ADAPTIVE CONSTRAINT INTEGRATION FOR SIMULTA-NEOUSLY OPTIMIZING CRYSTAL STRUCTURES WITH MULTIPLE TARGETED PROPERTIES

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ABSTRACT

In materials science, finding crystal structures that have targeted properties is crucial. While recent methodologies such as Bayesian optimization and deep generative models have made some advances on this issue, these methods often face difficulties in adaptively incorporating various constraints, such as electrical neutrality and targeted properties optimization, while keeping the desired specific crystal structure. To address these challenges, we have developed the Simultaneous Multi-property Optimization using Adaptive Crystal Synthesizer (SMOACS), which utilizes state-of-the-art property prediction models and their gradients to directly optimize input crystal structures for targeted properties simultaneously. SMOACS enables the integration of adaptive constraints into the optimization process without necessitating model retraining. Thanks to this feature, SMOACS has succeeded in simultaneously optimizing targeted properties while maintaining perovskite structures, even with models trained on diverse crystal types. We have demonstrated the band gap optimization while meeting a challenging constraint, that is, maintaining electrical neutrality in large atomic configurations up to 135 atom sites, where the verification of the electrical neutrality is challenging. The properties of the most promising materials have been confirmed by density functional theory calculations.

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1 INTRODUCTION

We address the challenge of simultaneously optimizing multiple material properties while preserving
 specific crystal structures and ensuring electrical neutrality. To achieve this, we have developed a
 methodology that leverages property prediction models and their gradients to facilitate the discovery
 of materials with multiple desired properties. This approach allows for the adaptive application
 of constraints, such as electrical neutrality and specific crystal structures, without necessitating
 retraining. As a result, our method enables the optimization of large atomic configurations to obtain
 specific properties while ensuring electrical neutrality and preserving specific crystal structures.

Materials design is crucial for various advancing technologies, e.g., enhancing efficiency or reducing 041 the cost of solar cells. The goal of materials design is to identify materials that simultaneously satisfy 042 multiple property criteria, for instance, in terms of band gap and formation energy, while meeting 043 other requirements, such as electrical neutrality. Furthermore, during the design process, it is often 044 desirable to focus on specific promising systems, such as perovskite structures for next-generation 045 solar cells (Green et al., 2014). In the exploration of specific crystal structures, elemental sub-046 stitution-blending different elements-is commonly employed. For instance, blended perovskite 047 structures might have alternative compositions such as $AA'BB'X_2X'X''_3$, which are derived from 048 the standard ABX_3 format of a perovskite unit cell. Computational experiments involving these complex compositions often require larger systems that combine multiple unit cells. Consequently, the critical aspects of material design include 1) the ability to optimize multiple properties simulta-051 neously, 2) the adaptive incorporation of various constraints, such as electrical neutrality or specific crystal structures, and 3) the ability to optimize large atomic configurations. In summary, we need 052 to solve the problem of simultaneously optimizing multiple properties while preserving a specific crystal structure and ensuring electrical neutrality.

Advances in computational techniques have tremendously accelerated material design, with Density
Functional Theory (DFT) becoming a standard tool for rapid property validation. Recent developments in machine learning have enabled faster property predictions through deep learning models
trained on DFT-generated data. Models such as Crystalformer (Taniai et al., 2024), a transformerbased model (Vaswani et al., 2017), and ALIGNN (Choudhary & DeCost, 2021), a Graph Neural
Network (GNN)-based model, significantly facilitate the screening process (Choubisa et al., 2023).

060 Bayesian optimization, such as Gaussian Process and Tree-structured Parzen estimator (TPE) 061 (Watanabe, 2023), is commonly employed in material design (Ozaki et al., 2020a; Boyar et al., 2024; 062 Zhai et al., 2024). A key advantage of Bayesian optimization is its capability to perform inverse in-063 ference, therefore enabling the prediction of crystal structures from given properties. Recently, deep 064 generative models designed to synthesize crystal structures, such as FTCP (Ren et al., 2022), have gained much attention due to their potential to discover new stable materials. Additionally, there 065 are methods that leverage large language models (Ding et al., 2024; Gruver et al., 2024), Generative 066 Flow Networks (AI4Science et al., 2023), reinforcement learning (Govindarajan et al., 2024), or 067 flow matching (Miller et al., 2024) to synthesize new crystal structures. 068

069 Despite these advances, many challenges remain. Firstly, research using deep generative models 070 primarily aims to identify stable materials, and only a limited number of studies focus on optimizing both stability and key properties, such as the band gap, which is crucial for maximizing solar cell 071 efficiency. Secondly, deep generative models are often built with specialized architectures, making 072 it difficult to adopt the latest property prediction models for their prediction branches. This lack of 073 flexibility in model architectures can hinder the improvement of prediction accuracy for generated 074 materials. Thirdly, current generative models require retraining for targeted properties optimization 075 within specific crystal structures, which is often the case in practice, such as perovskite structures 076 for solar cells. Finally, verifying the electrical neutrality in large atomic configurations is compli-077 cated due to the combinatorial explosion resulting from the possible multiple oxidation numbers for 078 many atomic species. Nevertheless, ensuring electrical neutrality is essential for proposing realistic 079 materials.

080 To address these challenges, we have developed a framework, the Simultaneous Multi-property 081 Optimization using Adaptive Crystal Synthesizer (SMOACS). SMOACS can employ various property prediction models as far as their gradients can be computed and optimizes input crystal struc-083 tures directly to achieve target properties through the backpropagation technique (Fig. 1(left)). This 084 approach enables accurate prediction of multiple properties and simultaneous optimization by uti-085 lizing several recently developed pre-trained models for predicting different material properties. 086 Unlike methods using normalizing flows that require architectural constraints for invertibility, our method imposes no such restrictions on these models. When newer models become available in 087 088 the future, improved prediction accuracy will be achieved by incorporating them into our approach. Additionally, by managing the optimization range and utilizing special loss functions, we facilitate 089 targeted properties optimization within specific crystal structures, avoiding retraining. Moreover, 090 by imposing constraints via combinations of oxidation numbers, our method ensures the electrical 091 neutrality of any proposed materials, even in large atomic configurations where verifying electrical 092 neutrality is difficult due to combinatorial explosion. The generalizability of SMOACS enables it to 093 adopt various prediction models and optimize various properties. 094

SMOACS is the first method that directly optimizes the space of crystal structures using a 095 gradient-based approach. We achieve this by making the entire crystal structure differentiable, 096 which involves decomposing it into various components and representing atomic species as 097 atomic distributions. Unlike traditional methods that convert crystal structures into latent 098 variables (Ren et al., 2022)-thereby entangling their elements-our approach maintains the independence of each component. This independence facilitates the preservation of crystal structures 100 and ensures electrical neutrality by precisely specifying the atoms at each site. Furthermore, unlike 101 generative models that probabilistically generate materials satisfying certain conditions, our method 102 can inherently guarantee electrical neutrality and the preservation of crystal structures. Moreover 103 we can add additional constraints as long as they are differentiable.

