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# Prioritizing Samples in Reinforcement Learning with Reducible Loss

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## Abstract

1 Most reinforcement learning algorithms take advantage of an experience replay  
2 buffer to repeatedly train on samples the agent has observed in the past. This  
3 prevents catastrophic forgetting, however simply assigning equal importance to  
4 each of the samples is a naive strategy. In this paper, we propose a method to  
5 prioritize samples based on how much we can learn from a sample. We define  
6 the learn-ability of a sample as the steady decrease of the training loss associated  
7 with this sample over time. We develop an algorithm to prioritize samples with  
8 high learn-ability, while assigning lower priority to those that are hard-to-learn,  
9 typically caused by noise or stochasticity. We empirically show that our method  
10 is more robust than random sampling and also better than just prioritizing with  
11 respect to the training loss, i.e. the temporal difference loss, which is used in vanilla  
12 prioritized experience replay.

## 13 1 Introduction

14 Deep reinforcement learning has shown great promise in recent years, particularly with its ability to  
15 solve difficult games such as Go Silver et al. [2016], chess Silver et al. [2018], and Atari Mnih et al.  
16 [2015]. However, online Reinforcement Learning (RL) suffers from sample inefficiency because  
17 updates to network parameters take place at every time-step with the data being discarded immediately.  
18 One of the landmarks in the space of online RL learning has been Deep Q Learning (DQN) Mnih  
19 et al. [2015], where the agent learns to achieve human-level performance in Atari 2600 games. A key  
20 feature of that algorithm was the use of batched data for online learning. Observed transitions are  
21 stored in a buffer called the *experience replay* Lin [2004], from which one randomly samples batches  
22 of transitions for updating the RL agent. This way, the agent is trained on previously visited samples  
23 to prevent catastrophic forgetting.

24 Instead of randomly sampling from the experience replay, we propose to sample based on the *learn-*  
25 *ability* of the samples. We consider a sample to be learnable if there is a potential for reducing the  
26 agent's loss with respect to that sample. We term the amount by which we can reduce the loss of a  
27 sample to be its *reducible loss* (ReLo). This is different from vanilla prioritization in Schaul et al.  
28 [2016] which just assigns high priority to samples with high loss, which can potentially lead to  
29 repeated sampling of data points which can not be learned from due to noise.

30 In our paper, we first briefly describe the current methods for prioritization while sampling from the  
31 buffer, followed by the intuition for reducible loss in reinforcement learning. We demonstrate the  
32 performance of our approach empirically on the DeepMind Control Suite Tassa et al. [2018], MinAtar  
33 Young and Tian [2019] and Arcade Learning Environment Bellemare et al. [2013] benchmarks. These  
34 experiments show how prioritizing based on the reducible loss is a more robust approach compared  
35 to just the loss term Schaul et al. [2016] used in Hessel et al. [2017] and that it can be integrated  
36 without adding any additional computational complexity.



Figure 1: Performance difference between vanilla PER and ReLo aggregated across 21 benchmarks, from DMC, MinAtar and ALE suites with 5 runs each, based on proposals from Agarwal et al. [2021]. ReLo clearly outperforms PER with a higher interquartile mean (IQM) and median as well as a lower optimality gap.

## 2 Background and Related Work

In Reinforcement Learning (RL), an agent is tasked with maximizing the expected total reward it receives from an environment via interaction with it. This problem is formulated using a Markov Decision Process (MDP) Bellman [1957] that is described by  $\langle \mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ , where  $\mathcal{S}$ ,  $\mathcal{A}$ ,  $\mathcal{R}$  and  $\mathcal{P}$  represent the state space, the action space, the reward function, and the transition function of the environment, respectively. The objective of RL is to learn an optimal policy  $\pi^*$ , which is a mapping from states to actions that maximizes the expected discounted sum of rewards it receives from the environment, that is

$$\pi^* = \operatorname{argmax}_{\pi} \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t r_t \mid S_t = s, A_t = a \right], \quad (1)$$

where  $\gamma \in [0, 1]$  is the discount factor. Action value methods obtain a policy by learning the action value ( $Q^{\pi}(s_t, a_t)$ ) of a policy which is the expected return by taking action  $a_t$  in state  $s_t$  and then following the policy  $\pi$  to choose further actions. This is done using the Bellman equation, which defines a recursive relationship in terms of the  $Q$  value function, as follows

$$Q^{\pi}(s_t, a_t) = r_t + \gamma \operatorname{argmax}_a Q^{\pi}(s_{t+1}, a) \quad (2)$$

The difference between the left and right sides of Eq. 2 is called the temporal difference error (TD error), and  $Q$  value methods minimize the TD error of the learned  $Q$  function  $Q^{\theta}$  (implemented as a neural network) using stochastic gradient descent. That is, the loss for the  $Q$  network is

$$L_{\theta} = (Q^{\theta}(s_t, a_t) - (r_t + \gamma \operatorname{argmax}_a Q^{\theta}(s_{t+1}, a)))^2. \quad (3)$$

We can then use the  $Q$  value to implicitly represent a policy by choosing actions with high  $Q$  values. While this is easy in discrete control tasks which have a small action space, it can be difficult in continuous action spaces because finding the action that maximizes the  $Q$  value can be an optimization problem in itself. This can be computationally expensive to do at every instant, so recent methods alleviate this problem through an actor network  $\mu_{\theta}$  that learns the action that produces the maximum  $Q$  value through stochastic gradient ascent, that is

$$\mu_{\theta} = \operatorname{argmax}_{\theta} Q^{\theta}(s_t, \mu_{\theta}(s_t)). \quad (4)$$

The loss for the  $Q$  network in Eq. 3 is then modified so that the  $\operatorname{argmax}$  is evaluated using the actor network,

$$L_{\theta} = (Q^{\theta}(s_t, a_t) - (r_t + \gamma Q^{\theta}(s_{t+1}, \mu_{\theta}(s_t))))^2 \quad (5)$$

### 2.1 Experience Replay

Online RL algorithms perform updates immediately after observing a transition. However, these not only make learning inefficient but also lead to catastrophic forgetting as some transitions can be sparsely visited. To eliminate this problem, Lin [2004] introduced experience replay, which stores the observed transitions and provides an interface to sample batches of transitions. This has been successfully used in DQN Mnih et al. [2015] to play Atari 2600 games.

