

# Supplementary Materials: The Name of the Title is Hope

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## 1 PROOF OF THEOREM 1

Suppose the SDE in Eq. 1 is amenable with  $\mathbf{u}(X)$  and  $M(t)$  satisfying (2) and (3). Applying the Itô's formula, we have for each  $k$

$$d\left(\frac{u^k(X_t)}{1-t}\right) = \left(\frac{u^k}{(1-t)^2} - \frac{\nabla u^k \cdot \mathbf{F}(X_t)}{(1-t)^2} + \frac{1}{2(1-t)} \text{tr}\left(G(X_t, t)^T D^2 u^k G(X_t, t)\right)\right) dt + \frac{\nabla u^k \cdot G(X_t, t)}{1-t} dW$$

The amenability condition Eq. 3 implies that

$$d\left(\frac{u^k(X_t)}{1-t}\right) = \frac{M^k(t)}{1-t} dW.$$

The functional equation Eq. 4 follows from integrating the above equation over  $[0, t]$ .

From Eq. 4, we find the covariance matrix of  $\mathbf{u}(X_t)$  is

$$(1-t)^2 \int_0^t \frac{M(\tau)M(\tau)^T}{(1-\tau)^2} d\tau$$

which converges to 0 as  $t \rightarrow 0-$ . This fact and the amenability condition in Eq. 2 jointly lead to the conclusion that

$$\lim_{t \rightarrow 1-} X_t = 0 \text{ almost surely.}$$

## 2 JUSTIFICATION OF AMENABILITY OF THE LINEAR A-BRIDGE

In the linear A-Bridge (Eq. 6),  $\mathbf{F} = X - Y$  and  $\mathbf{G} = \lambda\sqrt{1-t}\mathbf{I}$ . Take  $\mathbf{u}(X) = X - Y$ . Then  $D\mathbf{u}(X) = \mathbf{I}$ , and  $D^2\mathbf{u} = 0$ . The both sides of the first equation in Eq. 3 is equal to  $X - Y$ , while the left-hand-side of the second is equal to  $\lambda\sqrt{1-t}\mathbf{I}$  which depends on  $t$  only.

## 3 ADDITIONAL QUALITATIVE COMPARISON

Reference CycleGAN CUT DCLGAN Ours GT



Figure 1: Additional qualitative comparison between our linear A-Bridge and other methods on the Cityscapes datasets. GT stands for Ground Truth.



Figure 2: Additional diversity samples of our linear A-Bridge on the Edges2Shoes dataset.

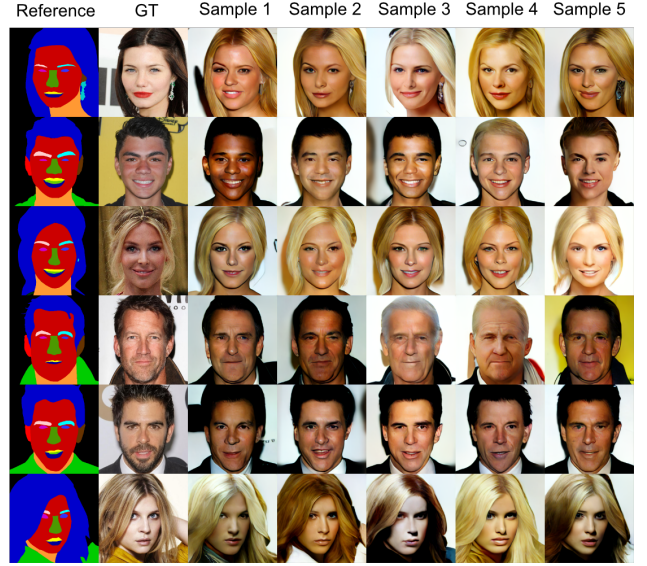


Figure 3: Additional diversity samples of our linear A-Bridge on the CelebAMask-HQ dataset.



Figure 4: Additional qualitative comparison between our linear A-Bridge and other methods on the Edges2Shoes, Face2Comics, and CelebAMask-HQ datasets. GT stands for Ground Truth.