.2	96.7	1019.1	114.3	1285.7	160.5
	±105.7	±10.3	±15.3	±6.3	±44.3	±12.2	±33.7	±13.6	±21.9	±4.3
PUMA1703531	23906.2	142.9	562.2	94.9	599.0	72.1	429.9	112.9	409.8	78.8
	±105.7	±10.3	±8.4	±4.9	±32.8	±11.0	±12.3	±9.4	±12.0	±4.9
PUMA1901700	23906.2	142.9	1516.1	115.9	1665.9	129.7	1312.1	156.9	1617.1	205.0
	±105.7	±10.3	±16.6	±4.9	±64.7	±17.9	±29.9	±13.8	±25.1	±8.6
PUMA2401004	23906.2	142.9	1954.4	130.0	1971.8	147.8	1983.4	311.3	1760.1	168.9
	±105.7	±10.3	±19.4	±6.2	±59.9	±18.7	±51.6	±25.6	±26.7	±8.8
PUMA2602702	23906.2	142.9	977.2	100.0	956.4	109.4	843.4	121.7	930.1	156.2
	±105.7	±10.3	±13.5	±4.7	±53.3	±19.3	±25.6	±11.3	±19.1	±6.6
PUMA2801100	23906.2	142.9	686.9	97.5	705.7	79.0	534.2	92.7	516.0	78.7
	±105.7	±10.3	±9.9	±5.1	±33.6	±10.0	±16.2	±8.0	±12.2	±5.1
PUMA2901901	23906.2	142.9	944.4	100.4	919.2	103.2	809.4	131.6	888.2	138.2
	±105.7	±10.3	±12.5	±4.6	±40.2	±18.0	±26.7	±14.1	±18.2	±6.6
PUMA3200405	23906.2	142.9	2189.2	119.6	2191.5	134.7	1918.5	142.3	2336.1	259.1
	±105.7	±10.3	±20.5	±6.9	±77.4	±22.2	±31.4	±11.2	±29.7	±14.2
PUMA3603710	23906.2	142.9	2884.1	119.2	3088.6	149.1	2484.2	140.7	2870.4	166.1
	±105.7	±10.3	±24.6	±3.5	±103.3	±29.8	±33.8	±6.1	±33.0	±3.9
PUMA3604010	23906.2	142.9	1432.5	105.9	1442.3	120.6	1262.1	122.7	1448.6	194.0
	±105.7	±10.3	±14.5	±3.8	±42.7	±17.0	±22.8	±7.7	±24.0	±10.5
PUMA5101301	23906.2	142.9	1474.7	108.3	1498.6	101.8	1394.5	203.4	1392.9	153.2
	±105.7	±10.3	±16.4	±6.5	±58.3	±15.8	±32.3	±15.2	±24.4	±8.0
PUMA5151255	23906.2	142.9	2239.7	130.3	2274.1	124.3	2079.0	178.5	2123.0	172.8
	±105.7	±10.3	±21.4	±7.2	±79.8	±16.4	±37.8	±16.9	±29.4	±12.5

Table 25. Squared Errors (with standard deviations). Id Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
PUMA0101301	107.2	547.2	500.3	106.7	112.5
	±5.9	±14.5	±40.8	±10.7	±6.6
PUMA0800803	107.2	446.1	571.7	120.3	107.2
	±5.9	±13.4	±55.5	±13.5	±6.4
PUMA1304600	107.2	408.1	426.3	120.8	109.8
	±5.9	±12.9	±40.9	±16.7	±6.4
PUMA1703529	107.2	435.3	426.3	134.9	110.9
	±5.9	±13.3	±36.8	±19.3	±6.5
PUMA1703531	107.2	584.0	677.4	111.4	108.1
	±5.9	±14.9	±53.8	±8.6	±6.6
PUMA1901700	107.2	395.1	443.6	119.1	110.4
	±5.9	±12.8	±42.0	±13.4	±6.4
PUMA2401004	107.2	369.3	329.0	109.6	107.5
	±5.9	±12.4	±32.2	±15.2	±6.3
PUMA2602702	107.2	467.8	472.0	146.0	109.2
	±5.9	±13.7	±45.0	±16.6	±6.4
PUMA2801100	107.2	543.7	558.2	117.8	110.8
	±5.9	±14.5	±42.1	±13.6	±6.5
PUMA2901901	107.2	485.2	464.4	126.5	110.8
	±5.9	±13.9	±38.9	±16.5	±6.5
PUMA3200405	107.2	329.1	301.0	122.9	108.4
	±5.9	±11.7	±37.8	±12.5	±6.3
PUMA3603710	107.2	300.3	293.3	85.7	108.8
	±5.9	±11.3	±43.4	±8.5	±6.4
PUMA3604010	107.2	399.9	386.5	129.8	111.3
	±5.9	±12.9	±32.5	±14.1	±6.5
PUMA5101301	107.2	396.1	369.5	139.2	107.2
	±5.9	±12.7	±47.2	±16.3	±6.3
PUMA5151255	107.2	330.7	280.3	139.1	107.8
	±5.9	±11.8	±43.7	±15.4	±6.3

Table 26. Squared Error (with standard deviations). Sum Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	24896.5	3215.6	10104.5	1708.4	9642.4	1786.5	10263.6	3245.0	7632.6	2034.3
	±526.6	±198.2	±224.8	±88.2	±1335.2	±812.4	±553.9	±324.2	±266.6	±82.8
PUMA0800803	24896.5	3215.6	12272.6	2263.3	12433.7	3064.4	14411.4	4356.0	10115.3	2199.8
	±526.6	±198.2	±259.8	±106.9	±1894.4	±1212.6	±2215.9	±1524.3	±330.9	±118.5
PUMA1304600	24896.5	3215.6	12120.8	1812.5	11921.0	2081.1	13247.7	2884.2	12740.9	3563.4
	±526.6	±198.2	±256.2	±100.0	±1621.6	±755.3	±2037.6	±719.6	±399.9	±185.2
PUMA1703529	24896.5	3215.6	12838.6	1805.1	13531.6	2312.8	23113.3	6541.3	13386.5	3221.7
	±526.6	±198.2	±266.7	±102.1	±2620.2	±1701.4	±4924.2	±4081.2	±395.3	±147.8
PUMA1703531	24896.5	3215.6	9061.0	1801.3	10780.0	2611.4	7140.4	4292.3	4327.4	1710.4
	±526.6	±198.2	±212.9	±88.6	±1525.7	±755.5	±584.3	±473.2	±212.9	±94.4
PUMA1901700	24896.5	3215.6	13529.0	2218.4	16867.9	3205.4	13848.3	4556.5	13439.0	3091.4
	±526.6	±198.2	±278.5	±111.0	±2285.1	±956.0	±2074.6	±1715.9	±412.2	±176.3
PUMA2401004	24896.5	3215.6	15690.2	3323.8	12290.1	2493.8	NA	NA	14873.8	3085.1
	±526.6	±198.2	±309.9	±149.6	±1532.5	±591.9	NA	NA	±430.0	±171.5
PUMA2602702	24896.5	3215.6	11788.9	1642.1	12902.9	2684.5	13206.5	2797.6	11656.5	3066.2
	±526.6	±198.2	±252.4	±82.8	±2428.1	±1818.0	±851.2	±390.7	±368.5	±158.8
PUMA2801100	24896.5	3215.6	10658.4	1754.2	12538.7	2648.4	9076.9	3191.2	6842.2	2483.3
	±526.6	±198.2	±237.3	±92.6	±2033.0	±1364.8	±1023.3	±636.7	±281.6	±153.9
PUMA2901901	24896.5	3215.6	11494.9	1741.4	9753.9	2134.0	13258.2	2941.7	10340.4	3165.1
	±526.6	±198.2	±247.7	±96.2	±1364.1	±939.0	±1759.8	±796.4	±339.4	±176.9
PUMA3200405	24896.5	3215.6	15740.1	2388.3	16015.6	2612.2	15100.4	2439.5	17778.8	3758.6
	±526.6	±198.2	±314.0	±123.7	±1910.9	±754.6	±1151.0	±483.9	±472.0	±211.4
PUMA3603710	24896.5	3215.6	17655.8	2756.2	17234.3	4363.1	14934.9	2884.7	16438.2	4355.5
	±526.6	±198.2	±340.1	±140.5	±1885.8	±1173.0	±944.3	±437.5	±465.3	±221.7
PUMA3604010	24896.5	3215.6	11373.5	1971.8	14373.0	3585.1	15183.5	6053.7	9585.6	2336.1
	±526.6	±198.2	±243.6	±100.6	±2347.7	±799.9	±2446.4	±1818.7	±318.2	±133.7
PUMA5101301	24896.5	3215.6	12946.4	2040.5	16033.9	3618.1	14648.9	2980.5	12533.1	2613.3
	±526.6	±198.2	±269.3	±99.5	±2164.8	±1039.4	±1320.7	±477.4	±389.9	±143.4
PUMA5151255	24896.5	3215.6	15451.4	2277.8	14946.2	2335.3	18403.9	4421.7	17045.8	3732.4
	±526.6	±198.2	±299.0	±111.6	±1995.7	±1283.8	±2462.0	±1386.8	±455.8	±210.5

TABLE 27. Squared Errors (with standard deviations). Marg1 Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	65664.1	3075.7	13618.3	5141.4	12947.3	4395.2	14169.5	8114.8	4949.6	2036.7
	±856.9	±207.6	±265.2	±171.8	±1779.9	±1258.0	±710.6	±552.1	±257.0	±109.7
PUMA0800803	65664.1	3075.7	17302.2	5898.5	23152.7	9272.9	22604.8	8350.0	9168.6	2488.8
	±856.9	±207.6	±307.3	±197.0	±3745.8	±2955.5	±2651.9	±1504.3	±322.7	±140.7
PUMA1304600	65664.1	3075.7	17227.7	6608.7	20876.1	7232.8	13908.9	5583.9	7834.3	2652.8
	±856.9	±207.6	±317.2	±217.4	±2272.5	±1183.0	±1901.8	±1061.2	±336.7	±154.9
PUMA1703529	65664.1	3075.7	18583.7	7148.5	18695.9	7183.1	21515.9	10223.1	8709.0	2650.2
	±856.9	±207.6	±331.2	±232.3	±3023.3	±2228.9	±4189.6	±3443.0	±328.2	±153.8
PUMA1703531	65664.1	3075.7	11527.0	4025.8	14654.2	4099.9	13799.8	9634.8	3623.6	1572.5
	±856.9	±207.6	±234.9	±140.2	±2447.6	±841.7	±920.3	±816.6	±212.2	±85.9
PUMA1901700	65664.1	3075.7	20261.4	5898.7	24119.9	6681.5	15818.3	4194.7	14173.4	4738.1
	±856.9	±207.6	±340.4	±205.9	±3161.8	±1720.6	±2428.3	±1077.2	±427.6	±165.0
PUMA2401004	65664.1	3075.7	20905.0	6110.6	25347.4	9196.5	NA	NA	14298.2	2593.0
	±856.9	±207.6	±355.5	±219.8	±3640.7	±2861.7	NA	NA	±454.6	±156.8
PUMA2602702	65664.1	3075.7	16674.9	7245.2	21024.3	9251.2	14273.5	7653.5	5933.2	2502.9
	±856.9	±207.6	±316.2	±228.0	±2925.9	±1894.1	±1014.7	±734.0	±302.6	±150.8
PUMA2801100	65664.1	3075.7	14301.5	5872.8	14027.9	4455.6	11696.7	7521.4	5157.7	2406.6
	±856.9	±207.6	±283.8	±197.2	±2102.9	±708.5	±1756.1	±1579.4	±274.7	±148.7
PUMA2901901	65664.1	3075.7	16288.1	6994.3	15740.0	7336.8	14045.6	7836.8	5810.7	2504.9
	±856.9	±207.6	±309.5	±221.6	±1973.1	±1226.4	±1511.6	±1276.2	±293.7	±151.8
PUMA3200405	65664.1	3075.7	24070.3	5824.6	25938.7	6332.2	20598.9	4370.2	13952.7	2838.4
	±856.9	±207.6	±385.7	±211.4	±2629.4	±1481.1	±1450.7	±796.4	±432.2	±163.2
PUMA3603710	65664.1	3075.7	27996.4	3696.8	26465.1	4220.0	28859.3	4474.6	25595.0	5328.0
	±856.9	±207.6	±412.8	±175.5	±2164.4	±1013.1	±1207.7	±443.4	±636.8	±242.6
PUMA3604010	65664.1	3075.7	15549.5	5083.6	21388.8	9090.9	14231.8	6216.5	7884.6	2332.7
	±856.9	±207.6	±286.7	±173.8	±3358.4	±2540.9	±3120.2	±2530.2	±321.5	±123.7
PUMA5101301	65664.1	3075.7	17886.8	6799.0	17689.4	6431.7	18006.2	7912.6	8521.4	2602.1
	±856.9	±207.6	±328.1	±222.8	±2057.8	±1296.3	±1483.5	±1077.3	±352.3	±153.2
PUMA5151255	65664.1	3075.7	21460.6	5928.5	22491.7	4344.7	23194.9	5064.6	15497.7	2792.9
	±856.9	±207.6	±353.2	±213.6	±2941.4	±984.4	±2668.4	±1066.6	±471.0	±167.7