Since Eqs. 3 and 5 do not require that the states and actions are generated from the current policy, algorithms trained this way are called off-policy RL algorithms. During training, data is collected from the environment and stored in a replay buffer from which mini-batches are sampled to be trained on.

70 A naive method of sampling is to uniformly sample all data in the buffer, however, this is inefficient  
 71 because not all data is necessarily equally important. Schaul et al. [2016] proposes Prioritized  
 72 Experience Replay (PER), that samples points with probabilities proportional to their TD error –  
 73 which has been shown to have a positive effect on performance by efficiently replaying samples that  
 74 the model has not yet learned, i.e., data points with high TD error. Each transition in the replay buffer  
 75 is assigned a priority  $p_i$ , and the transitions are sampled based on this priority. To ensure that data  
 76 points, even with low TD error, are sampled sometimes by the agent, instead of greedy sampling  
 77 based on TD error, the replay buffer in PER stochastically samples points with probability  $P_i$ .

$$P_i = \frac{p_i^\alpha}{\sum_j p_j^\alpha} \quad (6)$$

78 where  $\alpha \in [0, 1)$  is a hyper-parameter introduced to smoothen out very high TD errors. Setting  $\alpha$  to 0  
 79 makes it equivalent to uniform sampling. Since sampling points non-uniformly changes the expected  
 80 gradient of a mini-batch, PER corrects for this by using importance sampling (IS) weights  $w$

$$w_i = \left( \frac{p_{uniform}}{P_i} \right)^\beta \quad (7)$$

81 where  $\beta \in [0, 1]$  controls the amount by which the change in gradient should be corrected and  
 82  $p_{uniform} = \frac{1}{N}$  where  $N$  is the number of samples in the replay buffer. The loss attributed to each  
 83 sample is weighed by the corresponding  $w_i$  before the gradient is computed. In practice,  $\beta$  is either  
 84 set to 0.5 or linearly annealed from 0.4 to 1 during training.

85 While PER was initially proposed as an addition to DQN-style agents, Hou et al. [2017] have shown  
 86 that PER can be a useful strategy for improving performance in Deep Deterministic Policy Gradients  
 87 (DDPG) Lillicrap et al. [2016]. Another recent strategy to improve sample efficiency was to introduce  
 88 losses from the transition dynamics along with the TD error as the priority Oh et al. [2022]. Although  
 89 this has shown improvements, it involves additional computational complexity since it also requires  
 90 learning a reward predictor and transition predictor for the environment. Our proposal does not  
 91 require training additional networks and hence is similar in computational complexity to vanilla PER.  
 92 This makes it very simple to integrate into any existing algorithm. Wang and Ross [2019] propose  
 93 an algorithm to dynamically reduce the replay buffer size during training of SAC so that the agent  
 94 prioritizes recent experience while also ensuring that updates performed using newer data are not  
 95 overwritten by updates from older data. However, they do not distinguish between points based on  
 96 learn-ability and only assume that newer data is more useful for the agent to learn.

## 97 2.2 Target Networks

98 In Eqs. 3 and 5, the target action value depends not only on the rewards but also on the value of the  
 99 next state, which is not known. So, the value of the next state is approximated by feeding the next  
 100 state to the same network used for generating the current  $Q$  values. As mentioned in DQN Mnih  
 101 et al. [2015], this leads to a very unstable target for learning due to the frequent updates of the  $Q$   
 102 network. To alleviate this issue, Mnih et al. [2015] introduce target networks, where the target  $Q$   
 103 value is obtained from a lagging copy of the  $Q$  network used to generate the current  $Q$  value. This  
 104 prevents the target from changing rapidly and makes learning much more stable. So Eqs. 3 and 5 can  
 105 be suitably modified to

$$L_\theta = (Q^\theta(s_t, a_t) - (r_t + \gamma \underset{a}{\operatorname{argmax}} Q^{\theta_{tgt}}(s_{t+1}, a)))^2 \quad (8)$$

106 and

$$L_\theta = (Q^\theta(s_t, a_t) - (r_t + \gamma Q^{\theta_{tgt}}(s_{t+1}, \mu_\theta(s_t))))^2, \quad (9)$$

107 respectively, where  $\theta_{tgt}$  are the parameters of the target network, which are updated at a low frequency.

108 Mnih et al. [2015] copies the entire training network  $\theta$  to the target network, whereas Haarnoja et al.  
 109 [2018] performs a soft update, where the new target network parameters are an exponential moving  
 110 average (with a parameter  $\tau$ ) of the old target network parameters and the online network parameters.