We demonstrated that SMOACS could effectively utilize both GNN-based models and transformerbased models, outperforming FTCP, deep generative models, and TPE, Bayesian optimization. We demonstrated the band gap optimization within perovskite structures without retraining, using models trained on the MEGNet dataset (Chen et al., 2019), which includes various types of crystals.

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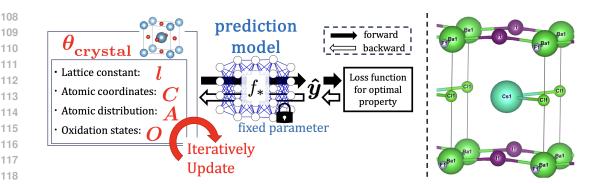


Figure 1: (left) Overview of the SMOACS framework. (right) An example of an optimized perovskite structure with a 4.02 eV band gap, verified at 3.96 eV through DFT calculations. Visualization was done with VESTA (Momma & Izumi, 2011).

Additionally, we demonstrated the optimization for large atomic configurations with as many as 135 atom sites while ensuring electrical neutrality. Furthermore, the validity of the proposed materials was verified through DFT calculations.

2 **RELATED WORKS**

Property prediction model. In recent years, much research has actively focused on predicting the 131 properties of materials using DFT-generated data (Davariashtiyani & Kadkhodaei, 2023; Merchant 132 et al., 2023; Yang et al., 2024a). There are two primary approaches involving deep learning. The first 133 approach utilizes GNNs (Chen et al., 2019; Park & Wolverton, 2020; Louis et al., 2020; Schmidt 134 et al., 2021; Lin et al., 2023), such as ALIGNN. The main advantage of using GNNs is their ability 135 to graphically represent crystal structures, thereby considering inter-atomic relationships in more 136 physically meaningful ways. The second approach employs transformers (Ying et al., 2021; Yan 137 et al., 2022), such as Crystalformer, which are known for their promising performance in the field 138 of computer vision and natural language processing (Brown et al., 2020; Dosovitskiy et al., 2021). 139

Deep generative models. Deep generative models, including language models, for producing new 140 stable materials have been emerging in the last years (Xie et al., 2022; Lyngby & Thygesen, 2022; 141 Sultanov et al., 2023; Yang & Mannodi-Kanakkithodi, 2022). However, only a few studies explored 142 material properties and stability at the same time. Studies such as FTCP (Ren et al., 2022), Matter-143 Gen (Zeni et al., 2023), and UniMat (Yang et al., 2024b) focused on optimizing properties including 144 band gap and material stability. They are generative models and thus conduct property optimizations 145 within the framework of generative modeling. FTCP, based on Variational Autoencoders (Kingma, 146 2013), encodes crystal structures into latent variables. It employs prediction branches to predict 147 properties from these variables. MatterGen and UniMat are diffusion models and employ classifierfree guidance (Ho & Salimans, 2021) to generate materials with specific properties. Although 148 methods exist to constrain condition-free models for generating specific outputs (Wu et al., 2024) 149 , no research has implemented these techniques for crystal structures. 150

151 Bayesian Optimization. Black-box optimization, including Bayesian optimization, is widely used 152 in materials science (Song et al., 2024). Numerous studies in materials science apply Bayesian optimization to predict crystal and molecular structures from target properties (Boyar et al., 2024; 153 Zhai et al., 2024; Khatamsaz et al., 2023). One representative method of Bayesian optimization 154 widely utilized in materials science research is the Gaussian process (GP) (Lu et al., 2022). However, 155 as the Gaussian process only handles continuous values, its ability to manage categorical variables 156 like elements is questionable. The recently proposed Tree-structured Parzen Estimator (TPE), which 157 is capable of handling categorical variables and multi-objective optimization (Ozaki et al., 2020b; 158 2022) and has been utilized in materials science (Ozaki et al., 2020a), could be a better choice. 159

Gradient based approach. Gradient-based approaches that aim to optimize design variables toward 160 desired properties using deep learning-based predictors and their gradients have been applied across 161 a wide range of fields. For example, they have been used to optimize designs for dynamics

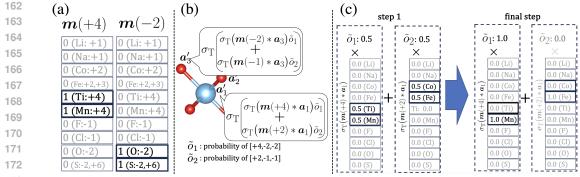




Figure 2: (a) SMOACS enforces site-specific restrictions on the types of elements to maintain elec-trical neutrality. The oxidation masks, labeled m(+4) and m(-2), correspond to elements with oxidation numbers of +4 and -2, respectively. The values 0 and 1 indicate the values of the mask. Parentheses indicate the elements and their potential oxidation numbers at each position. (b) Atomic distributions at each site considering two potential oxidation numbers. Here, we consider two possi-ble patterns of oxidation number combinations: [+4, -2, -2] and [+2, -1, -1] for three sites. The atomic distributions at each site are computed by taking the weighted sum of the probabilities of these patterns. (c) Change in the atomic distribution at site No.1 in the optimization process. The numbers in the grids indicate the probabilities of elements in parentheses. As a result of the opti-mization, the TiO₂-type oxidation pattern [+4, -2, -2] is selected, with Mn chosen as the element that achieves a +4 oxidation number.

of physical systems (Allen et al., 2022; Hwang et al., 2022), image manipulation (Xia et al., 2022), metamaterials (Bordiga et al., 2024), and chemical compositions (Fujii et al., 2024) to achieve target performance. These methods require that the chain rule of differentiation connects from the input to the output. While there is the study that apply this technique by mapping crystal structures into latent spaces (Xie et al., 2022), there are no studies that apply it directly within the space of crystal structures.

3 SMOACS

In SMOACS, the crystal structure $\theta_{crystal}$ is divided into four learnable parameters: lattice constant l, coordinates of N atomic sites C, elements e, and an oxidation state configuration parameter o (Fig. 1(left)).

$$\boldsymbol{\theta}_{\text{crystal}} = \{\boldsymbol{l}, \boldsymbol{C}, \boldsymbol{e}, \boldsymbol{o}\} \tag{1}$$

$$\boldsymbol{l} \in \mathbb{R}^{6}, \ \boldsymbol{C} \in \mathbb{R}^{N \times 3}, \ \boldsymbol{e} \in \mathbb{R}^{N}, \ \boldsymbol{o} \in \mathbb{R}^{D}$$
 (2)

The lattice constant l comprises the crystallographic axes lengths a, b, c and the angles between these axes α, β, γ . The oxidation state configuration parameter o denotes the probabilities for D patterns of oxidation number combinations determined by initial crystal structures, further described in Sec-tion 3.1. The l and C, being continuous variables, can be optimized directly through backpropaga-tion technique (Ren et al., 2020; Fujii et al., 2023). However, this technique cannot be used for the elements e since they are being discrete and categorical. Therefore, instead of directly handling the elements e, we employ a technique where an element at site n is represented by the atomic distribu-tion a_n ($a_n \in \mathbb{R}^K, A \in \mathbb{R}^{N \times K}, A_{i,:} = (a_i)^\top$) (Konno et al., 2021; Fujii et al., 2024). Here, Krepresents the highest atomic number considered. Since we are dealing with atomic numbers from 1 to 98, K = 98. When an element with the atomic number k occupies site n, a_n becomes a one-hot vector with the element k set to 1 and 0 at all others. Please refer to Section A.5 for a discussion on the general applicability of using atomic distributions in various property prediction models.