Table 28. Squared Errors (with standard deviations). Marg2 Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	597654.9	3571.4	13058.7	3275.2	12947.8	2686.7	12159.6	4872.7	8158.8	2002.2
	±2643.1	±258.6	±212.1	±139.8	±1344.0	±534.8	±483.3	±382.6	±265.3	±53.1
PUMA0800803	597654.9	3571.4	16513.8	2093.3	19653.0	2554.7	16523.9	2745.4	13274.0	2113.7
	±2643.1	±258.6	±218.7	±57.2	±2044.3	±517.1	±1327.2	±339.2	±308.9	±116.5
PUMA1304600	597654.9	3571.4	16512.9	2471.6	19550.9	2443.4	11209.4	1850.4	13004.8	2832.2
	±2643.1	±258.6	±252.0	±127.2	±2103.0	±766.4	±1090.8	±619.7	±372.3	±154.7
PUMA1703529	597654.9	3571.4	18223.1	2469.7	17509.2	3273.1	18088.1	3634.6	15570.9	3262.1
	±2643.1	±258.6	±267.3	±128.6	±2257.0	±1119.2	±2026.1	±846.3	±389.0	±123.8
PUMA1703531	597654.9	3571.4	10469.3	3760.6	12319.8	5015.3	11123.4	7551.8	4729.1	1786.9
	±2643.1	±258.6	±200.1	±146.2	±1748.9	±1444.0	±695.8	±633.9	±217.2	±115.7
PUMA1901700	597654.9	3571.4	20648.5	2575.2	25415.7	3731.7	15792.2	2338.8	18722.1	3023.7
	±2643.1	±258.6	±263.9	±85.9	±2715.8	±839.2	±1412.7	±719.2	±419.4	±103.2
PUMA2401004	597654.9	3571.4	23552.1	2736.6	28577.1	5984.3	NA	NA	21406.2	2722.5
	±2643.1	±258.6	±297.3	±138.8	±3444.6	±2504.7	NA	NA	±461.3	±150.8
PUMA2602702	597654.9	3571.4	15417.7	2472.0	17737.0	2903.3	12490.9	3014.8	12100.4	2944.3
	±2643.1	±258.6	±233.6	±106.5	±2589.0	±1479.7	±582.1	±353.3	±340.3	±139.3
PUMA2801100	597654.9	3571.4	13627.0	3034.9	15786.7	3214.9	10505.2	3516.3	8371.2	2902.2
	±2643.1	±258.6	±244.3	±135.3	±2597.8	±1419.6	±965.0	±728.6	±310.7	±178.4
PUMA2901901	597654.9	3571.4	14875.2	2616.8	15098.1	3670.4	12787.3	3334.6	10742.9	2868.6
	±2643.1	±258.6	±242.2	±128.8	±1833.4	±1328.1	±1298.7	±783.1	±319.5	±157.8
PUMA3200405	597654.9	3571.4	28252.4	2907.8	28794.9	3166.4	21533.9	2275.9	27848.0	3658.7
	±2643.1	±258.6	±353.8	±139.2	±2598.2	±1216.4	±947.7	±436.1	±583.4	±218.5
PUMA3603710	597654.9	3571.4	33268.3	2874.8	36198.8	3875.9	25180.8	2377.2	27205.8	2303.0
	±2643.1	±258.6	±358.4	±141.9	±2310.7	±1074.2	±642.8	±247.8	±497.9	±140.2
PUMA3604010	597654.9	3571.4	14244.6	2455.6	15963.1	3455.2	12436.4	4212.1	10295.9	2073.2
	±2643.1	±258.6	±214.9	±123.9	±1570.4	±813.0	±1549.7	±1214.8	±309.5	±131.9
PUMA5101301	597654.9	3571.4	17418.0	2445.3	18585.4	2733.3	14108.6	3130.0	14449.9	2557.0
	±2643.1	±258.6	±239.2	±97.4	±1980.0	±722.1	±720.0	±452.5	±365.9	±113.3
PUMA5151255	597654.9	3571.4	24694.4	2627.0	26043.6	3246.3	23306.6	4012.0	23083.9	3001.6
	±2643.1	±258.6	±296.5	±126.8	±2410.3	±918.1	±1799.3	±1065.6	±458.9	±155.0

Table 29. Squared Errors (with standard deviations). Id Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
PUMA0101301	2680.2	15540.2	20191.3	3486.7	2738.3
	±147.2	±380.4	±2841.2	±396.5	±171.6
PUMA0800803	2680.2	13761.1	13153.1	4435.7	2701.3
	±147.2	±363.5	±2622.8	±1003.9	±160.4
PUMA1304600	2680.2	13594.4	20043.4	5018.9	2729.4
	±147.2	±361.8	±2638.0	±2324.5	±165.1
PUMA1703529	2680.2	13502.4	10246.0	4174.4	2736.0
	±147.2	±361.4	±1979.8	±1681.4	±161.9
PUMA1703531	2680.2	16585.6	21135.7	3336.8	2716.7
	±147.2	±390.2	±3283.1	±499.2	±172.9
PUMA1901700	2680.2	12647.7	22102.9	3468.7	2702.0
	±147.2	±352.4	±3586.2	±1376.7	±159.3
PUMA2401004	2680.2	11795.3	8553.0	NA	2635.9
	±147.2	±343.1	±1790.3	NA	±158.0
PUMA2602702	2680.2	14423.8	12298.0	2602.7	2670.9
	±147.2	±370.3	±2152.6	±384.0	±164.4
PUMA2801100	2680.2	15642.4	15246.7	5985.2	2748.1
	±147.2	±381.3	±2409.8	±1344.1	±167.5
PUMA2901901	2680.2	14649.7	11092.6	3400.2	2664.6
	±147.2	±372.4	±1482.9	±978.1	±163.3
PUMA3200405	2680.2	11154.8	10611.7	2968.6	2756.5
	±147.2	±334.4	±1679.4	±512.8	±161.9
PUMA3603710	2680.2	10040.7	9504.2	3296.2	2731.3
	±147.2	±322.4	±1724.7	±413.7	±160.6
PUMA3604010	2680.2	13625.9	10093.9	5440.1	2742.4
	±147.2	±362.5	±1664.1	±2001.7	±162.1
PUMA5101301	2680.2	13157.3	12595.6	3420.9	2698.9
	±147.2	±357.3	±2072.2	±582.7	±161.8
PUMA5151255	2680.2	11160.6	13465.7	2661.5	2738.0
	±147.2	±335.1	±2485.5	±698.7	±161.8

Table 30. Squared Error (with standard deviations). Sum Query. PUMS datasets. Lap Mechanism ($\epsilon = 0.1$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-1d	198.8	2.2	10.7	6.3	10.9	6.5	10.3	8.0	2.5	1.7
	±0.9	±0.1	±0.2	±0.2	±0.3	±0.2	±0.2	±0.2	±0.1	±0.1
Level01-1d	198.8	2.2	112.6	2.1	112.9	2.1	112.4	2.1	112.4	2.1
	±0.9	±0.1	±0.6	±0.1	±0.6	±0.1	±0.6	±0.1	±0.6	±0.1
Level16-1d	198.8	2.2	198.8	2.2	201.5	2.9	198.8	2.2	198.8	2.2
	±0.9	±0.1	±0.9	±0.1	±2.4	±0.3	±0.9	±0.1	±0.9	±0.1
Level32-1d	198.8	2.2	198.8	2.2	197.8	2.3	198.8	2.2	198.8	2.2
	±0.9	±0.1	±0.9	±0.1	±1.2	±0.1	±0.9	±0.1	±0.9	±0.1
SplitStairs-1d	198.8	2.2	136.1	2.4	136.6	2.4	136.1	2.4	117.8	5.1
	±0.9	±0.1	±0.8	±0.1	±0.9	±0.1	±0.8	±0.1	±0.7	±0.1
Stair-1d	198.8	2.2	198.5	2.2	198.3	2.6	198.5	2.2	198.6	2.2
	±0.9	±0.1	±0.9	±0.1	±1.6	±0.2	±0.9	±0.1	±0.9	±0.1
Step16-1d	198.8	2.2	139.4	2.3	138.8	2.4	139.4	2.3	107.1	2.2
	±0.9	±0.1	±0.8	±0.1	±0.9	±0.1	±0.8	±0.1	±0.7	±0.1
Step50-1d	198.8	2.2	139.4	2.3	138.8	2.3	139.4	2.3	107.1	2.2
	±0.9	±0.1	±0.8	±0.1	±0.9	±0.1	±0.8	±0.1	±0.7	±0.1

Table 31. Squared Errors (with standard deviations). Id Query. 1-d datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-1d	2.0	6.1	6.1	2.0	1.6
	±0.1	±0.2	±0.2	±0.1	±0.1
Level01-1d	2.0	2.0	1.9	2.0	2.0
	±0.1	±0.1	±0.1	±0.1	±0.1
Level16-1d	2.0	2.0	2.1	2.0	2.0
	±0.1	±0.1	±0.2	±0.1	±0.1
Level32-1d	2.0	2.0	2.1	2.0	2.0
	±0.1	±0.1	±0.1	±0.1	±0.1
SplitStairs-1d	2.0	2.1	2.0	2.0	2.0
	±0.1	±0.1	±0.1	±0.1	±0.1
Stair-1d	2.0	2.0	1.8	2.0	2.0
	±0.1	±0.1	±0.1	±0.1	±0.1
Step16-1d	2.0	2.1	2.1	2.0	2.0
	±0.1	±0.1	±0.1	±0.1	±0.1
Step50-1d	2.0	2.1	2.1	2.0	2.0
	±0.1	±0.1	±0.1	±0.1	±0.1