## 111 2.3 Off-Policy Algorithms

112 Off-policy algorithms are those that can learn a policy by learning from data not generated from  
 113 the current policy. This improves sample efficiency by reusing data collected by old versions of

114 the policy. This is in contrast to on-policy algorithms such as PPO Schulman et al. [2017], which  
 115 after collecting a batch of data and training on it, discard those samples and start data collection  
 116 from scratch. Recent state-of-the-art off-policy algorithms for continuous control include Soft Actor  
 117 Critic (SAC) Haarnoja et al. [2018] and Twin Delayed DDPG (TD3) Fujimoto et al. [2018]. SAC  
 118 learns two  $Q$  networks together and uses the minimum of the  $Q$  values generated by these networks  
 119 for the Bellman update equation to avoid over estimation bias. The  $Q$  target update also includes a  
 120 term to maximize the entropy of the policy to encourage exploration, a formulation that comes from  
 121 Maximum Entropy RL Ziebart et al. [2008]. TD3 is a successor to DDPG Lillicrap et al. [2016]  
 122 which addresses the overestimation bias present in DDPG in a similar fashion to SAC, by learning  
 123 two  $Q$  networks in parallel, which explains the “twin” in the name. It learns an actor network  $\mu$   
 124 following Eq. 4 to compute the maximum over  $Q$  values. TD3 proposes that the actor networks be  
 125 updated at a less frequent interval than the  $Q$  networks, which gives rise to the “delayed” name. In  
 126 discrete control, Rainbow Hessel et al. [2017] combines several previous improvements over DQN,  
 127 such as Double DQN van Hasselt et al. [2016], PER Schaul et al. [2016], Dueling DQN Wang et al.  
 128 [2016], Distributional RL Bellemare et al. [2017] and Noisy Nets Fortunato et al. [2018].

## 129 2.4 Reducible Loss

130 The work of Mindermann et al. [2022] proposes prioritized training for supervised learning tasks  
 131 based on focusing on data points that reduce the model’s generalization loss the most. Prioritized  
 132 training keeps a held-out subset of the training data to train a small capacity model,  $\theta_{ho}$  at the  
 133 beginning of training. During training, this hold-out model is used to provide a measure of whether a  
 134 data point could be learned without training on it. The loss of the hold-out model’s prediction,  $\hat{y}_{ho}$  on  
 135 a data point  $x$  could be considered an estimate of the remaining loss after training on data other than  
 136  $(x, y)$ , termed the *irreducible loss*. This estimate becomes more accurate as one increases the size of  
 137 the held-out dataset. The difference between the losses of the main model,  $\theta$ , and the hold-out model  
 138 on the actual training data is called the *reducible loss*,  $L_r$  which is used for prioritizing training data  
 139 in mini-batch sampling.

$$L_r = Loss(\hat{y} | x, \theta) - Loss(\hat{y} | x, \theta_{ho}) \quad (10)$$

140  $L_r$  can be thought of as a measure of information gain by also training on data point  $(x, y)$ .

## 141 3 Reducible Loss for Reinforcement Learning

142 While PER helps the agent to prioritize points that the model has not yet learned based on high TD  
 143 error, we argue that there are some drawbacks. Data points could have *high* TD error because they  
 144 are noisy or not learnable by the model. It might not be the case that a data point with high TD error  
 145 is also a sample that the model can actually learn or get a useful signal from. Instead of prioritization  
 146 based on the TD error, we propose that the agent should focus on samples that have higher *reducible*  
 147 TD error. This means that instead of the TD error, we should use a measure of how much the TD error  
 148 can be potentially decreased, as the priority  $p_i$  term in Eq. 6. We contend that this is better because  
 149 it means that the algorithm can avoid repeatedly sampling points that the agent has been unable to  
 150 learn from and can focus on minimizing error on points that are learnable, thereby improving sample  
 151 efficiency. Motivated by prioritized training, we propose a scheme of prioritization tailored to the RL  
 152 problem.

153 In contrast to supervised learning, the concepts of a hold-out dataset or model are not well defined in  
 154 the RL paradigm. In  $Q$  learning based RL methods, a good proxy for the hold-out model is the target  
 155 network used in the Bellman update in Eq. 8. Since the target network is only periodically updated  
 156 with the online model parameters and retains the performance of the agent on older data which are  
 157 trained with outdated policies. Schaul et al. [2022] demonstrates how the policies keep changing with  
 158 more training even when the agent receives close to optimal rewards. Thus, the target network can be  
 159 easily used as an approximation of the hold out model that was not trained on the sample. In this  
 160 way, we define the Reducible Loss (ReLo) for RL as the difference between the loss of the data point  
 161 with respect to the online network (with parameters  $\theta$ ) and with respect to the target network (with  
 162 parameters  $\theta_{tgt}$ ). So the Reducible Loss (ReLo) can be computed as

$$ReLo = L_\theta - L_{\theta_{tgt}} \quad (11)$$

163 When using ReLo as  $p_i$ , there are similarities in the sampling behavior of low priority points when  
 164 compared to PER. Data points that were not important under PER, i.e. they have low  $L_\theta$ , will also

165 remain unimportant in ReLo. This is because if  $L_\theta$  is low, then as per Eq. 11, ReLo will also be low.  
 166 This ensures that we retain the desirable behavior of PER, which is to not repeatedly sample points  
 167 that have already been learned.

168 However, there is a difference in sampling points that have high TD error. PER would assign high  
 169 priority to data points with high TD error, regardless of whether or not those data points are noisy  
 170 or unlearnable. For example, a data point can have a high TD error which continues to remain high  
 171 even after being sampled several times due to the inherent noise of the transition itself, but it would  
 172 continue to have high priority with PER. Thus, PER would continue to sample it, leading to inefficient  
 173 learning. But, its priority should be reduced since there might be other data points that are worth  
 174 sampling more because they have useful information which would enable faster learning. The ReLo  
 175 of such a point would be low because both  $L_\theta$  and  $L_{\theta_{tgt}}$  would be high. In case a data point is  
 176 forgotten, then the  $L_\theta$  would be higher than  $L_{\theta_{tgt}}$ , and the ReLo would ensure that these points are  
 177 revisited.

