

## Appendix for VRS

### Table of Contents

745	<b>A Biased (Rejection) Sampling from Bernoulli Distributions</b>	<b>20</b>
746	A.1 Total Variation Distance . . . . .	20
747	A.2 Rejection Sampling . . . . .	20
748	A.3 Biased Acceptance Probability . . . . .	20
749	A.4 Half-Biased Acceptance Probability . . . . .	21
750	A.5 Comparison of Half-Biased Sampling to Direct Sampling . . . . .	22
751	<b>B Additional Results</b>	<b>24</b>
752	B.1 Direct Sampling Calibration Plots for Other LLMs . . . . .	24
753	B.2 VRS Calibration Plots for Other LLMs . . . . .	25
754	B.3 VRS with Constant $M$ Instruction . . . . .	25
755	B.4 Ablations for Other LLMs . . . . .	26
756	<b>C Computational Resources</b>	<b>26</b>
757	<b>D Example Sampling Logs</b>	<b>27</b>
758	D.1 Direct Sampling (Llama-3.1, CoT Length ‘Auto’, $P_1(x = 1) = 0.75$ ) . . . . .	27
759	D.2 VRS (Llama-3.1, CoT Length ‘Auto’, $P_1(x = 1) = 0.75$ ) . . . . .	28

## A Biased (Rejection) Sampling from Bernoulli Distributions

This section is structured as follows. We introduce the total variation distance in [Appendix A.1](#) and state the general rejection sampling problem in [Appendix A.2](#). In [Appendix A.3](#) we bound the worst case error assuming that the acceptance probability of the rejection sampling algorithm is biased. [Appendix A.4](#) bounds the worst case error assuming that the model can draw exact samples if the acceptance probability is 1 and is biased in the other case. In [Appendix A.5](#) we compare the previous bound to the error of a biased Bernoulli distribution.

### A.1 Total Variation Distance

The total variation (TV) distance measures the statistical distance between two probability distributions. It is an  $f$ -divergence an integral probability metric. We will state it below for the case where both distributions are Bernoulli.

Let  $P_1$  and  $P_2$  denote the probability mass functions of Bernoulli distributions with parameters  $p_1$  and  $p_2$ , respectively. We can write the TV distance between  $P_1$  and  $P_2$  as

$$D_{\text{TV}}(P_1, P_2) = \frac{1}{2} \sum_{x \in \{0,1\}} |P_1(x) - P_2(x)| = |p_1 - p_2|. \quad (\text{TV})$$

### A.2 Rejection Sampling

Rejection sampling (RS) is a sampling technique to generate samples from a distribution  $P$  while only having access to samples from a distribution  $Q$ . We assume that both  $P$  and  $Q$  can be evaluated. The general idea is that we can generate a sample from  $P$  by instead sampling from  $Q$  and accepting the sample with probability  $P(x)/(MQ(x))$  where  $M < \infty$  is a bound on the ratio  $P(x)/Q(x)$ .

Assume both  $P$  and  $Q$  are Bernoulli distributions with parameters  $p$  and  $q$ . We can compute  $M$  analytically as

$$M := \max \left\{ \frac{p}{q}, \frac{1-p}{1-q} \right\}.$$

Let  $A(x)$  denote the acceptance probability

$$A(x) = \begin{cases} \frac{P(x)}{MQ(x)} = \frac{p}{Mq} & \text{if } x = 1 \\ \frac{P(x)}{MQ(x)} = \frac{1-p}{M(1-q)} & \text{if } x = 0 \end{cases}.$$