Table 32. Squared Error (with standard deviations). Sum Query. 1-d datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-1d	795.2	8.8	42.7	25.4	42.4	24.8	41.3	31.9	9.6	6.7
	±3.5	±0.4	±0.9	±0.8	±1.3	±1.1	±0.9	±0.9	±0.4	±0.4
Level01-1d	795.2	8.8	252.9	9.8	251.4	9.8	250.1	9.8	246.4	8.3
	±3.5	±0.4	±1.8	±0.5	±2.3	±0.6	±1.8	±0.5	±1.8	±0.4
Level16-1d	795.2	8.8	795.2	8.8	787.7	9.6	795.2	8.8	795.2	8.8
	±3.5	±0.4	±3.5	±0.4	±6.3	±0.8	±3.5	±0.4	±3.5	±0.4
Level32-1d	795.2	8.8	795.2	8.8	791.3	10.1	795.2	8.8	795.2	8.8
	±3.5	±0.4	±3.5	±0.4	±6.6	±0.8	±3.5	±0.4	±3.5	±0.4
SplitStairs-1d	795.2	8.8	534.9	9.5	534.2	9.4	534.8	9.5	496.4	16.8
	±3.5	±0.4	±3.0	±0.4	±4.0	±0.5	±3.0	±0.4	±2.9	±0.6
Stair-1d	795.2	8.8	788.9	8.8	788.5	10.3	788.9	8.8	791.2	8.8
	±3.5	±0.4	±3.5	±0.4	±6.6	±0.9	±3.5	±0.4	±3.5	±0.4
Step16-1d	795.2	8.8	557.7	9.3	553.8	9.5	557.7	9.3	485.9	10.4
	±3.5	±0.4	±3.1	±0.4	±4.2	±0.6	±3.1	±0.4	±4.3	±0.8
Step50-1d	795.2	8.8	557.7	9.3	558.7	9.8	557.7	9.3	421.4	8.7
	±3.5	±0.4	±3.1	±0.4	±4.2	±0.6	±3.1	±0.4	±2.6	±0.4

Table 33. Squared Errors (with standard deviations). Id Query. 1-d datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-1d	7.9	24.2	24.5	8.0	6.1
	±0.3	±0.7	±1.1	±0.3	±0.3
Level01-1d	7.9	9.2	8.7	8.0	8.0
	±0.3	±0.4	±0.5	±0.3	±0.3
Level16-1d	7.9	7.9	7.8	8.0	7.9
	±0.3	±0.3	±0.6	±0.3	±0.3
Level32-1d	7.9	7.9	7.8	8.0	7.9
	±0.3	±0.3	±0.6	±0.3	±0.3
SplitStairs-1d	7.9	8.5	8.7	8.0	7.9
	±0.3	±0.4	±0.5	±0.3	±0.3
Stair-1d	7.9	7.9	7.0	8.0	7.9
	±0.3	±0.3	±0.6	±0.3	±0.3
Step16-1d	7.9	8.5	8.9	8.0	7.9
	±0.3	±0.4	±0.5	±0.3	±0.3
Step50-1d	7.9	8.5	8.2	8.0	7.9
	±0.3	±0.4	±0.5	±0.3	±0.3

Table 34. Squared Error (with standard deviations). Sum Query. 1-d datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-1d	19881.2	221.2	1068.5	633.9	1127.8	676.4	1033.3	796.6	235.6	164.6
	±88.4	±8.9	±23.3	±21.0	±44.9	±41.2	±23.5	±22.5	±9.7	±9.0
Level01-1d	19881.2	221.2	1855.9	411.3	1841.0	412.4	1704.9	435.4	1297.2	207.0
	±88.4	±8.9	±26.7	±17.1	±36.7	±24.0	±25.7	±17.6	±21.0	±10.2
Level16-1d	19881.2	221.2	15398.7	208.7	15426.4	205.1	15392.9	208.7	15431.7	207.1
	±88.4	±8.9	±63.7	±10.3	±88.4	±14.3	±63.7	±10.3	±64.7	±10.2
Level32-1d	19881.2	221.2	19474.4	217.5	19480.1	235.4	19476.2	217.7	19541.2	218.6
	±88.4	±8.9	±82.9	±8.5	±144.6	±20.8	±82.9	±8.5	±83.2	±8.6
SplitStairs-1d	19881.2	221.2	11138.3	247.3	11121.5	257.9	11123.5	248.3	11862.4	299.9
	±88.4	±8.9	±62.4	±10.9	±90.3	±18.9	±62.3	±11.0	±65.6	±11.5
Stair-1d	19881.2	221.2	18450.3	218.5	18449.4	233.5	18449.8	218.5	18805.1	227.6
	±88.4	±8.9	±83.0	±9.3	±125.1	±14.9	±83.0	±9.3	±84.5	±9.8
Step16-1d	19881.2	221.2	10057.4	234.0	10128.1	240.9	10026.5	235.0	10069.1	207.1
	±88.4	±8.9	±50.2	±11.7	±70.4	±17.2	±49.9	±11.8	±52.2	±10.2
Step50-1d	19881.2	221.2	13929.6	232.0	13848.4	240.1	13929.3	232.3	16780.0	305.4
	±88.4	±8.9	±76.7	±10.0	±109.4	±15.1	±76.8	±10.1	±83.1	±11.4

Table 35. Squared Errors (with standard deviations). Id Query. 1-d datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-1d	198.0	606.0	663.1	201.2	150.7
	±8.4	±18.0	±35.5	±8.6	±6.8
Level01-1d	198.0	390.2	400.7	201.2	199.1
	±8.4	±14.2	±20.3	±8.6	±8.5
Level16-1d	198.0	198.3	205.4	201.2	199.1
	±8.4	±8.4	±11.9	±8.6	±8.5
Level32-1d	198.0	198.0	192.3	201.2	199.0
	±8.4	±8.4	±16.1	±8.6	±8.5
SplitStairs-1d	198.0	220.4	217.9	201.2	199.1
	±8.4	±9.2	±13.1	±8.6	±8.5
Stair-1d	198.0	198.4	206.5	201.2	198.6
	±8.4	±8.4	±12.9	±8.6	±8.4
Step16-1d	198.0	221.2	221.5	201.2	199.1
	±8.4	±9.2	±13.8	±8.6	±8.5
Step50-1d	198.0	211.9	220.9	201.2	198.9
	±8.4	±8.8	±13.0	±8.6	±8.5

Table 36. Squared Error (with standard deviations). Sum Query. 1-d datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	33.3	3.7	8.3	2.3	8.8	2.5	9.6	6.0	3.2	1.6
	±0.5	±0.2	±0.2	±0.1	±0.3	±0.2	±0.3	±0.3	±0.1	±0.1
Level01-2d	33.3	3.7	30.0	3.3	29.8	3.3	36.9	4.0	36.5	4.0
	±0.5	±0.2	±0.4	±0.2	±0.8	±0.3	±0.5	±0.2	±0.5	±0.2
Level16-2d	33.3	3.7	33.3	3.7	33.4	3.8	37.0	4.0	33.3	3.7
	±0.5	±0.2	±0.5	±0.2	±0.6	±0.2	±0.5	±0.2	±0.5	±0.2
Level32-2d	33.3	3.7	33.3	3.7	33.7	3.8	37.0	4.0	33.3	3.7
	±0.5	±0.2	±0.5	±0.2	±0.6	±0.2	±0.5	±0.2	±0.5	±0.2
SplitStairs-2d	33.3	3.7	30.8	3.4	30.3	3.4	37.0	4.0	32.4	3.6
	±0.5	±0.2	±0.4	±0.2	±0.7	±0.3	±0.5	±0.2	±0.5	±0.2
Stair-2d	33.3	3.7	33.3	3.7	35.2	3.6	37.0	4.0	33.5	3.7
	±0.5	±0.2	±0.5	±0.2	±0.8	±0.2	±0.5	±0.2	±0.5	±0.2
Step16-2d	33.3	3.7	31.0	3.4	30.3	3.8	36.7	4.0	30.9	3.4
	±0.5	±0.2	±0.4	±0.2	±0.8	±0.3	±0.5	±0.2	±0.4	±0.2
Step50-2d	33.3	3.7	31.0	3.4	30.7	3.5	36.8	4.0	31.0	3.4
	±0.5	±0.2	±0.4	±0.2	±0.9	±0.3	±0.5	±0.2	±0.4	±0.2

Table 37. Squared Errors (with standard deviations). Marg1 Query. 2-d datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	32.8	3.6	8.2	2.3	8.7	2.5	8.7	5.5	3.1	1.5
	±0.5	±0.2	±0.2	±0.1	±0.3	±0.2	±0.3	±0.3	±0.1	±0.1
Level01-2d	32.8	3.6	29.7	3.2	30.5	3.3	35.9	3.8	35.6	3.8
	±0.5	±0.2	±0.4	±0.1	±0.8	±0.3	±0.5	±0.2	±0.5	±0.2
Level16-2d	32.8	3.6	32.8	3.6	33.4	3.6	35.9	3.8	32.8	3.6
	±0.5	±0.2	±0.5	±0.2	±0.6	±0.2	±0.5	±0.2	±0.5	±0.2
Level32-2d	32.8	3.6	32.8	3.6	33.7	3.9	35.9	3.8	32.8	3.6
	±0.5	±0.2	±0.5	±0.2	±0.6	±0.2	±0.5	±0.2	±0.5	±0.2
SplitStairs-2d	32.8	3.6	36.3	4.6	37.3	4.7	24.6	3.9	21.8	3.6
	±0.5	±0.2	±0.5	±0.2	±0.9	±0.4	±0.4	±0.2	±0.4	±0.2
Stair-2d	32.8	3.6	32.8	3.6	33.7	3.8	35.9	3.8	32.8	3.6
	±0.5	±0.2	±0.5	±0.2	±0.7	±0.3	±0.5	±0.2	±0.5	±0.2
Step16-2d	32.8	3.6	35.8	4.4	34.7	4.8	27.2	3.9	25.5	3.7
	±0.5	±0.2	±0.5	±0.2	±0.9	±0.4	±0.4	±0.2	±0.4	±0.2
Step50-2d	32.8	3.6	35.8	4.4	37.0	4.7	27.2	3.8	25.5	3.7
	±0.5	±0.2	±0.5	±0.2	±1.1	±0.5	±0.4	±0.2	±0.4	±0.2

Table 38. Squared Errors (with standard deviations). Marg2 Query. 2-d datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	330.1	3.6	9.7	4.6	10.9	5.4	12.4	9.1	4.3	2.3
	±1.5	±0.2	±0.2	±0.2	±0.4	±0.3	±0.3	±0.3	±0.1	±0.1
Level01-2d	330.1	3.6	139.0	3.1	139.9	3.0	137.0	3.0	139.1	4.1
	±1.5	±0.2	±0.7	±0.1	±1.4	±0.3	±0.7	±0.1	±0.8	±0.2
Level16-2d	330.1	3.6	330.1	3.6	334.5	4.0	330.8	3.6	330.1	3.6
	±1.5	±0.2	±1.5	±0.2	±2.0	±0.2	±1.5	±0.2	±1.5	±0.2
Level32-2d	330.1	3.6	330.1	3.6	333.1	3.7	330.8	3.6	330.1	3.6
	±1.5	±0.2	±1.5	±0.2	±1.9	±0.2	±1.5	±0.2	±1.5	±0.2
SplitStairs-2d	330.1	3.6	158.9	3.3	159.7	3.6	153.5	3.3	201.3	9.7
	±1.5	±0.2	±1.0	±0.1	±1.7	±0.3	±1.0	±0.1	±1.4	±0.3
Stair-2d	330.1	3.6	328.8	3.6	333.3	3.9	329.4	3.6	344.7	6.5
	±1.5	±0.2	±1.5	±0.2	±2.3	±0.3	±1.5	±0.2	±1.5	±0.3
Step16-2d	330.1	3.6	168.6	4.8	170.3	5.1	165.8	6.6	164.0	3.6
	±1.5	±0.2	±1.0	±0.2	±2.0	±0.4	±1.0	±0.2	±1.0	±0.2
Step50-2d	330.1	3.6	168.6	4.8	170.9	5.1	165.6	6.6	164.0	3.6
	±1.5	±0.2	±1.0	±0.2	±2.0	±0.4	±1.0	±0.2	±1.0	±0.2