### 178 3.1 Implementation

179 The probability of sampling a data point is related to the priority through Eq. 6 and requires the  
 180 priority to be non-negative. Since  $Q$  value methods use the mean-squared error (MSE) loss, the  
 181 priority is guaranteed to be non-negative. However, ReLo computes the difference between the MSE  
 182 losses and it does not have the same property. Hence, we should create a mapping  $f_{map}$  for the  
 183 ReLo error that is monotonically increasing and non-negative for all values. In practice, we found  
 184 that clipping the negative values to zero, followed by adding a small  $\epsilon$  to ensure samples had some  
 185 minimum probability, worked well. That is,  $p_i = \max(\text{ReLo}, 0) + \epsilon$ . This is not the only way  
 186 we can map the negative values and we have studied one other mapping in Sec. 4.4. ReLo is not  
 187 computationally expensive since it does not require any additional training. It only involves one  
 188 additional forward pass of the states through the target network. This is because the Bellman backup  
 189 (i.e., the right hand side of Eq. 2) is the same for  $L_\theta$  and  $L_{\theta_{tgt}}$ . The only additional term that needs to  
 190 be computed for ReLo is  $Q_{tgt}(s_t, a_t)$  to compute  $L_{\theta_{tgt}}$ .

191 In our implementation, we saw a negligible change in the computational time between PER and ReLo.  
 192 ReLo also does not introduce any additional hyper-parameters that need to be tuned and works well  
 193 with the default hyper-parameters of  $\alpha$  and  $\beta$  in vanilla PER. An important point to note is that ReLo  
 194 does not necessarily depend on the exact loss formulation given in Eq. 8 and can be used with the loss  
 195 function  $L_\theta^{alg}$  of any off-policy  $Q$  value learning algorithm. In order to use ReLo, we only have to  
 196 additionally compute  $L_{\theta_{tgt}}^{alg}$  with respect to the target network parameters  $\theta_{tgt}$ . Our experiments also  
 197 show that ReLo is robust to the target network update mechanism, whether it is a hard copy of online  
 198 parameters at a fixed frequency (as in DQN Mnih et al. [2015], and Rainbow Hessel et al. [2017])  
 199 or if the target network is an exponential moving average of the online parameters (as in Soft Actor  
 200 Critic Haarnoja et al. [2018]).

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#### Algorithm 1 Computing ReLo for prioritization

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Given off-policy algorithm  $A$  with loss function  $L^{alg}$ , online  $Q$  network parameters  $\theta$ , target  $Q$   
 network parameters  $\theta_{tgt}$ , replay buffer  $B$ , max priority  $p_{max}$ , ReLo mapping  $f_{map}$ , epsilon priority  
 $\epsilon$ , training timesteps  $T$ , gradient steps per timestep  $T_{grad}$ , batch size  $b$ .

**for**  $t$  in  $1, 2, 3, \dots, T$  **do**

  Get current state  $s_t$  from the environment

  Compute action  $a_t$  from the agent

  Store the transition  $\langle s_t, a_t, r_t, s_{t+1} \rangle$  in the replay buffer  $B$  with priority  $p_{max}$ .

**for** steps in  $1, 2, 3, \dots, T_{grad}$  **do**

    Sample minibatch of size  $b$  from replay buffer

    Compute the loss  $L_\theta^{alg}$  and update the agent parameters  $\theta$

    Compute  $L_{\theta_{tgt}}^{alg}$  and calculate ReLo as per Eq. 11

    Update priorities of the samples in mini-batch with the newly computed ReLo values as  
 $f_{map}(\text{ReLo}_i) + \epsilon$

**end for**

  Update target network following the original RL algorithm  $A$

**end for**

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201 **4 Results**

202 We study the effectiveness of ReLo on several continuous and discrete control tasks. For continuous  
 203 control, we evaluate on 9 environments from the DeepMind Control (DMC) benchmark Tassa et al.  
 204 [2018] as they present a variety of challenging robotic control tasks, with high dimensional state and  
 205 action spaces. For discrete control, we use the MinAtar suite Young and Tian [2019] which consists  
 206 of visually simpler versions of games from the Arcade Learning Environment (ALE) Bellemare  
 207 et al. [2013]. The goal of MinAtar is to provide a benchmark that does not require the vast amounts  
 208 of compute needed for the full ALE evaluation protocol, which involves training for 200M frames  
 209 usually for 5 runs per game. This can be prohibitively expensive for researchers and thereby the  
 210 MinAtar benchmark reduces the barriers present in studying deep RL research. We include scores on  
 211 a few games from the ALE benchmark for a reduced number of steps to observe if there are signs of  
 212 improvement when using ReLo over PER. We provide full training curves for each environment in  
 213 the supplementary material.

214 In addition to the per environment scores and training curves, we report metrics aggregated across  
 215 environments based on recommendations from Agarwal et al. [2021] in Fig. 2. They treat performance  
 216 across runs as a random variable and suggest that authors report statistical measures on these random  
 217 variables. The mean and the median in Fig. 2 are the respective measures of the random variables.  
 218 The interquartile mean (IQM) computes the mean of the middle 50% of runs while the optimality gap  
 219 is a measure of how far an algorithm is from optimal performance aggregated across environments<sup>1</sup>.  
 220 In the DMC benchmark, the optimal score for each environment is 1000, while we use the highest  
 221 reported scores for each environment from the MinAtar paper for calculating the optimality gap for  
 222 the benchmark. For the ALE benchmark, we normalize the scores of each game with respect to  
 223 reported random and human level scores, i.e.  $\text{norm score} = \frac{\text{score} - \text{random}}{\text{human} - \text{random}}$ .

224 We also aggregated the normalized scores across benchmarks and show the IQM and optimality gap  
 225 of ReLo and PER in Fig. 1. The scores are aggregated across 21 environments (9 from DMC, 5 from  
 226 MinAtar, and 7 from ALE) and 5 seeds. We can clearly see that ReLo has a significantly higher IQM  
 227 with a smaller interval. This highlights the generality of ReLo since it performs better than PER  
 228 across a diverse set of tasks.

