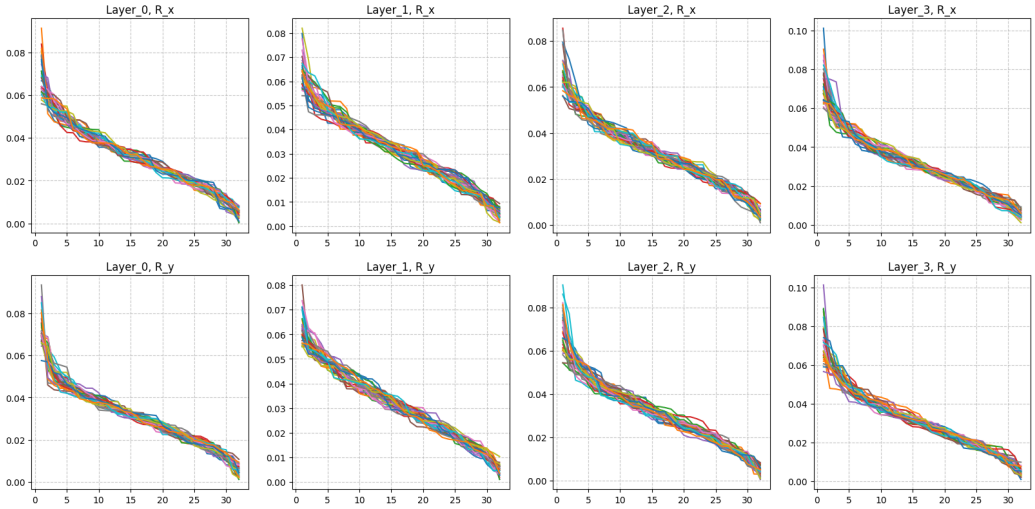
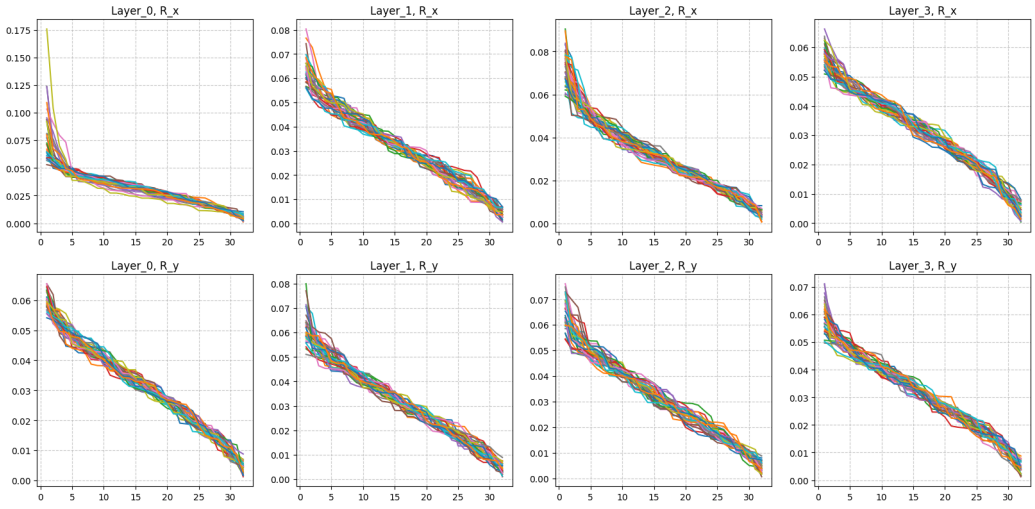
Reduced-rank Factorized Fourier Neural Operator

Supplementary Material A. The Normalized Sorted Eigenvalues of the FFNO



(a) Darcy Flow



(b) Kolmogorov Flow

Figure S1: Normalized eigenvalues of the FFNO spectral kernels $R_x^{(\ell)}, R_y^{(\ell)}$ after 1000 training epochs on (a) Darcy Flow and (b) Kolmogorov Flow. The top row presents the eigenvalues for the x-dimension kernels $R_x^{(\ell)}$, while the bottom row for the y-dimension kernels $R_y^{(\ell)}$, across different layers ℓ.

