

1 A Supplementary Material

2 A.1 High-level pseudocode for MUelim algorithm

Algorithm 1 MUelim motor unit decomposition

Require: Input data $\mathbf{X} \in \mathbb{R}^{C \times N}$ (EMG signals: C channels, N samples), sampling frequency f_s , extension factor R , lag τ , window size L , and other parameters (e.g., $max_bss_iterations$, $min_new_sources$, $frac_data_fit$, $sil_threshold$, etc.)

Ensure: Decomposed spike trains $\mathbf{s}_j(k)$

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1: procedure MUELIM( $\mathbf{X}$ )
2:   Step 1: Preprocessing and data augmentation
3:   Divide  $\mathbf{X}$  into  $W$  non-overlapping windows of size  $L$  ( $\mathbf{X}_{binned} \in \mathbb{R}^{W \times C \times L}$ )
4:   Extend the data using  $R$  lagged versions of each channel with lag  $\tau$  to form  $\mathbf{X}_{ext}$  (Eq. 3)
5:   Step 2: Iterative blind source separation (BSS)
6:   for  $i = 1$  to  $max\_bss\_iterations$  do
7:     Randomly sample  $frac\_data\_fit$  of windowed data for SPD computation.
8:     Compute SPD matrices (e.g., covariance, cospectral, kernel).
9:     Perform whitening (with optional dimension reduction) (Eq. 5).
10:    Perform approximate joint diagonalization by minimizing Eq. 6.
11:    Compute forward and backward filters from diagonal and whitening filters (Eqs. 7- 8)
12:    Compute  $\gamma_j(k)$  (source power) of full  $\mathbf{X}_{ext}$  (Eq. 9).
13:    Iterative filter refinement:
14:      Perform peak detection on  $\gamma_j(k)$  for each source.
15:      Update each filter using detected peaks (Eq. 10).
16:      Orthogonalize and normalize filters (Eqs. 11-12).
17:      Repeat until convergence.
18:      Cluster peaks into signal and noise using kmeans and save threshold.
19:      Compute SIL for each source and remove those below threshold.
20:      Evaluate and retain only unique sources based on:
21:        Spatial similarity (cosine similarity between filters)
22:        Temporal similarity (percentage of coincident spike timings)
23:      if optional peel-off step is enabled then
24:        Subtract the influence of identified sources from the data.
25:      end if
26:      if stopping criteria are met (e.g.,  $n\_new\_sources < min\_new\_sources$ ) then
27:        Break the loop.
28:      end if
29:    end for

30:   Inference mode
31:   Perform extend-lag procedure on input data  $\mathbf{X}$  (Eq. 3).
32:   Compute  $\gamma_j(k)$  (source power) of  $\mathbf{X}_{ext}$  (Eq. 9).
33:   Perform peak detection on  $\gamma_j(k)$  to identify spikes.
34:   Retain peaks that exceed saved kmeans clustering threshold.

35:   Output
36:   Return spike trains  $\mathbf{s}_j(k)$ 
37: end procedure
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3 A.2 Supplementary Tables