229 **4.1 DMC**

230 In the continuous control tasks, Soft Actor Critic (SAC) Haarnoja et al. [2018] is used as the base  
 231 off-policy algorithm to which we add ReLo. SAC has an online and an exponential moving average  
 232 target  $Q$  network which we use to generate the ReLo priority term as given in Eq. 11. For comparison,  
 233 we also include SAC with vanilla PER to showcase the differences in performance characteristics  
 234 of PER and ReLo. The results are given in Table 1 and Fig. 2. On 6 of the 9 environments, ReLo  
 235 outperforms the baseline SAC as well as SAC with PER. There is also a general trend where PER  
 236 leads to worse performance when compared to the baseline algorithm, in line with previous work  
 237 by Wang and Ross [2019] who show that the addition of vanilla PER to SAC hurts performance.  
 238 However, this is not the case when using ReLo as a prioritization scheme. This trend in performance  
 239 is visible in the aggregated scores in Fig. 2 where ReLo has a higher mean, median and IQM score  
 240 along with a lower optimality gap when compared to SAC and SAC with PER.

Table 1: Comparison of PER and ReLo on the DMC benchmark

	Baseline	PER	ReLo
cheetah run	761.9 ± 112.3	<b>831.9 ± 38.9</b>	660.3 ± 141.2
finger spin	966.7 ± 29.3	975.4 ± 6.7	<b>978.8 ± 14.4</b>
hopper hop	<b>264.7 ± 37.8</b>	217.4 ± 113.7	247.8 ± 51.0
quadruped run	612.7 ± 143.9	496.4 ± 216.0	<b>833.9 ± 81.0</b>
quadruped walk	831.9 ± 74.3	766.3 ± 200	<b>942.6 ± 9.7</b>
reacher easy	<b>983.1 ± 2.7</b>	981.6 ± 6.3	979.1 ± 11.0
reacher hard	955.1 ± 38.5	935.1 ± 47.9	<b>956.8 ± 38.7</b>
walker run	759.1 ± 23.9	755.5 ± 64.3	<b>795.1 ± 42.5</b>
walker walk	943.7 ± 30.2	957.4 ± 8.2	<b>963.3 ± 5.0</b>

<sup>1</sup>Lower optimality gap is better.

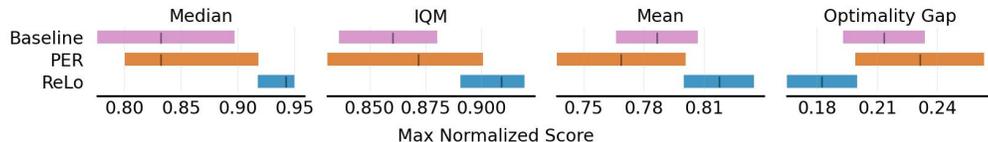


Figure 2: Metrics aggregated across 9 environments and 5 seeds in DMC based on proposed metrics from Agarwal et al. [2021]

## 241 4.2 MinAtar

242 In the MinAtar benchmark, we use DQN Mnih et al. [2015] as a baseline algorithm and compare its  
 243 performance with PER and ReLo on the 5 environments in the benchmark. DQN does not have a  
 244 moving average target  $Q$  network and instead performs a hard copy of the online network parameters  
 245 to the target network at a fixed interval. Similar to the implementation of ReLo in SAC, we use  
 246 the online and hard copy target  $Q$  network in the ReLo equation for calculating priorities. The  
 247 results on the benchmark are given in Table 2 and Fig. 3. Vanilla PER performs poorly on Seaquest  
 248 and SpaceInvaders, with scores lower than the baseline DQN. These results are consistent with  
 249 observations by Obando-Ceron and Castro [2021] which analysed the effect of the components  
 250 of Rainbow in the MinAtar environment. In contrast, ReLo consistently outperforms PER and is  
 251 comparable to or better than the baseline. Our previous observation that ReLo tends to help improve  
 252 performance in situations where PER hurts performance is also true here.

Table 2: Comparison of PER and ReLo on the MinAtar benchmark

	Baseline	PER	ReLo
Asterix	$12.5 \pm 1.0$	<b><math>16.2 \pm 1.0</math></b>	$16.1 \pm 0.5$
Breakout	<b><math>9.4 \pm 0.2</math></b>	$8.9 \pm 0.7$	<b><math>9.4 \pm 0.8</math></b>
Freeway	$52.8 \pm 0.3$	$52.8 \pm 0.2$	<b><math>53.2 \pm 0.4</math></b>
Seaquest	$16.1 \pm 2.8$	$6 \pm 1.9$	<b><math>19.5 \pm 0.6</math></b>
Space Invaders	<b><math>45.4 \pm 1.6</math></b>	$37.4 \pm 4.4$	$39.4 \pm 3.1$

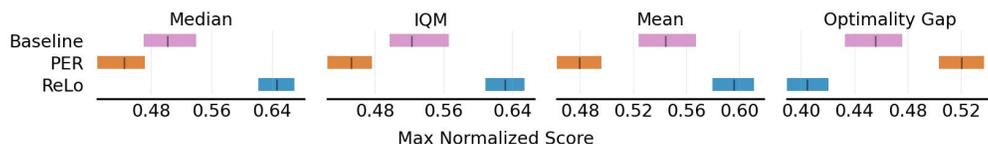


Figure 3: Metrics aggregated across 5 environments and 5 seeds in MinAtar based on proposed metrics from Agarwal et al. [2021]

## 253 4.3 ALE

254 As an additional test, we modified the Rainbow Hessel et al. [2017] algorithm, which uses PER by  
 255 default, to instead use ReLo as the prioritization scheme and compared it against vanilla Rainbow on  
 256 a subset of environments from the ALE benchmark. Instead of the usual 200M frames of evaluation,  
 257 we trained each agent for 2M frames to study if there are gains that can be observed in this compute-  
 258 constrained setting. As shown in Fig. 4 and Table 3, we see that Rainbow with ReLo achieves better  
 259 performance than vanilla Rainbow in nearly all the tested environments. These experiments show the  
 260 versatility of ReLo as a prioritization scheme.