216 3.1 MASKS TO MAINTAIN ELECTRICAL NEUTRALITY 217

218 To maintain electrical neutrality, we restrict the possible values of atomic distribution a_n by using 219 a mask that aligns with the possible oxidation numbers at site n. These possible oxidation numbers are determined from the initial structure. Here, we explain this using the rutile type structure as 220 an example. A typical material having this structure is titanium dioxide (TiO_2). The rutile TiO_2 221 contains one Ti site and two O sites, totaling three atomic sites. When titanium has an oxidation 222 number of +4 and the two oxygen atoms each have an oxidation number of -2, the total oxidation number is zero, thus achieving electrical neutrality. Therefore, to maintain electrical neutrality, we 224 can use an atomic distribution that includes only elements with a + 4 oxidation number, such as Ti 225 and Mn at the Ti site. At the O sites, we use those with an oxidation number of -2, such as O and S. 226 This ensures electrical neutrality regardless of the elements selected after optimization. The adjusted 227 atomic distribution a'_n , which considers oxidation numbers, is obtained by taking the element-wise 228 product of the learnable distribution a_n with the atomic mask m(s). 229

$$\boldsymbol{a}_n'(s) = \sigma(\boldsymbol{m}(s) \ast \boldsymbol{a}_n) \tag{3}$$

Here, m(s) is $m(s) \in \mathbb{R}^{K}$ and a mask that assigns a value of 1 to elements with the oxidation number s, and 0 to all others (Fig. 2(a)). σ is a normalization function that rescales all elements to the range [0,1], with their total sum normalized to 1.0. The asterisk denotes element-wise multiplication. This process is applied to all sites, yielding an atomic distribution A'(S) that reflects the oxidation numbers for all sites.

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$$\mathbf{A}'(\mathbf{S}) = \sigma_{\text{atom}}(\mathbf{A} * \mathbf{M}(\mathbf{S})) \tag{4}$$

$$M \in \mathbb{R}^{N \times K}, \ M(S)_{i,:} = (m(s_i))^{\top}, \ S_i = s_i, \ m(s_i) \in \{m(s_{\min}), ..., m(s), ..., m(s_{\max})\}$$
 (5)

Here, σ_{atom} is a function that normalizes values along elemental directions. s_i is the oxidation 242 number at site-i. The s_{\min} and s_{\max} respectively denote the minimum and maximum oxidation 243 numbers among all elements considered. To simultaneously consider different patterns of oxidation 244 number combinations, we introduce a learnable parameter o, which selects the optimal combination 245 of oxidation numbers. We illustrate this approach using CoF_2 and TiO_2 , both of which adopt the 246 rutile structure. The oxidation numbers differ at each atomic site, with CoF_2 exhibiting oxidation 247 numbers of [+2, -1, -1] and TiO₂ having [+4, -2, -2]. The o is a d-dimensional vector selecting 248 the best pattern from d patterns of oxidation number combinations. The o represents the probabilities 249 of each combination. For instance, when considering two patterns, such as those of CoF_2 and TiO_2 , 250 d equals 2. Using this framework, we can calculate the modified atomic distribution a'_n (Fig. 2(b)) considering multiple combination patterns as follows:

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$$\boldsymbol{a}_{n}'(\boldsymbol{a}_{n}, o_{d}) = \sum_{d=1}^{D} \boldsymbol{a}_{n,d}' = \sum_{d=1}^{D} \sigma(\boldsymbol{m}(s_{n,d}) \ast \boldsymbol{a}_{n}) o_{d}.$$
 (6)

256 Here, $m(s_{i,d}) \in \{m(s_{\min}), ..., m(s), ..., m(s_{\min})\}$. The property prediction models assume that each site contains a single element. Therefore, after optimization, it is desirable for the oxidation 258 state configuration parameter o and atomic distributions a'_n to become one-hot vectors. To guarantee that optimization will result in them becoming one-hot vectors, we normalize a'_n and o with the 260 temperature softmax function σ_T .

$$\sigma_T(z_i) = \frac{\exp\left(\frac{z_i}{T}\right)}{\sum_j \exp\left(\frac{z_j}{T}\right)} \tag{7}$$

$$\tilde{\boldsymbol{o}} = \sigma_T(\boldsymbol{o}) \tag{8}$$

$$\tilde{\boldsymbol{A}}_{n}\left(\boldsymbol{A}_{n},\boldsymbol{o},T\right) = \sigma_{T}\left(\sum_{d=1}^{D}\boldsymbol{A}_{d}'(\boldsymbol{S}_{d})\right) = \sigma_{T,\text{atom}}\left(\sum_{d=1}^{D}\sigma_{T,\text{atom}}(\boldsymbol{M}(\boldsymbol{S}_{d})*\boldsymbol{A})\tilde{o}_{d}\right)$$
(9)

$$(\boldsymbol{M}(\boldsymbol{S}_d) \in \mathbb{R}^{N \times K}, \ \boldsymbol{M}(\boldsymbol{S}_d)_{i,:} = \boldsymbol{m}(\boldsymbol{s}_{i,d})^{\top}, (\boldsymbol{S}_d)_i = s_{i,d}$$
 (10)

Here, $M(S_d)$ is an atomic mask of the *d*-th oxidation pattern. σ_T produces sharper distributions at lower temperatures *T*, ensuring that the parameters transition into one-hot vectors. For example, let σ_1 represent the probability of the TiO₂ type with oxidation numbers [+4, -2, -2], and σ_2 represent the CoF₂ type with oxidation numbers [+2, -1, -1]. If, at the end of optimization, o = (1, 0.) is achieved, the TiO₂ type is selected, resulting in a material with an oxidation pattern of [+4, -2, -2], as shown in Fig. 2(c).

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3.2 INITIALIZATION AND MULTIPLE PROPERTIES OPTIMIZATION

In SMOACS, optimizations begin with crystal structures from a dataset or those randomly generated the initial structures for the optimization process are obtained from two sources: an existing dataset and self-generated structures based on typical perovskites with modified lattice constants (for details, see Section A.6 and A.7). These crystal structures must satisfy electrical neutrality and generate D oxidation number patterns based on the compositions of initial crystal structures (see Section A.11 for details). The lattice constant l and atomic coordinates C are used directly as initial values. The atomic distribution A and the oxidation number pattern selection parameter o are initialized with a uniform distribution.

$$\theta_{\text{crystal}}^{\prime} = \tau\left(\{\boldsymbol{l}, \boldsymbol{C}, \tilde{\boldsymbol{A}}(\boldsymbol{A}, \boldsymbol{o}, T), \tilde{\boldsymbol{o}}(\boldsymbol{o}, T)\}\right)$$
(11)

$$\boldsymbol{l} \leftarrow -\eta_{\boldsymbol{l}} \frac{\partial \mathbf{L}}{\partial \boldsymbol{l}}, \quad \boldsymbol{C} \leftarrow -\eta_{\boldsymbol{C}} \frac{\partial \mathbf{L}}{\partial \boldsymbol{C}}, \quad \boldsymbol{A} \leftarrow -\eta_{\boldsymbol{A}} \frac{\partial \mathbf{L}}{\partial \boldsymbol{A}}, \quad \boldsymbol{o} \leftarrow -\eta_{\boldsymbol{o}} \frac{\partial \mathbf{L}}{\partial \boldsymbol{o}}. \tag{12}$$