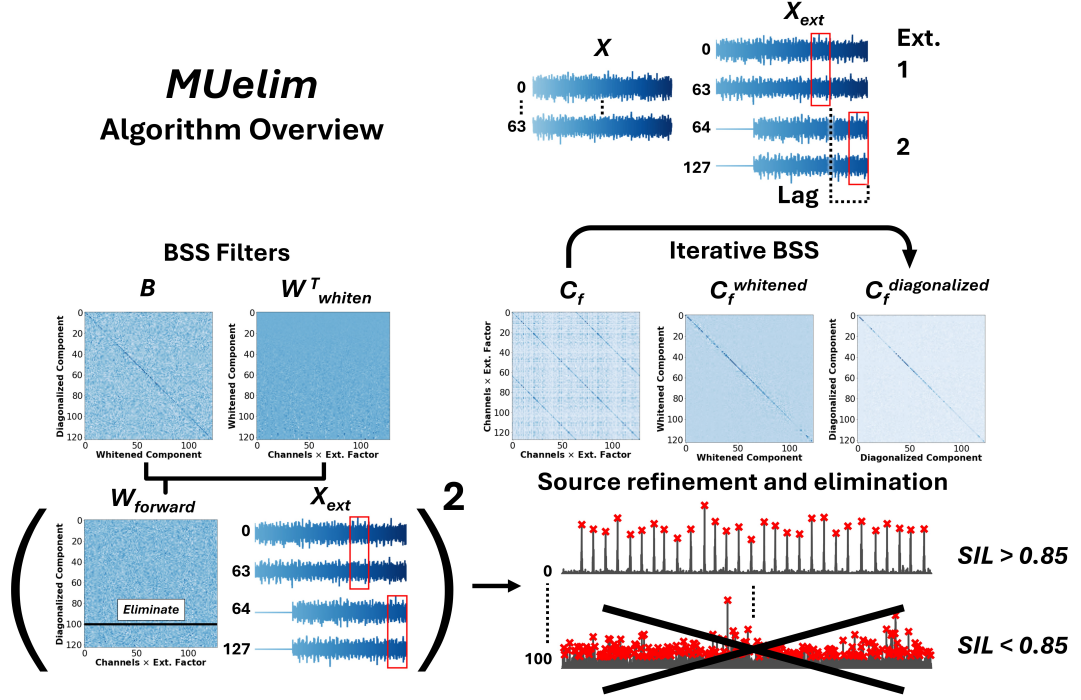
Supplementary Table 1: Comparison of motor unit decomposition methods on simulated EMG data

Method	Accuracy (%)	FP per source	FN per source
MUelim	99.99 ± 0.01	0.99 ± 0.14	0.03 ± 0.02