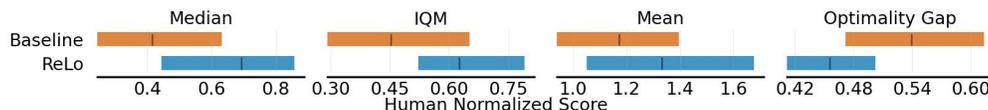


Figure 4: Metrics aggregated across 7 environments and 5 seeds in the ALE Benchmark based on proposed metrics from Agarwal et al. [2021]

Table 3: Comparison of Rainbow with PER and Rainbow with ReLo on the ALE benchmark

	Rainbow w/ PER	Rainbow w/ ReLo
Alien	1217.2 ± 207.2	<b>1544.0 ± 685.6</b>
Amidar	<b>445.3 ± 47.3</b>	393.7 ± 111.7
Assault	<b>2531.5 ± 444.7</b>	2506.9 ± 683.9
BankHeist	452.8 ± 131.2	<b>525.4 ± 201.3</b>
Frostbite	1842.0 ± 1450.5	<b>3366.4 ± 1613.7</b>
Jamesbond	663.0 ± 429.6	<b>851.0 ± 580.6</b>
Seaquest	1412.8 ± 402.6	<b>1755.2 ± 262.0</b>

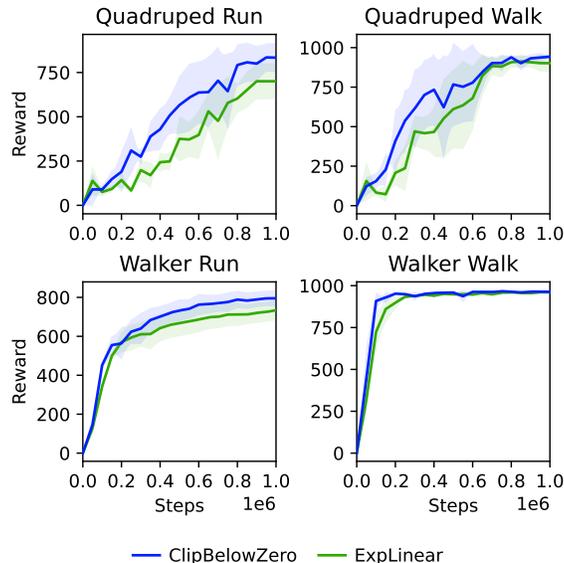


Figure 5: Comparison of different mapping functions from ReLo to  $p_i$  on a subset of environments from the DMC benchmark. Performance is evaluated for 10 episodes over 3 seeds.

#### 261 4.4 Mapping functions for ReLo

262 Prioritized experience replay buffers expect the priorities assigned to data points to be non-negative.  
 263 While the MSE version of the TD error used in vanilla PER satisfies this constraint, ReLo does not.  
 264 Therefore, there must be a non-negative, monotonically increasing mapping from ReLo to  $p_i$ . In the  
 265 main experiments above we clipped negative ReLo values to zero. Another mapping we tried was to  
 266 set  $p_i = e^{\text{ReLo}}$ , in which case the probability of sampling a data point  $P_i$ , from Eq. 6, corresponds  
 267 to the softmax over ReLo scores. However, for this choice the priority would explode if the ReLo  
 268 crossed values above 40 which happened occasionally during the initial stages of learning in Rainbow.  
 269 The second mapping function candidate was exponential when ReLo is negative and linear otherwise,  
 270 that is,

$$f_{ExpLinear} = \begin{cases} e^{\text{ReLo}} & \text{if ReLo} < 0 \\ \text{ReLo} + 1 & \text{otherwise} \end{cases} \quad (12)$$

271 The linear portion is shifted so that the mapping is smooth around  $\text{ReLo} = 0$ . As shown in Fig. 5,  
 272 ExpLinear performs worse compared to just clipping ReLo below zero. When the ReLo values during  
 273 training are analysed, we observe that the average of ReLo values (before the mapping) tends to be  
 274 positive, so clipping does not lead to a large loss in information.

#### 275 4.5 Analysis of TD Loss Minimization

276 To verify if using ReLo as a prioritization scheme leads to lower loss values during training, we  
 277 logged the TD error of each agent over the course of training and these loss curves are presented in  
 278 Figs. 6b and 6a. As we can see, ReLo does indeed lead to lower TD errors, empirically validating our