Here, $\eta_l, \eta_C, \eta_A, \eta_o$ denote the learning rates for each parameter. L denotes loss function $L(f_*(\theta'_{crystal}), y_{target})$ and f_* denotes a set of trained models. The τ is a function converting structures to inputs for f_* . We optimize structures by iteratively updating them using Eq. 11 and 12. During optimization, the temperature T of the softmax function starts high and is lowered towards the end, forcing \tilde{o} and \tilde{a}_n into one-hot vectors in the final stage. SMOACS optimizes multiple properties by incorporating various trained models or additional loss functions. Here, we aim to optimize the crystal structure by minimizing formation energy and targeting a specific band gap range, $y_{bg} \pm h_{bg}$, using trained models $f_* = \{f_{bg*}, f_{f*}\}$. Here, h_{bg} is an acceptable margin and f_{bg*} is the trained model predicting the band gap, and f_{f*} predicts the formation energy. We also set a strength parameter λ .

$$L_{bg}(y_{bg}, \hat{y}_{bg}) = \max(0, |y_{bg} - \hat{y}_{bg}| - h_{bg}), \ L_{f}(\hat{y}_{f}) = \hat{y}_{f}$$
(13)

$$\mathbf{L} = \mathbf{L}_{\mathrm{bg}}(y_{\mathrm{bg}}, f_{\mathrm{bg}*}(\theta_{\mathrm{crystal}}')) + \lambda \mathbf{L}_{\mathrm{f}}(f_{\mathrm{f}*}(\theta_{\mathrm{crystal}}'))$$
(14)

The influence of λ is discussed in Section A.12. Note that since the crystal structure changes during optimization, when using GNNs, we update the graph multiple times based on the current structure in the optimization process.

3.3 PRESERVATION OF SPECIFIC CRYSTAL STRUCTURES DURING OPTIMIZATION

Limiting the optimization variables and their range allows us to maintain specific crystal structures 312 during optimization. For instance, let us consider a typical perovskite structure, represented by the 313 chemical formula ABX_3 . It consists of five sites and adopts a crystal structure close to a cubic lattice. 314 The fractional coordinates for the five sites are as follows: (0.5, 0.5, 0.5) at the A site, (0.0, 0.0, 0.0)315 at the B site, and (0.5, 0.0, 0.0), (0.0, 0.5, 0.0), (0.0, 0.0, 0.5) at the three X sites. Note that devia-316 tions from these values are allowed, together with degrees of freedom related to the lattice constant 317 values. We optimize the structures within the range of small perturbations applied to typical per-318 ovskites. Specifically, first, only the a, b, and c of l are optimized, while α , β , and γ are fixed at 319 90°. Subsequently, the optimization range for the five sites is set close to their typical coordinates. 320 For example, we optimize the coordinates at the A site within the range $(0.5 \pm \epsilon, 0.5 \pm \epsilon, 0.5 \pm \epsilon)$, 321 where ϵ is a small constant. Following a previous work on the distortion of CaCu₃Ti₄O₁₂ (Božin et al., 2004), we set $\epsilon = 0.15$. We also specify possible patterns of oxidation number combinations. 322 Typically, some materials with perovskite structure such as $SrTiO_3$ exhibit oxidation numbers of 323 [+2, +4, -2] at the A, B, and X sites, respectively, while others such as $(CH_3NH_3)PbI_3$ exhibit

[+1, +2, -1]. Consequently, two oxidation number patterns are prepared for the perovskite structure: [+2, +4, -2] and [+1, +2, -1] for the A, B, and X sites, respectively. By specifying these variables and ranges for optimization, we are able to maintain the perovskite structure.

4 EXPERIMENTS

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We compare SMOACS's ability to propose material satisfying specified properties and constraints 331 with those from deep generative models and Bayesian optimization, represented by FTCP and TPE, 332 respectively. TPE was chosen over GP, as discussed in Section 2. Both SMOACS and TPE ran for 333 200 optimization steps. All models were trained on the MEGNet dataset. For further implementation 334 details, please refer to Section A.4. To demonstrate SMOACS's versatility across various property 335 prediction models, we conducted optimizations using ALIGNN and Crystalformer, GNN-based and 336 transformer-based models, respectively. We evaluated the optimized materials using three metrics: 337 whether they satisfied the specified criteria on the band gap range, formation energy, and validity of 338 crystal structure. Specifically, we judge that the formation energy criterion is satisfied if it is less 339 than -0.5 eV, and for the validity of crystal structure, following a previous research (Xie et al., 2022), we adopted two criteria: all interatomic distances being at least 0.5 Å and maintaining electrical 340 neutrality. For assessing electrical neutrality, we consider the material electrically neutral if the sum 341 of possible oxidation numbers for atoms at each site equals zero. Please refer to Section A.9 for 342 details. For experiments utilizing models trained on datasets other than MEGNet, please refer to 343 Section A.13. 344

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4.1 LEVERAGING THE LATEST RESEARCH ACHIEVEMENTS IN PROPERTY PREDICTION

First, we experimented with the performance of property prediction models that could be adopted in systems such as SMOACS, Bayesian optimization (TPE), or FTCP. In principle, Bayesian optimization—a type of black-box optimization—and SMOACS can adopt a broad range of property prediction models. Meanwhile, FTCP—a generative model—employs a property prediction branch within its architecture. Therefore, unlike SMOACS and Bayesian optimization, FTCP cannot use ALIGNN or Crystalformer for property predictions. The results are shown in Table 1.

Crystalformer demonstrated the highest prediction accuracy among the models in Table 1. ALIGNN ranked second, whereas the prediction branches of FTCP exhibited the lowest performance. This result confirmed an advantage of SMOACS and TPE: their ability to incorporate state-of-the-art property prediction models, such as Crystalformer and ALIGNN, allowing for highly accurate material property predictions.

4.2 SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES REGARDLESS OF THE CRYSTAL STRUCTURE

We tested the ability to optimize band gaps to target values. In this experiment, we optimized both the band gap and formation energy simultaneously, regardless of the crystal structure. The margins for the targets of band gaps were set to To ensure a fair comparison of optimization methods, we fixed the margin for all band gaps at ± 0.04 eV. For optimization results where the predictor's error is used as the margin, please refer to Section A.14. We conducted experiments with SMOACS using Crystalformer and ALIGNN, respectively. We utilized Crystalformer as a predictor for TPE. We used three objective functions for TPE: band gap, formation energy, and electrical neutrality. FTCP selected data from the MEGNet dataset close to the target band gap and with formation energy less

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Table 1: Comparison of property prediction models. This table compares models trained on the MEGNet dataset and presents Mean Absolute Error (MAE) scores for formation energy (**E_form**) and band gap on test data in the MEGNet dataset. Lower scores are better across all metrics.