SCD	99.99 ± 0.01	0.93 ± 0.13	0.03 ± 0.02
MUEdit	98.98 ± 0.07	0.00 ± 0.00	3.06 ± 0.20

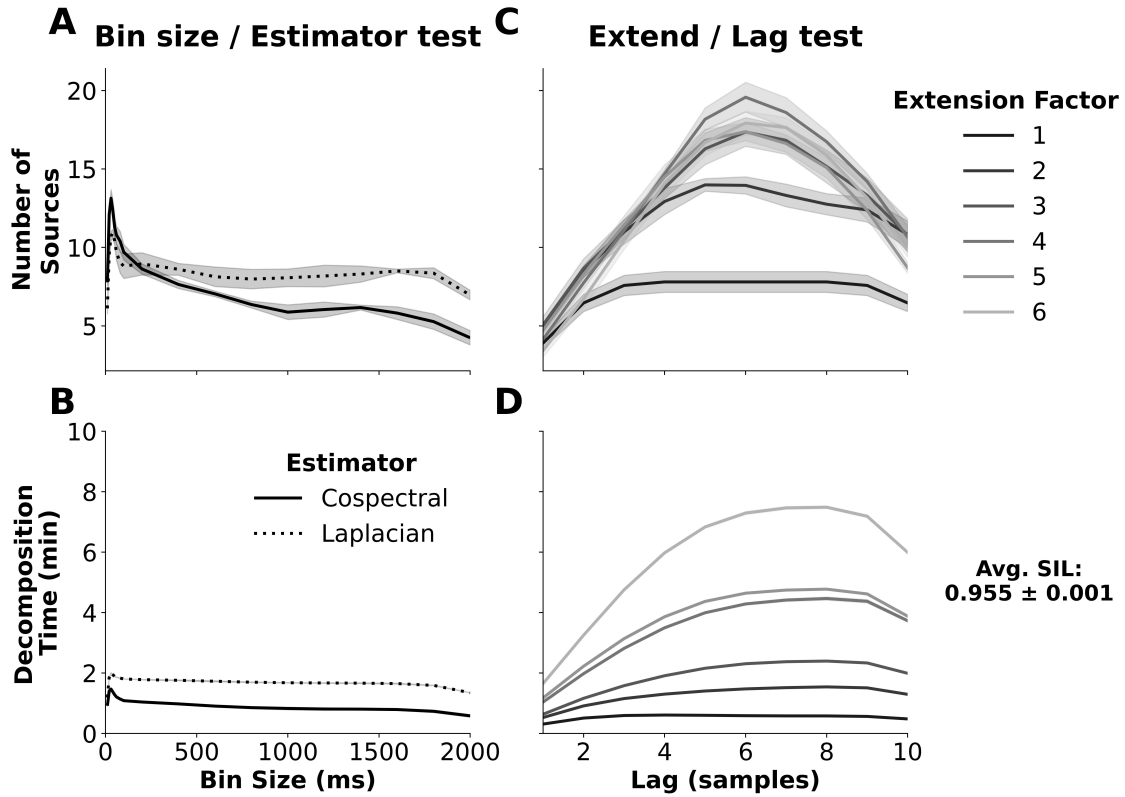
Supplementary Table 2: Speed comparison between MUelim and alternative methods

