A Near-Optimal Algorithm for Debiasing Trained Machine Learning Models

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Abstract

We present a scalable post-processing algorithm for debiasing trained models, including deep neural networks (DNNs), which we prove to be near-optimal by bounding its excess Bayes risk. We empirically validate its advantages on standard benchmark datasets across both classical algorithms as well as modern DNN architectures and demonstrate that it outperforms previous post-processing methods while performing on par with in-processing. In addition, we show that the proposed algorithm is particularly effective for models trained at scale where post-processing is a natural and practical choice.

1 Introduction

Background. Machine learning is increasingly applied to critical decisions which can have a lasting impact on individual lives, such as for credit lending [Bruckner, 2018], medical applications [Deo, 2015], and criminal justice [Brennan et al., 2009]. Consequently, it is imperative to understand and improve the degree of bias of such automated decision-making.

Unfortunately, despite the fact that bias (or "fairness") is a central concept in our society today, it is difficult to define it in precise terms. In fact, as people perceive ethical matters differently depending on a plethora of factors including geographical location or culture [Awad et al., 2018], no universally-agreed upon definition for bias exists. Moreover, bias may depend on the application and might even be ignored in favor of accuracy when the stakes are high, such as in medical diagnosis [Kleinberg et al., 2016]. As such, it is not surprising that several measures of bias have been introduced, such as statistical parity [Dwork et al., 2012, Zafar et al., 2017a], equality of opportunity [Hardt et al., 2016], and equalized odds [Hardt et al., 2016, Kleinberg et al., 2016], and these are not generally mutually compatible [Chouldechova, 2017, Kleinberg et al., 2016].

Let $\mathcal{X}$ be an instance space and let $\mathcal{Y} = \{0, 1\}$ be the target set in a binary classification problem. In the fair classification setting, we may further assume the existence of a sensitive attribute $s : \mathcal{X} \to \{1, \ldots, K\}$, where $s(x) = k$ if and only if $x \in X_k$ for some total partition $\mathcal{X} = \cup_k X_k$. For example, $\mathcal{X}$ might correspond to the set of job applicants while s indicates their sex. Then, a commonly used criterion for fairness is to require similar mean outcomes across the sensitive attribute (a.k.a. statistical parity) [Dwork et al., 2012, Zafar et al., 2017a, Mehrabi et al., 2019]:

Definition 1 (Statistical Parity)**.** *Let $\mathcal{X}$ be an instance space and $\mathcal{X} = \cup_k X_k$ be a total partition of $\mathcal{X}$. A predictor $h : \mathcal{X} \to [0, 1]$ satisfies ϵ statistical parity across all groups $X_1, \ldots, X_K$ if:*

$$\max_{k \in [K]} \mathbb{E}_{\boldsymbol{x}}[h(\boldsymbol{x}) \,|\, \boldsymbol{x} \in X_k] - \min_{k \in [K]} \mathbb{E}_{\boldsymbol{x}}[h(\boldsymbol{x}) \,|\, \boldsymbol{x} \in X_k] \leq \epsilon,$$

where $[K]$ denotes the set $\{1, \ldots, K\}$.

Our main contribution is to derive a near-optimal recipe for debiasing models, including deep neural networks, according to Definition 1. Specifically, we formulate the task of debiasing learned models

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as a regularized optimization problem that is solved efficiently using the projected SGD method. We show how the algorithm produces thresholding rules with randomization near the thresholds, where the width of randomization is controlled by a regularization hyperparamter. We also prove that randomization near the threshold is, in general, necessary for Bayes risk consistency. In Appendix D, we show how the proposed algorithm can be modified to handle a weaker notion of bias as well.

Besides the theoretical guarantees, we empirically validate the proposed algorithm on benchmark datasets across both classical algorithms as well as modern DNN architectures. Our experiments demonstrate that the proposed algorithm significantly outperforms previous post-processing methods and performs competitively with in-processing (Section 5). While we focus on binary sensitive attributes in the experiments, our algorithm and its guarantees continue to hold for non-binary attributes as well.

In addition, we show that the proposed algorithm is particularly effective for models trained at scale where post-processing is a natural and practical choice. Qualitatively speaking, for a fixed downstream task D, such as predicting facial attributes in the CelebA dataset [Liu et al., 2015], we say that the model is "trained at scale" if it is both: (1) heavily overparameterized, and (2) pretrained on large datasets before fine-tuning on the downstream task D. We show that the impact of debiasing models on their performance using the proposed algorithm can be improved with scale.

Remark. Because "bias" is a societal concept that cannot be reduced to metrics such as statistical parity [Chouldechova, 2017, Dixon et al., 2018, Selbst et al., 2019], our conclusions do not necessarily pertain to "fairness" in its broader sense. Rather, they hold for the narrow technical definition of statistical parity. Similarly, we conduct experiments on standard benchmark datasets, such as CelebA [Liu et al., 2015] and COCO [Lin et al., 2014], which are commonly used in the literature, as a way of validating the technical claims of this paper. Our experiments are, hence, not to be interpreted as an endorsement of those visions tasks, such as predicting facial attributes.

2 Related Work

Algorithms for fair machine learning can be broadly classified into three groups: (1) pre-processing methods, (2) in-processing methods, and (3) post-processing methods [Zafar et al., 2019].

Preprocessing algorithms transform the data into a different representation such that any classifier trained on it will not exhibit bias. This includes methods for learning a fair representation [Zemel et al., 2013, Lum and Johndrow, 2016, Bolukbasi et al., 2016, Calmon et al., 2017, Madras et al., 2018, Kamiran and Calders, 2012], label manipulation [Kamiran and Calders, 2009], data augmentation [Dixon et al., 2018], or disentanglement [Locatello et al., 2019].

On the other hand, in-processing methods constrain the behavior of learning algorithms in order to control bias. This includes methods based on adversarial training [Zhang et al., 2018] and constraint-based classification, such as by incorporating constraints on the decision margin [Zafar et al., 2019] or features [Grgić-Hlača et al., 2018]. Agarwal et al. [2018] showed that the task of learning an unbiased classifier could be reduced to a *sequence* of cost-sensitive classification problems, which could be applied to any black-box classifier. One caveat of the latter approach is that it requires solving a linear program (LP) and retraining classifiers, such as neural networks, *many* times before convergence.

The algorithm we propose in this paper is a post-processing method, which can be justified theoretically [Corbett-Davies et al., 2017, Hardt et al., 2016, Menon and Williamson, 2018, Celis et al., 2019]. Fish et al. [2016] and Woodworth et al. [2017] fall under this category. However, the former only provides generalization guarantees without consistency results while the latter proposes a two-stage approach that requires changes to the original training algorithm. Kamiran et al. [2012] also proposes a post-processing algorithm, called Reject Option Classifier (ROC), without any theoretical guarantees. In contrast, our algorithm is Bayes consistent and does not alter the original classification method. In Celis et al. [2019] and Menon and Williamson [2018], instance-dependent thresholding rules are also learned. However, our algorithm also learns to *randomize* around the threshold (Figure 1(a)) and this randomization is *key* to our algorithm both theoretically as well as experimentally (Appendix B and Section 5). Hardt et al. [2016] learns a randomized post-processing rule but our proposed algorithm outperforms it in all of our experiments (Section 5). Also, [Wei et al., 2019] is a post-processing method but it requires solving a non-linear optimization problem (for the dual variables) via ADMM and provides guarantees for approximate fairness only.

Woodworth et al. [2017] showed that the post-processing approach can be suboptimal. Nevertheless, the latter result does not contradict the statement that our post-processing rule is near-optimal because we assume that the original classifier outputs a score (i.e. a monotone transformation of an approximation to the posterior $p(\mathbf{y} = 1 \,|\, \mathbf{x})$ such as margin or softmax output) whereas Woodworth et al. [2017] assumed that the post-processing rule had access to the binary predictions only.

We argue that the proposed algorithm has distinct advantages, particularly for deep neural networks (DNNs). First, stochastic convex optimization methods can scale well to massive amounts of data [Bottou, 2010], which is often the case in deep learning today. Second, the guarantees provided by our algorithm hold w.r.t. the *binary* predictions instead of using a proxy, such as the margin as in some previous works [Zafar et al., 2017b, 2019]. Third, unlike previous reduction methods that would require retraining a deep neural network several times until convergence [Agarwal et al., 2018], which can be prohibitively expensive, our algorithm does not require retraining. Also, post-processing can be the *only* available option, such as when using machine learning as a service with out-of-the-box predictive models or due to various other constrains in data and computation [Yang et al., 2020b].

3 Near-Optimal Algorithm for Statistical Parity

Notation. We reserve boldface letters for random variables (e.g. $\mathbf{x}$), small letters for instances (e.g. x), capital letters for sets (e.g. X), and calligraphic typeface for universal sets (e.g. the instance space $\mathcal{X}$). Given a set S, $1_S(x) \in \{0,\, 1\}$ is its characteristic function. Also, we denote $[N] = \{1, \ldots, N\}$ and $[x]^+ = \max\{0,\, x\}$. We reserve $\eta(x)$ for the Bayes regressor: $\eta(x) = p(\mathbf{y} = 1 \,|\, \mathbf{x} = x)$.

Algorithm. Given a classifier outputting a probability score $\hat{p}(\mathbf{y} = 1 \,|\, \mathbf{x} = x)$, let $f(x) = 2\hat{p}(\mathbf{y} = 1 \,|\, \mathbf{x} = x) - 1$. We refer to $f(x) \in [-1,\, +1]$ as the classifier's output. Our goal is to post-process the predictions made by the classifier to control statistical parity with respect to a sensitive attribute $s : \mathcal{X} \to [K]$ according to Definition 1. To this end, instead of learning a deterministic rule, we consider *randomized* prediction rules $h(\mathbf{x})$, where $h(\mathbf{x})$ is the probability of predicting the positive class given $f(\mathbf{x})$ and $s(\mathbf{x})$. Note that we have the Markov chain: $\mathbf{x} \to (s(\mathbf{x}),\, f(\mathbf{x})) \to h(\mathbf{x})$.

A simple approach of achieving ϵ statistical parity is to output a constant prediction in each subpopulation, which is clearly suboptimal in general. As such, there is a tradeoff between accuracy and fairness. The approach we take in this work is to modify the original classifier such that the original predictions are matched as much as possible while satisfying the fairness constraints. Minimizing the probability of altering the binary predictions of the original classifier can be achieved by maximizing the inner product $\mathbb{E}_{\mathbf{x}}[h(\mathbf{x}) \cdot f(\mathbf{x})]$ (cf. Appendix B). However, maximizing this objective alone leads to *deterministic* thresholding rules which have a major drawback as illustrated in the following example.

