DRTR: Distance-Aware Graph Representation Learning

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1. Introduction

Graph Neural Networks (GNNs) commonly assume fixed-hop propagation and static edge structures, limiting their robustness in noisy and incomplete graphs. We propose **DRTR** (In Figure 1), an adaptive multi-hop graph learning framework that dynamically calibrates node-wise distances and reconstructs local topologies during training, leading to more expressive and robust representations.

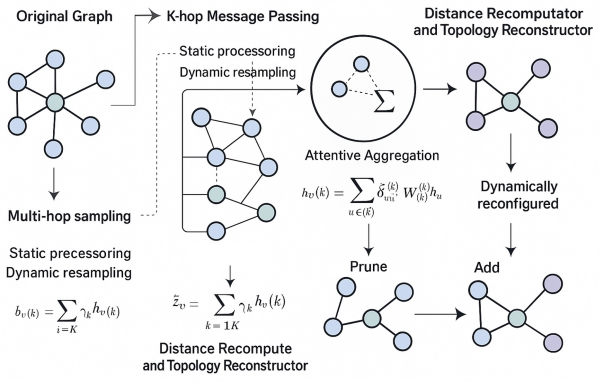

Fig. 1: Distance-Aware Graph Learning

2. Related Works

Adaptive Structure Learning in GNNs Conventional Graph Neural Networks (GNNs) such as GCN [1], GraphSAGE [2], and GAT [3] adopt fixed-hop message passing schemes and pre-defined neighbor aggregation strategies. However, recent studies recognize that structural connections in graphs often suffer from noise, imbalance, or redundancy [4]. To address this, various works have explored structure learning [5, 6, 7], which attempts to refine or reconstruct the graph topology during training. Nonetheless, many such methods rely on end-to-end optimization or learn a static edge mask, which lacks the flexibility to respond to evolving node semantics during multi-hop propagation.

Distance-aware Message Passing Several studies have attempted to incorporate distance or importance-based filtering into message passing. For instance, Personalized PageRank (PPR)-based methods [8, 9] reweight neighbor contributions based on precomputed transition probabilities. Meanwhile, GAMLP [10] and ImprovingTE [11] explicitly leverage multi-hop propagation, with hop-specific encoders and contextual subgraph tokens. However, these approaches still depend on fixed-hop sampling, which may not fully capture varying node proximities or information decay

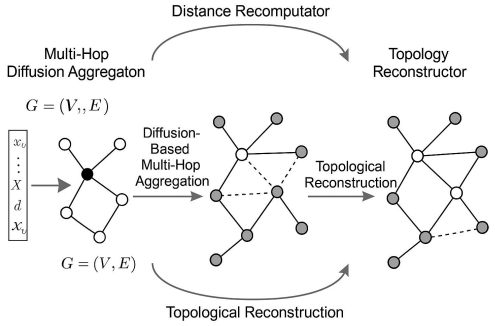

Fig. 2: An instance of The Distance Computation and Topology Reconstruction

across hops. Unlike prior work, DRTR dynamically recomputes node distances during training using the proposed *Distance Recomputator*, allowing for more context-aware and adaptive neighborhood construction.
In constrast, DRTR (an instance in 2) bridges the gap between dynamic topology learning, relevance-aware aggregation, and distance-sensitive diffusion, providing a unified framework that extends beyond existing adaptive GNN approaches.

3. Method

Given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, node features $\mathbf{X}$, and depth K, DRTR performs adaptive K-hop attention-based propagation:

$$\mathbf{h} * v^{(k)} = \sum * u \in \mathcal{N}^{(k)}(v) \alpha\_vu^{(k)} \mathbf{W}^{(k)} \mathbf{x} * u, \quad (1)$$

with attention coefficients:

$$\alpha_{vu}^{(k)} = \frac{\exp\left(\text{LeakyReLU}(\mathbf{a}^\top [\mathbf{W}\mathbf{x}_v \| \mathbf{W}\mathbf{x}_{u'}])/\tau_k\right)}{\sum_{u' \in \mathcal{N}^{(k)}(v)} \exp\left(\text{LeakyReLU}(\mathbf{a}^\top [\mathbf{W}\mathbf{x}_v \| \mathbf{W}\mathbf{x}_{u'}])/\tau_k\right)}. \quad (2)$$

Final embeddings aggregate across all hops:

$$\mathbf{z}_v = \sum_{k=1}^{K} \gamma_k \cdot \mathbf{h}_v^{(k)}, \quad \text{where } \sum_{k=1}^{K} \gamma_k = 1. \quad (3)$$

3.1 Distance Recomputator (DR)

We dynamically prune distant neighbors:

$$d_{vu}^{(k)} = \|\mathbf{x}_v - \mathbf{x}_u\|_2^2 + \lambda_k \delta_{vu}^{(k)}, \quad d_{vu}^{(k)} > \alpha \Rightarrow \text{prune}. \quad (4)$$

3.2 Topology Reconstructor (TR)

We augment the topology by adding latent edges:

$$\mathcal{E} \leftarrow \mathcal{E} \cup \{(v, u) \mid \|\mathbf{x}_v - \mathbf{x}_u\|_2^2 < \beta\}. \quad (5)$$

4. Experiments

To assess the broader applicability of DRTR, we extend our evaluation to two real-world downstream tasks beyond node classification:

Task 1: Recommendation (Link Prediction) We apply DRTR to a recommendation setting based on the MovieLens-100K dataset, where user–item interactions are modeled as a bipartite graph. The goal is to predict missing user–item links.

We follow a standard link prediction pipeline using node embeddings from a DRTR-augmented GraphSAGE model. Evaluation metrics include AUC and Average Precision (AP):

Model	AUC	AP
GraphSAGE	93.1	91.7
GraphSAGE+GDRA	94.0	92.5
GraphSAGE+GKHDA	94.3	92.8
GraphSAGE+GKHDDRA	**95.1**	**93.6**

Table 1: Link prediction results on MovieLens-100K. DRTR improves both ranking metrics.

Task 2: Molecular Property Prediction We evaluate DRTR on the ZINC-12K dataset for molecular graph regression, using a GCN-based backbone. Each molecule is represented as a graph with atoms as nodes and bonds as edges. The task is to predict molecular properties (e.g., solubility, logP).

We report Mean Absolute Error (MAE, ↓) across three targets:

Model	logP	QED	SA
GCN	0.423	0.218	0.387
GCN+GDRA	0.401	0.205	0.375
GCN+GKHDA	0.395	0.203	0.372
GCN+GKHDDRA	**0.383**	**0.197**	**0.366**

Table 2: Molecular property prediction on ZINC-12K (MAE ↓). DRTR enhances representation in dense chemical graphs.

Summary These results show DRTR's ability to:

- Improve ranking and predictive performance in sparse user–item graphs (recommendation).
- Preserve fine-grained local-global dependencies in dense, noisy molecular graphs.

This validates DRTR's generality across structural domains—sparse bipartite graphs, dense chemical graphs, and traditional citation networks.

5. Conclusion

DRTR introduces dynamic distance recalibration and topology reconstruction, providing robust graph representations that outperform traditional GNNs in various downstream tasks.

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