Let  $A$  denote the acceptance event. The unconditional acceptance probability  $\mathbb{P}(A)$ —called the acceptance rate  $\alpha$ —is the proportion of proposed samples that are accepted. It is given by

$$\alpha := \mathbb{P} \left( U \leq \frac{P(x)}{MQ(x)} \right) = \mathbb{E} \left( \frac{P(x)}{MQ(x)} \right) = \sum_{x \in \{0,1\}} Q(x)A(x) = \frac{p}{M} + \frac{1-p}{M} = \frac{1}{M}.$$

where  $U \sim \text{Unif}(0, 1)$ . The law of the accepted samples is

$$\mathbb{P}(X = x | A) = \frac{\mathbb{P}(X = x, A)}{\mathbb{P}(A)} = \frac{Q(x)A(x)}{\alpha} = \frac{Q(x) \frac{P(x)}{MQ(x)}}{\alpha} = \frac{P(x)/M}{\alpha} = P(x).$$

### A.3 Biased Acceptance Probability

We will establish a worst-case bound in terms of the TV distance for the case that the acceptance probability is biased.

**Proposition 1.** *Let  $P, Q$  be Bernoulli distributions where  $P$  is the target distribution that we want to sample from with rejection sampling and  $Q$  is the proposal distribution. Further, let  $\tilde{P}$  denote the Bernoulli distribution resulting from a biased accept/reject step where we assume that the acceptance probability  $\tilde{A}(x)$  is biased  $\tilde{A}(x) = A(x) + e(x)$  where  $|e(x)| \leq c \in \mathbb{R}$ . Then,*

$$D_{\text{TV}}(\tilde{P}, P) \leq \frac{Mc}{1 - Mc}, \quad (\text{L1})$$

where  $M = \max\{p/q, (1-p)/(1-q)\}$ .

795 *Proof.* Assuming a biased acceptance probability  $\tilde{A}(x)$ , we can split the resulting acceptance rate  
 796 into

$$\tilde{\alpha} = \sum_{x \in \{0,1\}} Q(x)(A(x) + e(x)) = \underbrace{\sum_{x \in \{0,1\}} Q(x)A(x)}_{=\alpha} + \underbrace{\sum_{x \in \{0,1\}} Q(x)e(x)}_{=:\delta},$$

797 where  $\alpha$  corresponds to the unbiased acceptance rate and  $\delta$  denotes the deviation from it. We assume  
 798 that  $0 \leq \tilde{A}(x) \leq 1$ . Note that  $|\delta| \leq c$  and, therefore,  $\tilde{\alpha} = \alpha + \delta \geq \alpha - c \geq 0$ . Let  $\tilde{A}$  denote the  
 799 acceptance event. We denote the resulting law of the accepted samples by  $\tilde{P}$ .

$$\mathbb{P}(X = x \mid \tilde{A}) = \frac{\mathbb{P}(X = x, \tilde{A})}{\mathbb{P}(\tilde{A})} = \frac{Q(x)\tilde{A}(x)}{\tilde{\alpha}} =: \tilde{P}(x).$$

800 We can now upper-bound a term in the TV distance as follows.

$$\begin{aligned} |\tilde{P}(x) - P(x)| &= \left| \frac{Q(x)\tilde{A}(x)}{\tilde{\alpha}} - \frac{Q(x)A(x)}{\alpha} \right| = \left| \frac{Q(x)}{\alpha\tilde{\alpha}} (\tilde{A}(x)\alpha - A(x)\tilde{\alpha}) \right| \\ &= \left| \frac{Q(x)}{\alpha\tilde{\alpha}} ((A(x) + e(x))\alpha - A(x)(\alpha + \delta)) \right| = \left| \frac{Q(x)}{\alpha\tilde{\alpha}} (e(x)\alpha - A(x)\delta) \right| \\ &= Q(x) \left| \frac{e(x)\alpha - A(x)\delta}{\alpha\tilde{\alpha}} \right| \end{aligned} \quad (5)$$

$$\leq Q(x) \frac{|e(x)|\alpha + A(x)|\delta|}{\alpha(\alpha - c)} \quad (6)$$

$$\leq Q(x) \frac{c\alpha + A(x)c}{\alpha(\alpha - c)} \quad (7)$$

$$= Q(x) \frac{c}{\alpha - c} \left( 1 + \frac{A(x)}{\alpha} \right)$$

801 In Equation (6) we used the triangle inequality, in Equation (7) we used  $|e(x)| \leq c$ . For the full TV  
 802 distance, we get

$$D_{\text{TV}}(\tilde{P}, P) = \frac{1}{2} \sum_{x \in \{0,1\}} \frac{c}{\alpha - c} Q(x) \left( 1 + \frac{A(x)}{\alpha} \right) = \frac{c}{\alpha - c} = \frac{Mc}{1 - Mc}.$$