Example 1 (Randomization is necessary)**.** *Suppose that $\mathcal{X} = \{-1, 0, 1\}$ where $p(\boldsymbol{x} = -1) = 1/2$, $p(\boldsymbol{x} = 0) = 1/3$ and $p(\boldsymbol{x} = 1) = 1/6$. Let $\eta(-1) = 0$, $\eta(0) = 1/2$ and $\eta(1) = 1$. In addition, let $\boldsymbol{s} \in \{0, 1\}$ be a sensitive attribute, where $p(\boldsymbol{s} = 1 | \boldsymbol{x} = -1) = 1/2$, $p(\boldsymbol{s} = 1 | \boldsymbol{x} = 0) = 1$, and $p(\boldsymbol{s} = 1 | \boldsymbol{x} = 1) = 0$. Then, the Bayes optimal prediction rule $h^\star(x)$ subject to statistical parity ($\epsilon = 0$) satisfies: $p(h^\star(\boldsymbol{x}) = 1 | \boldsymbol{x} = -1) = 0$, $p(h^\star(\boldsymbol{x}) = 1 | \boldsymbol{x} = 0) = 7/10$ and $p(h^\star(\boldsymbol{x}) = 1 | \boldsymbol{x} = 1) = 1$.*

As a result, randomization close to the threshold is *necessary* in the general case to achieve Bayes risk consistency. This conclusion in Example 1 remains true with approximate fairness ($\epsilon < 12/70$). In this work, we propose to achieve this by minimizing the following *regularized* objective for some hyperparameter $\gamma > 0$:

$$(\gamma/2)\, \mathbb{E}_{\mathbf{x}}[h(\mathbf{x})^2] \;-\; \mathbb{E}_{\mathbf{x}}[h(\mathbf{x}) \cdot f(\mathbf{x})]. \tag{4}$$

We prove in Appendix A that this regularization term leads to randomization around the threshold, which is critical, both theoretically (Section 4 and Appendix B) and experimentally (Section 5). Informally, γ controls the width of randomization as illustrated in Figure 1.

Let $\mathcal{X} = \cup_k X_k$ be a total partition of the instance space according to the sensitive attribute $s : \mathcal{X} \to [K]$. Denote a finite training sample by $\mathcal{S} = \{(x_1, y_1), \ldots, (x_N, y_N)\}$ and write $S_k = \mathcal{S} \cap X_k$. For each group S_k, the fairness constraint in Definition 1 over the training sample can be written as:

$$\frac{1}{|S_k|} \Big| \sum_{x_i \in S_k} (h(x_i) - \rho) \Big| \;\leq\; \frac{\epsilon}{2}, \tag{5}$$

for some hyper-parameter $\rho \in [0,\, 1]$. Precisely, if the optimization variables $h(x_i)$ satisfy the constraint (5), then Definition 1 holds over the training sample by the triangle inequality. Conversely,

Algorithm 1: Pseudocode of the Proposed Algorithm.

Input: $\gamma > 0; \rho \in [0,\ 1]; \epsilon \geq 0;\ f : \mathcal{X} \to [-1, 1]; s : \mathcal{X} \to [K]$

Output: Prediction rule: $h_\gamma(x)$

Training: Initialize $(\lambda_1, \mu_1), \ldots, (\lambda_K, \mu_K)$ to zeros. Then, repeat until convergence:

1. Sample an instance $\mathbf{x} \sim p(x)$
2. Perform the updates:

$$\lambda_{s(\mathbf{x})} \leftarrow [\lambda_{s(\mathbf{x})} - \eta g_{\lambda_{s(\mathbf{x})}}]^+, \quad \mu_{s(\mathbf{x})} \leftarrow [\mu_{s(\mathbf{x})} - \eta g_{\mu_{s(\mathbf{x})}}]^+ \tag{1}$$

where:

$$g_{\lambda_{s(\mathbf{x})}} = \frac{\epsilon}{2} + \rho + \frac{\partial}{\partial \lambda_{s(\mathbf{x})}} \xi_\gamma\big(f(\mathbf{x}) - (\lambda_{s(\mathbf{x})} - \mu_{s(\mathbf{x})})\big)$$

$$g_{\mu_{s(\mathbf{x})}} = \frac{\epsilon}{2} - \rho + \frac{\partial}{\partial \mu_{s(\mathbf{x})}} \xi_\gamma\big(f(\mathbf{x}) - (\lambda_{s(\mathbf{x})} - \mu_{s(\mathbf{x})})\big).$$

and:

$$\xi_\gamma(w) = \frac{w^2}{2\gamma} \cdot \mathbb{I}\{0 \leq w \leq \gamma\} \ + \ \big(w - \frac{\gamma}{2}\big) \cdot \mathbb{I}\{w > \gamma\} \tag{2}$$

Prediction: Given an instance x in the group X_k, predict the label $+1$ with probability $h_\gamma(x)$, where:

$$h_\gamma(x) = \big[\min\{1,\ (f(x) - \lambda_k + \mu_k)/\gamma\big]^+ \tag{3}$$

if Definition 1 holds, then the constraint (5) also holds where:

$$2\rho = \max_{k \in [K]} \mathbb{E}_{\mathbf{x}}[h(\mathbf{x}) \,|\, \mathbf{x} \in S_k] \ + \ \min_{k \in [K]} \mathbb{E}_{\mathbf{x}}[h(\mathbf{x}) \,|\, \mathbf{x} \in S_k].$$

Therefore, to learn the post-processing rule $h(x)$, we solve the optimization problem:

$$\begin{aligned} \min_{0 \leq h(x_i) \leq 1} \quad & \sum_{x_i \in \mathcal{S}} (\gamma/2)\, h(x_i)^2 \ - \ f(x_i)\, h(x_i) \\ \text{s.t.} \quad & \forall k \in [K] : \Big| \sum_{x_i \in S_k} (h(x_i) - \rho) \Big| \ \leq \ \epsilon_k, \end{aligned} \tag{6}$$

in which $\epsilon_k = |S_k|\, \epsilon/2$ for all $k \in [K]$. Using Lagrange duality we show in Appendix A that solving the above optimization problem is equivalent to Algorithm 1. In Appendix D, we show that if $\epsilon = 0$, an alternative formulation can be used to minimize the same objective while satisfying the fairness constraint but *without* introducing a hyperparameter ρ. To reiterate, $\rho \in [0, 1]$ is tuned via a validation dataset and $\gamma > 0$ is a hyperparameter that controls randomization.

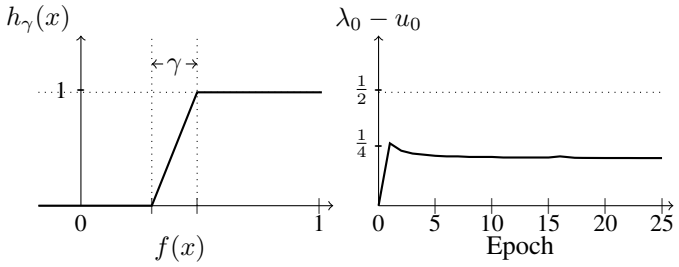
4 Theoretical Analysis

Our first theoretical result is to show that Algorithm 1 satisfies the desired fairness guarantees.

Theorem 1 (Correctness)**.** *Let $h_\gamma : \mathcal{X} \to [0, 1]$ be the randomized predictor in Equation 3 learned by applying the update rules in Equation 1 on a fresh sample of size N until convergence with learning rates satisfying the Robbins and Monro condition [Robbins and Monro, 1951]. Then, h_γ satisfies ϵ statistical parity on the training sample. Moreover, with a probability of at least $1 - \delta$, the following bound on bias holds w.r.t. the underlying distribution:*

$$\max_{k \in [K]} \mathbb{E}[h(\boldsymbol{x}) \,|\, \boldsymbol{x} \in X_k] \ - \ \min_{k \in [K]} \mathbb{E}[h(\boldsymbol{x}) \,|\, \boldsymbol{x} \in X_k] \leq \epsilon + 8\sqrt{\frac{2 \log \frac{eN}{2}}{N}} + 2\sqrt{\frac{\log \frac{2K}{\delta}}{N}}. \tag{7}$$

Proof. The proof is in Appendix A. We make use of strong duality, which holds by Slater's condition [Boyd and Vandenberghe, 2004]. The update rules correspond to the projected SGD method on the dual problem. This establishes the guarantee on the training sample. For the underlying distribution, we bound the Rademacher complexity [Bousquet et al., 2003] of the function class $\mathcal{H}_\gamma$ of Figure 1(a) by that of 0-1 thresholding rules over $\mathbb{R}$, from which a generalization bound is derived. □



(a) Decision rule (b) Convergence

Figure 1: (a) The learned post-processing rule $h_\gamma(x)$ in Equation 3 as a function of the classifier's score $f(x)$ over one subpopulation. Randomization is applied when $h_\gamma(x) \in (0, 1)$. (b) The value of $\lambda_0 - u_0$ is plotted against the number of epochs in projected SGD applied to the random forests classifier. The classifier is trained on the Adult dataset to implement statistical parity with respect to the sex attribute (cf. Section 5). We observe fast convergence in agreement with Proposition 1.

The following guarantee shows that the randomized prediction rule converges to the Bayes optimal unbiased classifier if the original classifier is Bayes consistent.

Theorem 2. *Let $h^\star = \arg\min_{h \in \mathcal{H}_\epsilon} \mathbb{E}[h(\boldsymbol{x}) \neq \boldsymbol{y}]$, where $\mathcal{H}_\epsilon$ is the set of binary predictors on $\mathcal{X}$ that satisfy fairness on the training sample according to Definition 1 for $\epsilon \geq 0$. Let $h_\gamma : \mathcal{X} \to [0, 1]$ be the randomized rule in Algorithm 1. If h_γ is trained on a fresh data of size N, then there exists a value of $\rho \in [0, 1]$ independent of N such that the following holds with a probability of at least $1 - \delta$:*

$$\mathbb{E}[\mathbb{I}\{h_\gamma(\boldsymbol{x}) \neq \boldsymbol{y}\}] \leq \mathbb{E}[\mathbb{I}\{h^\star(\boldsymbol{x}) \neq \boldsymbol{y}\}] + 2\gamma + \frac{8(2 + \frac{1}{\gamma})}{N^{\frac{1}{3}}} + \mathbb{E}|2\eta(\boldsymbol{x}) - 1 - f(\boldsymbol{x})| + 4\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}},$$

where $\eta(x) = p(\boldsymbol{y} = 1 | \boldsymbol{x} = x)$ is the Bayes regressor and K is the number of groups X_k.

Proof. The full proof is in Appendix B. First, we show that minimizing the probability of error can be achieved by maximizing $\mathbb{E}[f(\mathbf{x}) \cdot (2\eta(\mathbf{x}) - 1)]$. We use the regularized loss instead, which is strongly convex. Using Lipschitz continuity of the decision rule when $\gamma > 0$ (cf. Figure 1(a)), and the robustness framework of Xu and Mannor [2012], we prove a generalization bound and proceed with a series of inequalities to establish the main theorem. □

Thus, if the original classifier is Bayes consistent, namely $\mathbb{E}\,|2\eta(\mathbf{x}) - 1 - f(\mathbf{x})| \to 0$ as the sample size goes to infinity, and if $N \to \infty$, $\gamma \to 0^+$ and $\gamma N^{\frac{1}{3}} \to \infty$, then $\mathbb{E}[h_\gamma(\mathbf{x}) \neq \mathbf{y}] \xrightarrow{P} \mathbb{E}[h^\star(\mathbf{x}) \neq \mathbf{y}]$. Hence, Algorithm 1 converges to the *optimal* prediction rule subject to the fairness constraints.