Table 39. Squared Errors (with standard deviations). Id Query. 2-d datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-2d	3.1	13.5	13.6	3.7	2.9
	±0.1	±0.3	±0.5	±0.2	±0.1
Level01-2d	3.1	3.3	3.3	4.0	3.1
	±0.1	±0.1	±0.3	±0.2	±0.1
Level16-2d	3.1	3.1	3.0	4.0	3.1
	±0.1	±0.1	±0.2	±0.2	±0.1
Level32-2d	3.1	3.1	3.0	4.0	3.1
	±0.1	±0.1	±0.2	±0.2	±0.1
SplitStairs-2d	3.1	4.1	3.9	4.0	3.1
	±0.1	±0.2	±0.3	±0.2	±0.1
Stair-2d	3.1	3.1	3.1	4.0	3.1
	±0.1	±0.1	±0.2	±0.2	±0.1
Step16-2d	3.1	3.9	4.1	4.0	3.3
	±0.1	±0.2	±0.3	±0.2	±0.1
Step50-2d	3.1	3.9	4.1	4.0	3.3
	±0.1	±0.2	±0.4	±0.2	±0.1

Table 40. Squared Error (with standard deviations). Sum Query. 2-d datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	133.3	14.9	33.2	9.1	34.5	9.4	31.4	21.1	12.3	6.5
	±1.9	±0.7	±0.7	±0.4	±2.6	±1.5	±2.0	±1.8	±0.5	±0.3
Level01-2d	133.3	14.9	104.5	11.4	104.9	12.0	146.7	15.5	144.3	15.7
	±1.9	±0.7	±1.5	±0.5	±3.0	±1.0	±2.2	±0.7	±2.0	±0.6
Level16-2d	133.3	14.9	133.3	14.9	137.1	14.9	147.9	16.1	143.2	15.6
	±1.9	±0.7	±1.9	±0.7	±3.3	±1.2	±2.1	±0.7	±2.0	±0.7
Level32-2d	133.3	14.9	133.3	14.9	133.5	15.1	147.9	16.1	133.3	14.9
	±1.9	±0.7	±1.9	±0.7	±3.2	±1.1	±2.1	±0.7	±1.9	±0.7
SplitStairs-2d	133.3	14.9	122.6	13.6	123.5	13.7	148.0	16.1	137.0	14.8
	±1.9	±0.7	±1.7	±0.6	±3.0	±1.0	±2.1	±0.7	±1.9	±0.6
Stair-2d	133.3	14.9	133.2	14.8	132.8	16.1	147.7	16.1	137.5	15.1
	±1.9	±0.7	±1.9	±0.7	±4.5	±1.6	±2.1	±0.7	±1.9	±0.7
Step16-2d	133.3	14.9	123.8	13.7	124.2	15.7	148.1	16.2	142.9	15.5
	±1.9	±0.7	±1.8	±0.6	±4.2	±1.7	±2.1	±0.7	±2.0	±0.7
Step50-2d	133.3	14.9	123.8	13.7	120.1	13.9	147.4	16.0	123.8	13.6
	±1.9	±0.7	±1.8	±0.6	±4.3	±1.6	±2.1	±0.7	±1.8	±0.6

Table 41. Squared Errors (with standard deviations). Marg1 Query. 2-d datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	131.4	14.3	32.9	9.1	35.9	8.6	33.2	22.1	11.8	6.2
	±1.8	±0.6	±0.7	±0.4	±2.5	±1.2	±1.9	±1.7	±0.5	±0.3
Level01-2d	131.4	14.3	103.5	11.2	107.4	11.8	144.2	15.5	140.8	14.9
	±1.8	±0.6	±1.4	±0.5	±3.0	±1.0	±2.1	±0.7	±1.9	±0.6
Level16-2d	131.4	14.3	131.4	14.3	131.1	15.0	143.7	15.3	140.4	15.1
	±1.8	±0.6	±1.8	±0.6	±3.1	±1.1	±2.0	±0.7	±2.0	±0.7
Level32-2d	131.4	14.3	131.4	14.3	129.5	14.3	143.7	15.3	131.4	14.3
	±1.8	±0.6	±1.8	±0.6	±3.2	±1.1	±2.0	±0.7	±1.8	±0.6
SplitStairs-2d	131.4	14.3	144.3	18.5	151.0	19.2	98.6	15.5	79.5	13.5
	±1.8	±0.6	±2.0	±0.8	±3.6	±1.5	±1.7	±0.7	±1.5	±0.6
Stair-2d	131.4	14.3	131.2	14.3	136.7	16.1	143.5	15.3	132.4	14.3
	±1.8	±0.6	±1.8	±0.6	±4.8	±1.7	±2.0	±0.7	±1.9	±0.6
Step16-2d	131.4	14.3	143.1	17.7	146.9	19.0	108.6	15.5	89.5	14.8
	±1.8	±0.6	±2.0	±0.8	±4.8	±2.0	±1.8	±0.7	±1.6	±0.7
Step50-2d	131.4	14.3	143.1	17.7	146.5	19.1	109.0	15.5	101.9	15.0
	±1.8	±0.6	±2.0	±0.8	±5.2	±1.9	±1.8	±0.7	±1.7	±0.7

Table 42. Squared Errors (with standard deviations). Marg2 Query. 2-d datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	1320.4	14.3	38.9	18.5	43.6	19.6	43.3	32.7	16.6	8.9
	±5.9	±0.6	±0.8	±0.6	±2.8	±2.2	±2.2	±2.1	±0.5	±0.4
Level01-2d	1320.4	14.3	281.3	12.6	286.1	13.4	270.3	11.8	277.1	15.3
	±5.9	±0.6	±2.0	±0.6	±4.3	±1.2	±2.1	±0.6	±2.2	±0.7
Level16-2d	1320.4	14.3	1320.3	14.3	1331.2	16.0	1323.0	14.4	2483.6	29.6
	±5.9	±0.6	±5.9	±0.6	±10.2	±1.3	±5.9	±0.6	±15.7	±1.8
Level32-2d	1320.4	14.3	1320.4	14.3	1343.8	16.0	1323.0	14.4	1320.4	14.3
	±5.9	±0.6	±5.9	±0.6	±10.2	±1.2	±5.9	±0.6	±5.9	±0.6
SplitStairs-2d	1320.4	14.3	620.8	13.3	618.9	14.9	600.4	13.3	981.1	46.0
	±5.9	±0.6	±3.8	±0.6	±6.6	±1.0	±3.8	±0.6	±7.2	±1.8
Stair-2d	1320.4	14.3	1300.6	14.4	1305.3	17.7	1302.2	14.4	1713.8	52.9
	±5.9	±0.6	±5.8	±0.6	±14.3	±1.9	±5.9	±0.6	±9.3	±2.3
Step16-2d	1320.4	14.3	674.5	19.0	673.4	19.5	662.2	26.4	1269.5	27.7
	±5.9	±0.6	±4.0	±0.7	±9.5	±1.7	±4.1	±0.9	±10.9	±1.7
Step50-2d	1320.4	14.3	674.5	19.0	663.2	19.0	663.4	26.5	657.9	14.5
	±5.9	±0.6	±4.0	±0.7	±10.3	±1.8	±4.1	±0.9	±4.0	±0.7

TABLE 43. Squared Errors (with standard deviations). Id Query. 2-d datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-2d	12.4	53.8	60.2	14.8	11.0
	±0.5	±1.3	±5.4	±1.2	±0.5
Level01-2d	12.4	15.1	14.6	16.1	13.0
	±0.5	±0.6	±1.4	±0.7	±0.5
Level16-2d	12.4	12.4	13.6	16.1	12.4
	±0.5	±0.5	±1.0	±0.7	±0.5
Level32-2d	12.4	12.4	13.5	16.1	12.4
	±0.5	±0.5	±1.0	±0.7	±0.5
SplitStairs-2d	12.4	16.4	17.1	16.2	12.1
	±0.5	±0.7	±1.2	±0.7	±0.5
Stair-2d	12.4	12.4	14.7	16.1	12.4
	±0.5	±0.5	±1.4	±0.7	±0.5
Step16-2d	12.4	15.7	15.5	16.0	12.3
	±0.5	±0.7	±1.6	±0.7	±0.5
Step50-2d	12.4	15.7	18.9	16.0	13.0
	±0.5	±0.7	±2.1	±0.7	±0.5

TABLE 44. Squared Error (with standard deviations). Sum Query. 2-d datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	3333.2	371.8	829.4	227.4	483.9	105.7	898.3	460.5	304.9	163.2
	±47.1	±16.8	±17.9	±10.2	±79.2	±39.9	±39.4	±32.3	±11.8	±7.8
Level01-2d	3333.2	371.8	1363.7	219.6	1339.7	230.6	1805.8	360.8	1433.1	275.5
	±47.1	±16.8	±23.3	±10.4	±80.1	±37.0	±35.7	±19.8	±30.6	±13.4
Level16-2d	3333.2	371.8	3179.0	353.9	3153.5	354.3	3696.4	403.3	3686.8	403.1
	±47.1	±16.8	±45.0	±16.2	±63.8	±23.7	±52.4	±17.1	±52.2	±17.1
Level32-2d	3333.2	371.8	3311.1	369.9	3311.5	361.1	3698.6	404.0	3687.2	403.2
	±47.1	±16.8	±46.8	±16.8	±74.9	±24.4	±52.4	±17.2	±52.2	±17.1
SplitStairs-2d	3333.2	371.8	2927.4	322.8	2868.5	324.6	3693.3	407.5	3666.8	401.4
	±47.1	±16.8	±41.1	±14.7	±79.1	±29.9	±54.3	±17.9	±51.8	±17.0
Stair-2d	3333.2	371.8	3291.8	364.2	3291.3	367.4	3696.7	403.7	3681.4	402.8
	±47.1	±16.8	±46.6	±16.5	±76.5	±26.6	±52.4	±17.1	±52.0	±17.0
Step16-2d	3333.2	371.8	2854.0	317.0	3037.9	402.3	3692.5	394.1	3599.4	394.6
	±47.1	±16.8	±40.8	±14.5	±96.2	±40.8	±54.2	±17.1	±51.6	±17.4
Step50-2d	3333.2	371.8	3094.0	342.3	3209.5	382.7	3687.0	397.7	3677.2	402.1
	±47.1	±16.8	±44.0	±15.7	±106.5	±40.2	±52.3	±16.7	±52.1	±17.0