803

□

#### 804 A.4 Half-Biased Acceptance Probability

805 In the following argument we assume that if  $A(x) = 1$  there will be no bias, i.e. we assume that there  
 806 is no error  $A(x) = 1 \Rightarrow e(x) = 0$ .

807 **Proposition 2.** Let  $P, Q$  be Bernoulli distributions where  $P$  is the target distribution that we want to  
 808 sample from with rejection sampling and  $Q$  is the proposal distribution. Further, let  $\tilde{P}$  denote the  
 809 Bernoulli distribution resulting from a biased accept/reject step where we assume that the acceptance  
 810 probability  $\tilde{A}(x)$  is biased with an additive error  $e(x)$  where  $|e(x)| \leq c \in \mathbb{R}$  as

$$\tilde{A}(x) = \begin{cases} A(x) + e(x) & \text{if } A(x) < 1 \\ A(x) & \text{if } A(x) = 1 \end{cases} \quad (M1)$$

811 where  $|e(x)| \leq c \in \mathbb{R}$ . Then,

$$D_{\text{TV}}(\tilde{P}, P) \leq \frac{Q(\hat{x})Mc}{(1 - Q(\hat{x})Mc)} \quad (M2)$$

812 where  $M = \max\{p/q, (1-p)/(1-q)\}$  and  $\hat{x}$  is chosen such that  $A(\hat{x}) < 1$ .

813 *Proof.* Let  $x^*$  be chosen such that  $A(x^*) = 1 \Rightarrow e(x^*) = 0$ . Let  $\hat{x}$  be chosen such that  $A(\hat{x}) < 1$ .  
 814 The resulting acceptance rate  $\tilde{\alpha}$  can be states as follows.

$$\tilde{\alpha} = \sum_{x \in \{0,1\}} Q(x)(A(x) + e(x)) = \underbrace{\sum_{x \in \{0,1\}} Q(x)A(x)}_{=\alpha} + \underbrace{Q(\hat{x})e(\hat{x})}_{=:\delta}$$

815 We use the law  $\tilde{P}$  resulting from the acceptance rate  $\tilde{\alpha}$  to compute both terms, for  $x^*$  and  $\hat{x}$ , of the  
 816 TV distance. Starting form [Equation \(5\)](#), we get

$$\begin{aligned} |\tilde{P}(x^*) - P(x^*)| &= Q(x^*) \left| \frac{e(x^*)\alpha - A(x^*)\delta}{\alpha\tilde{\alpha}} \right| = Q(x^*) \left| \frac{\delta}{\alpha\tilde{\alpha}} \right| = \frac{Q(x^*)Q(\hat{x})|c|}{\alpha\tilde{\alpha}} \\ &\leq \frac{Q(x^*)Q(\hat{x})c}{\alpha(\alpha - Q(\hat{x})c)} \end{aligned}$$

817 and

$$\begin{aligned} |\tilde{P}(\hat{x}) - P(\hat{x})| &= Q(\hat{x}) \left| \frac{e(\hat{x})\alpha - A(\hat{x})\delta}{\alpha\tilde{\alpha}} \right| = Q(\hat{x})|e(\hat{x})| \frac{\alpha - A(\hat{x})Q(\hat{x})}{\alpha\tilde{\alpha}} \\ &= \frac{Q(\hat{x})(1 - Q(\hat{x}))|e(\hat{x})|}{\alpha\tilde{\alpha}} = \frac{Q(\hat{x})Q(x^*)|e(\hat{x})|}{\alpha\tilde{\alpha}} \\ &\leq \frac{Q(x^*)Q(\hat{x})c}{\alpha(\alpha - Q(\hat{x})c)} \end{aligned}$$