Convergence Rate. As we show in Appendix A, the update rules in Equation 1 perform a projected stochastic gradient descent on the following optimization problem:

$$\min_{\mu, \lambda \geq 0} F = \mathbb{E}_{\mathbf{x}}\Big[\epsilon\,(\lambda_{s(\mathbf{x})} + \mu_{s(\mathbf{x})}) + \rho\,(\lambda_{s(\mathbf{x})} - \mu_{s(\mathbf{x})}) + \xi_\gamma(f(\mathbf{x}) - (\lambda_{s(\mathbf{x})} - \mu_{s(\mathbf{x})}))\Big], \tag{8}$$

where ξ_γ is given by Equation 2. The following proposition shows that the post-processing rule can be efficiently computed. In practice, we observe fast convergence as demonstrated in Figure 1(b).

Proposition 1. *Let $\mu^{(0)} = \lambda^{(0)} = 0$ and write $\mu^{(t)}, \lambda^{(t)} \in \mathbb{R}^K$ for the value of the optimization variables after t updates defined in Equation 1 for some fixed learning rate $\alpha_t = \alpha$. Let $\bar{\mu} = (1/T)\sum_{t=1}^T \mu^{(t)}(x)$ and $\bar{\lambda} = (1/T)\sum_{t=1}^T \lambda^{(t)}(x)$. Then,*

$$\mathbb{E}[\bar{F}] - F^\star \leq (1 + \rho + \epsilon)^2\alpha + \frac{||\mu^\star||_2^2 + ||\lambda^\star||_2^2}{2T\alpha}, \tag{9}$$

where $\bar{F} : \mathbb{R}^K \times \mathbb{R}^K \to \mathbb{R}$ is the objective function in (8) using the averaged solution $\bar{\mu}$ and $\bar{\lambda}$ while $F^\star$ is its optimal value. In particular, $\mathbb{E}[\bar{F}] - F^\star = \mathcal{O}(\sqrt{K/T})$ when $\alpha = \mathcal{O}(\sqrt{K/T})$.

The proof of Proposition 1 is in Appendix C. As shown in Figure 1(a), the hyperparameter γ controls the width of randomization around the thresholds. A large value of γ may reduce the accuracy of the classifier. On the other hand, γ cannot be zero because randomization around the threshold is, in general, necessary for Bayes risk consistency as shown earlier in Example 1.

5 Experiments

Baselines and Experimental Setup. We compare against three post-processing methods: (1) the algorithm of Hardt et al. [2016] (2) the shift inference method, first introduced in Saerens et al. [2002] and used more recently in Wang et al. [2020b], and (3) the Reject Option Classifier (ROC) [Kamiran et al., 2012]. We also include the reduction approach of Agarwal et al. [2018] to compare the performance against in-processing rules. We briefly review each of these methods next.

The post-processing method of Hardt et al. [2016] is a randomized post-processing rule. It was originally developed for equalized odds and equality of opportunity. Nevertheless, it can be modified to accommodate other criteria, such as statistical parity [Agarwal et al., 2018, Dudik et al., 2020].

The shift inference rule, on the other hand, is a post-hoc correction that views bias as a shift in distribution, hence the name. It is based on the identity $r(\mathbf{y}|\mathbf{s},\mathbf{x}) \propto q(\mathbf{y}|\mathbf{s},\mathbf{x}) \cdot r(\mathbf{y},\mathbf{s})/q(\mathbf{y},\mathbf{s})$, which holds for any two distributions r and q on the product space of labels $\mathbf{y}$, sensitive attributes $\mathbf{s}$, and instances $\mathbf{x}$ if they share the same marginal $r(\mathbf{x}) = q(\mathbf{x})$ [Wang et al., 2020b]. By equating, $q(\mathbf{y}|\mathbf{s},\mathbf{x})$ with the classifier's output based on the biased distribution and $r(\mathbf{y}|\mathbf{s},\mathbf{x})$ with the unbiased classifier, the predictions of the classifier $q(\mathbf{y}|\mathbf{s},\mathbf{x})$ can be post-hoc corrected for bias by multiplying its probability score with the ratio $p(\mathbf{y})p(\mathbf{s})/p(\mathbf{y},\mathbf{s})$.

The reject option classifier (ROC) proposed by Kamiran et al. [2012] is a deterministic thresholding rule. It enumerates all possible values of some tunable parameter θ up to a given precision, where $\theta = 0$ corresponds to the original classifier. Candidate thresholds are then tested on the data.

Finally, the reduction approach of Agarwal et al. [2018] is an in-processing method that can be applied to black-box classifiers but it requires retraining the model several times. More precisely, let h be a hypothesis in the space $\mathcal{H}$ and M be a matrix, Agarwal et al. [2018] showed that minimizing the error of h subject to constraints of the form $M\mu(h) \leq c$, where $\mu(h)$ is a vector of conditional moments on h of a particular form, can be reduced (with some relaxation) to a sequence of cost-sensitive classification tasks for which many algorithms can be employed.

We use the implementations of Hardt et al. [2016] and Agarwal et al. [2018] in the FairLearn software package [Dudik et al., 2020]. The training data used for the post-processing methods is always a fresh sample, i.e. different from the data used to train the original classifiers. Specifically, we split the data that was not used in the original classifier into three subsets of equal size: (1) training data for the post-processing rules, (2) validation for hyperparameter selection, and (3) test data. The value of the hyper-parameter θ of the ROC algorithm is chosen in the grid $\{0.01,\ 0.02,\ \ldots,\ 1.0\}$. In the proposed algorithm, the parameter γ is chosen in the grid $\{0.01,\ 0.02,\ 0.05,\ 0.1,\ 0.2\}$ while ρ is chosen in the gird $\mathbb{E}[\mathbf{y}] \pm \{0,\ 0.05,\ 0.1\}$. All hyper-parameters are selected based on a separate validation dataset. For the in-processing approach, we used the Exponentiated Gradient method as proposed by Agarwal et al. [2018] with its default settings in the FairLearn package (e.g. max iterations of 50).

Tabular Data. We evaluate performance on two real-world datasets, namely the Adult income dataset [Kohavi, 1996] and the Default of Credit Card Clients (DCCC) dataset [Yeh and Lien, 2009], both from the UCI Machine Learning Repository [Blake and Merz, 1998]. The Adult dataset contains 48,842 records with 14 attributes each and the goal is to predict if the income of an individual exceeds \$50K per year. The DCCC dataset contains 30,000 records with 24 attributes, and the goal is to predict if a client will default on their credit card payment. We set sex as a sensitive attribute. In DCCC, we introduce bias to the training set to study the case in which bias shows up in the training data only (e.g. due to the data curation process) but the test data remains unbiased (cf. [Torralba and Efros, 2011] and [de Vries et al., 2019] who discuss similar observations in common benchmark datasets). Specifically, if $s(\mathbf{x}) = y(\mathbf{x})$ we keep the instance and otherwise drop it with probability 0.5.

We train four classifiers: (1) random forests with depth 10, (2) k-NN with $k = 10$, (3) a two-layer neural network with 128 hidden nodes, and (4) logistic regression whose parameter C is fine-tuned from a grid of values in a logarithmic scale between 10^{-4} and 10^{4} using 10-fold cross validation. The learning rate in our algorithm is fixed to $10^{-1}(K/T)^{1/2}$, where T is the number of steps, and $\epsilon = 0$.

Table 1 (Top and Middle) shows the bias on *test* data after applying each post-processing method. The column marked as "original" corresponds to the original classifier without alteration. As shown in the table, the shift-inference method does not succeed at controlling statistical parity while ROC can fail when the original classifier's output is concentrated on a few points because it does not randomize.

Table 1: A comparison of four post-processing methods and the reduction approach of Agarwal et al. [2018] on 3 datasets. The classifiers are random forests (RF), k-NN, MLP, logisitic regression (LR), ResNet50 trained from scratch (R50/S), ResNet50 pretrained on ImageNet (R50/I), MobileNet trained from scratch (MN/S) and MobileNet pretrained on ImageNet (MN/I). Values in **bold** correspond to cases where debiasing **fails**. ROC may fail in k-NN and in neural networks because debiasing them can require randomization. Original bias in the dataset is provided in the leftmost column.

		Bias					
Dataset	Classifier	*Original*	Proposed	Hardt, 2016	Shift Inference	ROC	Reduction
ADULT (Bias = .19)	RF	*.38*	.01	.01	**.16**	.02	.01
	kNN	*.24*	.02	.01	**.08**	**.08**	.01
	MLP	*.29*	.01	.02	**.10**	.02	.01
	LR	*.39*	.01	.02	**.10**	.01	.01
DCCC (Bias = .21)	RF	*.07*	.01	.01	**.09**	.02	.01
	kNN	*.10*	.01	.01	**.18**	.02	.01
	MLP	*.13*	.01	.01	**.12**	.02	.01
	LR	*.12*	.01	.01	**.13**	.01	.01
CELEBA (Bias = .33)	R50/S	*.43*	.01	.01	**.38**	**.08**	$\star$
	R50/I	*.40*	0.02	.01	**.35**	**.15**	$\star$
	MN/S	*.35*	.01	.01	**.24**	.01	$\star$
	MN/I	*.38*	.002	.002	**.34**	**.10**	$\star$

CelebA Dataset. Our second set of experiments builds on the task of predicting the "attractiveness" attribute in the CelebA dataset [Liu et al., 2015]. We reiterate that we do not endorse the usage of vision models for such tasks, and that we report these results because they exhibit sex-related bias. CelebA contains 202,599 images of celebrities annotated with 40 binary attributes, including sex. We use two standard architectures: ResNet50 [He et al., 2016] and MobileNet [Howard et al., 2017], trained from scratch or pretrained on ImageNet ILSVRC2012 [Deng et al., 2009]. We resize images to 224×224 and train with a fixed learning rate of 0.001 until the validation error converges. We present the bias results in Table 1 (bottom). We observe that randomization is indeed necessary: ROC and Shift Inference both fail at debiasing the neural networks because they do not learn to randomize when most scores produced by neural networks are concentrated around the set $\{-1, +1\}$.

Impact on Test Accuracy. As shown in Table 2, the proposed algorithm has a much lower impact on the test accuracy compared to Hardt et al. [2016] and even improves the test accuracy in DCCC because bias was introduced in DCCC to the training data only as discussed earlier. The tradeoff curves between accuracy and bias for both the proposed algorithm and Hardt et al. [2016] are shown in Figure 2 (LEFT). Also, for a comparison with in-processing rules, we observe that the post-processing algorithm performs competitively with the reduction approach of Agarwal et al. [2018].

