APPENDIX

A QUERY DECOMPOSITION OF DIFFERENT QUERY TYPES

Figure 4 provides the query decomposition of different query types considered in our empirical study as well as previous literature in the area.

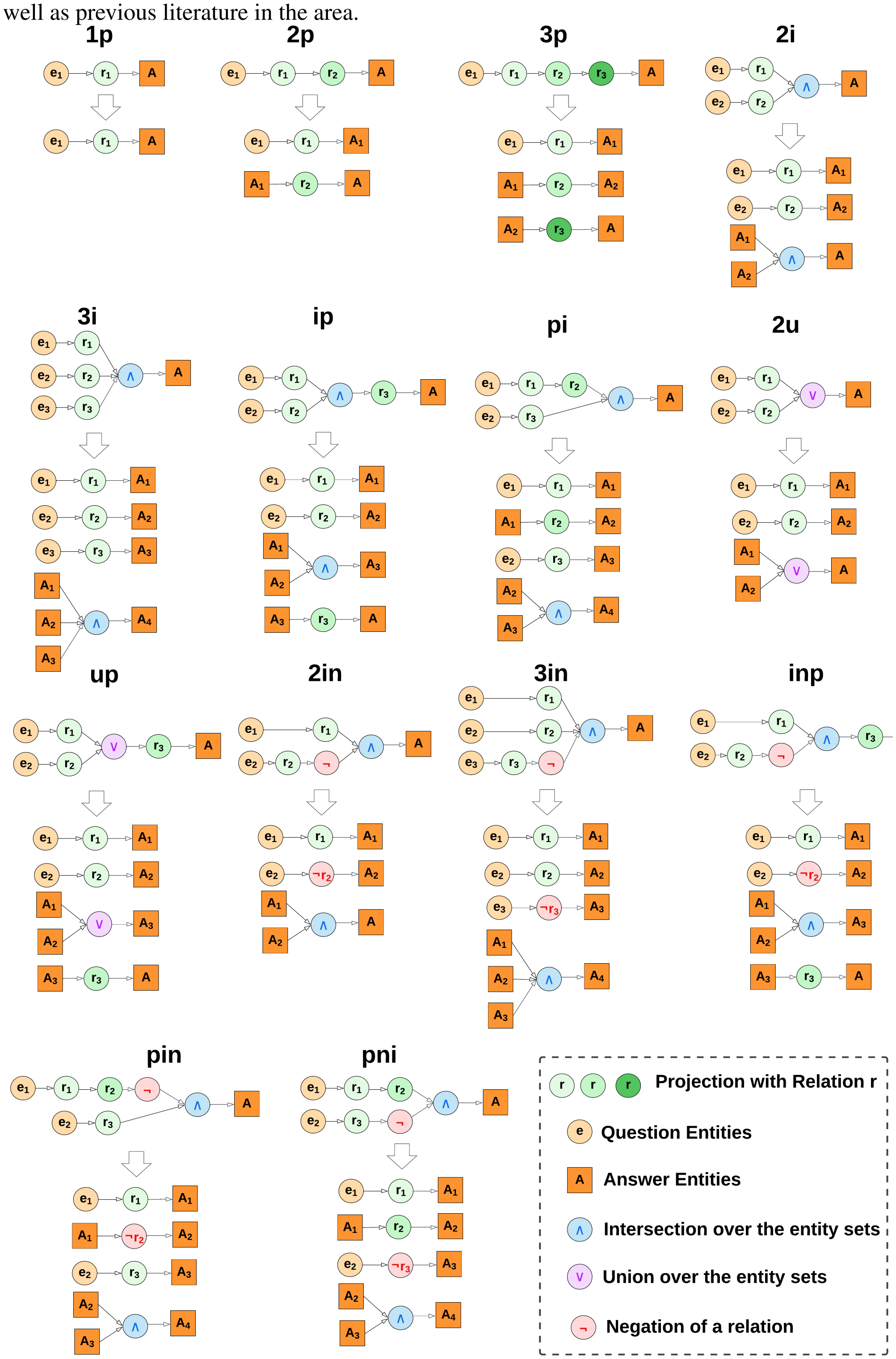

Figure 4: Query Decomposition of different query types considered in our experiments.

B PROMPT TEMPLATES OF DIFFERENT QUERY TYPES

The prompt templates for full complex logical queries with multiple operations and decomposed elementary logical queries with single operation are provided in Tables 5 and 6, respectively.

