Near-Exact Recovery for Sparse-View CT via Data-Driven Methods

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Motivation and Problem Setup

Scientific Machine Learning

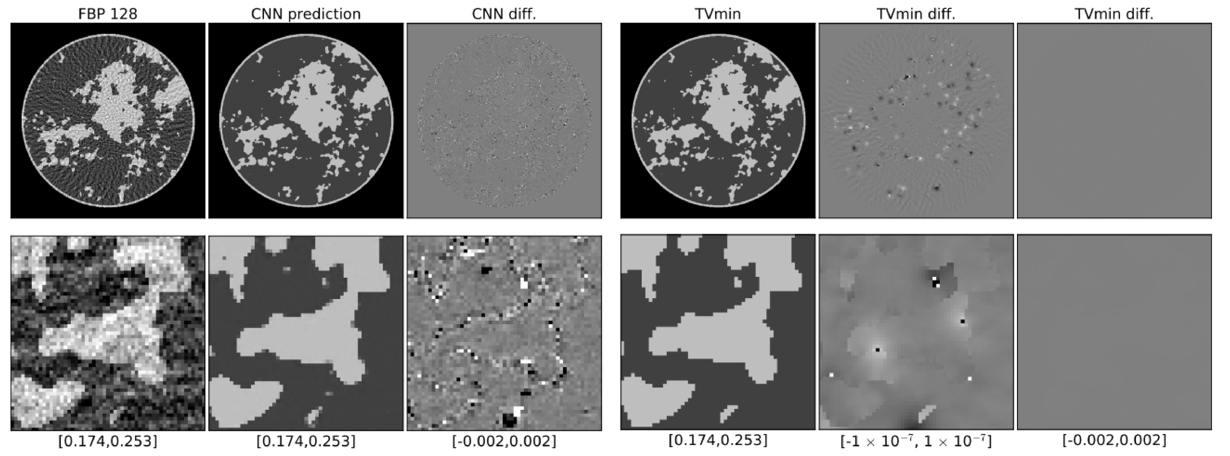
- scientific computing (model-based) meets machine learning (data-driven)
- applies to many problems of the natural sciences, e.g. the important setting of inverse problems in medical imaging: $\mathbf{y} = \mathbf{F}\mathbf{x} + e$
- a key concept is the error decomposition of reconstruction methods

$$\|\mathbf{x} - \mathrm{Rec}(\mathbf{y})\| \leq \underbrace{\|\mathbf{x} - \mathrm{Rec}(\mathbf{F}\mathbf{x})\|}_{\text{accuracy} \to \text{this work}} + \underbrace{\|\mathrm{Rec}(\mathbf{F}\mathbf{x}) - \mathrm{Rec}(\mathbf{y})\|}_{\text{robustness} \to \text{[Genzel et al., 2020]}}$$

The AAPM Grand Challenge

Starting Point:

- lack of evidence for the reliability of deep-learning based solutions
- post-processing of filtered-backprojections (FBPs) with U-Nets may not yield satisfactory results in CT reconstruction (Figure from [Sidky et al., 2020])



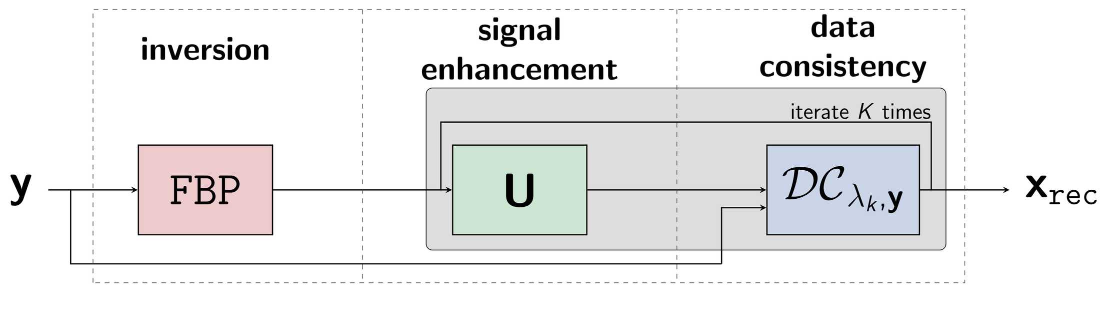
Challenge Setup:

> The goal of the challenge was to *"identify the state-of-the-art in solving the CT inverse problem with data-driven techniques"*. [Sidky et al., 2021]

- synthetic images comparable to mid-plane breast CT [Sidky et al., 2021]
- fanbeam CT sinograms and FBPs provided
- unknown fanbeam geometry

