

Table 1: Results without or with the meta-formulation applied to the physics component, supporting our theory that the physics component does not suffer from the type of un-identifiability as the neural component (although the meta-formulation does still moderately improve the accuracy of its estimation).

	MSE of z_p	MSE of x (Pre)
without meta formulation	9.17(1.33)e-3	8.82(2.03)e-2
with meta formulation	5.63(0.42)e-3	3.37(0.22)e-2

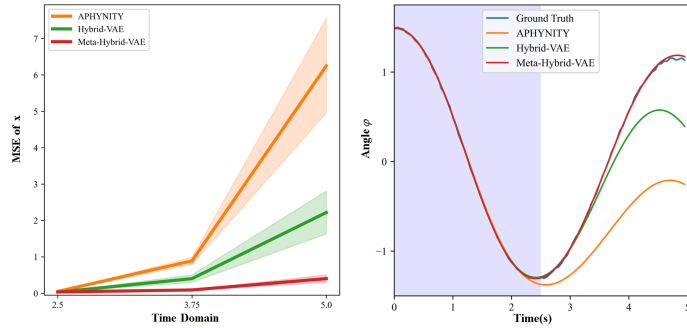


Fig 1: Left: The presented meta-hybrid-VAE significantly improved the prediction (y-axis: MSE of pendulum position) over longer time intervals beyond the training interval (0-2.5ms). Right: an example of predicted pendulum position over time.

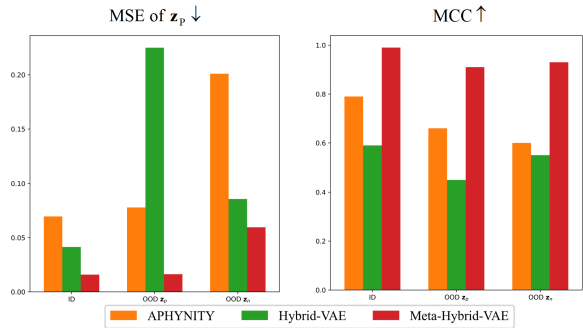


Fig 2: Identifiability results in in-distribution (ID), out-of-distribution physics component (OoD- z_p), and OoD neural component (OoD- z_n) test settings.

Table 2: The presented meta-formulation is able to improve the identifiability of a general non-hybrid VAE, similar to the identifiable-VAE (i-VAE) constructed using known class labels in [6].

	VAE	iVAE	meta-VAE
MCC	0.67 (0.04)	0.91 (0.03)	0.88 (0.00)

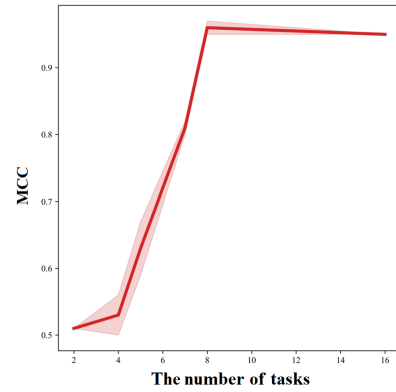


Fig 3: Empirical verification of the identifiability condition formulated in Theorem 1-iv: for a 3-dimensional parameter vector to be identified in the neural component of the hybrid model, with a Gaussian assumption of 2-dimensional sufficient statistics, a minimum of $3 \cdot 2 + 1 = 7$ distinct “tasks” (i.e., distinct parameters generating data) is needed to identify the parameters.

Table 3: The identifiability results of the presented meta-hybrid-VAE are minimally affected by the number of parameters (on Pendulum), as long as the theoretical condition for identifiability (see Fig 3) is met.

Number of parameter	MSE of z_p	MCC
2	1.28(0.01)e-2	0.97(0.01)
3	3.62(0.01)e-2	0.99(0.00)
4	1.59(0.07)e-2	0.99(0.00)

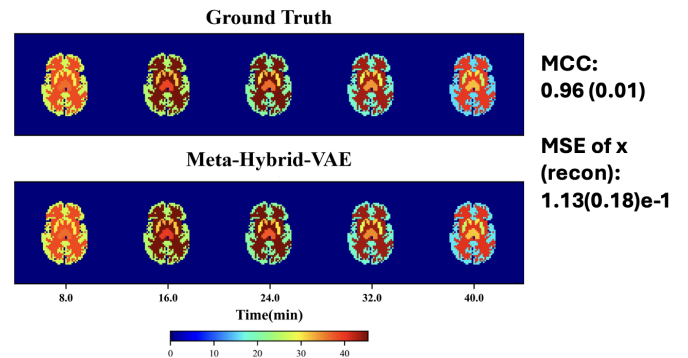


Fig 4: Identifiability results on PET images where x 's are generated from pixel-level 2-tissue compartment models, with 4 unknown kinetics parameters in each region of interest (ROI) with 5 ROIs in total. The hybrid model uses 1-tissue compartment model as the prior physics, which does not share any physics parameter with the data-generating kinetic model. The identifiability of the hybrid model is thus measured with the MCC metric.