Table 45. Squared Errors (with standard deviations). Marg1 Query. 2-d datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	3283.8	356.4	821.8	228.5	650.6	147.9	898.6	468.2	291.4	154.5
	±46.1	±15.9	±16.8	±9.7	±98.1	±41.6	±36.9	±30.1	±11.3	±7.3
Level01-2d	3283.8	356.4	1345.2	219.2	1296.5	206.0	1781.8	359.2	1398.4	264.8
	±46.1	±15.9	±21.9	±9.5	±76.4	±34.6	±33.6	±17.7	±28.9	±12.5
Level16-2d	3283.8	356.4	3129.3	342.2	3094.7	354.0	3592.6	383.0	3583.0	382.4
	±46.1	±15.9	±43.9	±15.2	±60.9	±23.0	±50.0	±16.7	±49.9	±16.7
Level32-2d	3283.8	356.4	3258.6	354.4	3365.3	381.5	3589.9	382.4	3583.4	382.5
	±46.1	±15.9	±45.7	±15.8	±76.7	±26.2	±50.1	±16.7	±49.9	±16.7
SplitStairs-2d	3283.8	356.4	3378.9	472.8	3460.6	521.1	2496.5	393.4	2006.2	356.3
	±46.1	±15.9	±48.3	±19.6	±101.2	±44.1	±44.6	±17.7	±38.2	±15.5
Stair-2d	3283.8	356.4	3263.7	356.8	3281.3	369.4	3594.4	383.2	3554.7	383.2
	±46.1	±15.9	±45.9	±16.1	±77.9	±28.9	±50.1	±16.7	±49.4	±16.7
Step16-2d	3283.8	356.4	3259.0	405.0	3346.3	419.0	2739.2	397.4	2238.9	372.4
	±46.1	±15.9	±47.1	±18.3	±106.5	±39.5	±46.5	±18.3	±39.2	±16.7
Step50-2d	3283.8	356.4	3574.7	442.9	3734.3	459.1	2716.8	385.5	2240.0	372.5
	±46.1	±15.9	±51.1	±20.2	±121.2	±45.7	±44.5	±17.3	±39.3	±16.7

Table 46. Squared Errors (with standard deviations). Marg2 Query. 2-d datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
Level00-2d	33009.3	358.7	973.2	462.3	628.1	230.7	1171.9	771.7	411.4	221.1
	±146.9	±15.4	±18.9	±15.7	±80.9	±51.1	±41.0	±37.7	±13.7	±10.6
Level01-2d	33009.3	358.7	1815.7	367.7	1981.2	462.2	1716.6	427.0	1493.4	290.4
	±146.9	±15.4	±24.3	±14.5	±101.3	±66.5	±28.2	±18.3	±27.7	±14.3
Level16-2d	33009.3	358.7	20740.6	319.2	20784.7	316.8	20633.6	316.0	21079.8	419.2
	±146.9	±15.4	±90.7	±13.9	±128.9	±19.3	±90.1	±13.8	±100.5	±18.3
Level32-2d	33009.3	358.7	30426.8	333.5	30995.1	362.3	30442.5	331.9	32278.1	419.3
	±146.9	±15.4	±124.4	±14.6	±205.0	±22.2	±124.6	±14.6	±142.7	±18.3
SplitStairs-2d	33009.3	358.7	11784.0	334.9	11908.5	325.0	11304.8	334.2	12703.8	403.3
	±146.9	±15.4	±72.0	±14.6	±146.7	±31.0	±73.8	±15.1	±87.9	±17.5
Stair-2d	33009.3	358.7	28790.5	354.2	28682.7	369.2	28727.4	358.2	37500.2	605.3
	±146.9	±15.4	±129.8	±15.1	±220.7	±26.8	±129.7	±15.6	±204.4	±38.3
Step16-2d	33009.3	358.7	10758.2	463.8	10818.2	458.5	10647.5	636.2	10298.1	305.9
	±146.9	±15.4	±60.9	±18.1	±133.8	±40.1	±65.0	±22.5	±67.0	±15.0
Step50-2d	33009.3	358.7	16789.7	475.1	17644.5	489.9	16504.9	658.7	20888.5	444.8
	±146.9	±15.4	±98.9	±18.6	±239.6	±44.4	±99.3	±22.3	±123.7	±18.6

Table 47. Squared Errors (with standard deviations). Id Query. 2-d datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
Level00-2d	310.7	1343.7	1084.0	450.9	268.9
	±13.1	±32.6	±171.2	±26.4	±12.1
Level01-2d	310.7	717.3	836.6	392.6	326.6
	±13.1	±24.7	±111.0	±18.8	±13.7
Level16-2d	310.7	312.5	304.3	402.3	312.0
	±13.1	±13.1	±19.0	±17.2	±13.2
Level32-2d	310.7	310.3	311.6	403.1	312.0
	±13.1	±13.1	±23.2	±17.2	±13.2
SplitStairs-2d	310.7	436.9	465.0	405.5	299.4
	±13.1	±17.7	±39.7	±17.9	±12.4
Stair-2d	310.7	311.0	307.5	402.6	311.8
	±13.1	±13.1	±22.0	±17.2	±13.2
Step16-2d	310.7	419.7	465.2	393.4	300.4
	±13.1	±17.2	±44.7	±17.3	±12.7
Step50-2d	310.7	393.1	414.4	398.5	300.7
	±13.1	±16.3	±40.1	±16.8	±12.7

Table 48. Squared Error (with standard deviations). Sum Query. 2-d datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	31.7	3.8	18.2	3.1	18.2	3.1	20.7	3.8	14.7	3.3
	±0.5	±0.2	±0.3	±0.1	±0.3	±0.1	±0.5	±0.2	±0.3	±0.2
PUMA0800803	31.7	3.8	25.6	5.8	25.5	5.7	25.6	3.6	22.5	3.8
	±0.5	±0.2	±0.4	±0.3	±0.4	±0.3	±0.6	±0.2	±0.4	±0.2
PUMA1304600	31.7	3.8	26.3	3.7	26.2	3.6	25.1	3.8	24.1	3.9
	±0.5	±0.2	±0.4	±0.2	±0.4	±0.2	±0.7	±0.3	±0.4	±0.1
PUMA1703529	31.7	3.8	25.6	4.7	25.6	4.6	24.7	3.8	22.8	3.6
	±0.5	±0.2	±0.4	±0.2	±0.4	±0.2	±0.5	±0.2	±0.4	±0.2
PUMA1703531	31.7	3.8	18.0	2.4	18.0	2.4	23.0	3.7	18.9	4.5
	±0.5	±0.2	±0.3	±0.1	±0.3	±0.1	±0.5	±0.2	±0.3	±0.2
PUMA1901700	31.7	3.8	27.4	4.6	27.4	4.5	24.0	3.8	20.9	3.6
	±0.5	±0.2	±0.4	±0.2	±0.5	±0.2	±0.6	±0.2	±0.4	±0.2
PUMA2401004	31.7	3.8	29.4	6.9	29.2	6.8	24.2	3.9	21.5	3.6
	±0.5	±0.2	±0.5	±0.3	±0.5	±0.3	±0.7	±0.3	±0.4	±0.2
PUMA2602702	31.7	3.8	25.5	5.2	25.3	5.1	24.2	3.8	20.4	3.7
	±0.5	±0.2	±0.4	±0.2	±0.4	±0.2	±0.6	±0.2	±0.4	±0.2
PUMA2801100	31.7	3.8	20.3	3.9	20.3	3.9	22.3	3.7	18.8	4.0
	±0.5	±0.2	±0.3	±0.2	±0.4	±0.2	±0.5	±0.2	±0.3	±0.2
PUMA2901901	31.7	3.8	23.3	4.6	23.2	4.6	25.2	3.7	22.8	4.3
	±0.5	±0.2	±0.4	±0.2	±0.4	±0.2	±0.6	±0.2	±0.4	±0.2
PUMA3200405	31.7	3.8	29.6	4.1	29.5	4.1	28.9	3.7	26.5	3.9
	±0.5	±0.2	±0.5	±0.2	±0.5	±0.2	±0.6	±0.2	±0.5	±0.2
PUMA3603710	31.7	3.8	31.3	4.7	31.4	4.8	29.6	4.0	27.6	4.0
	±0.5	±0.2	±0.5	±0.2	±0.5	±0.2	±0.7	±0.3	±0.5	±0.2
PUMA3604010	31.7	3.8	26.6	4.0	26.9	4.1	23.0	3.7	20.0	3.4
	±0.5	±0.2	±0.4	±0.2	±0.5	±0.2	±0.6	±0.2	±0.4	±0.2
PUMA5101301	31.7	3.8	28.2	6.0	27.8	5.9	25.8	3.8	23.2	3.6
	±0.5	±0.2	±0.4	±0.3	±0.5	±0.3	±0.6	±0.2	±0.4	±0.2
PUMA5151255	31.7	3.8	31.6	5.4	31.3	5.4	25.5	4.0	23.0	3.6
	±0.5	±0.2	±0.5	±0.2	±0.5	±0.3	±0.6	±0.2	±0.4	±0.2

Table 49. Squared Errors (with standard deviations). Marg1 Query. PUMS datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	83.3	3.7	26.2	7.5	26.3	7.6	27.2	5.5	16.5	3.1
	±0.8	±0.2	±0.4	±0.3	±0.4	±0.3	±0.5	±0.3	±0.3	±0.1
PUMA0800803	83.3	3.7	44.3	7.6	43.8	7.5	51.8	4.3	44.4	5.9
	±0.8	±0.2	±0.5	±0.3	±0.5	±0.3	±0.8	±0.2	±0.5	±0.2
PUMA1304600	83.3	3.7	46.8	6.2	46.5	6.2	52.4	4.6	47.2	5.2
	±0.8	±0.2	±0.5	±0.2	±0.5	±0.3	±1.0	±0.3	±0.5	±0.2
PUMA1703529	83.3	3.7	44.5	6.5	44.2	6.4	46.8	4.1	41.2	5.7
	±0.8	±0.2	±0.5	±0.2	±0.5	±0.3	±0.7	±0.2	±0.5	±0.2
PUMA1703531	83.3	3.7	26.6	9.8	26.6	9.7	21.2	5.7	16.6	4.2
	±0.8	±0.2	±0.4	±0.3	±0.4	±0.3	±0.4	±0.3	±0.3	±0.1
PUMA1901700	83.3	3.7	48.7	5.8	48.8	5.8	51.1	4.2	43.7	5.6
	±0.8	±0.2	±0.5	±0.2	±0.6	±0.2	±0.8	±0.2	±0.5	±0.2
PUMA2401004	83.3	3.7	55.1	5.7	54.7	5.6	74.9	4.2	62.4	5.2
	±0.8	±0.2	±0.5	±0.2	±0.6	±0.3	±1.2	±0.3	±0.6	±0.2
PUMA2602702	83.3	3.7	42.2	6.7	42.2	6.7	49.9	4.5	40.9	3.7
	±0.8	±0.2	±0.5	±0.2	±0.5	±0.3	±0.7	±0.3	±0.5	±0.2
PUMA2801100	83.3	3.7	30.6	7.9	30.7	8.0	31.3	5.2	25.3	5.3
	±0.8	±0.2	±0.4	±0.3	±0.4	±0.3	±0.5	±0.3	±0.4	±0.2
PUMA2901901	83.3	3.7	37.7	7.6	37.5	7.5	37.2	4.7	27.6	4.0
	±0.8	±0.2	±0.5	±0.3	±0.5	±0.3	±0.7	±0.3	±0.4	±0.2
PUMA3200405	83.3	3.7	57.3	5.8	57.0	5.8	61.6	4.1	56.7	5.4
	±0.8	±0.2	±0.6	±0.2	±0.6	±0.3	±0.9	±0.3	±0.6	±0.2
PUMA3603710	83.3	3.7	63.4	5.0	63.1	5.0	70.1	4.4	65.1	4.8
	±0.8	±0.2	±0.6	±0.2	±0.7	±0.2	±1.0	±0.3	±0.7	±0.2
PUMA3604010	83.3	3.7	47.3	5.0	46.9	4.9	53.4	4.3	41.2	4.7
	±0.8	±0.2	±0.5	±0.2	±0.6	±0.2	±0.8	±0.2	±0.5	±0.2
PUMA5101301	83.3	3.7	50.8	6.1	50.6	6.2	68.5	4.2	62.5	5.0
	±0.8	±0.2	±0.5	±0.2	±0.6	±0.3	±1.0	±0.3	±0.6	±0.2
PUMA5151255	83.3	3.7	59.6	4.8	59.5	4.9	73.8	4.3	69.2	4.6
	±0.8	±0.2	±0.6	±0.2	±0.6	±0.2	±0.9	±0.2	±0.7	±0.2