Table 5: Full Prompt Templates of Different Query Types.

Type	Logical Query	Template for Full Prompts
Context	$\mathcal{N}_k(q_\tau[Q_\tau])$	Given the following (h,r,t) triplets where entity h is related to entity t by relation r; $(h_1, r_1, t_1), (h_2, r_2, t_2), (h_3, r_3, t_3), (h_4, r_4, t_4), (h_5, r_5, t_5), (h_6, r_6, t_6), (h_7, r_7, t_7), (h_8, r_8, t_8)$
1p	$\exists X.r_1(X, e_1)$	Which entities are connected to e_1 by relation r_1?
2p	$\exists X.r_1(X, \exists Y.r_2(Y, e_1)$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1. Then, what are the entities connected to E by relation r_2?
3p	$\exists X.r_1(X, \exists Y.r_2(Y, \exists Z.r_3(Z, e_1)$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1 and the set of entities F is connected to entities in E by relation r_2. Then, what are the entities connected to F by relation r_3?
2i	$\exists X.[r_1(X, e_1) \wedge r_2(X, e_2)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1 and the set of entities F is connected to entity e_2 by relation r_2. Then, what are the entities in the intersection of set E and F, i.e., entities present in both F and G?
3i	$\exists X.[r_1(X, e_1) \wedge r_2(X, e_2) \wedge r_3(X, e_3)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, the set of entities F is connected to entity e_2 by relation r_2 and the set of entities G is connected to entity e_3 by relation r_3. Then, what are the entities in the intersection of set E, F and G, i.e., entities present in all E, F and G?
ip	$\exists X.r_3(X, \exists Y.[r_1(Y, e_1) \wedge r_2(Y, e_2)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, F is the set of entities connected to entity e_2 by relation r_2, and G is the set of entities in the intersection of E and F. Then, what are the entities connected to entities in set G by relation r_3?
pi	$\exists X.[r_1(X, \exists Y.r_2(Y, e_2)) \wedge r_3(X, e_3)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, F is the set of entities connected to entities in E by relation r_2, and G is the set of entities connected to entity e_2 by relation r_3. Then, what are the entities in the intersection of set F and G, i.e., entities present in both F and G?
2u	$\exists X.[r_1(X, e_1) \vee r_2(X, e_2)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1 and F is the set of entities connected to entity e_2 by relation r_2. Then, what are the entities in the union of set F and G, i.e., entities present in either F or G?
up	$\exists X.r_3(X, \exists Y.[r_1(Y, e_1) \vee r_2(Y, e_2)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1 and F is the set of entities connected to entity e_2 by relation r_2. G is the set of entities in the union of E and F. Then, what are the entities connected to entities in G by relation r_3?
2in	$\exists X.[r_1(X, e_1) \wedge \neg r_2(X, e_2)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1 and F is the set of entities connected to entity e_2 by any relation other than relation r_2. Then, what are the entities in the intersection of set E and F, i.e., entities present in both F and G?
3in	$\exists X.[r_1(X, e_1) \wedge r_2(X, e_2) \wedge \neg r_3(X, e_3)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, F is the set of entities connected to entity e_2 by relation r_2, and F is the set of entities connected to entity e_3 by any relation other than relation r_3. Then, what are the entities in the intersection of set E and F, i.e., entities present in both F and G?
inp	$\exists X.r_3(X, \exists Y.[r_1(Y, e_1) \wedge \neg r_2(Y, e_2)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, and F is the set of entities connected to entity e_2 by any relation other than relation r_2. Then, what are the entities that are connected to the entities in the intersection of set E and F by relation r_3?
pin	$\exists X.[r_1(X, \exists Y.\neg r_2(Y, e_2)) \wedge r_3(X, e_3)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, F is the set of entities connected to entities in E by relation r_2, and G is the set of entities connected to entity e_2 by any relation other than relation r_3. Then, what are the entities in the intersection of set F and G, i.e., entities present in both F and G?
pni	$\exists X.[r_1(X, \exists Y.\neg r_2(Y, e_2)) \wedge \neg r_3(X, e_3)]$	Let us assume that the set of entities E is connected to entity e_1 by relation r_1, F is the set of entities connected to entities in E by any relation other than r_2, and G is the set of entities connected to entity e_2 by relation r_3. Then, what are the entities in the intersection of set F and G, i.e., entities present in both F and G?

C ANALYSIS OF LOGICAL REASONING PERFORMANCE USING HITS METRIC

Tables 7 and 8 present the HITS@K=3 results of baselines and our model. HITS@K indicates the accuracy of predicting correct candidates in the top-K results.

