Symbolic Regression for PDEs using Pruned Differentiable Programs

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Motivation

- Current solutions of PDE numerical solvers are challenging to interpret.
- Limited exploration of Symbolic Regression to get analytical expressions for a system of PDEs

Our Contribution

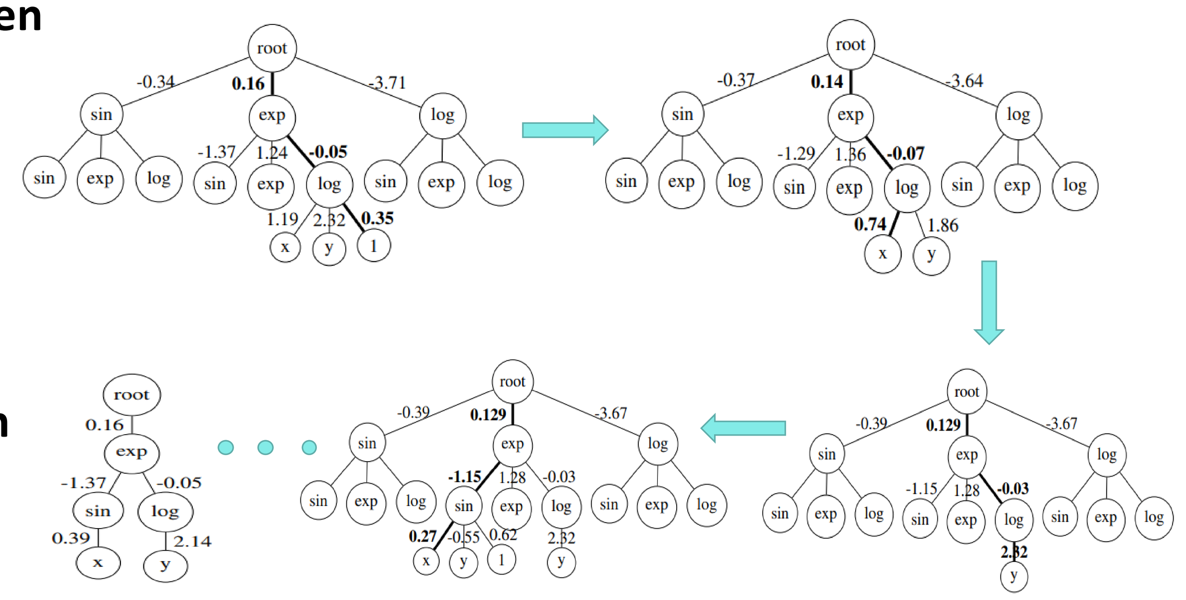
- End to End framework for obtaining mathematical expression for a system of PDEs using Differentiable Program Architecture (DPA)
- Components of Framework
  - Extracting Expressions based on Differentiable Program Architecture
  - Pruning based on Depth First Search (DFS)

Methodology

PDE Solver → Data Generation with PDE Solver → Regression with DPA → DFS based Pruning

Algorithm: Depth First Search Pruning Strategy of Differentiable Program Architecture

Global initialize W ← Unpruned DPA weights, loss ← SCORE (W)
Function DFS (node)
Initialize *visited* ← *node*, *children* ← children of node sorted by absolute value
if *children* is *None* **then**
    W' ← PRUNE (*node*)
    W' ← FINETUNE (*W'*)
    finetuned-loss ← SCORE (*W'*)
    if finetuned-loss ≤ loss **then**
        loss ← finetuned-loss
        W ← *W'*
    end if
    Return *W*, loss
end if
for all *child* in *children* **do**
    if *child* not in *visited* **then**
        DFS(*child*)
    end if
end for



Example Case Studies*

Diffusion Equation

Governing Equations

$u_t = u_{xx} - e^{-t} sin(\pi x)(1-\pi^2)$
$u(x,0) = \sin(\pi x)$
$u(-1,t) = u(1,t) = 0$

Ground Truth Expression

$u_{true} = e^{-t}\sin(\pi x)$

DFS Pruned Expression

$u = (1.51x - 2.04\sin(-2.51x + 0.20t))(2.62x + 0.32t) + 3.67 sin(1.30 sin(0.37t + 1.63) + 3.13x) - 3.53$

Kovasznay Flow

Governing Equations

$u \cdot \nabla u + \nabla p = \nu \Delta u$ in $[0,1]^2$
$div(u) = 0$ in $[0,1]$

Ground Truth Expression

$u_{true} = 1 - e^{\lambda x}\cos(2\pi y)$
$v_{true} = \lambda e^{\lambda x}\sin(2\pi y)/2\pi$
$p_{true} = (1 - e^{2\lambda x})/2$