TABLE 50. Squared Errors (with standard deviations). Marg2 Query. PUMS datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	746.5	3.8	33.9	4.5	34.2	4.6	37.2	6.6	33.2	5.0
	±2.3	±0.2	±0.3	±0.2	±0.4	±0.2	±0.5	±0.3	±0.5	±0.3
PUMA0800803	746.5	3.8	68.5	4.3	68.8	4.4	76.0	9.1	74.9	6.7
	±2.3	±0.2	±0.5	±0.2	±0.6	±0.2	±0.8	±0.4	±0.7	±0.3
PUMA1304600	746.5	3.8	88.0	4.4	87.9	4.4	87.7	5.5	106.0	8.1
	±2.3	±0.2	±0.6	±0.2	±0.7	±0.2	±1.0	±0.3	±0.9	±0.4
PUMA1703529	746.5	3.8	73.3	3.7	73.1	3.7	71.5	5.3	77.9	5.3
	±2.3	±0.2	±0.5	±0.2	±0.6	±0.2	±0.7	±0.3	±0.7	±0.2
PUMA1703531	746.5	3.8	31.6	4.4	31.9	4.4	31.5	5.8	30.2	5.3
	±2.3	±0.2	±0.3	±0.1	±0.3	±0.1	±0.4	±0.1	±0.4	±0.1
PUMA1901700	746.5	3.8	91.3	3.9	91.8	4.0	88.8	5.0	104.5	9.8
	±2.3	±0.2	±0.6	±0.2	±0.7	±0.2	±0.9	±0.3	±0.9	±0.5
PUMA2401004	746.5	3.8	103.2	4.7	103.1	4.7	114.6	9.0	114.5	6.3
	±2.3	±0.2	±0.7	±0.2	±0.8	±0.2	±1.3	±0.5	±0.8	±0.3
PUMA2602702	746.5	3.8	65.1	3.6	65.1	3.7	67.3	7.0	70.3	8.8
	±2.3	±0.2	±0.5	±0.2	±0.5	±0.2	±0.7	±0.3	±0.7	±0.4
PUMA2801100	746.5	3.8	40.7	3.9	40.9	4.0	42.8	5.8	40.1	4.7
	±2.3	±0.2	±0.4	±0.2	±0.4	±0.2	±0.5	±0.3	±0.5	±0.2
PUMA2901901	746.5	3.8	54.3	3.4	54.2	3.4	56.5	6.2	57.0	6.5
	±2.3	±0.2	±0.4	±0.1	±0.5	±0.1	±0.7	±0.3	±0.6	±0.3
PUMA3200405	746.5	3.8	131.0	4.3	131.4	4.2	130.1	5.2	155.2	7.6
	±2.3	±0.2	±0.8	±0.2	±0.9	±0.2	±1.2	±0.2	±1.1	±0.5
PUMA3603710	746.5	3.8	158.4	4.4	158.3	4.4	157.1	5.6	191.7	9.2
	±2.3	±0.2	±0.9	±0.2	±1.0	±0.2	±1.3	±0.3	±1.2	±0.4
PUMA3604010	746.5	3.8	84.5	4.4	84.3	4.4	84.5	5.9	85.2	5.6
	±2.3	±0.2	±0.6	±0.2	±0.7	±0.2	±0.8	±0.3	±0.8	±0.2
PUMA5101301	746.5	3.8	95.7	4.3	95.2	4.2	104.7	7.3	109.1	6.6
	±2.3	±0.2	±0.6	±0.2	±0.7	±0.2	±1.0	±0.4	±0.8	±0.2
PUMA5151255	746.5	3.8	136.7	4.3	136.7	4.5	136.7	5.5	158.1	10.2
	±2.3	±0.2	±0.8	±0.2	±0.9	±0.2	±1.1	±0.3	±1.0	±0.5

Table 51. Squared Errors (with standard deviations). Id Query. PUMS datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
PUMA0101301	3.2	13.7	13.8	4.0	3.1
	±0.1	±0.4	±0.4	±0.2	±0.1
PUMA0800803	3.2	9.9	10.1	4.1	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1
PUMA1304600	3.2	8.8	8.9	4.3	3.3
	±0.1	±0.3	±0.3	±0.3	±0.1
PUMA1703529	3.2	9.2	9.5	4.1	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1
PUMA1703531	3.2	13.6	13.8	3.9	3.2
	±0.1	±0.4	±0.4	±0.2	±0.1
PUMA1901700	3.2	8.9	8.9	4.0	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1
PUMA2401004	3.2	8.2	8.1	3.9	3.3
	±0.1	±0.3	±0.3	±0.3	±0.1
PUMA2602702	3.2	9.7	9.8	4.1	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1
PUMA2801100	3.2	12.4	12.6	3.9	3.2
	±0.1	±0.3	±0.4	±0.2	±0.1
PUMA2901901	3.2	10.7	10.8	4.0	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1
PUMA3200405	3.2	7.4	7.2	4.0	3.3
	±0.1	±0.3	±0.3	±0.3	±0.1
PUMA3603710	3.2	6.7	6.7	3.9	3.3
	±0.1	±0.2	±0.3	±0.2	±0.1
PUMA3604010	3.2	9.1	9.3	3.9	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1
PUMA5101301	3.2	8.5	8.6	4.2	3.2
	±0.1	±0.3	±0.3	±0.3	±0.1
PUMA5151255	3.2	7.1	7.2	4.2	3.2
	±0.1	±0.3	±0.3	±0.2	±0.1

Table 52. Squared Error (with standard deviations). Sum Query. PUMS datasets. Gauss Mechanism ($\rho = 0.5$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	127.0	15.2	67.2	10.7	67.3	10.9	80.3	15.0	62.1	19.4
	±1.9	±0.6	±1.1	±0.4	±1.1	±0.5	±2.0	±1.0	±1.2	±0.6
PUMA0800803	127.0	15.2	92.9	22.0	91.8	21.3	94.7	14.9	89.6	19.5
	±1.9	±0.6	±1.5	±0.9	±1.6	±1.0	±2.3	±0.9	±1.5	±0.7
PUMA1304600	127.0	15.2	97.4	13.9	97.7	13.9	97.0	15.1	84.9	14.2
	±1.9	±0.6	±1.5	±0.7	±1.6	±0.7	±2.3	±0.9	±1.6	±0.6
PUMA1703529	127.0	15.2	92.3	16.3	91.5	15.4	97.5	15.4	77.8	14.2
	±1.9	±0.6	±1.5	±0.8	±1.5	±0.7	±2.3	±0.9	±1.5	±0.7
PUMA1703531	127.0	15.2	63.3	8.6	63.1	8.7	83.2	15.2	75.8	17.8
	±1.9	±0.6	±1.0	±0.4	±1.1	±0.4	±1.9	±0.9	±1.3	±0.7
PUMA1901700	127.0	15.2	102.1	17.1	101.4	16.3	91.4	15.1	78.0	14.4
	±1.9	±0.6	±1.6	±0.8	±1.7	±0.8	±2.4	±1.1	±1.5	±0.7
PUMA2401004	127.0	15.2	111.2	28.6	111.2	28.9	97.1	16.6	79.8	16.4
	±1.9	±0.6	±1.8	±1.1	±1.9	±1.3	±3.1	±1.5	±1.6	±0.8
PUMA2602702	127.0	15.2	89.4	18.2	88.6	17.8	95.0	14.9	81.5	22.4
	±1.9	±0.6	±1.4	±0.8	±1.5	±0.8	±2.1	±0.8	±1.5	±0.8
PUMA2801100	127.0	15.2	72.7	13.6	73.1	13.7	81.8	15.1	74.8	18.1
	±1.9	±0.6	±1.2	±0.6	±1.3	±0.7	±1.8	±0.9	±1.4	±0.7
PUMA2901901	127.0	15.2	82.6	16.3	82.5	16.1	91.2	15.1	71.0	14.4
	±1.9	±0.6	±1.4	±0.7	±1.4	±0.8	±2.1	±0.9	±1.3	±0.7
PUMA3200405	127.0	15.2	112.7	15.8	112.5	15.8	112.0	15.6	115.9	19.3
	±1.9	±0.6	±1.7	±0.8	±1.8	±0.8	±2.5	±1.1	±1.8	±0.7
PUMA3603710	127.0	15.2	119.5	18.1	120.1	18.3	114.7	15.2	112.0	16.9
	±1.9	±0.6	±1.8	±0.7	±1.9	±0.8	±2.6	±1.0	±1.8	±0.6
PUMA3604010	127.0	15.2	99.4	15.6	100.5	16.0	97.7	15.7	71.7	14.1
	±1.9	±0.6	±1.5	±0.6	±1.6	±0.7	±3.1	±1.2	±1.4	±0.6
PUMA5101301	127.0	15.2	104.3	23.4	103.7	23.2	95.6	14.8	77.9	14.3
	±1.9	±0.6	±1.7	±1.0	±1.7	±1.0	±2.3	±0.9	±1.4	±0.7
PUMA5151255	127.0	15.2	118.5	21.7	118.6	21.9	98.2	15.6	80.9	14.4
	±1.9	±0.6	±1.8	±1.0	±2.0	±1.1	±2.5	±1.1	±1.5	±0.7

Table 53. Squared Errors (with standard deviations). Marg1 Query. PUMS datasets. Gauss Mechanism ($\rho = 0.125$).