Impact of Scale. Models trained at scale transfer better and enjoy improved out-of-distribution robustness [Djolonga et al., 2021]. As these models are now often used in practice, we assess to which extent can these models be debiased while retaining high accuracy. We conduct 768 experiments on 16 deep neural networks architectures, pretrained on either ILSVRC2012, ImageNet-21k (a superset of ILSVRC2012 that contains 21k classes [Deng et al., 2009]), or JFT-300M (a proprietary dataset with 300M examples and 18k classes [Sun et al., 2017]). The 16 architectures are listed in Appendix E and include MobileNet [Howard et al., 2017], DenseNet [Huang et al., 2017], Big Trasnfer (BiT) models [Kolesnikov et al., 2020], and NASNetMobile [Zoph et al., 2018]. The classification tasks contain seven attribute prediction tasks in CelebA [Liu et al., 2015] as well as five classification tasks based on the COCO dataset [Lin et al., 2014]. We describe how the tasks were selected in Appendix E. The sensitive attribute is always sex in our experiments and all classification tasks are binary. Unless explicitly stated, we use $\epsilon = 0$. Moreover, in the COCO dataset, we follow the procedure of [Wang et al., 2020a] in inferring the sensitive attribute based on the image caption: we use images that contain either the word "woman" or the word "man" in their captions but not both.

In every task, we build a linear classifier on top of the pretrained features. Inspired by the HyperRule in [Kolesnikov et al., 2020], we train for 50 epochs with an initial learning rate of 0.003, which is dropped by factor of 10 after 20, 30, and 40 epochs. All images are resized to 224×224. For augmentation, we use random horizontal flipping and cropping, where we increase the dimension of the image to 248×248 before cropping an image of size 224×224 at random.

Table 2: A comparison of the test accuracy of the proposed algorithm against the algorithms of Hardt et al. [2016] and the reduction approach of Agarwal et al. [2018]. Both Shift Inference and ROC failed at debiasing all models (Table 1) so they are excluded from the comparison here.

Test Accuracy					
		Original	Proposed	Hardt, 2016	Reduction
ADULT	RF	*85.7 ± .1%*	84.4 ± .1%	81.0 ± .2%	83.9±.1%
	kNN	*86.8 ± .1%*	81.3 ± .2%	78.7 ± .2%	80.2±.1%
	MLP	*85.5 ± .2%*	83.5 ± .3%	79.7 ± .2%	83.5±.1%
	LR	*84.9 ± .2%*	83.0 ± .1%	79.4 ± .2%	83.3±.2%
DCCC	RF	*81.2 ± .2%*	81.8 ± .1%	80.6 ± .2%	81.4±.3%
	kNN	*79.6 ± .2%*	80.4 ± .2%	78.7 ± .1%	79.5±.1%
	MLP	*80.5 ± .1%*	81.3 ± .2%	78.8 ± .2%	81.3±.2%
	LR	*80.6 ± .2%*	81.7 ± .1%	78.3 ± .1%	80.5±.3%
CELEBA	R-S	*77.8%*	71.3%	65.9%	⋆
	R-I	*79.7%*	71.7%	67.5%	⋆
	M-S	*76.9%*	71.8%	66.4%	⋆
	M-I	*79.3%*	72.8%	67.5%	⋆

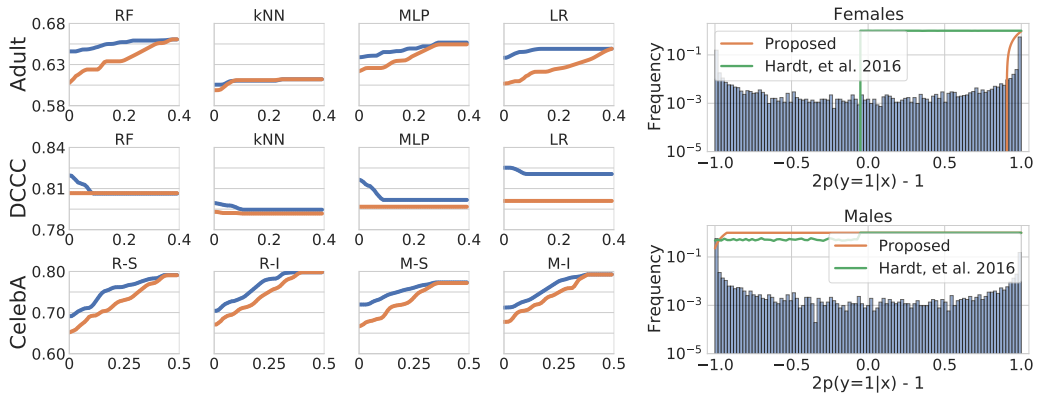


Figure 2: LEFT: The tradeoff curves are displayed for each classification problem, where blue curves are for the proposed algorithm and amber curves are for Hardt et al. [2016]. The x-axis is bias (Definition 1) while the y-axis is test accuracy. RIGHT: The distribution of the scores produced by ResNet50 trained from scratch are shown for both subpopulations. The curves correspond to $p(\mathbf{y} = 1|\mathbf{x})$ of Hardt et al. [2016] and the proposed algorithm when $\gamma = 0.1$ and $\rho = \mathbb{E}[\mathbf{y}]$.

Scaling up the Model Size. First, we examine the impact of over-parameterization in pretrained models on the effectiveness of the proposed post-processing algorithm. We fix the upstream dataset to ILSVRC2012 (8 models in total, cf. Appendix E) and aggregate the test error rates across tasks by placing them on a common scale using *soft ranking*. Specifically, we rescale all error rates in a given task linearly, so that the best error achieved is zero while the largest error is one. After that, we average the performance of each model across all tasks. Aggregated results are given in Figure 3. The impact of the proposed algorithm on test errors improves by scaling up the size of pretrained models.

Scaling up the Data. Second, we look into the impact of the size of the upstream data. We take the four BiT models ResNet50x1, ResNet50x3, ResNet101x1 and ResNet101x3, each is pretrained on either ILSVRC2012, ImageNet-21k, or JFT-300K [Kolesnikov et al., 2020]. For each model and every downstream task, we rank the upstream datasets according to the test error on the downstream task and report the average ranking. Figure 4 shows that pretraining each model on JFT-300M yields the best test accuracy when it is debiased using the proposed algorithm. To ensure that the improvement is not solely due to the data collection process, we pretrain ResNet50 on subsets of ImageNet-21k before fine-tuning on the 12 downstream tasks. Figure 5 shows, again, that the impact of the proposed post-processing rule on test errors improve when pretraining on large datasets.

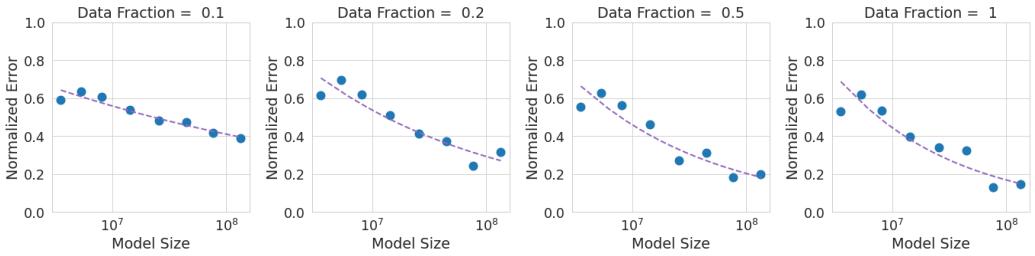


Figure 3: Aggregated performance of debiased DNN models pretrained on ILSVRC2012 across 12 classification tasks in CelebA and COCO (see Appendix E). The x-axis is the number of model parameters while the y-axis is the aggregated error rate across all tasks after normalization (see Section 5). Figures from left to right use 10%, 20%, 50%, & 100% of downstream data, respectively.

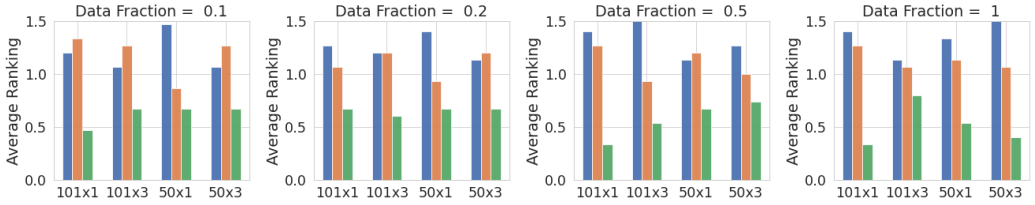


Figure 4: Aggregated performance of debiased Big Transfer (BiT) models pretrained on ILSVRC2012 (blue), ImageNet-21k (orange), or JFT-300M (green). The y-axis is the average ranking of each upstream dataset (lower is better) according to the test error rate on each of the 12 downstream classification tasks in Appendix E. Figures from left to right use 10%, 20%, 50%, & 100% of downstream data, respectively. In all models, pretraining on JFT-300K yields the best performance.

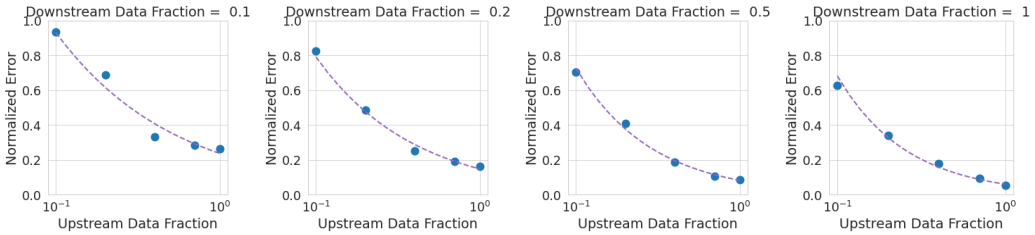


Figure 5: Aggregated performance of debiasing ResNet50 when pretrained on subsets of ImageNet-21k across 12 classification tasks in CelebA and COCO (see Appendix E). The x-axis is the fraction of ImageNet-21k used during pretraining while the y-axis follows the approach in Figure 3. Figures from left to right use 10%, 20%, 50%, & 100% of downstream data, respectively. The impact of the proposed post-processing algorithm on test errors improves when pretraining on large datasets.

6 Conclusion

The post-processing approach in fair classification enjoys many advantages. It can be applied to any classification algorithm and does not require retraining. In addition, it is sometimes the *only* option available, such as when using machine learning as a service with out-of-the-box predictive models [Obermeyer et al., 2019] or due to other constraints in data and computation [Yang et al., 2020a].

In this paper, we propose a near-optimal scalable post-processing algorithm for debiasing trained models according to statistical parity. In addition to its strong theoretical guarantees, we show that it outperforms previous post-processing methods on standard benchmark datasets across classical and modern machine learning models, and performs favorably with even in-processing methods. Finally, we show that the algorithm is particularly effective for models trained at scale, in which heavily overparameterized models are pretrained on large datasets before fine-tuning on the downstream task.

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A Proof of Theorem 1

A.1 Proof of Correctness on the Training Sample

Here we repeat the setup from Section 3 for completeness.

Suppose we have a binary classifier on the instance space $\mathcal{X}$. Let $f : \mathcal{X} \to [-1, +1]$ be its scores as described in Section 3. We would like to construct an algorithm for post-processing the predictions made by that classifier such that we control the bias with respect to a set of pairwise disjoint groups $X_1, \ldots, X_K \subseteq \mathcal{X}$ according to Definition 1. We assume that the output of the classifier $f : \mathcal{X} \to [-1, +1]$ is an estimate to $2\eta(x) - 1$, where $\eta(x) = p(\mathbf{y} = 1 | \mathbf{x} = x)$ is the Bayes regressor. This is not a strong assumption because many algorithms can be calibrated to provide probability scores [Platt et al., 1999, Guo et al., 2017] so the assumption is valid. We consider randomized rules $h(x)$ that post-process the original classifier's output $f(x)$ according to the sensitive attribute $s(x)$. Because randomization is sometimes necessary as demonstrated in Example 1, $h(x)$ is the probability of predicting the positive class when the instance is $x \in \mathcal{X}$.

