We propose a novel method that generalizes [1,2] to solve inverse problems for general differentiable forward operators and structured noise. The power of our approach stems from the flexibility of flow models which can be combined in a modular way to solve inverse problems via MAP inference.

**Compressed Sensing with Invertible Generative Models and Dependent Noise**

**MAP Formulation**
- We generalize Asim et al. [2] to the case of nonlinear forward operator $f(x)$ and general noise distribution $p_\delta$.
- $L_G(z; y, \beta) = -\log p_\Delta(y - f(G(z))) - \beta \log p(G(z))$

**Theoretical Analysis**
- **(Gaussian Denoising)** We show that when the log likelihood of the flow model is locally-concave around the ground truth signal, the L2 reconstruction error is bounded as $\|\hat{x} - x^*\| \leq \frac{1}{\beta} \|\delta\|$.
- **(Compressed Sensing)** We show that the worst-case reconstruction error for compressed sensing with Gaussian noise is bounded as $\mathbb{E}[\|\hat{x} - x^*\|] \leq \sqrt{\frac{2}{w(w(1)) + 1}}$, where $w(\cdot)$ is the Gaussian mean-width and $s(p) = \{x|\log p(x) \geq \rho\}$.

**Compressed Sensing with a GAN**
- Bora et al. [1] proposed to replace the sparsity prior with a GAN prior trained on the domain of interest. Since GANs are capable of modeling complex high-dimensional signals, they can be an effective learned prior for inverse problems.
- Thus the signal recovery is done by finding a point in the range of GAN that minimizes reconstruction loss: $L_{\text{Bora}}(z; y) = \|y - AG(z)\|^2 + \lambda \|z\|^2$
  where $G$ is the generator from a pre-trained GAN and $\lambda$ is a regularization coefficient.

**Compressed Sensing with a Flow Model**
- Asim et al. [2] pointed out that GAN prior does not generalize well to out-of-distribution samples, since it cannot represent any signal that’s outside its range. Thus they proposed to use a flow prior instead where the generator $G$ is invertible: $L(z; y) = \|y - AG(z)\|^2 + \gamma \|z\|^2$
- Since $G$ is invertible, it can represent any signal in its range.
- We show that this loss is related to a specific instance of our general formulation for Gaussian noise $\delta \sim \mathcal{N}(0, \gamma I)$.

**Experimental Results**

- **Denoising MNIST digits from CelebA-HQ faces and OOD images**
- **1-bit Compressed Sensing**

**Conclusion**
We propose a novel method that generalizes [1,2] to solve inverse problems for general differentiable forward operators and structured noise. The power of our approach stems from the flexibility of flow models which can be combined in a modular way to solve inverse problems via MAP inference.

**References**