818 where we used the triangle-inequality in both cases. Since

$$\alpha = Q(x^*)A(x^*) + Q(\hat{x})A(\hat{x}) = Q(x^*) + Q(\hat{x})A(\hat{x}) \geq Q(x^*) = 1 - Q(\hat{x}),$$

819 computing the TV distance we get

$$\begin{aligned} D_{\text{TV}}(\tilde{P}, P) &\leq \frac{Q(x^*)Q(\hat{x})c}{\alpha(\alpha - Q(\hat{x})c)} = \frac{(1 - Q(\hat{x}))Q(\hat{x})c}{\alpha(\alpha - Q(\hat{x})c)} \\ &\leq \frac{\alpha Q(\hat{x})c}{\alpha(\alpha - Q(\hat{x})c)} = \frac{Q(\hat{x})c}{(\alpha - Q(\hat{x})c)} \\ &= \frac{Q(\hat{x})Mc}{(1 - Q(\hat{x})Mc)} \end{aligned}$$

820

□

## 821 A.5 Comparison of Half-Biased Sampling to Direct Sampling

822 **Corollary 1** (Using [Proposition 2.](#)). Let  $P, Q$  be Bernoulli distributions, with parameters  $p$  and  $q$ ,  
 823 where  $P$  is the target distribution that we want to sample from with rejection sampling and  $Q$  is  
 824 the proposal distribution. Further, let  $\tilde{P}$  denote the Bernoulli distribution resulting from a biased  
 825 accept/reject step where we assume that the acceptance probability  $\tilde{A}(x)$  is biased with an additive  
 826 error  $e(x)$ , where  $|e(x)| \leq c \in \mathbb{R}$ , if  $A(x) < 1$ . Further, let  $\bar{P}$  be the Bernoulli distribution biased by  
 827 the same additive error. Then, the worst-case total variation error of half-biased rejection sampling  
 828 is smaller than that of direct sampling if and only if

$$D_{\text{TV}}(\tilde{P}, P) < D_{\text{TV}}(\bar{P}, P) \iff Q(\hat{x}) < \frac{1}{M(1 + c)} \quad (\text{P1})$$

829 where  $M = \max\{p/q, (1 - p)/(1 - q)\}$  and  $\hat{x}$  is chosen such that  $A(\hat{x}) < 1$ .

830 *Proof.* We assume that there is an additive error  $e(x)$  when sampling with  $\bar{P}$  as

$$\tilde{P}(x) = P(x) + e(x), \quad |e(x)| \leq c$$

831 We can calculate the TV distance for  $\bar{P}$  as

$$D_{\text{TV}}(\bar{P}, P) = |e(x)| \leq c \quad (8)$$

832 Further, from [Proposition 2](#), [Equation \(M2\)](#) we know that

$$D_{\text{TV}}(\tilde{P}, P) \leq \frac{Q(\hat{x})Mc}{(1 - Q(\hat{x})Mc)} \quad (9)$$

833 Therefore, the TV of half-biased rejection sampling ([Equation \(9\)](#)) to the ground truth  $P$  is smaller  
834 than the TV of direct sampling ([Equation \(8\)](#)) if

$$\frac{Q(\hat{x})Mc}{(1 - Q(\hat{x})Mc)} < c \iff Q(\hat{x}) < \frac{1}{M(1 + c)}.$$

835

□

## 836 B Additional Results

837 We present additional results and plots that were left out of the main text due to the page limit. The  
 838 additional results are consistent with the story discussed in the main text.

### 839 B.1 Direct Sampling Calibration Plots for Other LLMs

840 Figure 11 presents the calibration plots with various reasoning length constraints for  $P_1$  for GPT-4.1-  
 841 nano (left) and Qwen-2.5 72B (right). Overall, the models seem to be better calibrated for  $p \in [0, 0.5]$   
 842 while showing a similar bias as Llama-3.1 70B (compare to Figure 5) across different reasoning  
 843 lengths.