In a finite training sample of size N, which we will denote by $\mathcal{S} = \{(x_1, y_1), \ldots, (x_N, y_N)\}$, let $S_k = S \cap X_k$. For each group $X_k \subseteq \mathcal{S}$, the fairness constraint in Definition 1 over the training sample can be written as:

$$\frac{1}{|S_k|} \Big| \sum_{x_i \in S_k} (h(x_i) - \rho) \Big| \;\leq\; \frac{\epsilon}{2},$$

for some hyper-parameter $\rho > 0$. Precisely, if the optimization variables $h(x_i)$ satisfy the constraint (5), then Definition 1 holds by the triangle inequality. Conversely, if Definition 1 holds, then the constraint (5) also holds where:

$$2\rho = \max_{k \in [K]} \mathbb{E}_{\mathbf{x}}[h(\mathbf{x}) \,|\, \mathbf{x} \in X_k] \;+\; \min_{k \in [K]} \mathbb{E}_{\mathbf{x}}[h(\mathbf{x}) \,|\, \mathbf{x} \in X_k].$$

To learn h, we propose solving the following *regularized* optimization problem:

$$\begin{aligned} \min_{0 \leq h(x) \leq 1} \quad & \sum_{x_i \in \mathcal{S}} (\gamma/2)\, h(x_i)^2 \;-\; f(x_i)\, h(x_i) \\ \text{s.t.} \quad & \forall k \in [K] : \Big| \sum_{x_i \in S_k} (h(x_i) - \rho) \Big| \;\leq\; \epsilon_k, \end{aligned} \tag{10}$$

where $\gamma > 0$ is a regularization parameter and $\epsilon_k = |S_k|\, \epsilon/2$ for all $k \in [K]$.

Because the groups X_k are pairwise disjoint, the optimization problem in (10) decomposes into K separate suboptimization problems, one for each group X_k. Each sub-optimization problem can be written in the following general form, where $q_i = h(x_i)$:

$$\begin{aligned} \min_{0 \leq q_i \leq 1} \quad & \sum_{i=1}^{M} \frac{\gamma}{2} q_i^2 - f(x_i) q_i \\ \text{s.t.} \quad & \sum_{i=1}^{M} (z_i q_i - b) \leq \epsilon', \qquad -\sum_{i=1}^{M} (z_i q_i - b) \leq \epsilon', \end{aligned}$$

for some values of $M \in \mathbb{N}$ and $z_i, b \in \mathbb{R}$, where $\epsilon' = M\epsilon/2$.

Note: We introduce new symbols M, z_i and b to keep the subsequent analysis general. For (10), in particular, M would correspond to the size of the group S_k, $z_i = 1$, and $b = \rho$. Later in Appendix D, we show that another criterion of bias falls into this general form so the same analysis applies over there as well.

The Lagrangian of the convex optimization problem is:

$$\begin{aligned} L(q, \alpha, \beta, \lambda, \mu) = & \sum_{i=1}^{M} \big(\frac{\gamma}{2} q_i^2 - f(x_i) q_i \big) \\ & + \lambda \big(\sum_{i=1}^{M} (z_i q_i - b) - \epsilon' \big) - \mu \big(\sum_{i=1}^{M} (z_i q_i - b) + \epsilon' \big) + \sum_{i=1}^{M} \alpha_i (q_i - 1) - \sum_{i=1}^{M} \beta_i q_i. \end{aligned}$$

Taking the derivative w.r.t. q_i gives us:

$$q_i = \frac{1}{\gamma}\Big(f(x_i) - (\lambda - \mu)z_i - \alpha_i + \beta_i\Big).$$

Plugging this back, the dual problem becomes:

$$\begin{aligned} \min_{q,\lambda,\mu,\alpha,\beta} \quad & \sum_{i=1}^{M} \big(\frac{\gamma}{2}q_i^2 + b(\lambda - \mu)\big) + (\lambda + \mu)\epsilon' + \sum_{i=1}^{M}\alpha_i \\ \text{s.t.} \quad & q_i = \frac{1}{\gamma}\Big(f(x_i) - (\lambda - \mu)z_i - \alpha_i + \beta_i\Big) \\ & \lambda, \mu, \alpha_i, \beta_i \geq 0. \end{aligned}$$

Next, we eliminate variables. By eliminating β_i, we have:

$$\begin{aligned} \min_{q,\lambda,\mu,\alpha,\beta} \quad & \sum_{i=1}^{M} \big(\frac{\gamma}{2}q_i^2 + b(\lambda - \mu)\big) + (\lambda + \mu)\epsilon' + \sum_{i=1}^{M}\alpha_i \\ \text{s.t.} \quad & q_i - \frac{1}{\gamma}\Big(f(x_i) - (\lambda - \mu)z_i - \alpha_i\Big) \geq 0 \\ & \lambda, \mu, \alpha_i \geq 0. \end{aligned}$$

Equivalently:

$$\begin{aligned} \min_{q,\lambda,\mu,\alpha,\beta} \quad & \sum_{i=1}^{M} \big(\frac{\gamma}{2}q_i^2 + b(\lambda - \mu)\big) + (\lambda + \mu)\epsilon' + \sum_{i=1}^{M}\alpha_i \\ \text{s.t.} \quad & \alpha_i \geq f(x_i) - \gamma q_i - (\lambda - \mu)z_i \\ & \lambda, \mu, \alpha_i \geq 0. \end{aligned}$$

Next, we eliminate α_i to obtain:

$$\begin{aligned} \min_{q,\lambda,\mu} \quad & \sum_{i=1}^{M} \big(\frac{\gamma}{2}q_i^2 + b(\lambda - \mu)\big) + (\lambda + \mu)\epsilon' + \sum_{i=1}^{M}\big[f(x_i) - \gamma q_i - (\lambda - \mu)z_i\big]^+ \\ s.t. \quad & \lambda, \mu \geq 0. \end{aligned}$$

Finally, we eliminate the q_i variables. For a given optimal μ and λ, it is straightforward to observe that the minimizer $q^\star$ to $\gamma/2q^2 + [w - \gamma q]^+$ must lie in the set $\{0, w/\gamma, 1\}$. In particular, if $w/\gamma \leq 0$, then $q^\star = 0$. If $w/\gamma \geq 1$, then $q^\star = 1$. Note here that we make use of the fact that $\gamma > 0$.

So, the optimal value of $q^\star$ to $\gamma/2q^2 + [w - \gamma q]^+$ is:

$$\xi_\gamma(w) = \begin{cases} 0 & \frac{w}{\gamma} \leq 0 \\ \frac{w^2}{2\gamma} & 0 \leq \frac{w}{\gamma} \leq 1 \\ w - \frac{\gamma}{2} & \frac{w}{\gamma} \geq 1 \end{cases}$$

From this, the optimization problem reduces to:

$$\min_{\lambda,\mu \geq 0} \sum_{i=1}^{M} \Big(b(\lambda - \mu) + \epsilon'(\lambda + \mu) + \xi_\gamma(f(x_i) - (\lambda - \mu)z_i)\Big). \tag{11}$$

This is a differentiable objective function and can be solved quickly using the projected gradient descent method [Boyd and Mutapcic, 2008]. The projection step here is taking the positive parts of λ and μ. This leads to the update rules in Algorithm 1.

Finally, given λ and μ, the solution of q_i is a minimizer to;

$$\frac{\gamma}{2}q_i^2 + \big[f(x_i) - \gamma q_i - (\lambda - \mu)z_i\big]^+.$$

This solution is given by Equation (3). So, we have a ramp function. In the proposed algorithm, we have $z_i = 1$ and $b = \rho$ for all examples. The Robbins and Monro conditions on the learning rate schedule guarantee convergence to the optimal solution [Robbins and Monro, 1951]. This proves Theorem 1.

A.2 Generalization to Test Data

The previous section establishes the correctness of the proposed algorithm on the training sample. Therefore, upon termination, one has for every subpopulation X_k with $S_k \doteq \mathcal{S} \cap X_k$:

$$\frac{1}{S_k}\Big|\sum_{x_i \in S_k} h(x_i) - \rho\Big| \leq \frac{\epsilon}{2}.$$

This guarantee holds on the training sample. However, since the decision rule $h(x)$ is a ramp function of the form shown in Figure 1(a), which is learned according to a fresh training sample of size N, the bias guarantee generalizes to test data as well as shown next.

First, let $\hat{\mathcal{R}}(\mathcal{H}_\gamma)$ be the conditional Rademacher complexity of the hypothesis class that comprises of functions $h_\gamma : \mathbb{R} \to [0,\, 1]$ of the form:

$$h_\gamma(z) = \begin{cases} 0 & z \leq b \\ (1/\gamma)(z-b) & b < z < b+\gamma \\ 1 & z \geq b+\gamma \end{cases},$$

which are depicted in Figure 1(a). We show that $\hat{\mathcal{R}}(\mathcal{H}_\gamma)$ is bounded by the conditional Rademacher complexity of 0-1 thresholding rules over the real line $\mathbb{R}$. By definition, for a fixed training sample $\{z_1, \ldots, z_N\}$ [Bousquet et al., 2003]:

$$\hat{\mathcal{R}}(\mathcal{H}_\gamma) = \mathbb{E}_\sigma \sup_{h \in \mathcal{H}_\gamma} \frac{1}{N}\sum_{i=1}^{N} \sigma_i h(z_i). \tag{12}$$

Given fixed instances of the Rademacher random variables $\sigma_i \in \{-1, +1\}$, let $h^\star_\sigma(z)$ be the function that achieves the supremum inside the expectation. We note that if $\sum_i \sigma_i \mathbb{I}\{0 < h^\star_\sigma(z_i) < 1\} > 0$, then the 0-1 thresholding rule $h'(z) = \mathbb{I}\{z \geq b\}$ satisfies:

$$\frac{1}{N}\sum_{i=1}^{N} \sigma_i h'(z_i) \geq \frac{1}{N}\sum_{i=1}^{N} \sigma_i h^\star_\sigma(z_i).$$

Conversely, if $\sum_i \sigma_i \mathbb{I}\{0 < h^\star_\sigma(z_i) < 1\} \leq 0$, then the 0-1 thresholding rule $h'(z) = \mathbb{I}\{z \geq b+\gamma\}$ satisfies the above inequality. This shows that the conditional Rademacher complexity of 0-1 thresholding rules is, at least, as large as the conditional Rademacher complexity of $\mathcal{H}_\epsilon$. However, by classical counting results that relate the Rademacher complexity to the VC dimension, we conclude:

$$\mathcal{R}_n(\mathcal{H}_\gamma) \leq 2\sqrt{\frac{2\log\frac{en}{2}}{N}},$$

because the VC dimension of the 0-1 thresholding rules over the real line is 2. Thus, for any fixed subpopulation X_k, one has with a probability of at least $1-\delta$:

$$\Big|\mathbb{E}[h(\mathbf{x}) \,|\, \mathbf{x} \in X_k] - \rho\Big| \leq \frac{\epsilon}{2} + 2\mathcal{R}_n(\mathcal{H}_\gamma) + \sqrt{\frac{\log\frac{2}{\delta}}{N}} \leq \frac{\epsilon}{2} + 4\sqrt{\frac{2\log\frac{en}{2}}{N}} + \sqrt{\frac{\log\frac{2}{\delta}}{N}}.$$

In addition, by the union bound, we have with a probability of at least $1-\delta$, the following inequalities all hold simultaneously:

$$\forall k \in [K] \;:\; \Big|\mathbb{E}[h(\mathbf{x}) \,|\, \mathbf{x} \in X_k] - \rho\Big| \leq \frac{\epsilon}{2} + 4\sqrt{\frac{2\log\frac{en}{2}}{N}} + \sqrt{\frac{\log\frac{2K}{\delta}}{N}}.$$

Hence, with a probability of at least $1-\delta$:

$$\max_{k\in[K]} \mathbb{E}[h(\mathbf{x} \,|\, \mathbf{x} \in X_k] \;-\; \min_{k\in[K]} \mathbb{E}[h(\mathbf{x} \,|\, \mathbf{x} \in X_k] \leq \epsilon + 8\sqrt{\frac{2\log\frac{en}{2}}{N}} + 2\sqrt{\frac{\log\frac{2K}{\delta}}{N}}. \tag{13}$$

B Proof of Theorem 2

B.1 Optimal Unbiased Predictors

We begin by proving the following result, which can be of independent interest.