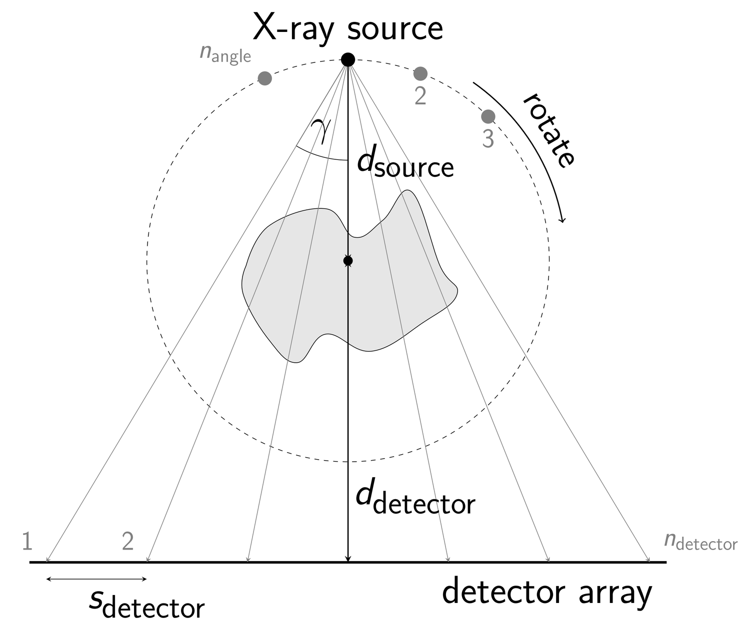
Our Approach:

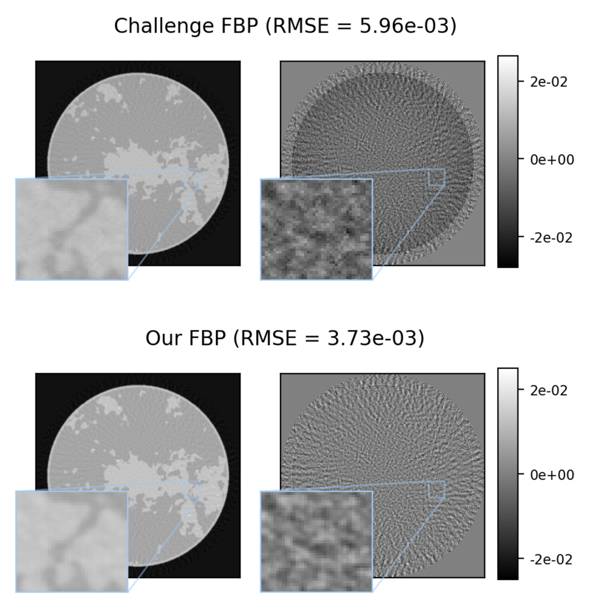
> High accuracy is possible if the forward model (estimated from the provided data) is explicitly incorporated into the solution map.



Methodology

Step 1: Data-Driven Geometry Identification





Data-driven estimation of fanbeam-geometry parameters $\boldsymbol{\theta}_{\mathrm{fan}}$:

$$\min_{\boldsymbol{\theta}_{\mathrm{fan}}} \frac{1}{M} \sum_{i=1}^{M} \|\mathbf{F}[\boldsymbol{\theta}_{\mathrm{fan}}](\mathbf{x}^i) - \mathbf{y}^i\|_2^2$$

Estimation of additional filtered-backprojection parameters $\boldsymbol{\theta}_{\mathrm{fbp}}$:

$$\min_{\boldsymbol{\theta}_{\mathrm{fbp}}} \frac{1}{M} \sum_{i=1}^{M} \|\mathbf{x}^i - \mathrm{FBP}[\boldsymbol{\theta}_{\mathrm{fan}}, \boldsymbol{\theta}_{\mathrm{fbp}}](\mathbf{y}^i)\|_2^2$$

> $\rightsquigarrow$ forward model $\mathbf{F} = \mathbf{F}[\boldsymbol{\theta}_{\mathrm{fan}}]$ and filtered-backprojection $\mathrm{FBP} = \mathrm{FBP}[\boldsymbol{\theta}_{\mathrm{fbp}}]$

Step 2: Pre-Training a U-Net

Empirical risk minimization learns the parameters $\tilde{\boldsymbol{\theta}}$ of a U-Net $\mathbf{U}[\tilde{\boldsymbol{\theta}}]$ that post-processes the FBP estimated in Step 1:

$$\min_{\boldsymbol{\theta}} \frac{1}{M} \sum_{i=1}^{M} \|\mathbf{x}^i - (\mathbf{U}[\tilde{\boldsymbol{\theta}}] \circ \mathrm{FBP})(\mathbf{y}^i)\|_2^2 + \mu \cdot \|\tilde{\boldsymbol{\theta}}\|_2^2$$