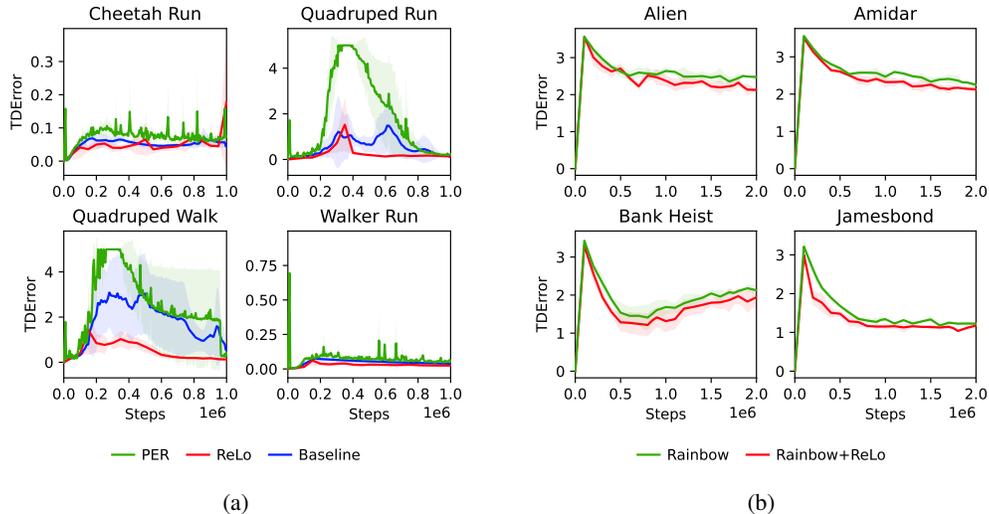


Figure 6: Comparison of temporal difference loss curves for a) DMC and b) ALE. ReLo achieves lower loss compared to the baseline and PER, showing that ReLo is able to prioritize samples with reducible loss. Dark line represents the mean and the shaded region is the standard deviation over 3 seeds.

279 claims that using ReLo helps the algorithm focus on samples where the loss can be reduced. Another  
 280 interesting point is that in Fig. 6a, SAC with PER has the highest reported TD errors throughout  
 281 training. This is due to PER prioritizing data points with high TD error, however, as we noted these  
 282 points need not necessarily be learnable. But since they have higher TD error, they repeatedly keep  
 283 getting sampled making the overall losses during training higher. ReLo addresses this issue and  
 284 is able to sample those data points which can be readily learned from, leading to the lowest TD errors  
 285 during training.

## 286 5 Conclusion

287 In this paper, we have proposed a new prioritization scheme for experience replay, Reducible Loss  
 288 (ReLo), which is based on the principle of frequently sampling data points that have potential for loss  
 289 reduction. We obtain a measure of the reducible loss through the difference in loss of the online model  
 290 and a hold-out model on a data point. In practice, we use the target network in  $Q$  value methods as a  
 291 proxy for a hold-out model.

292 ReLo avoids the pitfall that comes with naively sampling points based only on the magnitude of the  
 293 loss since having a high loss does not imply that the data point is actually learnable. While alleviating  
 294 this issue, ReLo retains the positive aspects of vanilla PER, thereby improving the performance of  
 295 deep RL algorithms. This has been empirically verified on both continuous and discrete control tasks  
 296 using a variety of algorithms: SAC, DQN, and Rainbow. It is very simple to implement, requiring  
 297 just the addition of a few lines of code to vanilla PER. It is also general and can be applied to any  
 298 off-policy algorithm and is agnostic to the choice of target network update mechanism. Since it  
 299 requires only one additional forward pass through the target network, the computational cost of ReLo  
 300 is minimal, and there is very little overhead in integrating it into an algorithm.

301 While the reducible loss can be intuitively reasoned about and has been tested empirically, future  
 302 work should theoretically analyse the sampling differences between ReLo and vanilla PER about the  
 303 kind of samples that they tend to prioritize or ignore. This deeper insight would allow us to find flaws  
 304 in how we approach non-uniform sampling in deep RL algorithms similar to work done in Fujimoto  
 305 et al. [2020].

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- 411

412 **A Implementation Details**

413 We build our experiments on top of existing implementations of SAC, DQN and Rainbow. For the  
 414 DeepMind Control Suite experiments, we modify Yarats and Kostrikov [2020], adding a prioritized  
 415 replay buffer and the ReLo version. We use an open source implementation of Rainbow<sup>2</sup> for the  
 416 Arcade Learning Environment and the DQN implementation from the MinAtar authors. Aside from  
 417 the collected frames and number of seeds, we have not modified any of the hyper-parameters from  
 418 these original implementations. The hyper-parameters as well as hardware and software used are  
 419 given in Table 4.

Table 4: Hyper-Parameters of all experiments

Environments	Algorithm	Algorithm Parameters	Hardware & Software
ALE	Rainbow	Frames = $2 \times 10^6$ seeds = 5  Remaining hyper-parameters same as Hessel et al. [2017]	Hardware- CPU: 6 Intel Gold 6148 Skylake GPU: 1 NVidia V100 RAM: 32 GB  Software- Pytorch: 1.10.0 Python: 3.8
DeepMind Control Suite	SAC	Frames = $1 \times 10^6$ seeds = 5  Remaining hyper-parameters same as Haarnoja et al. [2018]	Hardware- CPU: 6 Intel Gold 6148 Skylake GPU: 1 NVidia V100 RAM: 32 GB  Software- Pytorch: 1.10.0 Python: 3.8
MinAtar	DQN	Frames = $5 \times 10^6$ seeds = 5  Remaining hyper-parameters same as Mnih et al. [2015]	Hardware- CPU: 6 Intel Gold 6148 Skylake GPU: 1 NVidia V100 RAM: 32 GB  Software- Pytorch: 1.10.0 Python: 3.8

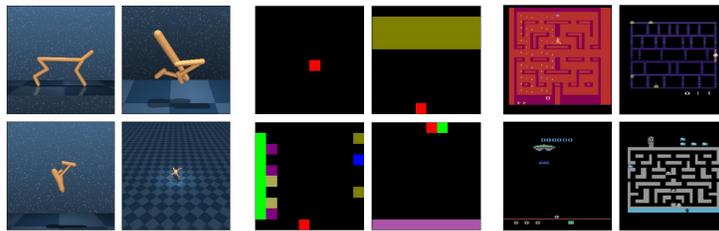


Figure 7: Visualization of a few environments from each benchmark. Left to right: DeepMind Control Suite, MinAtar, Arcade Learning Environment

420 **B DeepMind Control Suite**

421 We choose 9 environments from the DeepMind Control Suite Tassa et al. [2018] for testing the  
 422 performance of ReLo on continuous control tasks. Each agent was trained on proprioceptive inputs  
 423 from the environment for 1M frames with an action repeat of 1. The training curves for the baselines  
 424 and ReLo are given in Fig. 8.