	olsalg		nnlsalg		maxalg		seqalg		weightalg	
Dataset	Total	Max	Total	Max	Total	Max	Total	Max	Total	Max
PUMA0101301	333.4	15.0	92.7	30.4	93.2	30.6	88.7	24.1	46.7	12.9
	±3.0	±0.7	±1.4	±1.0	±1.5	±1.0	±2.2	±1.3	±1.1	±0.7
PUMA0800803	333.4	15.0	150.1	30.5	149.4	30.4	167.3	17.9	136.1	18.9
	±3.0	±0.7	±1.8	±1.0	±1.8	±1.1	±2.8	±1.1	±1.9	±0.7
PUMA1304600	333.4	15.0	162.7	25.6	162.4	25.2	166.2	18.4	135.8	16.2
	±3.0	±0.7	±1.9	±0.9	±2.0	±1.0	±2.8	±1.1	±1.9	±0.8
PUMA1703529	333.4	15.0	146.4	27.9	146.4	27.7	143.0	18.8	114.3	16.5
	±3.0	±0.7	±1.8	±1.0	±1.9	±1.0	±2.5	±1.1	±1.7	±0.7
PUMA1703531	333.4	15.0	88.6	38.5	89.0	38.9	64.2	27.6	39.5	13.1
	±3.0	±0.7	±1.4	±1.2	±1.5	±1.2	±1.6	±1.3	±0.9	±0.6
PUMA1901700	333.4	15.0	172.9	23.5	172.9	23.0	181.4	19.0	171.9	23.6
	±3.0	±0.7	±2.0	±0.9	±2.1	±0.9	±3.3	±1.1	±2.1	±0.8
PUMA2401004	333.4	15.0	200.6	23.7	197.8	23.2	274.7	17.8	235.5	19.8
	±3.0	±0.7	±2.0	±0.9	±2.2	±1.0	±5.0	±1.4	±2.5	±0.7
PUMA2602702	333.4	15.0	131.3	28.8	131.4	29.1	138.9	17.8	113.1	19.3
	±3.0	±0.7	±1.6	±1.0	±1.7	±1.1	±2.3	±1.0	±1.6	±0.6
PUMA2801100	333.4	15.0	103.3	32.8	103.9	33.2	94.0	22.4	72.0	17.4
	±3.0	±0.7	±1.5	±1.1	±1.5	±1.1	±1.8	±1.1	±1.3	±0.6
PUMA2901901	333.4	15.0	126.3	30.7	126.4	30.4	118.4	19.4	98.8	25.2
	±3.0	±0.7	±1.7	±1.0	±1.8	±1.1	±2.3	±1.1	±1.7	±0.9
PUMA3200405	333.4	15.0	208.9	24.4	207.9	24.3	217.0	17.2	202.7	23.2
	±3.0	±0.7	±2.2	±0.9	±2.4	±1.0	±3.4	±1.0	±2.4	±0.7
PUMA3603710	333.4	15.0	230.1	19.6	227.7	19.5	244.3	17.1	210.8	19.9
	±3.0	±0.7	±2.3	±0.8	±2.4	±0.8	±3.8	±1.0	±2.4	±0.7
PUMA3604010	333.4	15.0	168.2	22.2	167.9	21.8	187.5	17.2	180.1	24.1
	±3.0	±0.7	±1.9	±0.9	±2.0	±0.9	±4.0	±1.2	±2.2	±0.7
PUMA5101301	333.4	15.0	174.6	26.0	175.1	26.0	222.8	17.0	186.1	18.0
	±3.0	±0.7	±1.8	±1.0	±2.0	±1.0	±3.3	±1.1	±2.2	±0.7
PUMA5151255	333.4	15.0	210.4	20.1	210.0	19.4	256.9	15.9	222.7	15.8
	±3.0	±0.7	±2.1	±0.8	±2.2	±0.8	±3.8	±1.1	±2.4	±0.6

Table 54. Squared Errors (with standard deviations). Marg2 Query. PUMS datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	2985.8	15.2	113.2	19.9	113.6	20.0	125.5	30.6	117.4	23.0
	±9.0	±0.7	±1.3	±0.7	±1.4	±0.7	±2.2	±1.3	±2.3	±1.6
PUMA0800803	2985.8	15.2	205.7	14.9	206.3	15.0	228.1	34.9	215.7	17.6
	±9.0	±0.7	±1.8	±0.6	±1.9	±0.7	±2.9	±1.6	±2.3	±0.7
PUMA1304600	2985.8	15.2	267.2	14.9	266.7	14.9	257.3	19.8	287.8	28.2
	±9.0	±0.7	±2.0	±0.4	±2.1	±0.7	±2.7	±1.0	±2.9	±1.8
PUMA1703529	2985.8	15.2	209.3	14.4	209.5	14.8	200.2	18.2	232.9	21.6
	±9.0	±0.7	±1.8	±0.6	±1.9	±0.7	±2.4	±0.8	±2.7	±1.3
PUMA1703531	2985.8	15.2	91.7	11.8	92.1	11.9	85.9	15.3	90.3	17.9
	±9.0	±0.7	±1.0	±0.5	±1.1	±0.5	±1.6	±0.9	±1.4	±0.7
PUMA1901700	2985.8	15.2	281.8	15.8	282.6	15.7	271.6	19.6	331.1	35.3
	±9.0	±0.7	±2.1	±0.7	±2.2	±0.7	±3.1	±1.1	±3.4	±2.2
PUMA2401004	2985.8	15.2	331.5	18.6	330.8	18.5	370.7	38.5	355.6	26.7
	±9.0	±0.7	±2.4	±0.8	±2.6	±0.9	±4.9	±2.2	±2.9	±0.9
PUMA2602702	2985.8	15.2	174.4	12.7	174.8	12.6	175.4	22.7	171.2	16.3
	±9.0	±0.7	±1.5	±0.6	±1.6	±0.6	±2.1	±1.1	±1.9	±0.9
PUMA2801100	2985.8	15.2	123.4	15.7	124.1	15.8	124.4	20.8	128.2	19.0
	±9.0	±0.7	±1.3	±0.5	±1.3	±0.5	±1.7	±0.7	±1.7	±0.6
PUMA2901901	2985.8	15.2	159.0	14.4	159.9	14.5	158.5	21.5	157.3	19.1
	±9.0	±0.7	±1.5	±0.5	±1.6	±0.6	±2.1	±0.9	±1.9	±0.7
PUMA3200405	2985.8	15.2	418.3	15.2	419.7	15.2	412.5	19.1	498.0	44.6
	±9.0	±0.7	±2.7	±0.6	±2.9	±0.7	±3.8	±1.0	±4.1	±2.4
PUMA3603710	2985.8	15.2	497.3	17.5	496.9	17.6	486.4	24.8	591.7	41.3
	±9.0	±0.7	±3.0	±0.7	±3.1	±0.8	±4.3	±1.3	±4.7	±2.4
PUMA3604010	2985.8	15.2	268.2	16.3	268.5	16.4	269.2	21.7	286.9	22.8
	±9.0	±0.7	±2.1	±0.7	±2.2	±0.7	±3.9	±1.4	±3.0	±0.8
PUMA5101301	2985.8	15.2	283.7	15.9	285.3	16.3	303.7	32.4	314.7	41.2
	±9.0	±0.7	±2.0	±0.7	±2.2	±0.7	±3.2	±1.6	±2.8	±1.6
PUMA5151255	2985.8	15.2	407.9	16.9	409.8	16.9	406.0	23.7	456.2	38.9
	±9.0	±0.7	±2.6	±0.7	±2.8	±0.8	±3.9	±1.3	±3.7	±2.2

Table 55. Squared Errors (with standard deviations). Id Query. PUMS datasets. Gauss Mechanism ($\rho = 0.125$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
PUMA0101301	12.9	57.8	58.5	15.9	12.0
	±0.6	±1.5	±1.5	±0.9	±0.5
PUMA0800803	12.9	43.5	44.1	15.5	12.6
	±0.6	±1.3	±1.4	±0.9	±0.5
PUMA1304600	12.9	38.8	39.2	16.9	12.9
	±0.6	±1.2	±1.3	±1.0	±0.6
PUMA1703529	12.9	41.7	42.5	16.1	12.7
	±0.6	±1.2	±1.4	±0.9	±0.5
PUMA1703531	12.9	59.2	60.5	15.7	12.0
	±0.6	±1.5	±1.6	±0.9	±0.5
PUMA1901700	12.9	38.8	39.8	16.4	12.9
	±0.6	±1.2	±1.3	±1.1	±0.5
PUMA2401004	12.9	35.9	35.3	15.5	12.9
	±0.6	±1.1	±1.2	±1.1	±0.5
PUMA2602702	12.9	44.0	44.7	15.6	12.9
	±0.6	±1.3	±1.4	±0.9	±0.5
PUMA2801100	12.9	54.0	54.9	15.2	12.4
	±0.6	±1.4	±1.5	±0.8	±0.5
PUMA2901901	12.9	47.5	48.3	15.3	12.6
	±0.6	±1.3	±1.4	±0.9	±0.5
PUMA3200405	12.9	31.9	31.9	15.8	13.2
	±0.6	±1.1	±1.2	±0.9	±0.6
PUMA3603710	12.9	29.2	29.1	15.9	13.1
	±0.6	±1.0	±1.1	±1.1	±0.6
PUMA3604010	12.9	39.8	40.5	16.9	12.9
	±0.6	±1.2	±1.3	±1.2	±0.6
PUMA5101301	12.9	37.5	37.9	16.6	12.8
	±0.6	±1.2	±1.3	±1.0	±0.5
PUMA5151255	12.9	31.6	32.3	17.8	12.9
	±0.6	±1.1	±1.2	±1.1	±0.5

Table 56. Squared Error (with standard deviations). Sum Query. PUMS datasets. Gauss Mechanism ($\rho = 0.125$).

	olsalg		nnlsalg		maxalg		seqalg		weightalg	
Dataset	Total	Max	Total	Max	Total	Max	Total	Max	Total	Max
PUMA0101301	3174.8 ±47.5	380.7 ±16.2	1344.9 ±23.2	199.2 ±8.9	1505.8 ±158.4	251.8 ±47.2	1562.0 ±53.2	442.9 ±34.9	1207.2 ±25.7	399.7 ±16.2
PUMA0800803	3174.8 ±47.5	380.7 ±16.2	1762.3 ±30.3	389.7 ±17.2	1680.5 ±149.1	324.8 ±67.6	1818.6 ±90.6	415.2 ±45.0	1696.3 ±34.1	451.1 ±16.8
PUMA1304600	3174.8 ±47.5	380.7 ±16.2	1775.7 ±28.5	247.9 ±11.4	2110.7 ±184.6	343.6 ±86.6	1929.3 ±138.7	347.3 ±67.3	2028.5 ±35.5	467.0 ±17.5
PUMA1703529	3174.8 ±47.5	380.7 ±16.2	1812.6 ±29.4	292.8 ±13.4	1834.8 ±195.6	263.1 ±83.8	2186.2 ±160.6	389.2 ±75.4	1846.4 ±33.8	436.3 ±15.5
PUMA1703531	3174.8 ±47.5	380.7 ±16.2	1110.1 ±20.8	188.2 ±8.5	1168.5 ±142.7	219.9 ±83.6	1011.6 ±42.3	343.5 ±26.9	713.8 ±18.1	208.6 ±9.7
PUMA1901700	3174.8 ±47.5	380.7 ±16.2	1979.9 ±31.5	311.2 ±14.4	1908.6 ±229.9	269.1 ±74.5	2335.2 ±127.2	383.6 ±43.8	2363.4 ±39.9	540.7 ±18.1
PUMA2401004	3174.8 ±47.5	380.7 ±16.2	2281.1 ±38.2	639.7 ±25.8	2368.5 ±168.3	690.8 ±118.2	1842.9 ±180.6	395.6 ±81.7	1622.9 ±33.6	382.8 ±17.4
PUMA2602702	3174.8 ±47.5	380.7 ±16.2	1629.5 ±27.2	261.4 ±11.9	1806.0 ±189.2	287.5 ±106.0	1860.2 ±70.2	370.3 ±37.4	1740.1 ±31.8	451.8 ±17.0
PUMA2801100	3174.8 ±47.5	380.7 ±16.2	1319.0 ±23.7	230.1 ±10.3	1161.6 ±134.6	169.5 ±44.2	1431.0 ±91.7	443.3 ±61.5	806.3 ±19.9	226.8 ±10.2
PUMA2901901	3174.8 ±47.5	380.7 ±16.2	1589.1 ±26.7	252.8 ±11.5	1381.1 ±115.4	271.1 ±61.1	1804.1 ±85.5	389.6 ±46.2	1630.8 ±31.2	419.8 ±15.7
PUMA3200405	3174.8 ±47.5	380.7 ±16.2	2237.7 ±35.5	355.0 ±16.6	2238.9 ±316.3	433.1 ±138.5	2449.2 ±96.6	386.6 ±39.1	2337.5 ±41.9	546.3 ±20.9
PUMA3603710	3174.8 ±47.5	380.7 ±16.2	2529.8 ±40.0	400.5 ±16.6	2924.0 ±254.4	491.2 ±131.1	2202.8 ±59.8	353.6 ±24.5	2217.6 ±41.3	469.4 ±20.1
PUMA3604010	3174.8 ±47.5	380.7 ±16.2	1830.6 ±28.8	261.3 ±11.6	1827.3 ±203.0	312.6 ±97.0	2013.5 ±99.2	400.3 ±49.2	2156.8 ±37.0	481.7 ±16.5
PUMA5101301	3174.8 ±47.5	380.7 ±16.2	1959.8 ±32.3	404.0 ±18.1	1697.0 ±170.9	303.9 ±67.6	2176.5 ±106.1	438.7 ±45.2	2157.6 ±38.1	470.0 ±17.5
PUMA5151255	3174.8 ±47.5	380.7 ±16.2	2361.5 ±37.7	429.3 ±19.5	2671.4 ±270.5	472.6 ±135.9	2358.7 ±81.7	427.8 ±35.3	2053.0 ±47.1	494.5 ±26.5