Supplementary Material B. Learning curves

Figure S2, Figure S3 and Figure S4 present the loss curves of training and testing over epochs for all the models evaluated in the three datasets. The blue line denotes the training loss, while the orange line represents the testing loss. These plots generally demonstrate that losses stabilize by the end of 1000 epochs, indicating sufficient training duration for most models. However, several models show signs of overfitting: FNO, Geo-FNO, and WNO, particularly on the Navier-Stokes dataset, exhibit a rising testing loss after approximately 150 epochs, which triggered early stopping between epochs 250 and 300. Additionally, U-FNO also demonstrated an earlier convergence point, leading to early stopping around epochs 600 to 800. It is worth noting that our proposed R^2-FFNO consistently maintains closely aligned training and testing loss curves, indicating effective mitigation of overfitting and superior generalization compared to other models.

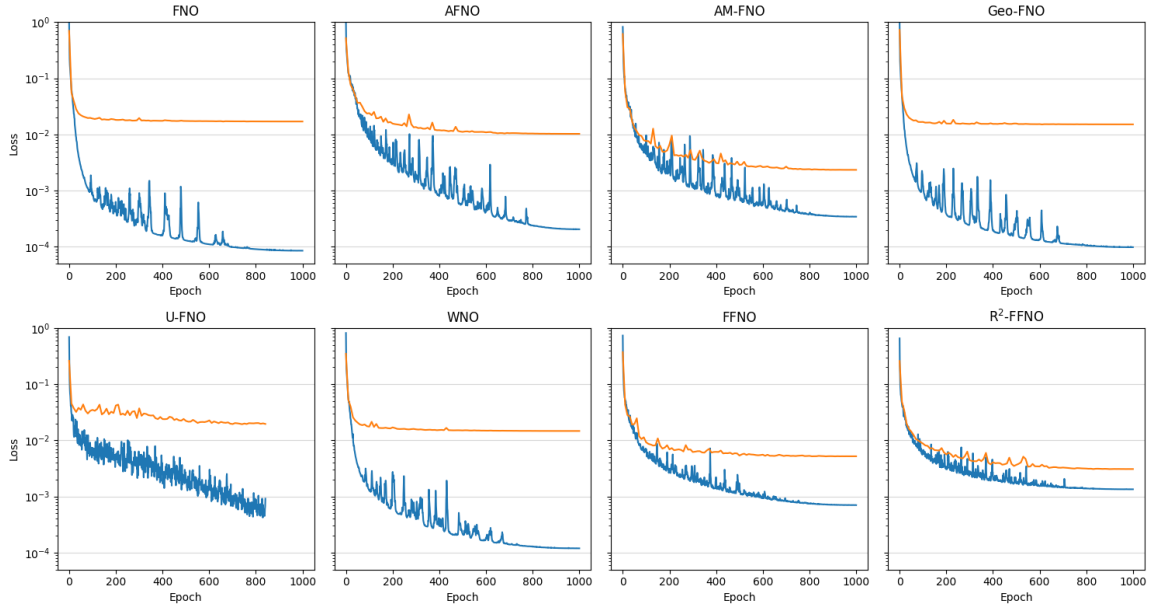


Figure S2: Training and testing loss curves over epochs of different models for the Darcy Flow datasets.