DFS Pruned Expression

$u = 1.01 + 0.99e^{-1.81x}\sin(6.28y - 1.57)$
$v = \sin(6.28y - 3.14)(0.29 - 0.54x + 0.46x^2 - 0.28x^3 + 0.12x^4$
$p = -2x^4 + 4.38x^3 - 3.75x^2 + 1.81x + 0.02$

Industrial Heat Exchanger

Governing Equations

For j = 1,2,3

$\frac{\partial T_{m_j}}{\partial \varphi} = NTU_{m_j}\left(T_j - T_{m_j}\right) + \frac{1}{Pe_{m_j}}\frac{\partial^2 T_{m_j}}{\partial z^2}$

$\frac{\partial T_j}{\partial z} = NTU_{m_j}\left(T_{m_j} - T_j\right)$

$T_j(\varphi, z = 0) = T_{in,j}$
$T_{m_1}(\varphi = 0, z) = T_{m_3}(\varphi = 1, 1 - z)$
$T_{m_1}(\varphi = 1, z) = T_{m_2}(\varphi = 0, 1 - z)$
$T_{m_2}(\varphi = 1, z) = T_{m_3}(\varphi = 0, z)$

Ground Truth Expression

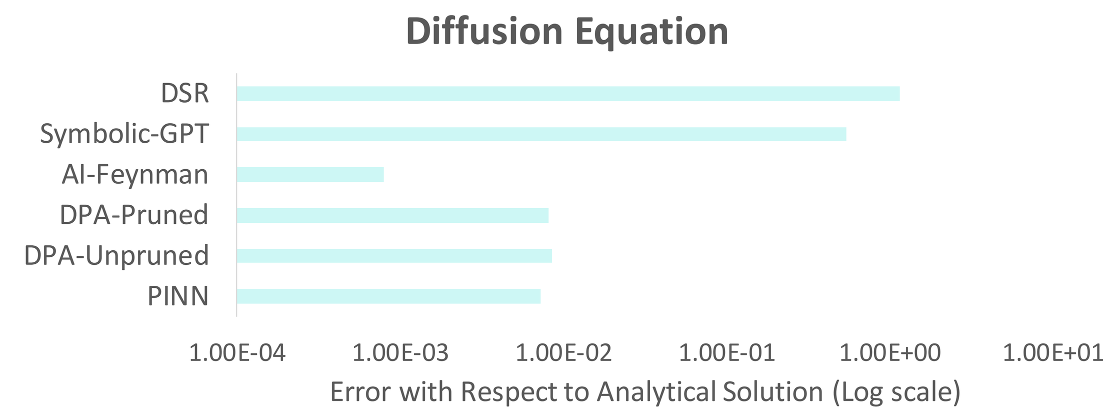
Finite Difference Based Numerical Solver

