A Appendix

A.1 Reference Control

The reference control for the (DI) obeys the PID control law

$$u_{nom}(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$$

where $e(t)$ is the error between the current state and the target state at time t. For our experiments, we use the gains $K_p^x = -1.2,\ K_p^v = -2.0,\ K_d^x = 0.1,\ K_d^v = 0.1$ with $K_i^x = K_i^v = 0$.

For (F16), we hand-design a reference control policy resembling takeoff. To test the PINN-CBF filtering abilities, we obstruct the trajectory with rectangular obstacles and place walls to restrict the safe spatial coordinates to a box. In Figure 8, filtering demonstrates agile planning and obstacle avoidance. The following is an example control policy for takeoff while banking.

$$A_T(t) = \frac{a}{8}t + 1, \quad P(t) = \frac{p}{4}\sin(2t), \quad Q(t) = \frac{q}{5}\cos(t), \quad x(0) = [0, 1, 2, 0, 0, \pi, 1]$$

A.2 3D Dubins Fixed-Wing Aircraft System Dynamics

Following [41], the dynamics of the Dubins fixed-wing aircraft (F16) system is given by:

$$\begin{aligned}
\dot{n} &= V_T \cos\psi \cos\theta, & \dot{e} &= V_T \sin\psi \cos\theta, \\
\dot{d} &= -V_T \sin\theta, & \dot{\phi} &= P + \sin\phi \tan\theta\, Q + \cos\phi \tan\theta\, R, \\
\dot{\theta} &= \cos\phi\, Q - \sin\phi\, R, & \dot{\psi} &= \frac{\sin\phi}{\cos\theta} Q + \frac{\cos\phi}{\cos\theta} R, \\
\dot{V_T} &= A_T, & R &= \frac{g_D}{V_T} \sin\phi \cos\theta
\end{aligned} \tag{13}$$

where $[n\ e\ d\ \phi\ \theta\ \psi\ V_T]^\top$ are the states and $[A_T\ P\ Q]^T$ are the controls. In particular, n, e, d are the position coordinates, V_T is the tangential velocity of the aircraft, ϕ, θ, ψ are the Euler coordinates orienting the plane, A_T is the tangential acceleration control input, P and Q are rotational control inputs, and g_D is gravitational acceleration.