Table 6: Decomposed Prompt Templates of Different Query Types.

Type	**Logical Query**	**Template for Decomposed Prompts**
Context	$\mathcal{N}_k(q_\tau[Q_\tau])$	Given the following (h,r,t) triplets where entity h is related to entity t by relation r; $(h_1, r_1, t_1), (h_2, r_2, t_2), (h_3, r_3, t_3), (h_4, r_4, t_4), (h_5, r_5, t_5), (h_6, r_6, t_6), (h_7, r_7, t_7), (h_8, r_8, t_8)$
1p	$\exists X.r_1(X, e_1)$	Which entities are connected to e_1 by relation r_1?
2p	$\exists X.r_1(X, \exists Y. r_2(Y, e_1)$	Which entities are connected to e_1 by relation r_1? Which entities are connected to any entity in [PP1] by relation r_2?
3p	$\exists X.r_1(X, \exists Y .r_2(Y, \exists Z. r_3(Z, e_1)$	Which entities are connected to e_1 by relation r_1? Which entities are connected to any entity in [PP1] by relation r_2? Which entities are connected to any entity in [PP2] by relation r_3?
2i	$\exists X.[r_1(X, e_1) \wedge r_2(X, e_2)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to e_2 by relation r_2? What are the entities in the intersection of entity sets [PP1] and [PP2]?
3i	$\exists X.[r_1(X, e_1) \wedge r_2(X, e_2) \wedge r_3(X, e_3)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to e_2 by relation r_2? Which entities are connected to e_3 by relation r_3? What are the entities in the intersection of entity sets [PP1], [PP2] and [PP3]?
ip	$\exists X.r_3(X, \exists Y.[r_1(Y, e_1) \wedge r_2(Y, e_2)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to e_2 by relation r_2? What are the entities in the intersection of entity sets [PP1] and [PP2]? What are the entities connected to any entity in [PP3] by relation r_3?
pi	$\exists X.[r_1(X, \exists Y.r_2(Y, e_2)) \wedge r_3(X, e_3)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to [PP1] by relation r_2? Which entities are connected to e_2 by relation r_3? What are the entities in the intersection of entity sets [PP2] and [PP3]?
2u	$\exists X.[r_1(X, e_1) \vee r_2(X, e_2)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to e_2 by relation r_2? What are the entities in the union of entity sets [PP1] and [PP2]?
up	$\exists X.r_3(X, \exists Y.[r_1(Y, e_1) \vee r_2(Y, e_2)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to e_2 by relation r_2? What are the entities in the union of entity sets [PP1] and [PP2]? Which entities are connected to any entity in [PP3] by relation r_3?
2in	$\exists X.[r_1(X, e_1) \wedge \neg r_2(X, e_2)]$	Which entities are connected to e_1 by any relation other than r_1? Which entities are connected to e_2 by any relation other than r_2? What are the entities in the intersection of entity sets [PP1] and [PP2]?
3in	$\exists X.[r_1(X, e_1) \wedge r_2(X, e_2) \wedge \neg r_3(X, e_3)]$	Which entities are connected to e_1 by any relation other than r_1? Which entities are connected to e_2 by any relation other than r_2? Which entities are connected to e_3 by any relation other than r_3? What are the entities in the intersection of entity sets [PP1], [PP2] and [PP3]?
inp	$\exists X.r_3(X, \exists Y.[r_1(Y, e_1) \wedge \neg r_2(Y, e_2)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to e_2 by any relation other than r_2? What are the entities in the intersection of entity sets [PP1], and [PP2]? What are the entities connected to any entity in [PP3] by relation r_3?
pin	$\exists X.[r_1(X, \exists Y.\neg r_2(Y, e_2)) \wedge r_3(X, e_3)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to entity set in [PP1] by relation r_2? Which entities are connected to e_2 by any relation other than r_3? What are the entities in the intersection of entity sets [PP2] and [PP3]?
pni	$\exists X.[r_1(X, \exists Y.\neg r_2(Y, e_2)) \wedge \neg r_3(X, e_3)]$	Which entities are connected to e_1 by relation r_1? Which entities are connected to any entity in [PP1] by any relation other than r_2? Which entities are connected to e_2 by relation r_3? What are the entities in the intersection of entity sets [PP2] and [PP3]?