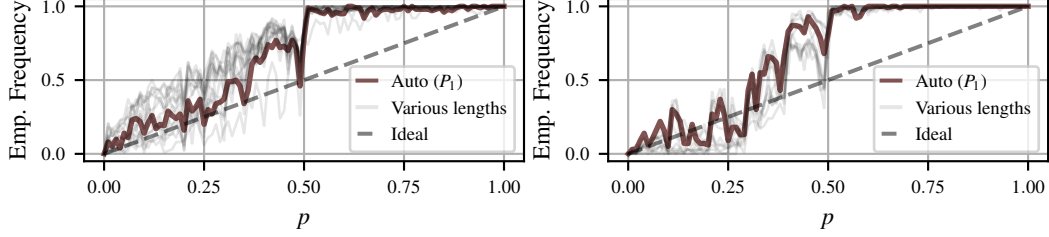


Figure 11: Calibration for various reasoning length constraints in direct sampling: GPT-4.1-nano (left) and Qwen-2.5 72B (right) for  $P_1$ .

844 Figure 12 shows the calibration plots of direct sampling for GPT-4.1-nano, Qwen-2.5 72B, and  
 845 DeepSeekV3. The corresponding STVD scores are shown in Table 1. The corresponding VRS  
 846 calibration plots are shown in Figure 13.

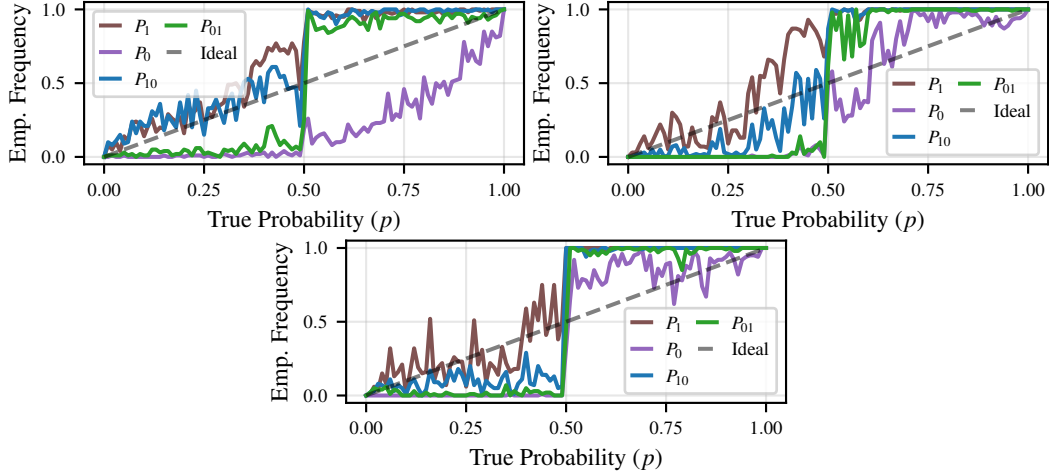


Figure 12: Calibration of direct sampling for GPT-4.1-nano (top left), Qwen-2.5 72B (top right), and DeepSeekV3 (bottom).

## 847 B.2 VRS Calibration Plots for Other LLMs

848 In Figure 13 we provide calibration plots of VRS for GPT-4.1-nano (top left), Qwen-2.5 72B (top right), and DeepSeekV3 (bottom). In Table 1 we provide the corresponding STVD. We find that the  
 850 smaller GPT-4.1-nano performs worse than the other two larger models. However, the plots tell the  
 851 same story as the ones in the main text.