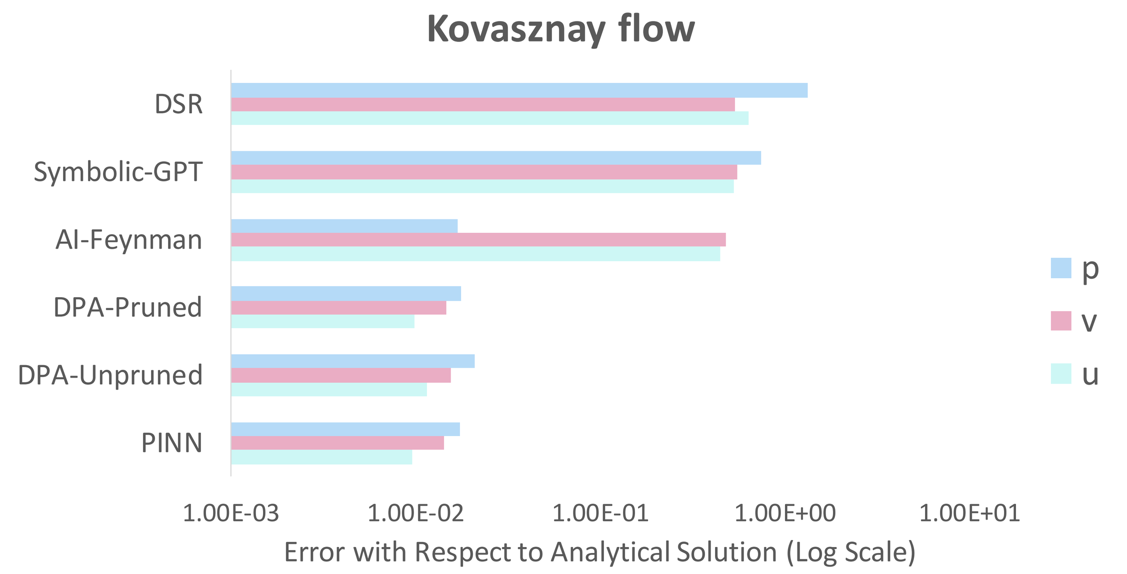
DFS Pruned Expression

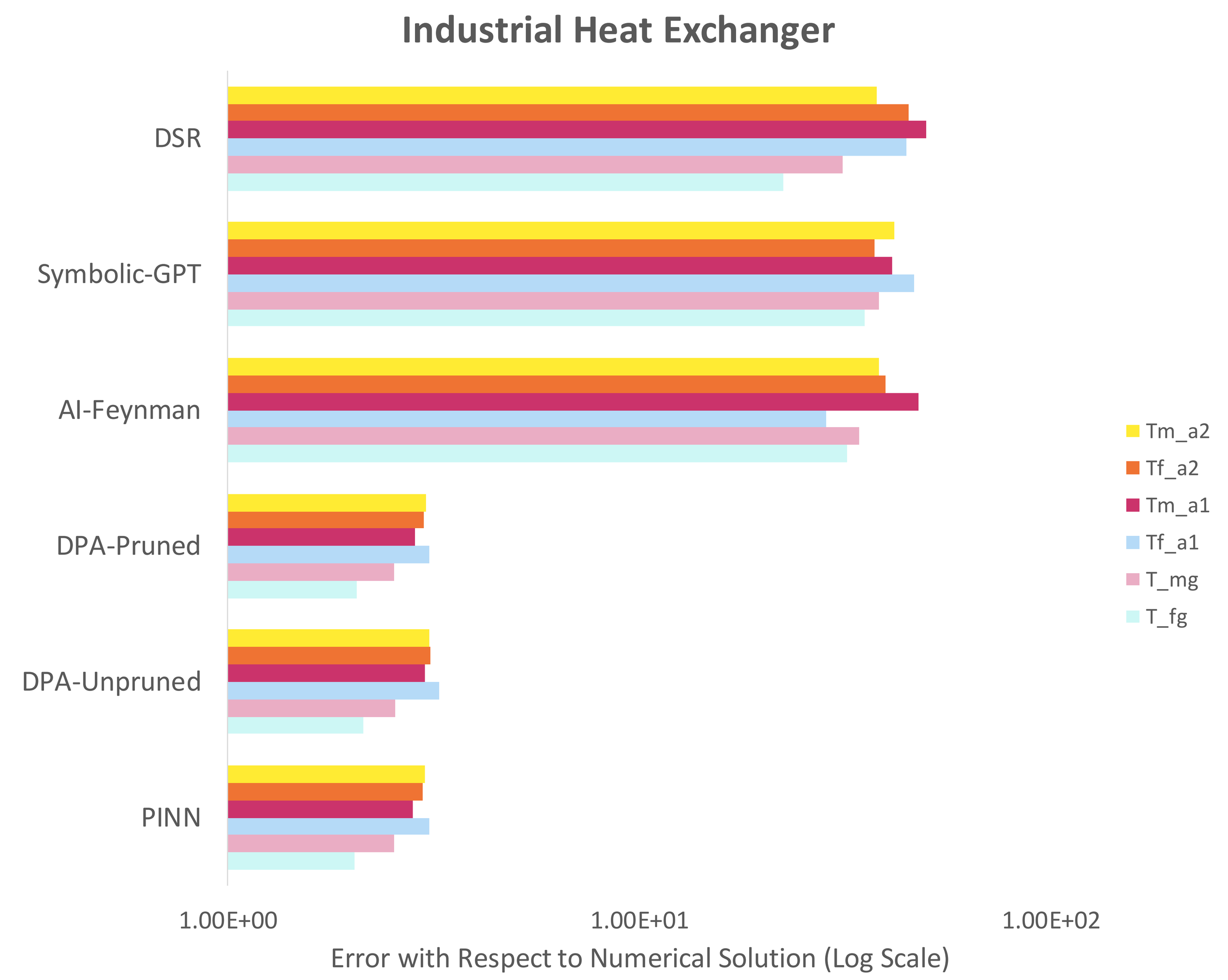
$T_{f_g} = 0.99\sin\left(0.22z + 0.83e^{-0.21z + 0.61\theta + 0.66}\right)$
$T_{m_g} = (0.52\sin(1.56\theta) + 1.18)(0.83e^{-0.44\theta} + 0.18z\theta - 0.23\theta^2 - 0.27\theta)$
$T_{f_{a1}} = (-0.11z + 0.77\theta - 1.59 + 0.27(z\theta))(0.03z + 0.22\theta - 0.60 + 0.37\theta^2)$
$T_{m_{a1}} = (-1.14\sin(0.07z + 0.88\theta - 1.06))(0.06z + 0.69\theta + 0.97)$
$T_{f_{a2}} = 0.86\sin(0.34z + 1.26\theta + 0.55)(-0.10z - 1.02\theta)(0.11z + 1.10\theta) + 0.96$
$T_{m_{a2}} = (0.13z + 1.30\sin(1.08\theta) + 1.25)(0.08z - 0.68\theta + 0.77)$

