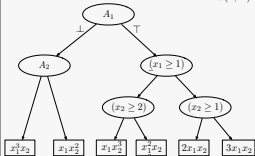
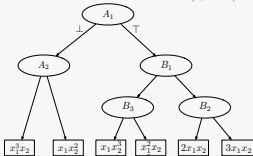


**a**

$$\varphi \stackrel{\text{def}}{=} \top, \chi \stackrel{\text{def}}{=} [x_1 \in [0, 2]] \wedge [x_2 \in [0, 3]]$$

**b** $w(\mathbf{x}, \mathbf{A})$ **c**

$$\varphi^*(\mathbf{x}, \mathbf{A} \cup \mathbf{B}) \stackrel{\text{def}}{=} \varphi \wedge \chi \wedge B_1 \leftrightarrow (x_1 \geq 1) \wedge \\ B_2 \leftrightarrow (x_2 \geq 1) \wedge B_3 \leftrightarrow (x_2 \geq 2)$$

**d** $w^*(\mathbf{x}, \mathbf{A} \cup \mathbf{B})$ **e**

Assignment	Range	w
{ A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [2, 3]]	x <sub>1</sub> <sup>3</sup> x <sub>2</sub>
{ A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [1, 2]]	x <sub>1</sub> <sup>3</sup> x <sub>2</sub>
{ A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , ¬B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [0, 1]]	x <sub>1</sub> <sup>3</sup> x <sub>2</sub>
{ A <sub>1</sub> , A <sub>2</sub> , ¬B <sub>1</sub> , B <sub>2</sub> , B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [2, 3]]	x <sub>1</sub> x <sub>2</sub> <sup>2</sup>
{ A <sub>1</sub> , A <sub>2</sub> , ¬B <sub>1</sub> , B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [1, 2]]	x <sub>1</sub> x <sub>2</sub> <sup>2</sup>
{ A <sub>1</sub> , A <sub>2</sub> , ¬B <sub>1</sub> , ¬B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [0, 1]]	x <sub>1</sub> x <sub>2</sub> <sup>2</sup>
{ A <sub>1</sub> , ¬A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [2, 3]]	x <sub>1</sub> <sup>2</sup> x <sub>2</sub>
⋮ [same as with {A <sub>1</sub> , A <sub>2</sub> }]	⋮	⋮
{ A <sub>1</sub> , ¬A <sub>2</sub> , ¬B <sub>1</sub> , ¬B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [0, 1]]	x <sub>1</sub> x <sub>2</sub> <sup>3</sup>
{ ¬A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [2, 3]]	2x <sub>1</sub> x <sub>2</sub>
{ ¬A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [1, 2]]	2x <sub>1</sub> x <sub>2</sub>
{ ¬A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , ¬B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [0, 1]]	2x <sub>1</sub> x <sub>2</sub>
{ ¬A <sub>1</sub> , A <sub>2</sub> , ¬B <sub>1</sub> , B <sub>2</sub> , B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [2, 3]]	2x <sub>1</sub> x <sub>2</sub>
{ ¬A <sub>1</sub> , A <sub>2</sub> , ¬B <sub>1</sub> , B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [1, 2]]	2x <sub>1</sub> x <sub>2</sub>
{ ¬A <sub>1</sub> , A <sub>2</sub> , ¬B <sub>1</sub> , ¬B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [0, 1]]	2x <sub>1</sub> x <sub>2</sub>
{ ¬A <sub>1</sub> , ¬A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , B <sub>3</sub> }	[x <sub>1</sub> ∈ [1, 2]], [x <sub>2</sub> ∈ [2, 3]]	3x <sub>1</sub> x <sub>2</sub>
⋮ [same as with {¬A <sub>1</sub> , A <sub>2</sub> }]	⋮	⋮
{ ¬A <sub>1</sub> , ¬A <sub>2</sub> , ¬B <sub>1</sub> , ¬B <sub>2</sub> , ¬B <sub>3</sub> }	[x <sub>1</sub> ∈ [0, 1]], [x <sub>2</sub> ∈ [0, 1]]	3x <sub>1</sub> x <sub>2</sub>