Invertible Learned Primal-Dual

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Computed tomography inverse problem

- We consider the task of recovering a 2D/3D image $x \in X$ from noisy tomographic data/sinogram $y \in Y$, where
$$y = T(x) + \delta(x)\,.$$

Learned iterative methods

- A machine learning based approach is the problem of finding a (non-linear) parametrized mapping $T_\theta^\dagger : Y \to X$ that satisfies the pseudo-inverse property: $T_\theta^\dagger(y) \approx x$.
- Supervised training:
$$\mathrm{L}(\theta) = \mathbb{E}_{(x,y)\sim\mathcal{D}}[\ell(T_\theta^\dagger(y), x)].$$
- Training and deploying such methods for 3D CT reconstruction is still a challenge.
- Aim in this work is to explore the idea of invertibility for reducing GPU memory requirements.

Learned Primal-Dual

- This Learned Primal-Dual (LPD)[1] architecture incorporates a forward operator into a deep neural network by unrolling a proximal primal-dual optimization scheme and replacing proximal operators with convolutional neural networks (CNNs).

Algorithm 1 LPD
1: Choose initial primal and dual variables $(x_0, u_0) = \texttt{init}(y)$, where $(x_0, u_0) \in (X^N, Y^N)$
2: **For** $i = 1, 2, \ldots, M$ **do**:
3: Dual update: $u_i = u_{i-1} + \Gamma^i\left(u_{i-1}, Tx^{(2)}_{i-1}, y\right)$
4: Primal update: $x_i = x_{i-1} + \Lambda^i\left(x_{i-1}, T^* u_i^{(1)}\right)$
5: **return** $x_M^{(1)}$

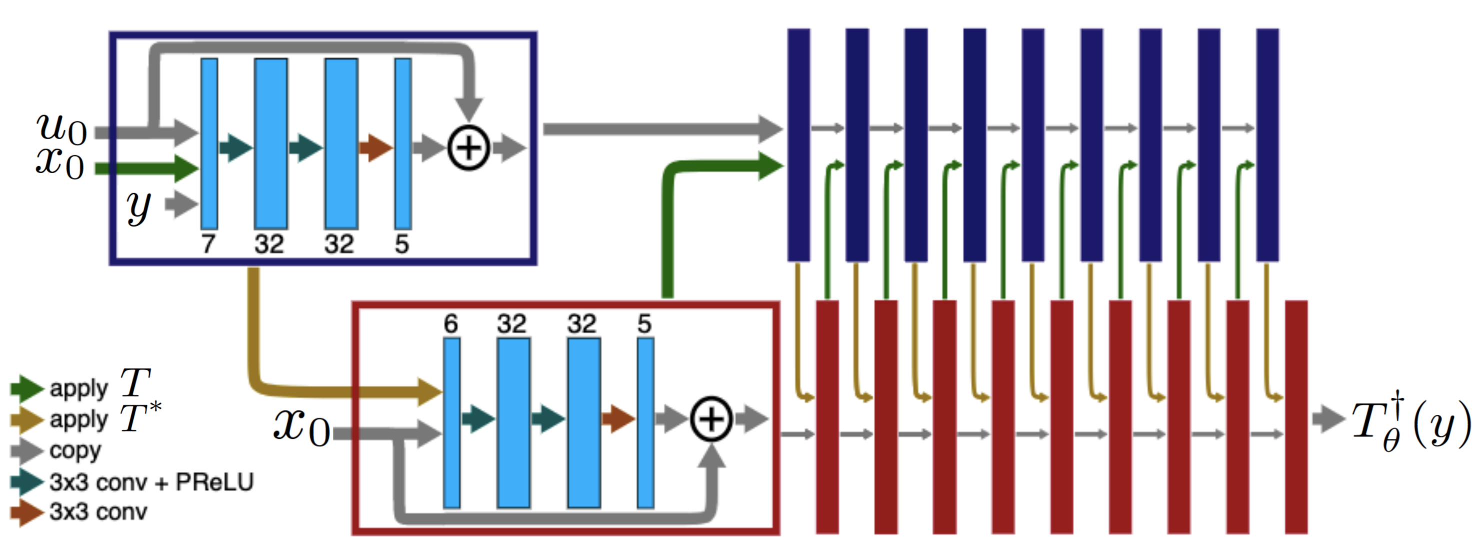


Figure 1: Learned Primal-Dual architecture.

Invertible Learned Primal-Dual

- A slight change in the LPD architecture is sufficient to make it invertible.
- One of the key benefits of invertible neural networks is that depth of the network can be increased, while maintaining a constant memory footprint.