Theorem 3. *Let $f^\star = \arg\min_{f:\mathcal{X}\to\{0,1\}} \mathbb{E}[\mathbb{I}\{f(\boldsymbol{x}) \neq \boldsymbol{y}\}]$ be the Bayes optimal decision rule subject to group-wise affine constraints of the form $\mathbb{E}[w_k(\boldsymbol{x}) \cdot f(\boldsymbol{x}) \,|\, \boldsymbol{x} \in X_k] = b_k$ for some fixed partition $\mathcal{X} = \cup_k X_k$. If $w_k : \mathcal{X} \to \mathbb{R}$ and $b_k \in \mathbb{R}$ are such that there exists a constant $c \in (0,1)$ in which $p(f(x) = 1) = c$ will satisfy all the affine constraints, then $f^\star$ satisfies $p(f^\star(x) = 1) = \mathbb{I}\{\eta(x) > t_k\} + \tau_k \mathbb{I}\{\eta(x) = t_k\}$, where $\eta(x) = p(\boldsymbol{y} = 1 | \boldsymbol{x} = x)$ is the Bayes regressor, $t_k \in [0,1]$ is a threshold specific to the group $X_k \subseteq \mathcal{X}$, and $\tau_k \in [0,1]$.*

Proof. Minimizing the expected misclassification error rate of a classifier f is equivalent to maximizing:

$$\begin{aligned}\mathbb{E}[f(\mathbf{x}) \cdot \mathbf{y} + (1 - f(\mathbf{x})) \cdot (1 - \mathbf{y})] &= \mathbb{E}\Big[\mathbb{E}_{\mathbf{x}}[f(\mathbf{x}) \cdot \mathbf{y} + (1 - f(\mathbf{x})) \cdot (1 - \mathbf{y})] \,\Big|\, \mathbf{x}\Big] \\ &= \mathbb{E}\Big[\mathbb{E}_{\mathbf{x}}[f(\mathbf{x}) \cdot (2\eta(\mathbf{x}) - 1)] \,\Big|\, \mathbf{x}\Big] + \mathbb{E}[1 - \eta(\mathbf{x})].\end{aligned}$$

Hence, selecting f that minimizes the misclassification error rate is equivalent to maximizing:

$$\mathbb{E}[f(\mathbf{x}) \cdot (2\eta(\mathbf{x}) - 1)]. \tag{14}$$

Instead of maximizing this directly, we consider the regularized form first. Writing $g(x) = 2\eta(x) - 1$, the optimization problem is:

$$\begin{aligned}\min_{0 \leq f(x) \leq 1} \quad & (\gamma/2)\mathbb{E}[f(\mathbf{x})^2] - \mathbb{E}[f(\mathbf{x}) \cdot g(\mathbf{x})] \\ \text{s.t.} \quad & \mathbb{E}[w(\mathbf{x}) \cdot f(\mathbf{x})] = b\end{aligned}$$

Here, we focused on one subset X_k because the optimization problem decomposes into K separate optimization problems, one for each X_k. If there exists a constant $c \in (0,1)$ such that $f(x) = c$ satisfies all the equality constraints, then Slater's condition holds so strong duality holds [Boyd and Vandenberghe, 2004]. Note that in the case of fair classification, this is always the case because having a fixed $f(x) = c$ yields a predictor that is independent of the instances so the fairness constraints are satisfied.

The Lagrangian is:

$$(\gamma/2)\mathbb{E}[f(\mathbf{x})^2] - \mathbb{E}[f(\mathbf{x}) \cdot g(\mathbf{x})] + \mu(\mathbb{E}[w(\mathbf{x}) \cdot f(\mathbf{x})] - b) + \mathbb{E}[\alpha(\mathbf{x})(f(\mathbf{x}) - 1)] - \mathbb{E}[\beta(\mathbf{x}) f(\mathbf{x})],$$

where $\alpha(x), \beta(x) \geq 0$ and $\mu \in \mathbb{R}$ are the dual variables.

Taking the derivative w.r.t. the optimization variable $f(x)$ yields:

$$\gamma f(x) = g(x) - \mu\, w(x) - \alpha(x) + \beta(x). \tag{15}$$

Therefore, the dual problem becomes:

$$\max_{\alpha(x),\beta(x)\geq 0} \quad -(2\gamma)^{-1}\, \mathbb{E}[(g(\mathbf{x}) - \mu\, w(\mathbf{x}) - \alpha(\mathbf{x}) + \beta(\mathbf{x}))^2] - b\mu - \mathbb{E}[\alpha(\mathbf{x})].$$

We use the substitution in Equation (15) to rewrite it as:

$$\begin{aligned}&\min_{\alpha(x),\beta(x)\geq 0} (\gamma/2)\, \mathbb{E}[f(\mathbf{x})^2] + b\mu + \mathbb{E}[\alpha(\mathbf{x})] \\ &\text{s.t.} \forall x \in \mathcal{X} : \gamma f(x) = g(x) - \mu\, w(x) - \alpha(x) + \beta(x).\end{aligned}$$

Next, we eliminate the multiplier $\beta(x)$ by replacing the equality constraint with an inequality:

$$\begin{aligned}&\min_{\alpha(x)\geq 0} (\gamma/2)\, \mathbb{E}[f(\mathbf{x})^2] + b\mu + \mathbb{E}[\alpha(\mathbf{x})] \\ &\text{s.t.} \forall x \in \mathcal{X} : g(x) - \gamma f(x) - \mu\, w(x) - \alpha(x) \leq 0.\end{aligned}$$

Finally, since $\alpha(x) \geq 0$ and $\alpha(x) \geq g(x) - \gamma f(x) - \mu w(x)$, the optimal solution is the minimizer to:

$$\min_{f:\mathcal{X}\to\mathbb{R}} \quad (\gamma/2)\mathbb{E}[f(\mathbf{x})^2] + b\mu + \mathbb{E}[\max\{0,\, g(\mathbf{x}) - \gamma f(\mathbf{x}) - \mu w(\mathbf{x})\}].$$

Next, let $\mu^\star$ be the optimal solution of the dual variable μ. Then, the optimization problem over f decomposes into separate problems, one for each $x \in \mathcal{X}$. We have:

$$f(x) = \arg\min_{\tau\in\mathbb{R}} \Big\{(\gamma/2)\tau^2 + [g(x) - \gamma\tau - \mu^\star\, w(x)]^+\Big\}.$$

Using the same argument in Appendix A, we deduce that $f(x)$ is of the form:

$$f(x) = \begin{cases} 0, & g(x) - \mu^\star\, w(x) \leq 0 \\ 1 & g(x) - \mu^\star\, w(x) \geq \gamma \\ (1/\gamma)\,(g(x) - \mu^\star\, w(x)) & \text{otherwise} \end{cases}$$

Finally, the statement of the theorem holds by taking the limit as $\gamma \to 0^+$. □

B.2 Excess Risk Bound

In this section, we write $\mathcal{D}$ to denote the underlying probability distribution and write $\mathcal{S}$ to denote the uniform distribution over the training sample (a.k.a. empirical distribution).

The parameter ρ stated in the theorem is given by:

$$\rho = (1/2)\,\big(\max_{k\in[K]} \mathbb{E}_{\mathbf{x}}[h^\star(\mathbf{x}) \,|\, \mathbf{x} \in X_k] + \min_{k\in[K]} \mathbb{E}_{\mathbf{x}}[h^\star(\mathbf{x}) \,|\, \mathbf{x} \in X_k]\big).$$

Note that, by definition, the optimal classifier $h^\star$ that satisfies ϵ statistical parity also satisfies the constraint in (6) with this choice of ρ. Hence, with this choice of ρ, $h^\star$ remains optimal among all possible classifiers.

Observe that the decision rule depends on x only via $f(x) \in [-1, +1]$. Hence, we write $\mathbf{z} = f(\mathbf{x})$. Since the thresholds are learned based on a fresh sample of data, the random variables $\mathbf{z}_i$ are i.i.d. In light of Equation 14, we would like to minimize the expectation of the loss $l(h_\gamma, \mathbf{x}) = -f(\mathbf{x}) \cdot h_\gamma(\mathbf{x}) = -\mathbf{z} \cdot q(\mathbf{z}) \doteq \zeta(\mathbf{z})$ for some function $q : [-1, +1] \to [0, 1]$ of the form shown in 1(a). Note that ζ is $2(1 + 1/\gamma)$-Lipschitz continuous within the same group and sensitive class. This is because the thresholds are always in the interval $[-1 - \gamma, 1 + \gamma]$; otherwise moving beyond this interval would not change the decision rule.