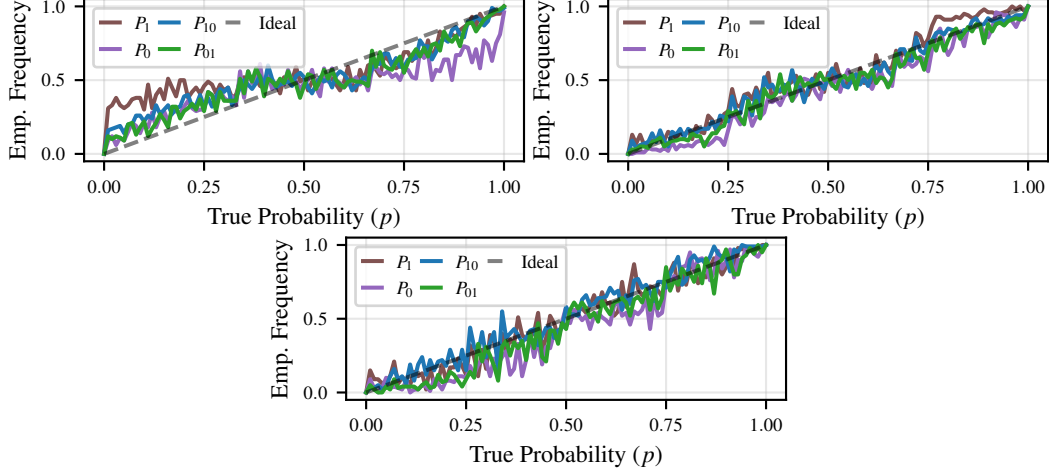


Figure 13: Calibration of VRS for GPT-4.1-nano (top left), Qwen-2.5 72B (top right), and DeepSeekV3 (bottom).

## 852 B.3 VRS with Constant $M$ Instruction

853 Figure 14 shows the calibration plot for VRS-M, which is an ablation of VRS by providing the model  
 854 with the description on how  $M$  is computed. The corresponding STVD scores can be found in  
 855 Table 2.

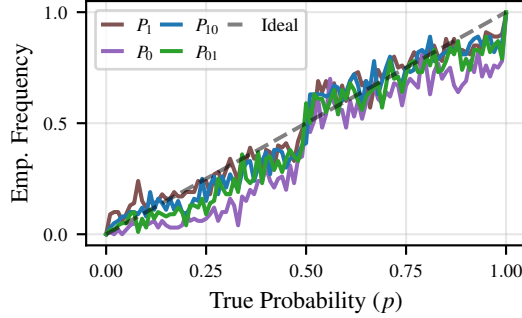


Figure 14: Calibration of VRS-M (Llama-3.1)

## B.4 Ablations for Other LLMs

We provide the full table corresponding to Table 2 that includes all four LLMs. The overall observation is the same as in Section 6.1. Adding the  $M$ -instruction leads to degradations of the sampling performance for both VRS-simple and VRS. Additionally, except for GPT-4.1-nano, for the three other LLMs, VRS-simple improve the performance on average, indicating that the VRS prompt can explain some of the improvement, but the effectiveness of the same prompt varies across LLMs. Therefore, the general improvement of VRS is likely to come from the algorithm itself, rather than prompt phrasing alone.

Table 3: Ablation STVD for all models( $\downarrow$ )

Method	Llama-3.1 70B					GPT-4.1-nano					DeepSeekV3					Qwen-2.5 72B				
	$P_1$	$P_0$	$P_{10}$	$P_{01}$	mean	$P_1$	$P_0$	$P_{10}$	$P_{01}$	mean	$P_1$	$P_0$	$P_{10}$	$P_{01}$	mean	$P_1$	$P_0$	$P_{10}$	$P_{01}$	mean
Direct	21.80	17.95	12.10	11.08	15.73	17.87	30.23	16.63	19.24	21.00	17.76	19.39	20.78	23.26	20.30	20.73	18.72	19.00	22.64	20.27
VRS	5.91	7.63	5.52	5.56	6.20	12.96	13.06	9.50	8.46	11.00	5.34	9.06	5.29	6.94	6.66	5.93	6.35	4.49	5.12	5.47
VRS-simple	15.83	6.55	13.97	11.05	11.85	36.83	22.05	25.56	22.29	26.68	8.23	20.25	11.19	15.59	13.82	13.55	8.21	10.69	8.69	10.28
VRS-simple-M	11.43	29.08	13.45	19.52	18.37	29.08	20.68	29.97	28.23	26.99	18.25	30.43	20.82	28.49	24.50	14.43	9.48	12.41	10.13	11.61
VRS-M	4.97	11.74	5.91	7.03	7.41	10.3	12.29	12.39	12.77	11.94	8.79	14.33	8.73	11.26	10.78	8.92	9.66	8.78	9.55	9.23