Table 7: Performance comparison study between LARK and the baseline, focusing on their efficacy of logical reasoning using HITS@K=1,3,10 scores. The rows correspond to the models and columns denote the different query structures of multi-hop projections, geometric operations, and compound operations. The best results for each query type in every dataset are highlighted in **bold** font.

Dataset	Variant	1p	2p	3p	2i	3i	ip	pi	2u	up
		HITS@1								
FB15k	**Llama2-7B**	74.6	26	18.5	59.9	47.7	2.4	5.7	5.8	5
	complex	**77.5**	37.9	26.3	67.4	54.6	8.2	20.7	20.7	17.6
	step	**77.5**	**41.8**	**28.1**	**70.2**	**57.3**	**10.3**	**24.3**	**22.8**	**17.8**
FB15k-237	**Llama2-7B**	77.2	28.5	17.7	10.9	22.6	10.8	8.7	10.5	13.2
	complex	**78.5**	30.8	19.3	41.1	38.1	9.6	18.7	24.2	14.0
	step	**78.5**	**34.3**	**21.3**	**43.2**	**40.2**	**11.7**	**22.2**	**27.9**	**14.2**
NELL995	**Llama2-7B**	86.4	28.3	19.6	10.2	24	8.6	3.5	1.5	15.9
	complex	**88.0**	30.9	21.7	44.1	41.6	7.4	8.2	3.3	17
	step	**88.0**	**34.3**	**24.0**	**46.1**	**43.8**	**9.5**	**9.8**	**8.9**	**17.3**
		HITS@3								
FB15k	**Llama2-7B**	74	53.4	34.6	18.2	36.4	44.7	39.4	35.7	77.1
	complex	**77.7**	**57.6**	37.9	68.5	61.3	39.6	84.8	82.9	81.7
	step	**77.7**	57.4	**40.1**	**69.4**	**62.5**	**48.4**	**91.2**	**92.7**	**82.6**
FB15k-237	**Llama2-7B**	75.9	42.6	25.7	12.6	25.9	43.6	35.1	42.9	53.8
	complex	**78.3**	**45.9**	28.1	47.2	43.7	38.7	75.6	89.4	57
	step	**78.3**	**45.9**	**29.8**	**48.2**	**44.6**	**47.3**	**80.0**	**93.6**	**57.6**
NELL995	**Llama2-7B**	85.6	42.9	28.7	11.8	27.6	34.6	14.1	5.7	65
	complex	**87.8**	**46.8**	31.6	50.7	47.9	29.8	32.9	13.2	69.4
	step	**87.8**	45.7	**33.5**	**51.3**	**48.7**	**38.1**	**39.6**	**35.8**	**70.3**
		HITS@10								
FB15k	**Llama2-7B**	73.6	53.9	35.7	18.1	36.3	44.6	39.5	35.7	77.1
	complex	**77.7**	**58.2**	39.1	68.2	61.4	39.5	85	82.9	81.7
	step	**77.7**	57.4	**46.0**	**69.4**	**62.5**	**48.2**	**91.2**	**84.7**	**82.6**
FB15k-237	**Llama2-7B**	75.2	43	26.5	12.6	25.9	43.6	35.1	42.9	53.8
	complex	**78.3**	**46.4**	29	47.3	43.8	38.7	75.6	89.4	57
	step	**78.3**	45.9	**34.1**	**48.2**	**44.6**	**47.3**	**80.0**	**93.6**	**57.6**
NELL995	**Llama2-7B**	84.9	43.4	29.2	11.8	27.6	34.6	14.1	5.7	65
	complex	**87.8**	**47.4**	32.2	50.8	48	29.8	32.9	13.2	69.4
	step	**87.8**	45.7	**38.3**	**51.3**	**48.7**	**38.1**	**39.6**	**35.8**	**70.3**

Table 8: Performance comparison between LARK and the baseline for negation query types using HITS@K=1,3,10 scores. The best results for each query type in every dataset are given in **bold** font.

