



Figure 12: Unconditional watermarked generation on ImageNet  $256 \times 256$ . Similar to Section 5.2, we consider a polytope constraint set,  $\mathcal{M} := \{x \in \mathbb{R}^d : c_i < a_i^\top x < b_i, \forall i \in [m]\}$ , where  $m = 100$ ,  $b = 1.2$ ,  $c = -1.2$ , and  $a_i$  orthogonal Gaussian random vectors, except in a much higher-dimensional space,  $d = 196608$ . For comparison, we also include the results of pretrained diffusion model, which, despite being indistinguishable, actually violate the polytope constraint, *i.e.*,  $x \notin \mathcal{M}$ . It is clear that our MDM-proj is capable of embedding invisible watermarks to high-resolution images.



Figure 13: Conditional watermarked generation on ImageNet  $256 \times 256$ . Specifically, we consider the JPEG restoration task, where, given degraded, low-quality inputs  $y$  (*c.f.*, upper-left), we wish to generate their corresponding clean images  $x$  (*c.f.*, upper-right) by modeling  $p(x|y)$ . It is clear that both MDM-dual and MDM-proj are capable of solving this conditional generation task, generating clean images that additionally embed invisible watermarks, *i.e.*,  $x \in \mathcal{M}$ .