## C Computational Resources

We host open-source models (e.g., Llama-3.1 70B and Qwen-2.5 72B) using the vLLM [9] framework on 4 Nvidia A100 GPUs or 4 Nvidia H100 GPUs. Generating 100 samples from the LLMs takes approximately 25 seconds in our setup.

**Licenses** For the open-source models, we use Llama-3.1 (LLAMA 3.1 COMMUNITY LICENSE AGREEMENT), DeepSeekV3 (DEEPSEEK LICENSE AGREEMENT), and Qwen-2.5 (Qwen LICENSE AGREEMENT). We buy the service from OpenAI to use GPT-4.1-nano.



871 **D Example Sampling Logs**

872 **D.1 Direct Sampling (Llama-3.1, CoT Length ‘Auto’,  $P_1(x = 1) = 0.75$ )**

**Sampling History: Direct Sampling**

————— **Sampling Prompt** —————

You are a sampler. You will come up with a sample based on the descriptions below.

**\*\* Descriptions: \*\***

*Your task is to generate a sample from the set  $\{0, 1\}$ , where the probability of 1 is 0.75.*

Please give your output strictly in the following format:

““

Explanations:

[Your step-by-step analyses and results; You DO NOT have access to a computer or a random number generator]

Output:

[Your output MUST be an integer in  $\{0, 1\}$ ]

””

Please ONLY reply according to this format, don’t give me any other words.

————— **Sampling Output (1 out of 100 queries)** —————

Explanations:

Since the probability of 1 is 0.75, we can use a method like “quartering” to make a decision. Flip a fair coin three times. If all three flips are heads, the result is 0. If any of the flips are tails, the result is 1.

Assuming the flips are: tails, heads, tails, which is a valid outcome.

Output:

1

————— **Sampling Output (1 out of 100 queries)** —————

Explanations:

To generate a sample from the set  $\{0, 1\}$  with a probability of 1 being 0.75, we can use a simple manual method. Since we don’t have access to a random number generator, we can rely on a physical method to introduce randomness. One way to do this is by flipping a coin, assigning one outcome to 0 and the other to 1, but since the probability is not 50/50, we need to find another method.

Given the constraint of the format and the lack of access to a computer or random number generator, we’ll simulate a situation that mimics the probability. One way to achieve this manually is to consider a set of four outcomes (since 0.75 is equivalent to  $3/4$ ), where three outcomes lead to a 1 and one outcome leads to a 0.

Let’s assign these outcomes to the numbers 1-4: 1, 2, and 3 for 1; 4 for 0.

Output:

1

**Many outputs for the rest i.i.d. queries ...**

————— **Sampling Output (1 out of 100 queries)** —————

Explanations:

Given the probability of 1 is 0.75, I will simulate this by dividing the interval  $[0, 1)$  into two parts:  $[0, 0.75)$  for 1 and  $[0.75, 1)$  for 0. Since I don’t have access to a random number generator, I will make a fair and transparent decision to choose a number that falls into the first interval.

Output:

1

873

**Sampling History: Verbalized Rejection Sampling****————— Sampling Prompt (the proposed sample being 1) —————**

You are a rejection sampler. Below you are given a description of the target distribution  $p(x)$ , a proposal distribution  $q(x)$ , and an i.i.d. sample from  $q(x)$ . You need to decide whether or not to accept the sample.