Metric	Variant	2in	3in	inp	pin	pni	2in	3in	inp	pin	pni	2in	3in	inp	pin	pni
		HITS@1					**HITS@3**					**HITS@10**				
FB15k	**Llama2-7B**	1.8	0.7	4.0	2.1	0.9	18.6	5.7	40.8	18.8	8.6	18.6	5.7	40.8	18.8	8.6
	complex	6.7	2.4	14.2	7.8	3.3	26.6	9.5	59.2	30.3	12.3	26.6	9.5	59.3	30.3	12.4
	step	**7.4**	**2.7**	**14.9**	**9.1**	**3.4**	**31.0**	**12.1**	**64.8**	**38.7**	**14.4**	**31.0**	**12.1**	**64.8**	**38.7**	**14.4**
FB15k-237	**Llama2-7B**	1.9	0.8	6.8	2.8	0.7	7.5	3.5	27.3	11.6	2.7	7.5	3.5	27.3	11.6	2.7
	complex	2.7	1.4	9.8	4.6	1	10.8	5.8	39.6	18.7	3.9	10.8	5.8	39.6	18.7	3.9
	step	**3.2**	**1.7**	**10.6**	**5.8**	**1.1**	**12.6**	**7.4**	**43.3**	**23.9**	**4.6**	**12.6**	**7.4**	**43.3**	**23.9**	**4.6**
NELL995	**Llama2-7B**	2.8	1.4	7.2	2.2	1.5	11.2	6	29.1	9.2	6.2	11.2	6	29.1	9.2	6.2
	complex	3.9	2.3	10.2	3.7	2.2	16.1	9.4	41.8	15.1	9	16.1	9.4	41.8	15.1	9
	step	**4.6**	**2.8**	**11.1**	**4.7**	**2.7**	**18.5**	**12.0**	**46.0**	**19.3**	**10.9**	**18.5**	**12.0**	**46.0**	**19.3**	**10.9**

D Algorithm

Algorithm for the LARK's procedure is provided in Algorithm 1.

Algorithm 1: LARK Algorithm

Input: Logical query q_τ, Knowledge Graph $\mathcal{G} : E \times R$;
Output: Answer entities V_τ;

```
# Query Abstraction: Map entity and relations to IDs
q_τ = Abstract(q_τ);
G = Abstract(G);
# Neighborhood Retrieval
N_k(q_τ[Q_τ]) = {(h, r, t)}, using Eq. (7)
# Query Decomposition
q_τ^d = Decomp(q_τ);
# Initialize Answer Cache ans = {};
for i ∈ 1 : length(q_τ^d) do
    # Replace Answer Cache in Question
    q_τ^d[i] = replace(q_τ^d[i], ans[i − 1]);
    ans[i] = LLM(q_τ^d[i]);
end
return ans[length(q_τ^d)]
```

Table 9: Details of the token distribution for various query types in different datasets. The columns present the mean, median, minimum (Min), and maximum (Max) values of the number of tokens in the queries of different query types. Column 'Cov' presents the percentage of queries (coverage) that contain less than 4096 tokens, which is the token limit of Llama2 model.

Dataset	FB15k					FB15k-237					NELL				
Type	Mean	Median	Min	Max	Cov	Mean	Median	Min	Max	Cov	Mean	Median	Min	Max	Cov
1p	70.2	61	58	10338	100	82.1	61	58	30326	99.9	81.7	61	58	30250	99.9
2p	331.2	106	86	27549	97.1	1420.9	140	83	130044	89.7	893.4	136	83	108950	90.9
3p	785.2	165	103	80665	91	3579.8	329	103	208616	75.7	3052.6	389	100	164545	73.7
2i	1136.7	276	119	20039	86.3	4482.8	636	119	60655	67.7	4469.3	680	119	54916	67.3
3i	2575.4	860	145	29148	68.4	8760.2	2294	145	85326	48.3	8979.4	2856	145	76834	44.8
ip	1923.8	1235	135	21048	67.4	4035.8	2017	131	32795	50.5	4838	2676	131	33271	43.6
pi	1036.8	455	140	10937	85.8	1255.6	343	141	45769	83.4	1535.3	435	135	21125	79.9
2u	1325.4	790	121	14703	80.8	2109.5	868	123	60655	68.9	2294.9	1138	125	23637	65.7
up	115.3	112	110	958	100	113.7	112	110	981	100	113.2	112	110	427	100
2in	1169.1	548	123	18016	84.9	5264.7	1116	128	60281	61.8	3496	774	124	58032	71.6
3in	4070.3	2230	159	28679	46.6	13695.8	8344	175	88561	25.9	12575.9	7061	164	88250	28.1
inp	629	112	110	73457	91.8	1949.4	394	110	115169	78.4	696.7	112	110	89660	93.8
pin	400.7	154	129	6802	95.8	1106.5	242	129	44010	87.2	418.1	131	129	24062	96.7
pni	345.9	129	127	7938	96.6	547.1	129	127	18057	95.1	289.3	129	127	17489	97.9

E Query Token Distribution in Datasets

The quantitative details of the query token's lengths is provided in Table 9 and their complete distribution plots are provided in Figure 5. From the results, we observe that the distribution of token lengths is positively-skewed for most of the query types, which indicates that the number of samples with high token lengths is small in number. Thus, small improvements in the LLMs' token limit can potentially lead to better coverage on most of the reasoning queries in standard KG datasets.