Let $\mathbf{h}_\gamma$ be the decision rule learned by the algorithm. Using Corollary 5 in [Xu and Mannor, 2012], we conclude that with a probability of at least $1 - \delta$:

$$\big|\mathbb{E}_{\mathcal{D}}[l(\mathbf{h}_\gamma, \mathbf{x})] - \mathbb{E}_{\mathcal{S}}[l(\mathbf{h}_\gamma, \mathbf{x})]\big| \leq \inf_{R\geq 1} \Big\{\big(\frac{4}{R}(1 + \frac{1}{\gamma}) + 2\sqrt{\frac{2(R + K)\log 2 + 2\log\frac{1}{\delta}}{N}}\Big\}.$$

Here, we used the fact that the observations $f(\mathbf{x})$ are bounded in the domain $[-1, 1]$ and that we can first partition the domain into groups X_k (K subsets) in addition to partitioning the interval $[-1, 1]$ into R smaller sub-intervals and using the Lipschitz constant. Choosing $R = N^{\frac{1}{3}}$ and simplifying gives us with a probability of at least $1 - \delta$:

$$\big|\mathbb{E}_{\mathcal{D}}[l(\mathbf{h}_\gamma, \mathbf{x})] - \mathbb{E}_{\mathcal{S}}[l(\mathbf{h}_\gamma, \mathbf{x})]\big| \leq \frac{4(2 + \frac{1}{\gamma})}{N^{\frac{1}{3}}} + 2\sqrt{\frac{2K + 2\log\frac{1}{\delta}}{N}}.$$

Define $\mathbf{h}^\star_\gamma$ to be the minimizer of:

$$(\gamma/2)\mathbb{E}[h(\mathbf{x})^2] - \mathbb{E}[h(\mathbf{x}) \cdot f(\mathbf{x})],$$

subject to the fairness constraints. Then, the same generalization bound above also applies to the decision rule $\mathbf{h}^\star_\gamma$ because the ϵ-cover (Definition 1 in [Xu and Mannor, 2012]) is independent of the

choice of the thresholds. By the union bound, we have with a probability of at least $1-\delta$, *both* of the following inequalities hold:

$$\left|\mathbb{E}_{\mathcal{D}}[l(\mathbf{h}_\gamma, \mathbf{x})] - \mathbb{E}_{\mathcal{S}}[l(\mathbf{h}_\gamma, \mathbf{x})]\right| \leq \frac{4(2+\frac{1}{\gamma})}{N^{\frac{1}{3}}} + 2\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}} \tag{16}$$

$$\left|\mathbb{E}_{\mathcal{D}}[l(\mathbf{h}^\star_\gamma, \mathbf{x})] - \mathbb{E}_{\mathcal{S}}[l(\mathbf{h}^\star_\gamma, \mathbf{x})]\right| \leq \frac{4(2+\frac{1}{\gamma})}{N^{\frac{1}{3}}} + 2\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}}. \tag{17}$$

In particular:

$$\begin{aligned}
\mathbb{E}_{\mathcal{D}}[l(\mathbf{h}_\gamma, \mathbf{x})] &\leq \mathbb{E}_{\mathcal{S}}[l(\mathbf{h}_\gamma, \mathbf{x})] + \frac{4(2+\frac{1}{\gamma})}{N^{\frac{1}{3}}} + 2\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}} \\
&\leq \mathbb{E}_{\mathcal{S}}[l(\mathbf{h}^\star_\gamma, \mathbf{x})] + \gamma + \frac{4(2+\frac{1}{\gamma})}{N^{\frac{1}{3}}} + 2\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}} \\
&\leq \mathbb{E}_{\mathcal{D}}[l(\mathbf{h}^\star_\gamma, \mathbf{x})] + \gamma + \frac{8(2+\frac{1}{\gamma})}{N^{\frac{1}{3}}} + 4\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}}.
\end{aligned}$$

The first inequality follows from Equation (16). The second inequality follows from the fact that $\mathbf{h}_\gamma$ is an empirical risk minimizer to the regularized loss. The last inequality follows from Equation (17).

Finally, we know that the thresholding rule $\mathbf{h}^\star_\gamma$ with width $\gamma > 0$ is, by definition, a minimizer to:

$$(\gamma/2)\mathbb{E}[h(\mathbf{x})^2] - \mathbb{E}[h(\mathbf{x}) \cdot f(\mathbf{x})]$$

among all possible bounded functions $h : \mathcal{X} \to [0, 1]$ subject to the desired fairness constraints. Therefore, we have:

$$(\gamma/2)\mathbb{E}[\mathbf{h}^\star_\gamma(\mathbf{x})^2] - \mathbb{E}[\mathbf{h}^\star_\gamma(\mathbf{x}) \cdot f(\mathbf{x})] \leq (\gamma/2)\mathbb{E}[\mathbf{h}^\star(\mathbf{x})^2] - \mathbb{E}[\mathbf{h}^\star(\mathbf{x}) \cdot f(\mathbf{x})]$$

Hence:

$$\mathbb{E}[l(\mathbf{h}^\star_\gamma, \mathbf{x})] = -\mathbb{E}[\mathbf{h}^\star_\gamma(\mathbf{x}) \cdot f(\mathbf{x})] \leq \gamma + \mathbb{E}[l(\mathbf{h}^\star, \mathbf{x})]$$

This implies the desired bound:

$$\mathbb{E}_{\mathcal{D}}[l(\tilde{\mathbf{h}}_\gamma, \mathbf{x})] \leq \mathbb{E}_{\mathcal{D}}[l(\mathbf{h}^\star, \mathbf{x})] + 2\gamma + \frac{8(2+\frac{1}{\gamma})}{N^{\frac{1}{3}}} + 4\sqrt{\frac{2K + 2\log\frac{2}{\delta}}{N}}.$$

Therefore, we have consistency if $N \to \infty$, $\gamma \to 0^+$ and $\gamma N^{\frac{1}{3}} \to \infty$. For example, this holds if $\gamma = O(N^{-\frac{1}{6}})$.

So far, we have assumed that the output of the original classifier coincides with the Bayes regressor. If the original classifier is Bayes consistent, i.e. $\mathbb{E}[|2\eta(\mathbf{x}) - 1 - f(\mathbf{x})|] \to 0$ as $N \to \infty$, then we have Bayes consistency of the post-processing rule by the triangle inequality.

C Proof of Proposition 1

Proof. Since $|\xi'_\gamma(w)| \leq 1$ (see Equation 2), the derivative squared in the stochastic loss in Equation 8 w.r.t. the optimization variable γ at a point $\mathbf{x}$ is bounded by $(1+\rho+\epsilon)^2$ at all rounds. The same holds for the other optimization variable μ. Therefore, the norm squared of the gradient w.r.t. (λ, μ) is bounded by $2(1+\rho+\epsilon)^2$. Following the proof steps of [Boyd and Mutapcic, 2008] and using the fact that projections are contraction mappings, one obtains:

$$\begin{aligned}
\sum_{t=1}^{T}\left(\mathbb{E}[F^{(t)}] - F^\star\right) &\leq \frac{||\mu^\star||_2^2 + ||\lambda^\star||_2^2 + 2(1+\rho+\epsilon)^2 T\alpha^2}{2\alpha} \\
&= (1+\rho+\epsilon)^2\alpha T + \frac{||\mu^\star||_2^2 + ||\lambda^\star||_2^2}{2\alpha}.
\end{aligned}$$

Dividing both sides by T, we have by Jensen's inequality $\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[F^{(t)}] \geq \mathbb{E}[F(\bar{\lambda},\bar{\mu})]$. Plugging this into the earlier results yields:

$$\mathbb{E}[\bar{F}] - F^{\star} \leq (1+\rho+\epsilon)^2\alpha + \frac{||\mu^{\star}||_2^2 + ||\lambda^{\star}||_2^2}{2T\alpha}.$$

□

D Extension to Other Criteria

D.1 Controlling the Covariance

The proposed algorithm can be adjusted to control bias according to other criteria as well besides statistical parity. For example, we demonstrate in this section how the proposed post-processing algorithm can be adjusted to control the *covariance* between the classifier's prediction and the sensitive attribute when both are binary random variables.

Let $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \{0,1\}$ be random variables. Let $C(\mathbf{a},\mathbf{b}) \doteq \mathbb{E}[\mathbf{a}\cdot\mathbf{b}] - \mathbb{E}[\mathbf{a}]\cdot\mathbb{E}[\mathbf{b}]$ be their covariance, and $C(\mathbf{a},\mathbf{b}\,|\,\mathbf{c})$ their covariance conditioned on $\mathbf{c}$:

$$C(\mathbf{a},\mathbf{b}\mid \mathbf{c}=c) = \mathbb{E}[\mathbf{a}\cdot\mathbf{b}\,|\,\mathbf{c}=c] - \mathbb{E}[\mathbf{a}\,|\,\mathbf{c}=c]\cdot\mathbb{E}[\mathbf{b}\,|\,\mathbf{c}=c]. \tag{18}$$

Then, one possible criterion for measuring bias is to measure the conditional/unconditional covariance between the classifier's predictions and the sensitive attribute when both are binary random variables. Because the random variables are binary, it is straightforward to show that achieving zero covariance implies independence. Hence, this is equivalent to statistical parity when $\epsilon = 0$. The advantage of this formulation, as will be shown next, is that it does not include a hyperparameter ρ. The disadvantage, however, is that it can only accommodate binary sensitive attributes.

Suppose we have a binary classifier on the instance space $\mathcal{X}$. We would like to construct an algorithm for post-processing the predictions made by that classifier such that we guarantee $|C\big(f(\mathbf{x}),\, 1_S(\mathbf{x})\,|\,\mathbf{x}\in X_k\big)| \leq \epsilon$, where $\mathcal{X} = \cup_k X_k$ is a total partition of the instance space. Informally, this states that the fairness guarantee with respect to the senstiive attribute $1_S : \mathcal{X} \to \{0,1\}$ holds within each subgroup X_k.

We assume, again, that the output of the classifier $f : \mathcal{X} \to [-1,+1]$ is an estimate to $2\eta(x)-1$, where $\eta(x) = p(\mathbf{y}=1|\mathbf{x}=x)$ is the Bayes regressor and consider randomized rules of the form:

$$h : \{0,1\}\times\{1,2,\ldots,K\}\times[-1,\,1] \to [0,1],$$

whose arguments are: (i) the sensitive attribute $1_S : \mathcal{X} \to \{0,1\}$, (ii) the sub-group membership $k : \mathcal{X} \to [K]$, and (iii) the original classifier's score $f(x)$. Because randomization is sometimes necessary as proved in Section 4, $h(x)$ is the probability of predicting the positive class when the instance is $x \in \mathcal{X}$.

Similar to before, if we have a training sample of size N, which we will denote by $\mathcal{S}$, we denote $S_k = \mathcal{S}\cap X_k$. The desired fairness constraint on the covariance can be written as:

$$\frac{1}{|S_k|}\Big|\sum_{x_i\in S_k}(1_S(i)-\rho_k)\,h(x_i)\Big| \;\leq\; \epsilon,$$

where $\rho_k = \mathbb{E}_{\mathbf{x}}[1_S(\mathbf{x})\,|\,\mathbf{x}\in X_k]$. This is because:

$$\begin{aligned}\frac{1}{|S_k|}\sum_{x_i\in S_k}(1_S(i)-\rho_k)\,h(x_i) &= \frac{1}{|S_k|}\sum_{x_i\in S_k}1_S(i)\,h(x_i) - \frac{\rho_k}{|S_k|}\sum_{x_i\in S_k}h(x_i)\\ &= \mathbb{E}[1_S(\mathbf{x})\cdot h(\mathbf{x})\,|\,\mathbf{x}\in S_k] - \mathbb{E}[1_S(\mathbf{x})|\,\mathbf{x}\in S_k]\cdot\mathbb{E}[h(\mathbf{x})\,|\,\mathbf{x}\in S_k]\\ &= C(h(\mathbf{x}),\,1_S(\mathbf{x})\,|\,\mathbf{x}\in S_k),\end{aligned}$$

where the expectation is over the training sample. Therefore, in order to learn h, we solve the regularized optimization problem:

$$\begin{aligned}\min_{0\leq h(x_i)\leq 1}\quad & \sum_{i=1}^{N}(\gamma/2)\,h(x_i)^2 \;-\; f(x_i)\,h(x_i)\\ \text{s.t.}\quad & \forall k\in[K]: \Big|\sum_{x_i\in S_k}(1_S(i)-\rho_k)\,h(x_i)\Big| \;\leq\; \epsilon_k\end{aligned} \tag{19}$$

where $\gamma > 0$ is a regularization parameter and $\epsilon_k = |S_k|\,\epsilon$. This is of the same general form analyzed in Section A. Hence, the same algorithm can be applied with $b = 0$ and $z_i = 1_S(i) - \rho_k$.