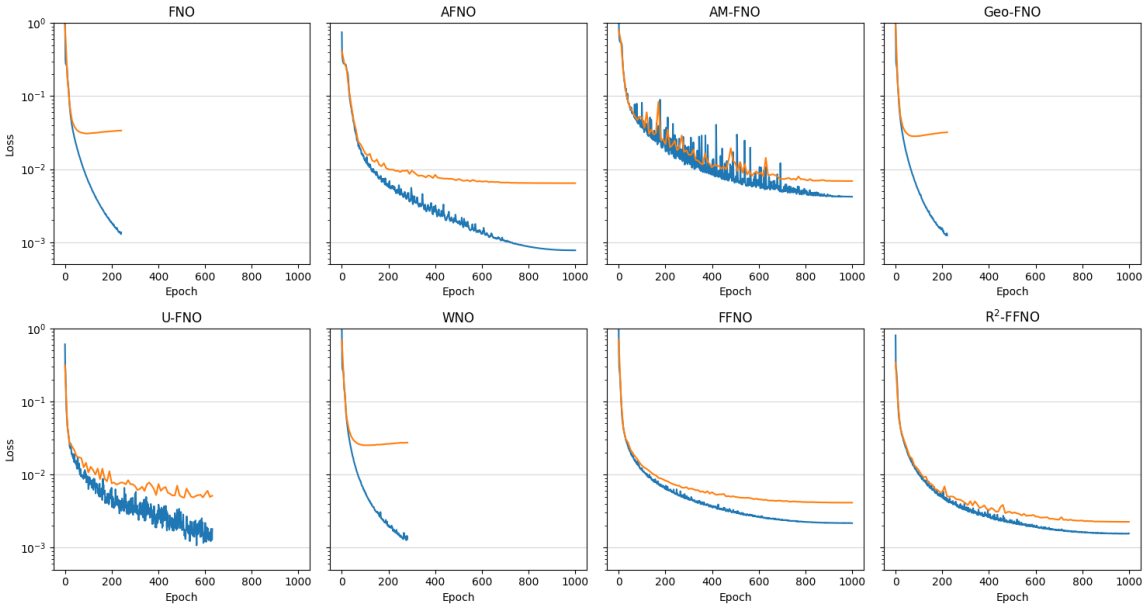


Figure S3: Training and testing loss curves over epochs of different models for the Navier-Stokes datasets.

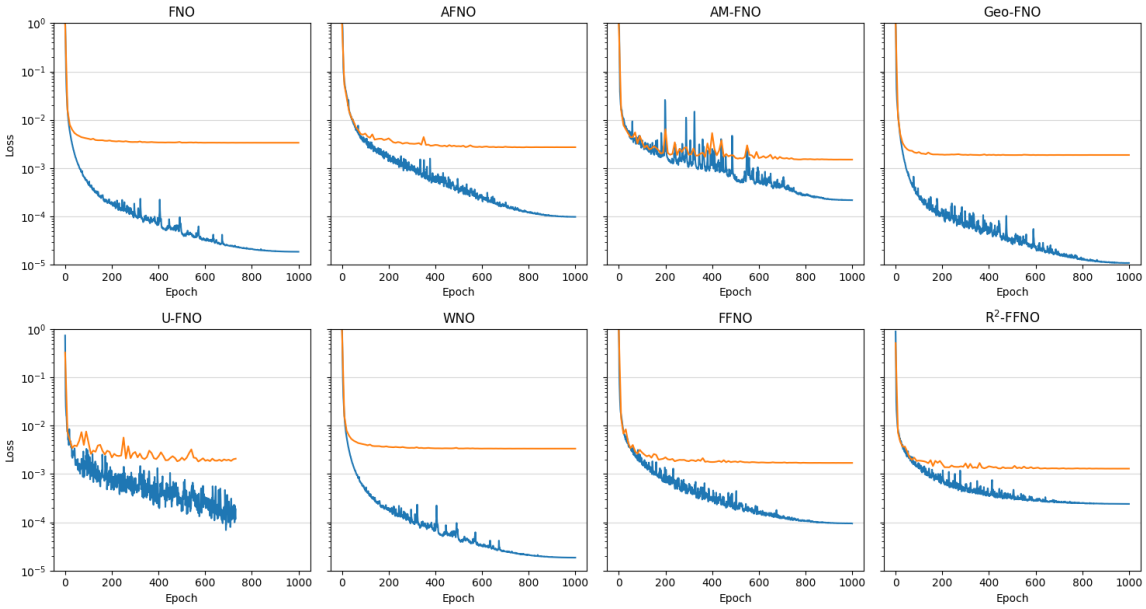


Figure S4: Training and testing loss curves over epochs of different modelss for the Kolmogorov Flow datasets.