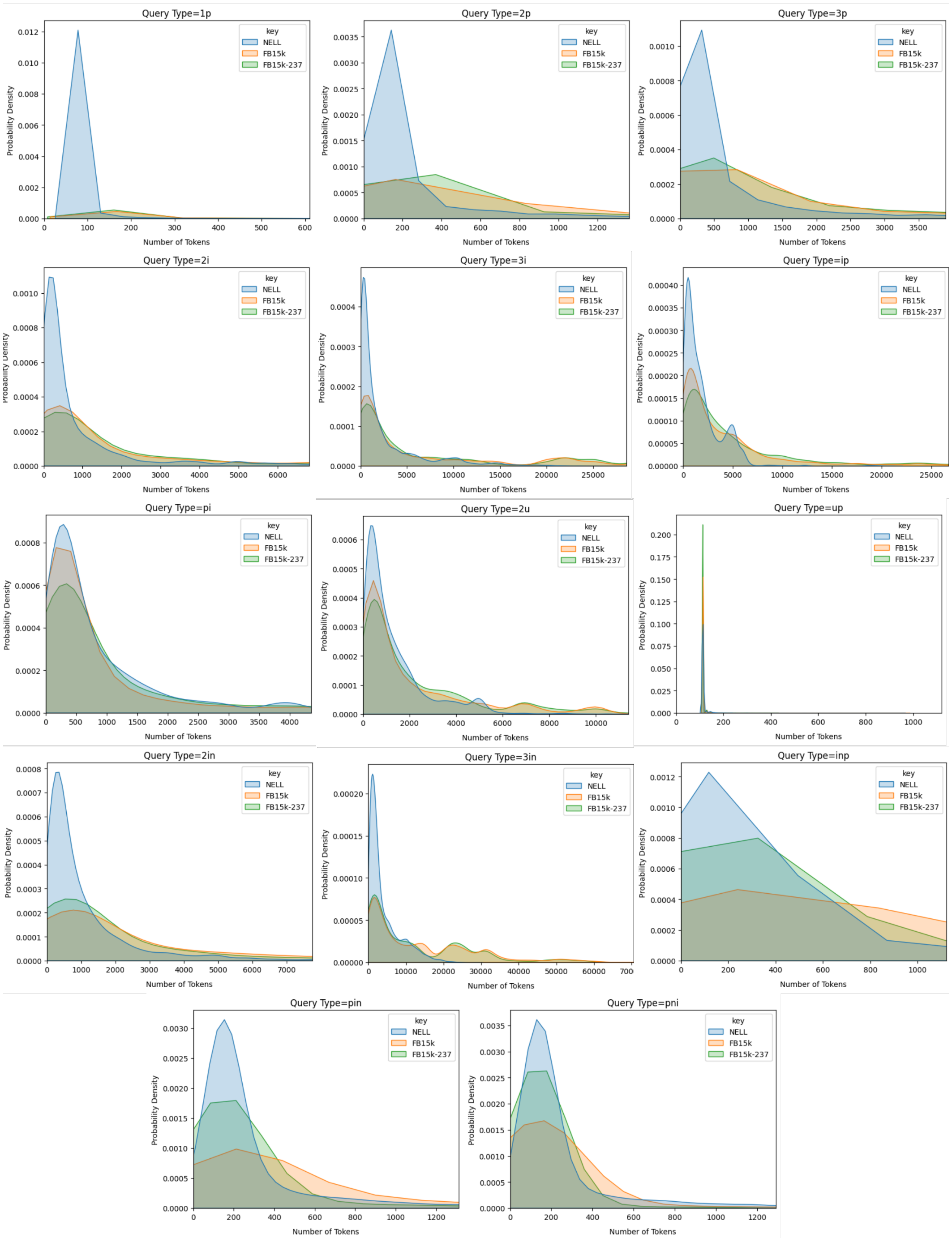

Figure 5: Probability distribution of the number of tokens in each query type. The figures contains 14 graphs for the 14 different query types. The x-axis and y-axis presents the number of tokens in the query and their probability density, respectively.