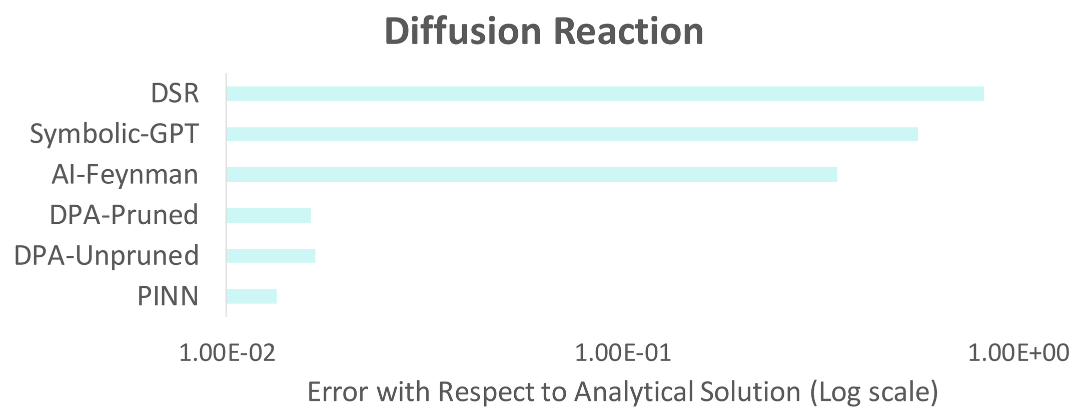
*Case studies Equations for Taylor-Green Vortex and Diffusion Reaction Equation can be referred through paper

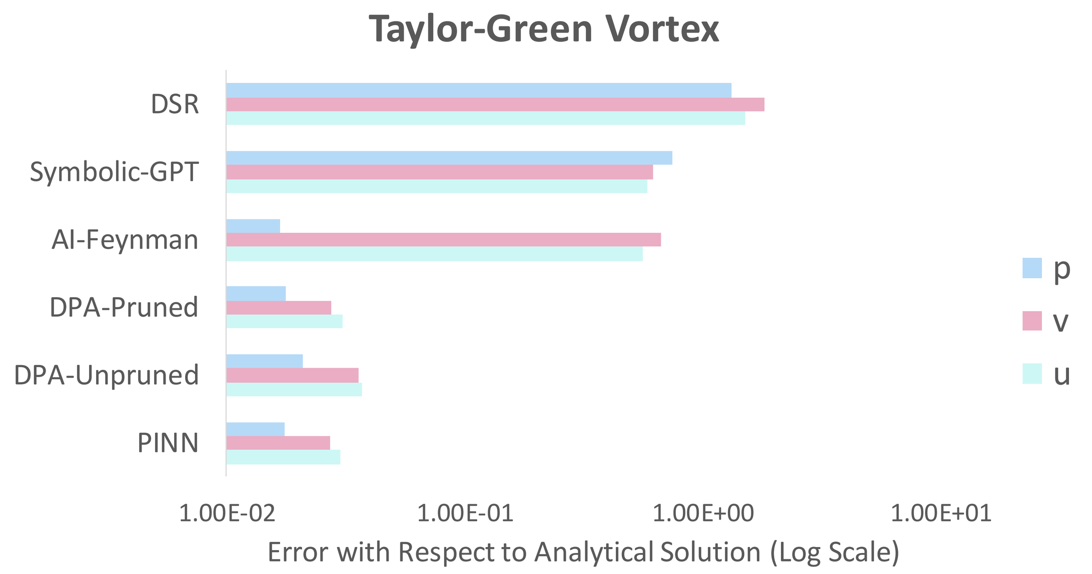
Comparison with State of the Art Methods











References

- Cui et al. "Differentiable Synthesis of Program Architectures" Advances in Neural Information Processing Systems 2021
- Majumdar et al. "Physics informed Symbolic Networks":DLDE Workshop in Neural Information Processing Systems 2022
- Jadhav et al: "Physics informed neural network for health monitoring of an air preheater. PHM Society European Conference, 7(1), 07 2022."

Find our paper here!