Experiment	Baseline Method	Comparison Method	Baseline Time (min.)	Comparison Time (min.)	Speed Factor	Time Reduction
Ramp	SCD	MUelim	14.6	3.1	5×	78.5%
	MUEdit		112.3		36×	97.2%
MVC	SCD	MUelim	9.2	0.7	12×	91.9%
	MUEdit		30.9		41×	97.6%

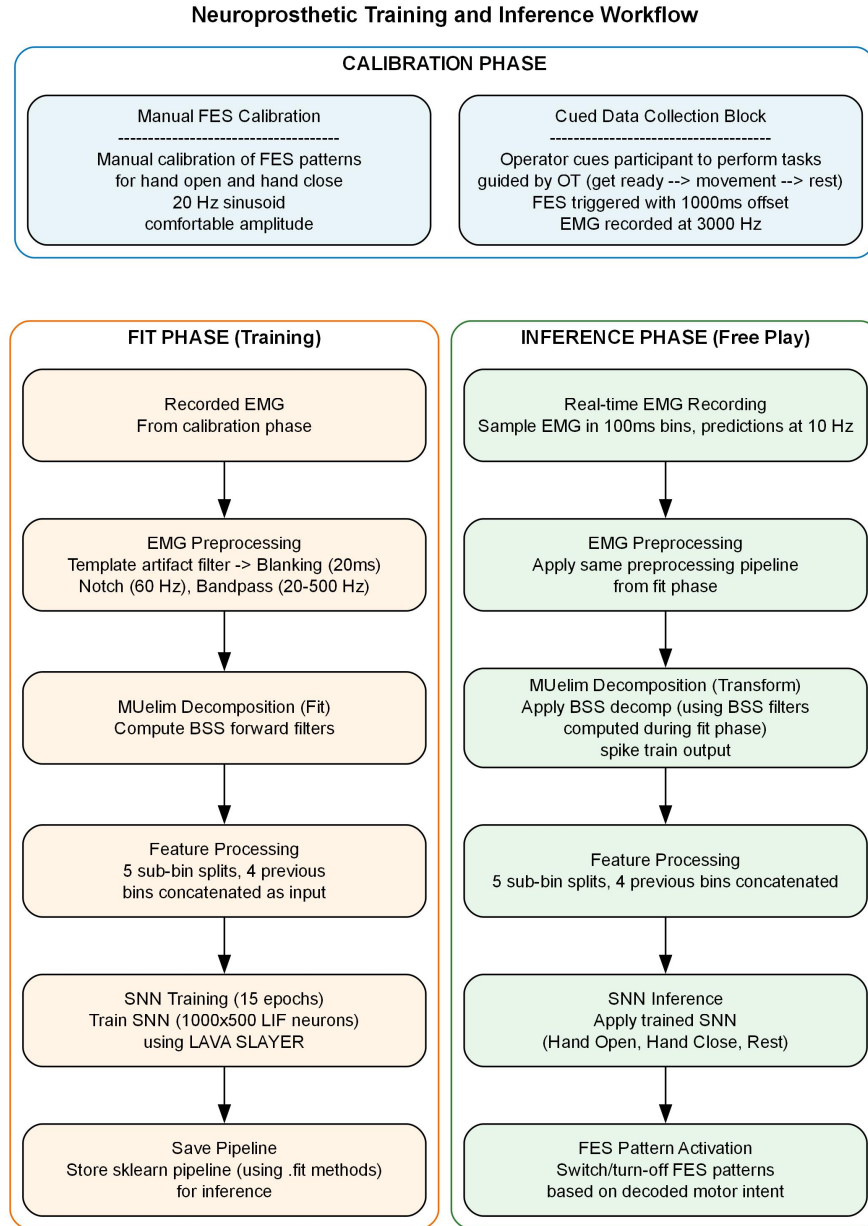
4 A.3 Supplementary Figures



Supplementary Figure 1: **Overview of the MUelim algorithm.** The input EMG data, X , is first embedded using an extend-lag procedure (X_{ext}) to capture temporal dependencies (a large 500ms lag is shown here for illustration only). The extended data is then windowed into non-overlapping bins for symmetric positive definite (SPD) matrix computation. An iterative second-order-statistics (SOS) blind source separation (BSS) method is used to extract candidate motor unit sources. SPD matrices, such as cospectral matrices (C_f), are computed from the binned and extended data, whitened to obtain C_f^{whitened} , and then jointly diagonalized to yield $C_f^{\text{diagonalized}}$. The resulting BSS filters consist of the diagonalization filters B and whitening filters W^T_{whiten} , which together form the forward filters W_{forward} . These filters project the extended data into source space, and the squared output yields the gamma source power. An iterative source refinement procedure is then performed using peak detection and silhouette score (SIL) to evaluate candidate sources, in which high signal-to-noise sources are retained, while those with SIL below a threshold are eliminated. This BSS process is repeated, with each iteration comparing new filters to previously found filters to retain only unique sources, until a stopping criterion is met. Refer to Supplementary Algorithm 1 for further details.