874 875	Prediction Model	Applicable Method	E_form MAE (eV)	Band Gap MAE (eV)
376	Prediction Branch of FTCP	FTCP	0.224	0.442
	ALIGNN	SMOACS, TPE	0.022	0.218
377	Crystalformer	SMOACS, TPE	<u>0.019</u>	<u>0.198</u>

378 Table 2: Experiments on optimizing band gaps. We define the success rate as the probability of 379 simultaneously satisfying three conditions: (A) the band gap is optimized within the target range, 380 (B) the formation energy is below -0.5 eV, and (C) the crystal structure is valid. C is achieved when two criteria are met simultaneously: (a) all inter-atomic distances are greater than 0.5 Å, and (b) the 381 structure is electrically neutral. S(Cry) and S(ALI) denote SMOACS utilizing the Crystalformer and 382 ALIGNN models, respectively. We evaluated each of the proposed materials using all evaluation 383 metrics, and the results were averaged over 256 samples. Higher scores are better across all metrics. 384 Augmented results are shown in Table A.2. 385

Target	method	success		(D)E	COSTD	(a)	(b)
BG (eV)	method	rate	(A)BG	(D)Lf	(C)STR	neut	0.5\AA
	S(Cry)	<u>0.328</u>	0.465	0.566	0.758	0.957	0.758
0.50	S(ALI)	0.055	0.062	0.867	0.867	0.949	0.867
± 0.04	TPE	0.004	0.945	0.059	0.066	0.070	0.910
	FTCP	0.000	0.004	1.000	0.719	0.746	0.906
	S(Cry)	<u>0.387</u>	0.543	0.672	0.824	0.980	0.824
1.50	S(ALI)	0.043	0.066	0.828	0.852	0.938	0.852
± 0.04	TPE	0.020	0.855	0.055	0.074	0.082	0.828
	FTCP	0.000	0.000	1.000	0.703	0.723	0.895
	S(Cry)	<u>0.383</u>	0.473	0.715	0.840	0.984	0.840
2.50	S(ALI)	0.051	0.059	0.809	0.793	0.898	0.793
± 0.04	TPE	0.023	0.711	0.098	0.051	0.055	0.816
	FTCP	0.004	0.004	1.000	0.695	0.707	0.902

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than -0.5 eV, subsequently encoding them into latent variables. Finally, after adding noise, we decoded them back into crystal structures for evaluation. We optimized and evaluated the structures based on the band gap and formation energy values predicted by their respective predictors.

405 The results are shown in Table 2. SMOACS significantly outperformed both TPE and FTCP in 406 terms of success rates. SMOACS consistently maintained electrical neutrality, except for extreme geometries causing NaN values during crystal vector calculations. While FTCP always met the re-407 quirements for formation energy, it struggled to achieve the target band gap, contributing to its lower 408 overall success rate. TPE achieved a high success rate in optimizing the band gap within the target 409 range, but it could not optimize formation energy well. SMOACS maintained a high overall success 410 rate as it achieved substantial success rates in both band gap and formation energy optimization. 411 SMAOCS can easily scale this computation and can optimize 2,048 samples simultaneously in just 412 a few minutes using a single A100 GPU. This allows us to repeat the optimization process multiple 413 times, enabling us to obtain a large number of successful optimization samples. Please refer to 414 Section A.6 for details, including the diversity of generated materials. 415

SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES WHILE PRESERVING 4.3 PEROVSKITE STRUCTURES

419 We optimized the band gap within the range of 0.5 to 2.5 eV while preserving the perovskite struc-420 ture. Besides the previously discussed metrics, we used three new criteria to confirm a structure's perovskite identity: internal coordinates, angles between crystal axes, and the tolerance factor. The 422 tolerance factor t serves as a metric to assess the suitability of atomic combinations for forming 423 perovskite structures (West, 2022). t is calculated based on the ionic radii r_A , r_B and r_X of each site in the perovskite structure ABX_3 and we employed a loss function to optimize the tolerance factor 424 alongside minimizing the band gap and formation energy. 425

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$$t = \frac{r_{\rm A} + r_{\rm X}}{\sqrt{2}(r_{\rm D} + r_{\rm X})} \tag{15}$$

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$$\sqrt{2(r_{\rm B} + r_{\rm X})}$$

430 $L_{\rm A} = |t - 0.9|$ (16)

$$\mathbf{L}_{t} = \begin{bmatrix} 0 & 0.0 \end{bmatrix}$$

$$\mathbf{I}_{t} = \begin{bmatrix} 0 & 0.0 \end{bmatrix}$$

432 Table 3: Experiments on optimizing various band gaps while preserving perovskite structures. The 433 "success rate" is the probability of simultaneously satisfying four criteria: (A) the band gap is opti-434 mized within the target range, (B) the formation energy is below -0.5 eV, (C) the crystal structure is valid, and (D) approximating a valid perovskite structure. Criteria (A), (B), and (C) are consistent 435 with those outlined in Table 2. The (D) is achieved when three criteria are met simultaneously: (c) 436 the tolerance factor t is between 0.8 and 1.0, (d) coordinates are within ± 0.15 of typical perovskite 437 structure coordinates, and (e) crystal axis angles are from 85° to 95°. The results are averaged over 438 256 samples. Higher scores are better across all metrics. Augmented results are shown in Table A.4. 439

Target	method	success	(A)BG	(B)F.	(C)STR	(a)	(b)	(D)PS	(c)	(d)	(e)
BG (eV)	memou	rate		(D)Ef	(C)51K	neut	0.5Å	(D)15	tole	angles	coord
	S(Cry)	<u>0.113</u>	0.477	0.410	0.965	1.000	0.965	0.500	0.500	1.000	1.000
$\begin{array}{c} 0.50 \\ \pm 0.04 \end{array}$	S(ALI)	0.090	0.211	0.535	1.000	1.000	1.000	0.500	0.500	1.000	1.000
	TPE	0.027	1.000	0.137	0.535	0.535	1.000	0.648	0.648	1.000	1.000
	FTCP	0.004	0.023	1.000	0.836	0.840	0.938	0.258	0.508	0.441	0.285
	S(Cry)	<u>0.148</u>	0.422	0.461	0.984	1.000	0.984	0.578	0.578	1.000	1.000
1.50	S(ALI)	0.070	0.219	0.652	1.000	1.000	1.000	0.629	0.629	1.000	1.000
± 0.04	TPE	0.023	0.992	0.281	0.293	0.293	1.000	0.523	0.523	1.000	1.000
	FTCP	0.000	0.016	1.000	0.895	0.906	0.965	0.242	0.547	0.418	0.320
	S(Cry)	<u>0.152</u>	0.285	0.516	0.988	1.000	0.988	0.625	0.625	1.000	1.000
2.50	S(ALI)	0.113	0.184	0.938	1.000	1.000	1.000	0.625	0.625	1.000	1.000
± 0.04	TPE	0.016	0.918	0.281	0.352	0.352	1.000	0.387	0.387	1.000	1.000
	FTCP	0.008	0.012	0.996	0.879	0.898	0.953	0.250	0.543	0.441	0.289

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If t is close to 1, the structure is likely a perovskite; if it is far from 1, it is not. Considering t values of 456 typical perovskite structures (BaCeO₃:0.857, SrTiO₃: 0.910 and BaTiO₃: 0.970), we established 457 a tolerance factor range of $0.8 \le t \le 1.0$ as the criterion for success. SMOACS optimized the 458 structures with the procedure outlined in Section 3.3. Due to the limited number of perovskite 459 structures in the MEGNet dataset, random perovskite configurations are used as initial values for 460 SMOACS and TPE. The optimization range for SMOACS and TPE is established as outlined in 461 Section 3.3. FTCP initially encoded typical perovskite structures from the MEGNet dataset into 462 latent variables. After adding noise, these latent variables are decoded back into crystal structures 463 for evaluation. Please refer to Section A.7 for details, including the diversity analysis. We evaluated 464 the structures based on the band gap and formation energy values predicted by their respective 465 predictors. The evaluation results are shown in Table 3.