Algorithm 2 iLPD
1: Choose initial primal and dual variables $(x_0, u_0) = \texttt{init}(y)$, where $(x_0, u_0) \in (X, Y)$
2: **For** $i = 1, 3, \ldots, 2M-1$ **do**:
3: Dual update: $\begin{cases} x_i = x_{i-1} \\ u_i = u_{i-1} + \Gamma^i(Tx_{i-1}, y) \end{cases}$
4: Primal update: $\begin{cases} x_{i+1} = x_i + \Lambda^i(T^* u_i) \\ u_{i+1} = u_i \end{cases}$
5: **return** x_{2M}

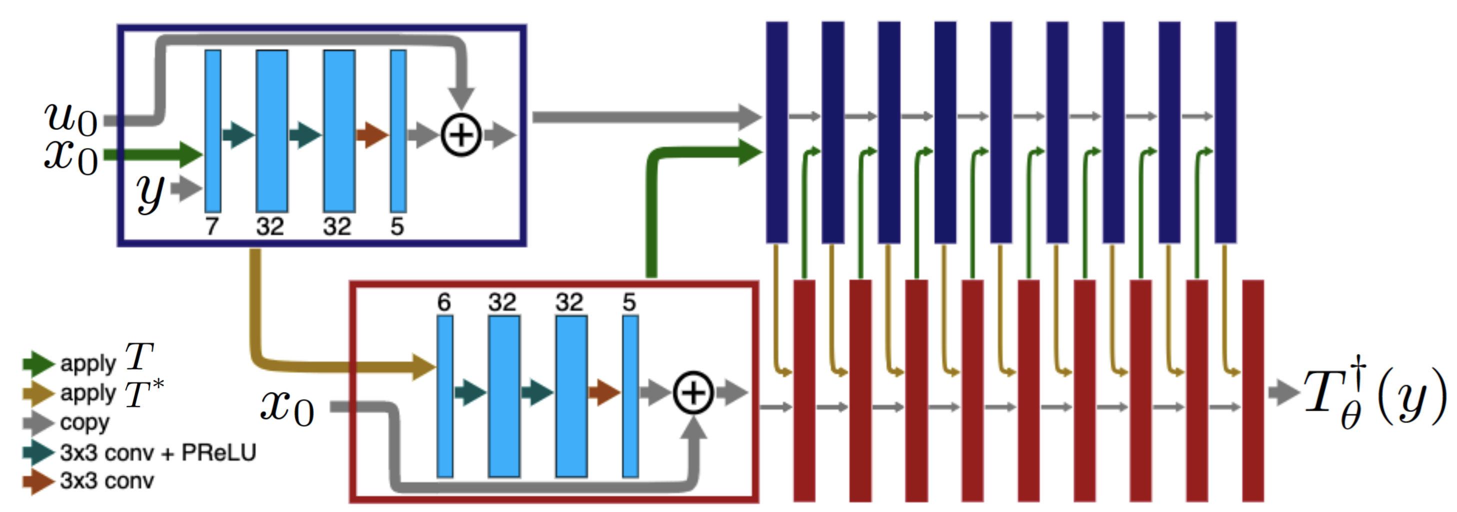


Figure 2: Invertible Learned Primal-Dual (iLPD) architecture.

- Implementation: github.com/JevgenijaAksjonova/invertible_learned_primal_dual.

