A SYNTHETIC DATASETS

In the supplementary materials, we provide further details on the three network models: BA model, PSO model, and Fitness model.

- **Barabási–Albert (BA) model**: The BA model generates networks through *preferential attachment*. Starting with a small fully connected graph, each new node added to the network connects to an existing node with probability $p \propto k$, where k is the degree (number of connections) of the existing node. This model reflects the *power-law degree distribution* found in many natural networks, where a few nodes dominate with many connections. The construction of the BA model used in this work is as follows:
  1. Generate an ER random network with the number of nodes $N_0 = 10$ and connection probability $q = 0.5$ as the initial network of the BA model.
  2. At every time step, a single new node is introduced to the network.
  3. The new node connects to n existing nodes in the network. The existing nodes are selected by the rule of preferential attachment, in which the nodes are selected with the probability proportional to their degree. Mathematically, the probability P_a that a new node added at time step t connects to an existing node a is given by:
  $$P_a = \frac{k_a}{\sum_{b=1}^{N_t} k_b},$$
  where k_a is the degree of node a, N_t is the number of the existing nodes at time step t, and k_b is the degree of node b.
  4. Iterate steps 2 and 3 until all nodes and edges are added.
- **Popularity-similarity-optimization (PSO) model**: The PSO model is designed to capture both popularity and similarity between nodes. Nodes are embedded in hyperbolic space, where each new node introduced at time step t is assigned a radial coordinate $r = \ln t$ and a random angular coordinate. The connection between the new node and existing nodes is determined by minimizing the product of the birth time s of the existing nodes and the angular distance θ_{st} between the new and existing nodes, such that nodes with the smallest $s\theta_{st}$ values are selected to connect. The radial distance corresponds to node popularity, while the angular distance represents their similarity. The parameters used in this work are set as $m = 5$, $L = 5$, $\gamma = 2.1$, $T = 0.4$, and $\zeta = 1$. This model effectively simulates social networks where both popularity and shared interests influence connections.
- **Fitness model**: In the Fitness model, each node is characterized by a *fitness value* η_i, reflecting its intrinsic ability to attract connections. The probability of a new node connecting to an existing node is given by:
  $$P_a = \frac{\eta_a k_a}{\sum_{b=1}^{N_t} \eta_b k_b}$$
  where η_a and η_b represent the fitness of nodes a and b, respectively, and k_a is the degree of node a at time t. The fitness values are drawn from a power-law distribution, $P(\eta) = \eta^{-3.5}$, emphasizing the heterogeneity in node attractiveness. This model is commonly used to simulate networks where both competitiveness and popularity influence connections, such as academic citation or business networks.