Table 57. Squared Errors (with standard deviations). Marg1 Query. PUMS datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	8334.3	374.7	1768.8	680.6	1876.2	828.8	1423.4	698.7	836.0	297.5
	±75.7	±16.7	±28.9	±22.2	±165.2	±133.0	±46.2	±38.8	±20.8	±13.0
PUMA0800803	8334.3	374.7	2421.6	763.4	2315.0	721.9	2582.6	687.0	1659.1	512.2
	±75.7	±16.7	±35.7	±25.1	±214.7	±160.4	±112.5	±70.1	±32.5	±15.5
PUMA1304600	8334.3	374.7	2479.2	761.8	3053.7	1120.9	2188.5	605.8	1685.0	338.0
	±75.7	±16.7	±35.0	±25.3	±261.0	±204.7	±125.5	±75.1	±31.9	±14.7
PUMA1703529	8334.3	374.7	2570.7	817.3	2759.4	787.0	2156.1	726.9	1712.5	691.7
	±75.7	±16.7	±38.0	±26.6	±234.6	±135.2	±194.0	±150.5	±31.5	±17.8
PUMA1703531	8334.3	374.7	1501.2	666.1	1703.8	763.7	1306.0	834.6	471.0	262.8
	±75.7	±16.7	±26.4	±20.7	±196.4	±155.8	±53.1	±45.6	±16.1	±11.2
PUMA1901700	8334.3	374.7	2816.2	726.2	2986.8	766.6	2980.6	679.5	1609.1	347.6
	±75.7	±16.7	±39.1	±24.8	±296.6	±194.0	±165.7	±89.8	±30.5	±14.8
PUMA2401004	8334.3	374.7	3427.8	666.8	3418.7	699.5	3659.6	474.0	3122.8	323.2
	±75.7	±16.7	±39.9	±23.9	±189.2	±129.1	±185.0	±87.9	±45.2	±14.4
PUMA2602702	8334.3	374.7	2097.6	876.0	2295.3	942.6	1511.3	676.2	936.0	320.3
	±75.7	±16.7	±34.3	±27.4	±233.6	±177.8	±64.0	±50.7	±23.6	±14.7
PUMA2801100	8334.3	374.7	1714.4	743.1	1583.8	649.3	1437.2	814.4	618.2	265.5
	±75.7	±16.7	±29.6	±23.6	±174.8	±140.0	±89.1	±76.2	±18.0	±11.4
PUMA2901901	8334.3	374.7	2105.3	878.9	2443.2	1240.6	1775.1	824.5	965.8	324.8
	±75.7	±16.7	±34.0	±27.3	±186.2	±160.8	±112.8	±96.2	±23.3	±14.2
PUMA3200405	8334.3	374.7	3527.2	678.6	3383.2	746.2	3425.1	462.4	2803.8	424.5
	±75.7	±16.7	±43.3	±24.3	±371.5	±244.4	±96.2	±33.0	±41.9	±15.5
PUMA3603710	8334.3	374.7	4395.0	529.9	4256.4	564.5	4535.3	443.2	3958.1	528.4
	±75.7	±16.7	±49.6	±20.2	±307.6	±155.5	±83.6	±28.1	±52.3	±20.0
PUMA3604010	8334.3	374.7	2504.2	687.9	2814.2	815.6	2576.0	603.8	1831.9	353.0
	±75.7	±16.7	±33.8	±23.4	±232.6	±158.9	±104.9	±61.8	±33.1	±14.6
PUMA5101301	8334.3	374.7	2589.2	789.0	2506.4	832.1	2385.2	570.9	1764.4	330.9
	±75.7	±16.7	±36.4	±26.4	±233.4	±171.8	±104.4	±63.5	±34.2	±15.0
PUMA5151255	8334.3	374.7	3529.9	619.3	3413.1	636.9	4129.0	454.4	3495.5	430.9
	±75.7	±16.7	±41.2	±22.9	±296.6	±188.5	±94.3	±38.2	±47.4	±16.4

Table 58. Squared Errors (with standard deviations). Marg2 Query. PUMS datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg Total	olsalg Max	nnlsalg Total	nnlsalg Max	maxalg Total	maxalg Max	seqalg Total	seqalg Max	weightalg Total	weightalg Max
PUMA0101301	74645.2	380.9	1885.3	369.2	1922.9	451.9	1785.6	466.7	1485.5	420.3
	±225.9	±17.8	±24.5	±14.3	±156.6	±86.7	±47.0	±32.7	±27.9	±16.8
PUMA0800803	74645.2	380.9	2763.8	338.4	2686.3	426.1	3098.6	516.6	2772.4	476.5
	±225.9	±17.8	±31.5	±14.7	±201.6	±124.7	±100.3	±46.4	±41.7	±18.0
PUMA1304600	74645.2	380.9	2726.3	332.3	3035.0	416.5	2268.1	390.9	2698.8	488.4
	±225.9	±17.8	±29.1	±14.3	±199.3	±121.3	±107.1	±64.0	±38.4	±18.1
PUMA1703529	74645.2	380.9	2898.4	320.5	3063.2	320.4	2738.5	514.2	2793.3	410.7
	±225.9	±17.8	±30.8	±14.3	±198.0	±85.1	±141.2	±87.3	±39.1	±14.3
PUMA1703531	74645.2	380.9	1246.1	329.1	1423.9	398.7	1011.5	387.6	761.2	200.4
	±225.9	±17.8	±17.6	±12.2	±132.4	±100.5	±32.7	±25.7	±17.0	±9.1
PUMA1901700	74645.2	380.9	3452.1	306.7	3623.8	361.0	3162.2	340.7	3456.1	439.4
	±225.9	±17.8	±34.4	±11.0	±248.2	±93.0	±112.7	±39.5	±45.7	±14.2
PUMA2401004	74645.2	380.9	4529.2	359.5	4575.7	365.6	4774.0	820.1	4157.3	470.0
	±225.9	±17.8	±39.6	±15.7	±187.7	±81.9	±219.7	±133.2	±49.2	±19.4
PUMA2602702	74645.2	380.9	2265.6	305.8	2539.8	385.1	1933.9	342.5	2182.2	448.6
	±225.9	±17.8	±27.9	±13.6	±204.4	±92.9	±55.6	±26.2	±34.2	±16.8
PUMA2801100	74645.2	380.9	1601.8	318.0	1568.5	295.5	1363.5	401.9	1030.2	227.7
	±225.9	±17.8	±22.3	±12.3	±143.6	±48.6	±68.5	±42.3	±20.9	±9.8
PUMA2901901	74645.2	380.9	2193.4	296.3	2100.6	304.0	1938.0	358.5	2016.3	425.9
	±225.9	±17.8	±26.3	±11.7	±131.7	±71.0	±70.9	±34.7	±32.1	±15.3
PUMA3200405	74645.2	380.9	4765.9	355.4	4221.0	296.7	4444.7	467.5	5518.1	642.2
	±225.9	±17.8	±40.9	±15.4	±272.2	±105.7	±94.0	±47.5	±66.4	±37.7
PUMA3603710	74645.2	380.9	6786.2	311.4	7413.6	462.8	6085.4	285.0	7136.2	403.9
	±225.9	±17.8	±48.8	±13.1	±345.2	±70.4	±70.2	±12.4	±63.6	±18.6
PUMA3604010	74645.2	380.9	2825.3	328.8	2927.2	324.0	2616.4	352.4	2899.9	400.2
	±225.9	±17.8	±27.2	±13.3	±193.1	±92.8	±74.0	±41.5	±40.6	±16.3
PUMA5101301	74645.2	380.9	3159.6	323.3	3111.8	424.3	3284.7	581.1	3193.0	457.3
	±225.9	±17.8	±32.5	±14.3	±188.5	±81.9	±108.0	±61.5	±43.2	±16.6
PUMA5151255	74645.2	380.9	4997.8	374.8	5258.1	444.1	5096.9	480.2	5444.9	579.4
	±225.9	±17.8	±42.0	±16.1	±313.8	±116.0	±86.8	±41.3	±67.9	±42.3

Table 59. Squared Errors (with standard deviations). Id Query. PUMS datasets. Gauss Mechanism ($\rho = 0.005$).

Dataset	olsalg	nnlsalg	maxalg	seqalg	weightalg
PUMA0101301	323.4	1623.3	1661.8	395.8	286.2
	±13.8	±38.5	±224.8	±26.2	±12.4
PUMA0800803	323.4	1379.3	1373.5	356.6	306.1
	±13.8	±35.7	±220.1	±38.9	±12.9
PUMA1304600	323.4	1297.6	1077.2	505.3	313.5
	±13.8	±34.7	±184.6	±78.3	±13.5
PUMA1703529	323.4	1346.2	1271.7	442.0	306.6
	±13.8	±35.4	±178.3	±75.3	±13.0
PUMA1703531	323.4	1747.3	1927.9	387.0	278.7
	±13.8	±39.5	±223.2	±28.0	±12.2
PUMA1901700	323.4	1242.3	1579.2	444.1	317.7
	±13.8	±33.9	±208.5	±60.1	±13.6
PUMA2401004	323.4	1138.8	1142.0	216.5	309.0
	±13.8	±32.6	±146.4	±42.8	±13.1
PUMA2602702	323.4	1457.8	1228.1	432.5	306.4
	±13.8	±36.7	±189.6	±35.6	±12.8
PUMA2801100	323.4	1651.1	2102.5	492.5	295.0
	±13.8	±38.8	±283.4	±50.5	±12.7
PUMA2901901	323.4	1493.7	1150.0	414.4	303.9
	±13.8	±37.1	±172.6	±51.1	±12.9
PUMA3200405	323.4	1058.1	965.0	422.6	321.7
	±13.8	±31.4	±256.9	±39.9	±13.6
PUMA3603710	323.4	952.9	967.8	379.0	331.0
	±13.8	±29.7	±155.3	±25.2	±14.1
PUMA3604010	323.4	1270.3	1090.5	470.3	327.4
	±13.8	±34.3	±262.5	±58.1	±13.8
PUMA5101301	323.4	1255.1	957.0	373.1	310.9
	±13.8	±34.2	±162.1	±46.9	±13.1
PUMA5151255	323.4	1047.3	838.4	378.7	328.4
	±13.8	±31.2	±159.5	±30.1	±13.9

Table 60. Squared Error (with standard deviations). Sum Query. PUMS datasets. Gauss Mechanism ($\rho = 0.005$).