Quantitative evaluation

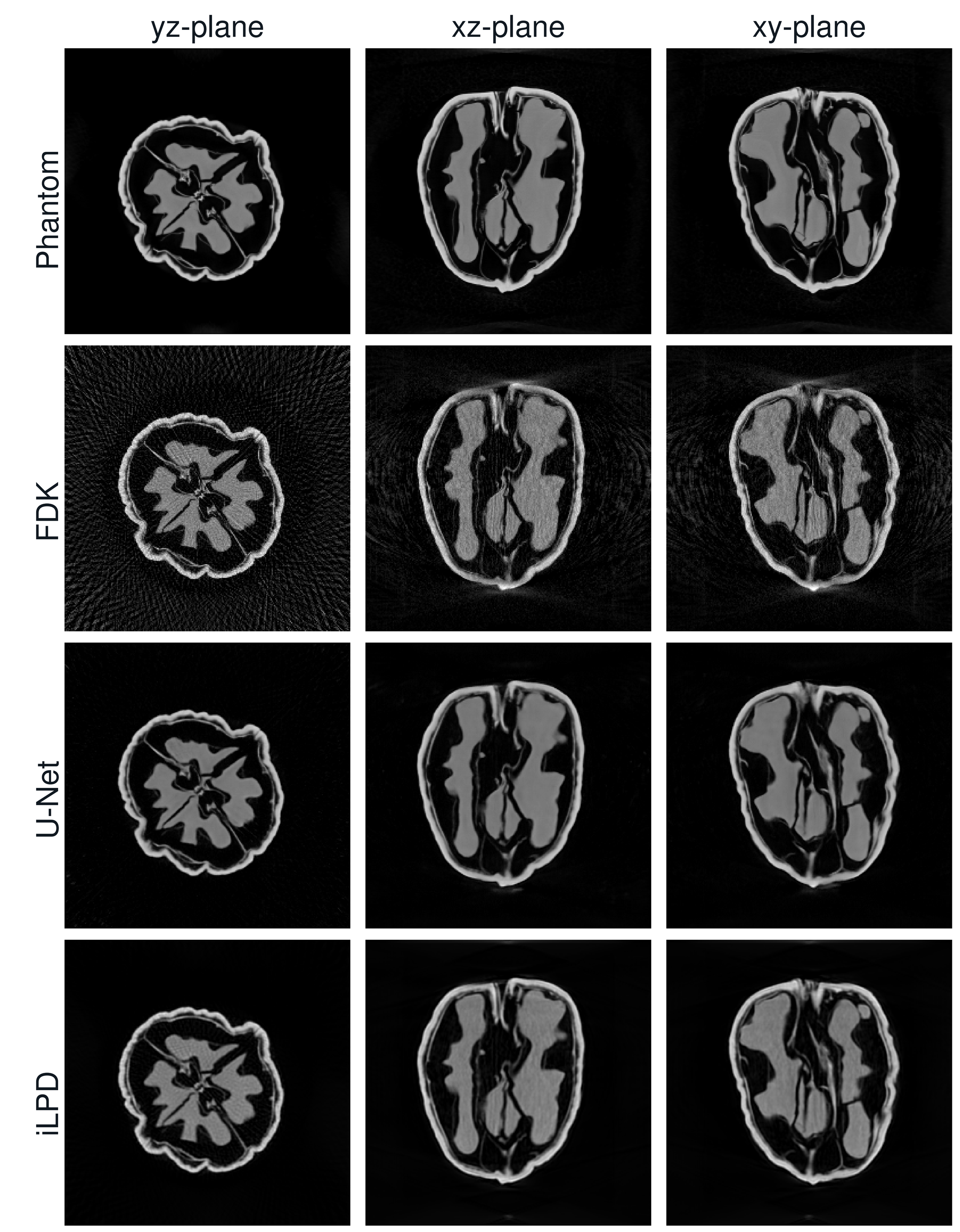
Table 1: Performance metrics for various reconstruction methods in 2D low-dose CT.

	LPD	iLPD-10	iLPD-20
PSNR	**47.05**	46.10	46.65
SSIM	**0.9997**	0.9996	0.9996
GPU Memory (MiB)	16554	**6268**	6280

Table 2: Performance metrics for various reconstruction methods in 3D sparse-angle CBCT.

	FDK[2]	nnFDK[4]	U-Net[3]	iLPD-20
PSNR	22.85	30.14	33.10	**34.68**
SSIM	0.30	0.76	0.74	**0.87**
Execution time (sec)	1.06	4.69	3.22	20.42

Qualitative evaluation



Conclusion and future work

- The proposed iLPD method requires significantly less GPU memory for training than the original LPD and therefore it is applicable for 3D CT reconstruction.
- Future work: extension to helical geometry.

References

[1] Jonas Adler and Ozan Öktem. “Learned primal-dual reconstruction”. In: *IEEE Transactions on Medical Imaging* 37.6 (2018), pp. 1322–1332.

[2] Lee A Feldkamp, Lloyd C Davis, and James W Kress. “Practical cone-beam algorithm”. In: *Josa a* 1.6 (1984), pp. 612–619.

[3] Kyong Hwan Jin et al. “Deep convolutional neural network for inverse problems in imaging”. In: *IEEE Transactions on Image Processing* 26.9 (2017), pp. 4509–4522.

[4] Marinus J Lagerwerf et al. “A computationally efficient reconstruction algorithm for circular cone-beam computed tomography using shallow neural networks”. In: *Journal of Imaging* 6.12 (2020), p. 135.