Step 3: Constructing an Iterative Scheme

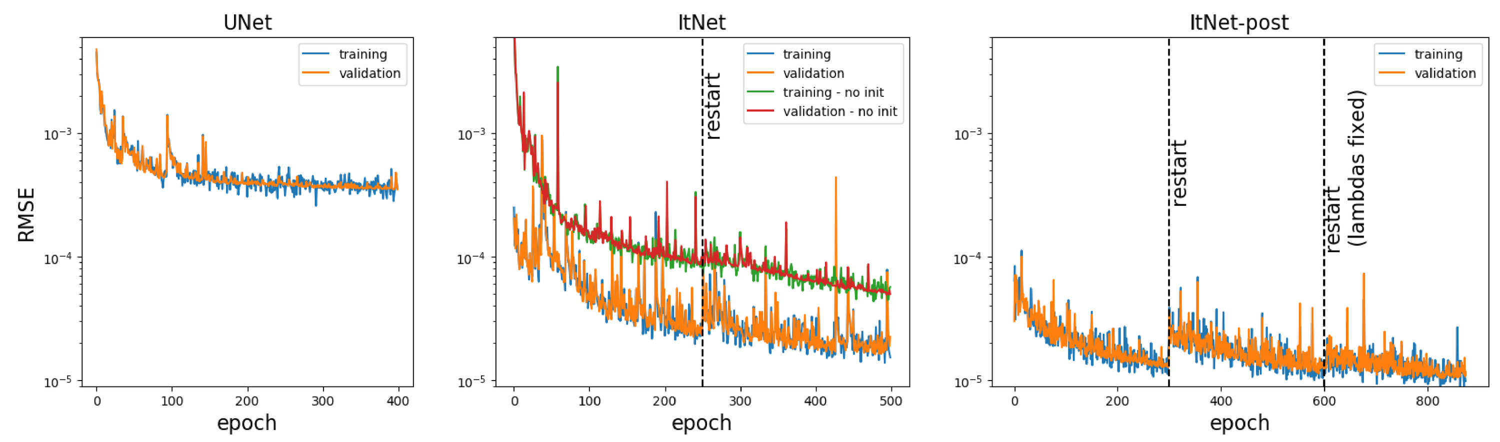
The iterative procedure

$$\mathrm{ItNet}_K[\boldsymbol{\theta}] \colon \mathbb{R}^m \to \mathbb{R}^N, \ \mathbf{y} \mapsto \left[\bigcirc_{k=1}^{K} \left(\mathcal{DC}_{\lambda_k,\mathbf{y}} \circ \mathbf{U}[\tilde{\boldsymbol{\theta}}_k]\right) \circ \mathrm{FBP}\right](\mathbf{y})$$

with learnable parameters $\boldsymbol{\theta} = \{\tilde{\boldsymbol{\theta}}_k, \lambda_k\}_{k=1}^{K}$ and data-consistency layer

$$\mathcal{DC}_{\lambda_k,\mathbf{y}} \colon \mathbb{R}^N \to \mathbb{R}^N, \ \mathbf{x} \mapsto \mathbf{x} - \lambda_k \cdot \mathrm{FBP}(\mathbf{F}\mathbf{x} - \mathbf{y})$$

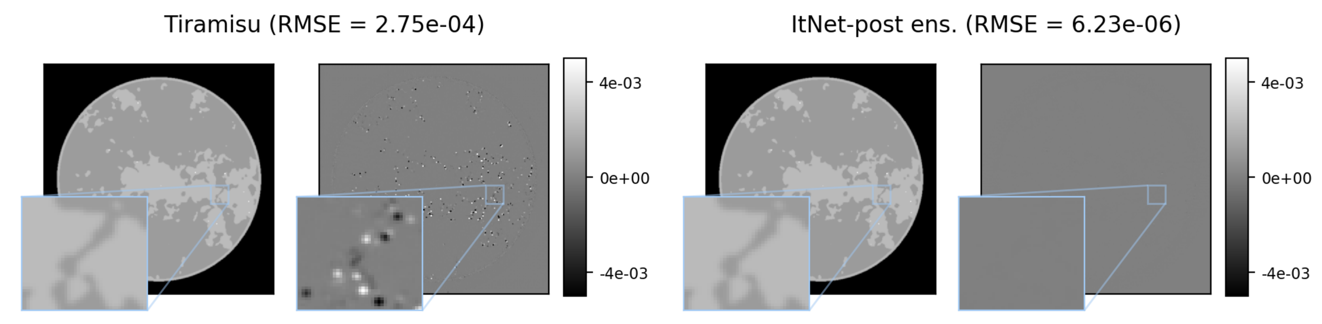
is trained analogous to Step 2.