SMOACS with Crystalformer significantly outperformed both TPE and FTCP in overall success rates while preserving perovskite structures. In terms of (d) coordinates and (e) angles, both SMOACS and TPE consistently meet the criteria because their optimization ranges are the same. The generative model (FTCP), which uses latent variables, fails to obtain specific structural features of perovskite. Note that this limitation occurs despite the use of latent variables based on typical perovskite structures. This seems to be attributed to the training dataset that includes mixed crystal types. It is noteworthy that SMOACS consistently ensures electrical neutrality.

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474 4.4 OPTIMIZING LARGE ATOMIC CONFIGURATIONS

We optimized large atomic configurations where calculating electrical neutrality is impractical. In systems containing many atoms, the calculation of electrical neutrality becomes infeasible due to combinatorial explosion. For example, a system containing 135 atoms, each with two possible oxidation numbers, results in about 4.3×10^{40} combinations. Therefore, including an objective function for electrical neutrality in the TPE is infeasible. We conducted experiments on $3 \times 3 \times 3$ perovskite structures containing 135 atom sites and compared SMOACS with TPE, not including the objective function for electrical neutrality (referred to as TPE(/N)). The results are shown in Table 4.

SMOACS successfully optimized large atomic configurations, while TPE(/N) failed due to its in ability to optimize the formation energy. The success of SMOACS likely stems from the utilization of information on gradients to optimize based on physics, enabling optimization even in large and complex systems. Furthermore, TPE was not able to evaluate electrical neutrality due to the compu-

Target	method	success	(A)BG		(C)STR	(a)	(b)	(D)PS	(c)	(d)	(e)
BG (eV)	method	rate	(A)DU		(C)SIK	neut	0.5Å	(D)F5	tole	angles	coord
$\begin{array}{c} 0.50 \\ \pm 0.04 \end{array}$	S(Cry)	0.156	0.734	0.547	0.968	1.00	0.969	0.570	0.570	1.000	1.000
	S(ALI)	<u>0.188</u>	0.234	0.812	0.687	1.00	0.688	0.789	0.789	1.000	1.000
	TPE(/N)	0.000	1.000	0.000	- !	N/A	1.000	0.609	0.609	1.000	1.000
1.50	S(Cry)	0.047	0.125	0.422	0.953	1.00	0.953	0.617	0.617	1.000	1.000
± 0.04	S(ALI)	<u>0.062</u>	0.086	0.867	0.726	1.00	0.727	0.586	0.586	1.000	1.00
± 0.04	TPE(/N)	0.000	0.141	0.000	- !	N/A	1.000	0.180	0.180	1.000	1.000
2.50 ± 0.04	S(Cry)	0.023	0.039	0.438	0.984	1.00	0.984	0.664	0.664	1.000	1.000
	S(ALI)	<u>0.102</u>	0.172	1.000	0.703	1.00	0.703	0.812	0.812	1.000	1.00
± 0.04	TPE(/N)	0.000	0.023	0.000	_ !	N/A	0.984	0.156	0.156	1.000	1.00

Table 4: Experiments optimizing for various band gaps while preserving $3 \times 3 \times 3$ perovskite structures. We include only TPE, showing better performance in Section 4.3, for comparison. Evaluation methods are based on those described in Table 3. Augmented results are shown in Table A.6.

tational cost of calculating it. Conversely, since SMOACS always maintains electrical neutrality, it is able to optimize properties while preserving this neutrality. Please refer to Section A.8 for details.

4.5 VERIFICATION BY DENSITY FUNCTIONAL THEORY

We used Density Functional Theory (see Section A.1 for details) to verify the band gaps of materials proposed by SMOACS. Among these materials, BaCsFCII (Fig. 1(right)), a perovskite structured for a 4.02 eV band gap, showed a DFT-calculated value of 3.96 eV. However, we also found discrepancies between the values the model predicted and the DFT calculated for other candidate materials. Detailed results can be found in the appendix, Section A.2.

513 5 CONCLUSIONS

We propose SMOACS, a framework that utilizes the latest high-accuracy property prediction models and their gradients to search for materials with targeted multiple properties. SMOACS can adaptively apply constraints such as electrical neutrality and specific crystal structures without re-training. SMOACS not only outperformed FTCP and TPE in optimizing multiple targeted properties simultaneously but also maintained electrical neutrality in large systems where calculating electri-cal neutrality is challenging due to combinatorial complexity. As a further potential application, SMOACS should facilitate the exploration of stable structures. Using a compositional formula and various structure candidates they could form, this method minimizes formation energy while main-taining the crystal structure, thus determining the most stable configuration for that formula (see Section A.3). However, the performance of SMOACS heavily depends on the accuracy of prop-erty prediction models. Using models based on DFT calculations that underestimate band gaps (see Section A.2) can lead to similar underestimations in the predictions. By adopting more accurate models trained on datasets that are large and developed with more accurate DFT, we may address these challenges.

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810 A APPENDIX

A.1 DETAILS OF THE SETTINGS IN DENSITY FUNCTIONAL THEORY

We used Density Functional Theory (DFT) to verify the band gaps of materials proposed by
SMOACS. We employed the Vienna Ab initio Simulation Package (VASP) (Kresse & Joubert, 1999)
version 5.4.4 with Perdew-Burke-Ernzerhof (PBE) exchange-correlation functional (Perdew et al.,
1996) and Projector Augmented Wave (PAW) pseudo-potentials (Blöchl, 1994) in all DFT calculations. We used the MPRelaxSet from PyMatGen (Ong et al., 2013) to generate input files: KPOINTS
and INCAR.

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A.2 BAND GAP DISCREPANCIES BETWEEN MACHINE LEARNING PREDICTED AND DFT CALCULATED

824 SMOACS heavily relies on the accuracy of property prediction models. However, we found dis-825 crepancies between the values model predicted and the DFT calculated (Fig.A.1). Furthermore, structures relaxed by the MPRelaxSet, which is a parameter set for structural relaxations with VASP 826 provided in PyMatGen, sometimes significantly differ from their proposed forms. There are two 827 possible reasons. First, DFT settings: MEGNet dataset comes from an older version of Materials 828 Project database (Jain et al., 2013). Materials in this database are sometimes updated, and calculation 829 conditions when the MEGNet dataset is created could be different from the current MPRelaxset. We 830 could not reproduce the band gap values in the MEGNet dataset with MPRelaxset. Second, MEG-831 Net dataset features: All models used in this work are trained on the MEGNet dataset, which is 832 comprised predominantly of stable materials. So, predicting unstable or physically inappropriate 833 structures with these models can lead to inaccurate predictions that may affect the proposed mate-834 rials. To address these issues, we may need a model trained on a large dataset that includes both 835 stable and unstable structures.