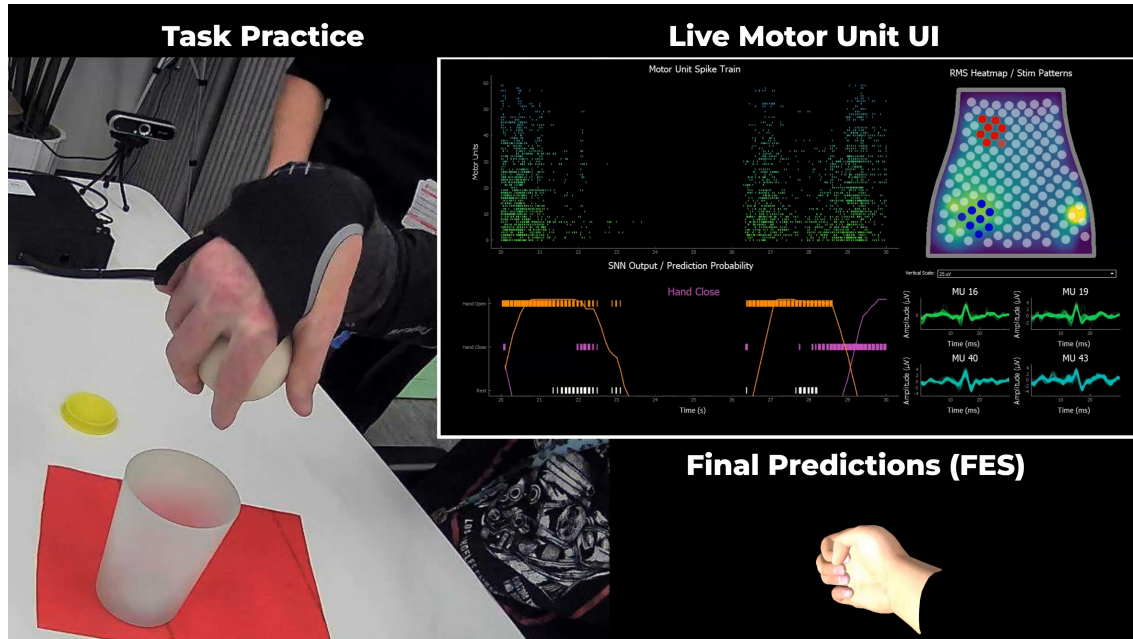


Supplementary Figure 2: **MUelim decomposition results on HD-EMG grid data in a healthy participant over a range of parameters.** **A-B.** Bin size / Estimator test: bin size for SPD matrix computation and SPD matrix estimator were varied to determine the effect on the number of sources (**A.**) and decomposition time (**B.**). **C-D.** Extend / Lag test: Extension factor and lag were varied over a range of values to assess the number of sources decomposed and decomposition time. All motor units had a silhouette (SIL) score above 0.85 with the average SIL of 0.955. An extension factor of 3-4 with lag between 4-8 yielded the most motor units.



Supplementary Figure 3: **Neuroprosthetic training and inference workflow.** The workflow consists of three phases. **Calibration Phase:** The operator manually calibrates functional electrical stimulation (FES) patterns for hand open and hand close movements using a 20 Hz sinusoidal waveform with comfortable amplitude. The participant is then cued to perform movements in blocks (get ready → movement → rest) while EMG is recorded with FES triggered at a 1,000 ms offset. **Fit Phase:** Recorded EMG data is preprocessed, followed by MUelim decomposition to compute BSS forward filters and extract motor unit spike trains. Motor unit spike trains are processed (5 sub-bin splits with 4 previous bins concatenated) and fed into a spiking neural network (SNN) for training using the LAVA framework (1,000×500 leaky integrate-and-fire neurons, 15 epochs). The complete pipeline follows sklearn convention with fit methods and is saved for inference. **Inference Phase:** During free play, real-time EMG is recorded at 3,000 Hz in 100ms bins (10 Hz prediction rate) as the participant attempts movements. The saved pipeline transforms the data through preprocessing, MUelim decomposition (applying BSS filters), and motor unit spike train processing. The trained SNN decodes motor intent from motor unit activity in real-time, triggering corresponding FES patterns (Hand Open, Hand Close, or Rest). The participant maintains volitional control of the neuroprosthetic while performing rehabilitation tasks guided by an occupational therapist.

5 A.4 Supplementary Video



Supplementary Video 1: **Intention driven functional electrical stimulation (FES) task practice by a participant with spinal cord injury (SCI).** Webcam video showing a SCI participant controlling FES to aid in rehabilitation tasks guided by an occupational therapist. Motor intent is decoded using a spiking neural network (SNN) from motor unit activity decomposed using the MUElim algorithm. The live spike train is shown in the motor unit UI. Below the spike train is the output of the SNN with each neuron output corresponding to a predicted movement class. The prediction probability is plotted on top calculated based on output neuron firing rate. The final prediction is shown in text above the plot, as well as in the virtual hand movement. This prediction corresponds to a movement-specific FES activation, shown in the top right of the UI on the sleeve heatmap. A set of electrodes contribute to a virtual patch for both a cathode and anode that was manually calibrated by the operator. Root mean square (RMS) EMG activity is shown on the sleeve heatmap to visualize muscle activity, although this signal is not used for decoding. Below the heatmap are plots of motor unit action potential (MUAP) waveforms identified via the spike triggered average of spike timings from the reconstructed EMG signal for four motor units.