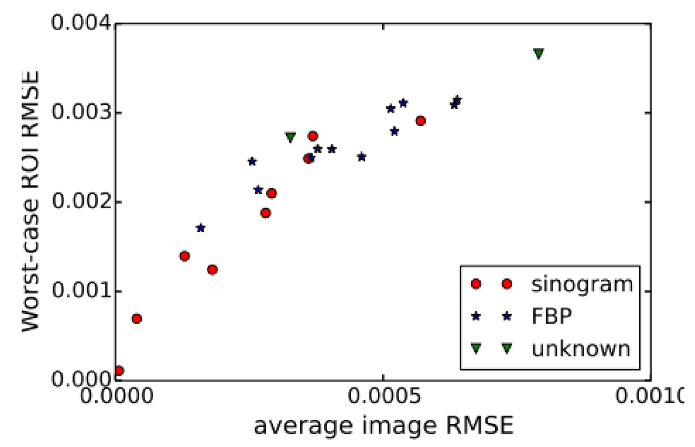


Results and Analysis

Winning the Challenge

> We were able to achieve near-exact recovery and win the AAPM challenge with a margin of about an order of magnitude compared to the runners-up.

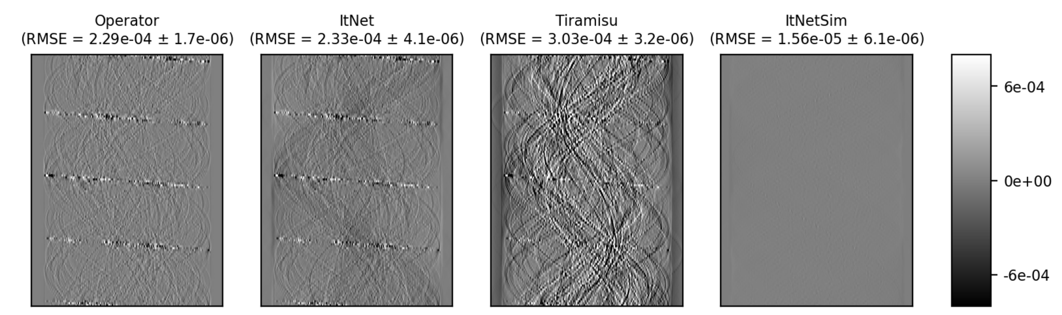




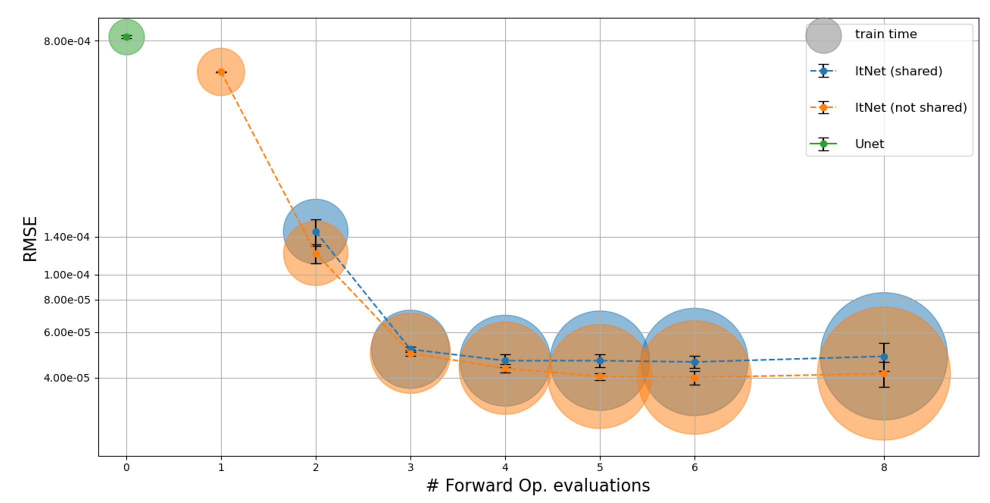
Username/team	s_1	s_2
Max/Robust-and-stable	6.37×10^{-6}	1.11×10^{-4}
TUM/YM&RH	3.99×10^{-5}	6.95×10^{-4}
cebel67/DEEP_UL	1.29×10^{-4}	1.39×10^{-3}
deepx/–	1.59×10^{-4}	1.71×10^{-3}
Haimiao/HBB	1.81×10^{-4}	1.24×10^{-3}

Results from the challenge report [Sidky et al., 2021]

Data Consistency



The Deeper the Better?



Conclusions

- end-to-end neural networks can achieve near-perfect accuracy
- our iterative scheme invokes the forward operator only 5 times (FBP 6 times), much less than classical model-based solvers
- careful training is more important than specific architecture details
- there is a sweetspot regarding depth at about $K = 5$ iterations, after which the improvement in accuracy is negligible and only training time increases

Further reading:

accuracy: `arxiv.org/abs/2106.00280`, `openreview.net/forum?id=IhI3ZhtZGUo`

robustness: `arxiv.org/abs/2011.04268`