The MEGNet dataset utilizes DFT calculations with PBE functionals that are known to underestimate band gaps. Consequently, when models trained on the MEGNet dataset are used in SMOACS, this tendency may be reflected in the predictions of the proposed materials. This issue may be addressed by constructing a dataset using more accurate band gap calculations, such as HSE06 hybrid functionals (Krukau et al., 2006) and adopting models trained on that dataset. It should be noted that the amount of data available with these accurate calculations is much more limited than for DFT-PBE.

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A.3 A POSSIBLE APPLICATION: IDENTIFYING THE MOST STABLE CRYSTAL STRUCTURE

846 Our method can optimize energy while specifying the base crystal structure. This property may 847 allow for identifying crystal structures based either solely on the chemical formula or on a combination of the chemical formula and physical properties. This is a Crystal Structure Prediction (CSP) 848 task (Ryan et al., 2018; Miller et al., 2024; Jiao et al., 2024). To verify if this is possible, we exper-849 imented to see if the crystal structure of metallic silicon with a zero band gap could be identified. 850 Initially, we extracted structures from the MEGNet dataset that contained only one atom besides Si, 851 using them as the initial structure. The atomic distribution was fixed with a one-hot vector indicat-852 ing silicon, and only the lattice constants were optimized. The target properties for optimization 853 were a zero band gap and formation energy minimization. We chose silicon structures from the 854 MEGNet dataset with a band gap of 0 eV as the reference and compared these with the optimized 855 structures that exhibited the lowest formation energy. Consequently, we identified structures close 856 to the reference among those optimized for the lowest formation energy. 857

The results are shown in Table A.1. The reference material of mp-34 is close to optimized candidate No. 2. Similarly, mp-1014212 is close to candidates from No. 4 to No. 12."

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- 861 A.4 IMPLEMENTATION DETAILS
- To demonstrate that SMOACS can utilize various property prediction models, we selected ALIGNN as a representative of the GNN-based models and Crystalformer as a representative of the

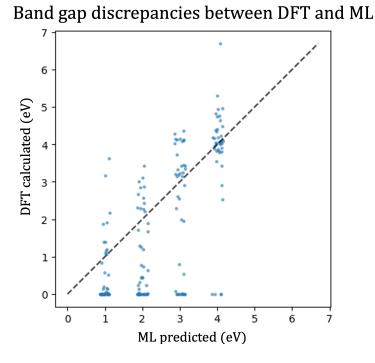


Figure A.1: The discrepancies between band gap values predicted the machine learning model (Crystalformer) and that of DFT calculated.

Table A.1: Reference Si materials (band gap 0 eV) and optimized candidates.

Materials	<i>a,b,c</i> (Å)	$lpha,eta,\gamma$ (°)	predicted formation energy (eV)
(Ref) mp-34	2.64, 2.64, 2.47	90.0, 90.0, 120	-
$\overline{(\text{Ref})}$ mp- $\overline{1014212}$	$\bar{2.66}, \bar{2.66}, \bar{2.66}$	$\begin{bmatrix} \bar{1}0\bar{9}.\bar{5}, \bar{1}0\bar{9}.\bar{5}, \bar{1}0\bar{9}.\bar{5} \end{bmatrix}$	
candidate-1	2.67, 2.67, 2.94	124.0, 124.0, 97.9	-0.367
candidate-2	$\overline{2.50, 2.50, 2.27}$	$[-\bar{89.9}, \bar{89.9}, \bar{134.0}]$	
candidate-3	$\bar{2.76}, \bar{2.76}, \bar{2.76}$	$1\bar{1}\bar{1}\bar{5}.\bar{3}, 1\bar{1}\bar{5}\bar{.}\bar{3}, 1\bar{1}\bar{5}\bar{.}\bar{3}$	
candidate-4	$\bar{2.72, 2.72, 2.72}$	$1\bar{1}\bar{1}\bar{5}.0, 1\bar{1}\bar{5}.0, 1\bar{1}\bar{5}.0$	
candidate-5	$\overline{2.71, 2.71, 2.71}$	$1\bar{1}\bar{4}.9, 1\bar{1}\bar{4}.9, 1\bar{1}\bar{4}.9$	
candidate-6	$\bar{2.71}, \bar{2.71}, \bar{2.71}$	$1\bar{1}\bar{1}\bar{4}.9, 1\bar{1}\bar{4}.9, 1\bar{1}\bar{4}.9$	
candidate-7	$\bar{2.71, 2.71, 2.71}$	$11\overline{14.9}, 1\overline{14.9}, \overline{114.9}$	
candidate-8	$\bar{2.73}, \bar{2.73}, \bar{2.73}$	$1\overline{1}\overline{1}\overline{5}.0, 1\overline{1}\overline{5}.0, \overline{1}\overline{1}\overline{5}.0$	
candidate-9	$\overline{2.69}, \overline{2.69}, \overline{2.69}$	$1\bar{1}\bar{1}\bar{4}.9, 1\bar{1}\bar{4}.9, 1\bar{1}\bar{4}.9$	
candidate-10	$\bar{2.68}, \bar{2.68}, \bar{2.68}$	114.8, 114.8, 114.8	
candidate-11	$\bar{2.64}, \bar{2.64}, \bar{2.64}$	$11\overline{1}4.\overline{3}, 1\overline{1}\overline{4}.\overline{3}, \overline{1}1\overline{4}.\overline{3}$	
candidate-12	2.55, 2.55, 2.55	$1\overline{1}\overline{1}\overline{3}.\overline{9}, 1\overline{1}\overline{3}.\overline{9}, \overline{1}\overline{1}\overline{3}.\overline{9}$	
candidate-13	$\bar{2.44}, \bar{2.43}, \bar{2.44}$	$\overline{68.8, 64.7, 111.1}$	
candidate-14	$\bar{2.42}, \bar{2.42}, \bar{2.42}$	$11\overline{1}2.\overline{3}, 1\overline{1}2.\overline{3}, \overline{1}1\overline{2}.\overline{3}$	
candidate-15	$\bar{2.42}, \bar{2.42}, \bar{2.42}$	$11\overline{1}2.\overline{3}, 1\overline{1}2.\overline{3}, \overline{1}1\overline{2}.\overline{3}$	

912 Transformer-based models. For both ALIGNN and Crystalformer, we utilized publicly available
 913 weights trained on the MEGNet dataset that predict band gaps and formation energies.