<sup>2</sup><https://github.com/Kaixhin/Rainbow>

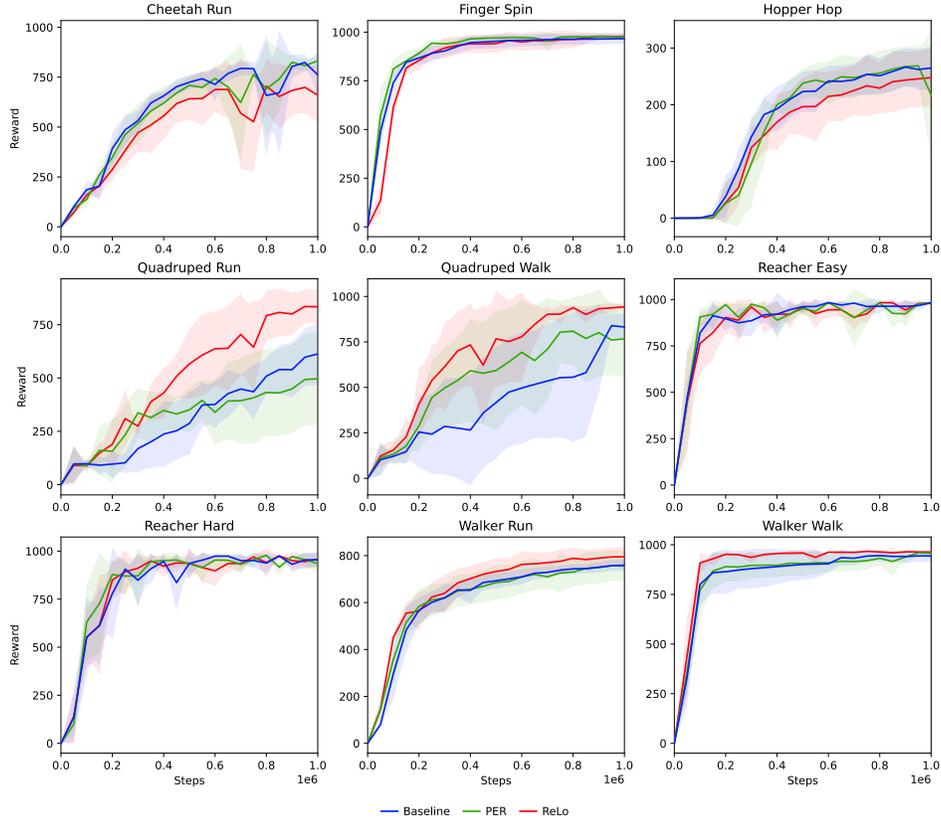


Figure 8: Training curves of environments from the DeepMind Control Suite. Performance is evaluated for 10 episodes over 5 random seeds.

### 425 C MinAtar

426 We evaluate the baselines against all 5 environments in the MinAtar suite Young and Tian [2019].  
 427 A visualization of a few environments from the suite is presented in Fig. 7. Each agent receives  
 428 the visual observations from the environment and is trained for 5M frames following the evaluation  
 methodology outlined in Young and Tian [2019]. The training curves are given in Fig. 9.

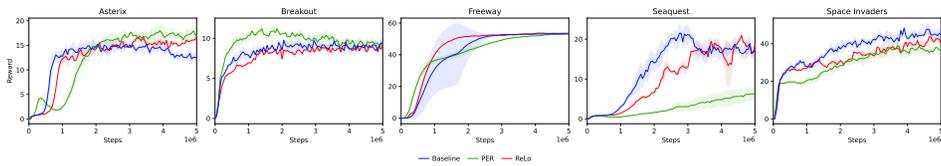


Figure 9: Training curves of environments from the MinAtar benchmark. Performance is evaluated using a running average over the last 1000 episodes over 5 random seeds.

429

### 430 D Arcade Learning Environment

431 We evaluate agents on a compute-constrained version of the Arcade Learning Environment Bellemare  
 432 et al. [2013], training each agent for 2M frames. We chose a subset of 7 games from the suite for our  
 433 evaluation. ReLo performs on par or better than vanilla PER Schaul et al. [2016] in each environment.  
 434 The training curves are given in Fig. 10.

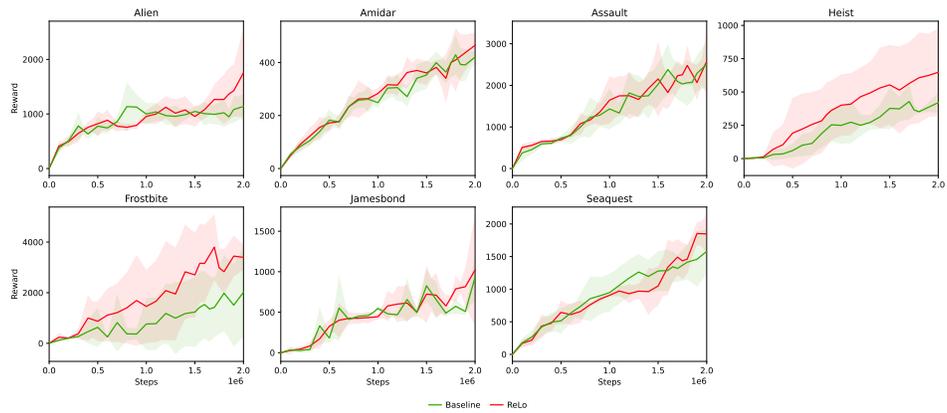


Figure 10: Training curves of 7 environments from the ALE benchmark. Performance is evaluated for 10 episodes over 5 random seeds.