D.2 Impossibility Result

The previous algorithm for controlling covariance requires that the subgroups X_k be known in advance. Indeed, our next impossibility result shows that this is, in general, necessary. In other words, a deterministic classifier $h : \mathcal{X} \to \{0, 1\}$ cannot be universally unbiased with respect to a sensitive class S across all possible known and unknown groups unless the representation $\mathbf{x}$ has zero mutual information with the sensitive attribute or if h is constant almost everywhere. As a corollary, the groups X_k have to be known *in advance*.

Proposition 2 (Impossibility result)**.** *Let $\mathcal{X}$ be the instance space and $\mathcal{Y} = \{0, 1\}$ be a target set. Let $1_S : \mathcal{X} \to \{0, 1\}$ be an arbitrary (possibly randomized) binary-valued function on $\mathcal{X}$ and define $\gamma : \mathcal{X} \to [0, 1]$ by $\gamma(x) = p(1_S(\boldsymbol{x}) = 1 \,|\, \boldsymbol{x} = x)$, where the probability is evaluated over the randomness of $1_S : \mathcal{X} \to \{0, 1\}$. Write $\bar{\gamma} = \mathbb{E}_{\boldsymbol{x}}[\gamma(\boldsymbol{x})]$. Then, for any binary predictor $h : \mathcal{X} \to \{0, 1\}$ it holds that*

$$\sup_{\pi:\,\mathcal{X}\to\{0,1\}} \Big\{ \mathbb{E}_{\pi(\boldsymbol{x})} \big| \mathcal{C}\big(h(\boldsymbol{x}), \gamma(\boldsymbol{x}) |\, \pi(\boldsymbol{x})\big) \big| \Big\} \geq \frac{1}{2}\, \mathbb{E}_{\boldsymbol{x}} |\gamma(\boldsymbol{x}) - \bar{\gamma}| \cdot \min\{\mathbb{E}f, 1 - \mathbb{E}f\}, \tag{20}$$

where $\mathcal{C}\big(f(\boldsymbol{x}), \gamma(\boldsymbol{x}) |\, \pi(\boldsymbol{x})\big)$ is defined in Equation 18.

Proof. Fix $0 < \beta < 1$ and consider the subset:

$$W = \{x \in \mathcal{X} :\ (\gamma(x) - \bar{\gamma}) \cdot (f(x) - \beta) > 0\},$$

and its complement $\bar{W} = \mathcal{X} \setminus W$. Since $f(x) \in \{0, 1\}$, the sets W and $\bar{W}$ are independent of β as long as it remains in the open interval $(0,\ 1)$. More precisely:

$$W = \begin{cases} \gamma(x) - \bar{\gamma} > 0 & \wedge \quad f(x) = 1 \\ \gamma(x) - \bar{\gamma} \leq 0 & \wedge \quad f(x) = 0 \end{cases}$$

Now, for any set $X \subseteq \mathcal{X}$, let p_X be the projection of the probability measure $p(x)$ on the set X (i.e. $p_X(x) = p(x)/p(X)$). Then, with a simple algebraic manipulation, one has the identity:

$$\mathbb{E}_{\mathbf{x}\sim p_X}[(\gamma(\mathbf{x}) - \bar{\gamma})\,(f(\mathbf{x}) - \beta)] = C(\gamma(\mathbf{x}), f(\mathbf{x}); \mathbf{x} \in X) + (\mathbb{E}_{\mathbf{x}\sim p_X}[\gamma] - \bar{\gamma}) \cdot (\mathbb{E}_{\mathbf{x}\sim p_X}[f] - \beta) \tag{21}$$

By definition of W, we have:

$$\begin{aligned} \mathbb{E}_{\mathbf{x}\sim p_W}[(\gamma(\mathbf{x}) - \bar{\gamma})(f(\mathbf{x}) - \beta)] &= \mathbb{E}_{\mathbf{x}\sim p_W}[|\gamma(\mathbf{x}) - \bar{\gamma}||f(\mathbf{x}) - \beta|] \\ &\geq \min\{\beta, 1 - \beta\}\mathbb{E}_{\mathbf{x}\sim p_W}|\gamma(\mathbf{x}) - \bar{\gamma}| \end{aligned}$$

Combining this with Equation (21), we have:

$$C(\gamma(\mathbf{x}), f(\mathbf{x}); \mathbf{x} \in W) \geq \min\{\beta, 1 - \beta\}\mathbb{E}_{\mathbf{x}\sim p_W}|\gamma(\mathbf{x}) - \bar{\gamma}| + (\mathbb{E}_{\mathbf{x}\sim p_W}[\gamma] - \bar{\gamma})(\beta - \mathbb{E}_{\mathbf{x}\sim p_W}[f]) \tag{22}$$

Since the set W does not change when β is varied in the open interval $(0,\ 1)$, the lower bound holds for any value of $\beta \in (0, 1)$. We set:

$$\beta = \bar{f} \doteq \frac{1}{2}\big(\mathbb{E}_{\mathbf{x}\sim p_W} f(\mathbf{x}) \ + \ \mathbb{E}_{\mathbf{x}\sim p_{\bar{W}}} f(\mathbf{x})\big) \tag{23}$$

Substituting the last equation into Equation (22) gives the lower bound:

$$\begin{aligned} &C(\gamma(\mathbf{x}), f(\mathbf{x}); \mathbf{x} \in W) \geq \\ &\quad \min\{\bar{f}, 1 - \bar{f}\} \cdot \mathbb{E}_{\mathbf{x}\sim p_W}|\gamma(\mathbf{x}) - \bar{\gamma}| + \frac{1}{2}(\mathbb{E}_{\mathbf{x}\sim p_W}[\gamma] - \bar{\gamma})\,\big(\mathbb{E}_{\mathbf{x}\sim p_W} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p_{\bar{W}}} f(\mathbf{x})\big) \end{aligned} \tag{24}$$

Repeating the same analysis for the subset $\bar{W}$, we arrive at the inequality:

$$\begin{aligned} &C(\gamma(\mathbf{x}), f(\mathbf{x}); \mathbf{x} \in \bar{W}) \\ \leq \quad &- \min\{\bar{f}, 1 - \bar{f}\}\,\mathbb{E}_{\mathbf{x}\sim p_{\bar{W}}}|\gamma(\mathbf{x}) - \bar{\gamma}| + \frac{1}{2}(\mathbb{E}_{\mathbf{x}\sim p_{\bar{W}}}[\gamma] - \bar{\gamma})\,\big(\mathbb{E}_{\mathbf{x}\sim p_W} f(\mathbf{x}) - \mathbb{E}_{\mathbf{x}\sim p_{\bar{W}}} f(\mathbf{x})\big) \end{aligned} \tag{25}$$

Writing $\pi(x) = 1_W(x)$, we have by the reverse triangle inequality:

$$\mathbb{E}_{\pi(\mathbf{x})} \left| \mathcal{C}\big(f(\mathbf{x}), \gamma(\mathbf{x});\ \pi(\mathbf{x})\big) \right| \geq \min\{\bar{f}, 1-\bar{f}\} \cdot \mathbb{E}_{\mathbf{x}} |\gamma(\mathbf{x}) - \bar{\gamma}|. \tag{26}$$

Finally:

$$2\bar{f} \geq p(\mathbf{x} \in W) \cdot \mathbb{E}_{\mathbf{x} \sim p_W} f(\mathbf{x}) \ + \ p(\mathbf{x} \in \bar{W}) \cdot \mathbb{E}_{\mathbf{x} \sim p_{\bar{W}}} f(\mathbf{x}) = \mathbb{E}[f].$$

Similarly, we have $2(1-\bar{f}) \geq 1 - \mathbb{E}[f]$. Therefore:

$$\min\{\bar{f},\ 1-\bar{f}\} \geq \frac{1}{2}\ \min\{\mathbb{E}f,\ 1-\mathbb{E}f\}.$$

Combining this with Equation (26) establishes the statement of the proposition. □

E Training at Scale Experiment Setup

E.1 Architectures

The 16 DNN models are:

1. **S-R50x1/1**: A BiT ResNet50 model pretrained on ILSRCV2012.
2. **M-R50x1/1**: A BiT ResNet50 model pretrained on ImageNet-21k.
3. **L-R50x1/1**: A BiT ResNet50 model pretrained on JFT-300M.
4. **S-R50x3/1**: A BiT ResNet50 model 3x wide, pretrained on ILSRCV2012.
5. **M-R50x3/1**: A BiT ResNet50 model 3x wide, pretrained on ImageNet-21k.
6. **L-R50x3/1**: A BiT ResNet50 model 3x wide, pretrained on JFT-300M.
7. **S-R101x1/1**: A BiT ResNet101 model pretrained on ILSRCV2012.
8. **M-R101x1/1**: A BiT ResNet101 model pretrained on ImageNet-21k.
9. **L-R101x1/1**: A BiT ResNet101 model pretrained on JFT-300M.
10. **S-R101x3/1**: A BiT ResNet101 model 3x wide pretrained on ILSRCV2012.
11. **M-R101x3/1**: A BiT ResNet101 model 3x wide pretrained on ImageNet-21k.
12. **L-R101x3/1**: A BiT ResNet101 model 3x wide pretrained on JFT-300M.
13. **MobileNetV2**: pretrained on ILSRCV2012 [Howard et al., 2017].
14. **DenseNet121**: pretrained on ILSRCV2012 [Huang et al., 2017].
15. **DenseNet169**: pretrained on ILSRCV2012 [Huang et al., 2017].
16. **NASNetMobile**: pretrained on ILSRCV2012 [Zoph et al., 2018].

Big Transfer (BiT) models are described in Kolesnikov et al. [2020].

E.2 Downstream Tasks

The downstream classification tasks are all in CelebA [Liu et al., 2015] and COCO datasets [Lin et al., 2014]. In CelebA, we choose seven attributes that are not immediately related to sex: (1) Smiling, (2) Young, (3) Attractiveness, (4) Narrow Eyes, (5) Oval Face, (6) Pale Skin, and (7) Pointy Nose. We reiterate that we conduct experiments on such vision tasks as a way of validating the technical claims of this paper. Our experiments are not to be interpreted as an endorsement of these visions tasks.

In COCO, we choose the most frequent five objects among images that contain individuals whose sex can be reliably identified from the image caption (see Section 5). The five objects are: Chair (Object ID: 56), Car (Object ID: 2), Handbag (Object ID: 26), Skateboard (Object ID: 36), Tennis Racket (Object ID: 38).