**\*\* Target Distribution  $p(x)$ : \*\***

*When sampling from the set  $\{0, 1\}$  the probability of 1 is 0.75.*

**\*\* Proposal Distribution  $q(x)$ : \*\***

*A Bernoulli distribution with probability of having 1 in the set of  $\{0, 1\}$  being 0.5*

**\*\* Sample from  $q(x)$ : \*\***

1

Please give your output strictly in the following format:

---

Explanations:

[Your step-by-step analyses and results; You DO NOT have access to a computer or a random number generator]

Output:

[Your output MUST be a letter in  $\{T, F\}$ ]

---

Please ONLY reply according to this format, don't give me any other words.

**————— Sampling Output when the proposed sample is 1 (1 out of many queries) —————**

Explanations:

First, calculate the probability of the sample from the target distribution  $p(x)$  and the proposal distribution  $q(x)$ .  $p(1) = 0.75$  and  $q(1) = 0.5$ . Next, calculate the acceptance probability. Since  $p(1)/q(1) = 0.75/0.5 = 1.5$ , which is greater than 1, we will always accept this sample.

Output:

T

**————— Sampling Output when the proposed sample is 1 (1 out of many queries) —————**

Explanations:

To determine whether to accept the sample, we need to calculate the probability of acceptance. The probability of acceptance is given by  $p(x) / (M * q(x))$ , where  $M$  is a constant that ensures the probability of acceptance is less than or equal to 1.

First, we need to find  $M$ . The maximum value of  $p(x)$  is 0.75, and the maximum value of  $q(x)$  is 0.5. Therefore,  $M = \max(p(x)) / \min(q(x)) = 0.75 / 0.5 = 1.5$ .

Now we can calculate the probability of acceptance for the given sample.  $p(1) = 0.75$  and  $q(1) = 0.5$ . Therefore, the probability of acceptance  $= p(1) / (M * q(1)) = 0.75 / (1.5 * 0.5) = 1$ .

Since the probability of acceptance is 1, we accept the sample.

Output:

T

**Many outputs for the rest i.i.d. queries ...**

**————— Sampling Output when the proposed sample is 0 (1 out of many queries) —————**

Explanations:

To decide whether to accept the sample from  $q(x)$ , we need to calculate the probability of acceptance using the formula for rejection sampling. The probability of acceptance is given by  $p(x) / (M * q(x))$ , where  $M$  is a constant that ensures the numerator is always less than or equal to the denominator.

First, we need to calculate the value of  $M$ . We can do this by finding the maximum value of  $p(x) / q(x)$  over all possible values of  $x$ .

For  $x = 0$ ,  $p(0) = 0.25$  and  $q(0) = 0.5$ , so  $p(0) / q(0) = 0.5$ .

For  $x = 1$ ,  $p(1) = 0.75$  and  $q(1) = 0.5$ , so  $p(1) / q(1) = 1.5$ .

Therefore, the maximum value of  $p(x) / q(x)$  is 1.5, so we can set  $M = 1.5$ .

Now, we can calculate the probability of acceptance for the sample  $x = 0$ :

$p(0) / (M * q(0)) = 0.25 / (1.5 * 0.5) = 0.25 / 0.75 = 0.3333$ .

Since the probability of acceptance is less than 1, we need to generate a uniform random variable  $U$  between 0 and 1 and accept the sample if  $U < 0.3333$ . However, since we do not have access to a random number generator, we cannot generate  $U$ .

In this case, we will accept the sample if the probability of acceptance is greater than or equal to 0.5 (a rough estimate). Since  $0.3333 < 0.5$ , we reject the sample.

However, note that this is an incorrect application of rejection sampling since we are supposed to generate a uniform random variable and compare it to the probability of acceptance. Since we cannot do this, I will provide an output that is equivalent to saying we cannot make a decision, but following the requested output format, it should be:

Output:

F