The number of optimization steps was 200 for both SMOACS and TPE. The softmax temperature T was linearly decayed from T = 0.01 at step 1 to T = 0.0001 at step 200. Unless otherwise specified, to prevent extreme crystal structures, the crystal axis lengths a, b, c were clipped to a range of 2 Å to 10 Å, and the angles α, β, γ were clipped to between 30° and 150°. The types of elements considered ranged from atomic numbers 1 to 98. Unless otherwise noted, the search range for TPE was aligned with SMOACS, with crystal axis lengths a, b, c ranging from 2 Å to 10 Å and angles α, β, γ from 30° to 150°. We set the strength parameter $\lambda = 1.0$.

In ALIGNN, bonds are defined using a graph structure. However, because the graph structure is non-differentiable, it cannot be optimized directly. Moreover, as the crystal structure is optimized, the nearest-neighbor atoms may change, potentially rendering the continued use of the same graph structure inappropriate. Therefore, we updated the graph structure multiple times during the optimization process. Considering that the learning rate decay follows a cosine schedule, we updated the graph several times according to a sine schedule, which is the integral of the cosine function.

SMOACS was implemented using PyTorch (Paszke et al., 2019); we used the web-available implementation of FTCP¹ and trained it on the MEGNet dataset. Optuna (Akiba et al., 2019) was used for TPE. We conducted optimizations using ALIGNN and Crystalformer, GNN-based, and transformer-based models, respectively. We used a NVIDIA A100 GPU. We utilized official codes and weights that are available online²³.

932 We trained FTCP from scratch using the MEGNet dataset. We tuned the hyperparameters, including the max_elms parameter (the number of types of atoms in the crystal), the max_sites param-933 eter (the number of atomic sites in the crystal), and the learning rate. As a result, max_elms, 934 max_sites, and the learning rate were set to 4, 20, and 0.0001, respectively. Note that the MEG-935 Net dataset contains data with a larger number of element types and sites than these settings, so we 936 did not utilize all 60,000 training samples; however, the reconstruction error score was better with 937 this setting. During inference, after testing several values for the standard deviation of the noise 938 added to the latent variables, we decided to sample from a normal distribution with a mean of 0 and 939 a standard deviation of 0.6.

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A.5 APPLICABILITY TO PROPERTY PREDICTION MODELS

943 This strategy of using atomic distributions discussed in Section 3 is widely applicable to various property prediction models. It readily supports formats such as Crystalformer, where one-hot vectors 944 representing elements are fed into the model. Next, we consider a scenario of using models such as 945 ALIGNN that require atomic representations as input. In this scenario, we treat the inner product 946 of the atomic distribution a_n and the *u*-dimensional representation vector for atoms r_{atom} ($r_{\text{atom}} \in$ 947 $\mathbb{R}^{K \times u}$) as the atomic representation. In either case, since the output is connected to the learnable 948 atomic distributions through the chain rule of differentiation, we are able to optimize the atomic 949 distribution through backpropagation. 950

A.6 DETAILS IN SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES REGARDLESS OF THE CRYSTAL STRUCTURE

954 We optimized the band gap regardless of the crystal structure and simultaneously minimized the 955 formation energy. We randomly selected initial crystal structures with up to 10 atomic sites from the MEGNet dataset for SMOACS and TPE, ensuring that each selected structure met the crite-956 rion of electrical neutrality. In SMOACS, we selected up to 10 possible oxidation number pat-957 terns based on the atom combinations in the initial crystal structure, all of which ensure overall 958 electrical neutrality. the learning rates were set as $\eta_l = 0.01, \eta_C = 0.02, \eta_A = \eta_O = 6.0$ 959 for SMOACS with Crystalformer. For SMOACS with ALIGNN, the learning rates were set as 960 $\eta_l = 0.008, \eta_C = 0.02, \eta_A = \eta_O = 0.0002$. The learning rates were decayed using a cosine 961 annealing schedule. 962

We updated the graph structure data in ALIGNN 32 times according to a sine schedule, which is the integral of the cosine function. Specifically, we reconstructed the graph structure based on the current crystal configuration at steps [4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 61, 65, 69, 74, 79, 83, 88, 93, 99, 104, 110, 116, 123, 129, 137, 146, 156, 169], as well as during the evaluation after optimization. Then, we constructed masks M_d corresponding to its oxidation pattern d. The atomic distribution A and the oxidation state configuration parameter o were initialized with a uniform distribution. The loss functions for the band gap and formation energy in SMOACS use

³https://github.com/omron-sinicx/crystalformer

^{970 &}lt;sup>1</sup>https://github.com/PV-Lab/FTCP

^{971 &}lt;sup>2</sup>https://github.com/usnistgov/alignn

972 Equation 14. TPE required separate settings for each objective: band gap, formation energy, and
973 electrical neutrality. The objectives for band gap and formation energy were adopted from Equation
974 13. Additionally, we implemented a binary objective function that assigns a value of 0 if electrical
975 neutrality is achieved and 1 otherwise:

$$L_{neutral} = \begin{cases} 0 & \text{electrical neutrality} \\ 1 & \text{otherwise} \end{cases}$$
(A.1)

981TPE used L_{bg} , L_f and $L_{neutral}$ as objective functions, respectively. FTCP selects initial data from982the training dataset where the band gap is close to the target and the formation energy is below -0.5983eV, and then uses an encoder to convert this into latent variables. Next, we add noise to these latent984variables using a normal distribution with a mean of 0 and a standard deviation of 0.6, then decode985them back into crystal structures for evaluation. Augmented results are shown in Table A.2. We also986evaluate the diversity of the proposed materials, as shown in Table A.3.

SMOACS consistently maintained electrical neutrality, provided that extreme geometries causing NaN values during crystal vector calculations did not occur. We calculate the crystal vectors from a, b, c, and α , β , γ . When the crystal axis lengths or angles are extremely large, computational errors can cause the value under the square root to become a very small negative number, resulting in NaN occurrences. Apart from this, SMOACS consistently maintained electrical neutrality. SMOACS utilizing ALIGNN achieved significantly lower scores compared to when using Crystalformer. We attribute this to the optimization difficulty arising from changes in the shape of the hypersurface of the loss function due to updates to the graph structure. SMOACS demonstrates the ability to generate highly diverse materials.

Table A.2: Experiments on optimizing for various targets of a band gap. The "success rate" is the probability of simultaneously satisfying three conditions: (A) the band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the crystal structure is valid. C is achieved when two criteria are met simultaneously: (a) all interatomic distances are greater than 0.5 Å, and (b) the structure is electrically neutral. S(Cry) and S(ALI) denote SMOACS utilizing the Crystalformer and ALIGNN models, respectively. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 256 samples. Higher scores are better across all metrics.

1041			1			1	
1042	Target	method	success	(A)BG	(B)E _f	(C)STR (a)	(b)
1043	BG (eV)		rate			neu	
1044		S(Cry)	<u>0.328</u>	0.465	0.566	0.758 0.95	
1045	0.50	S(ALI)	0.055	0.062	0.867	0.867 + 0.94	
1046	± 0.04	TPE	0.004	0.945	0.059	0.066 0.07	0 0.910
1047		FTCP	0.000	0.004	1.000	0.719 0.74	6 0.906
		S(Cry)	<u>0.340</u>	0.504	0.613	0.785 0.97	3 0.785
1048	1.00	S(ALI)	0.047	0.059	0.848	0.805 0.92	
1049	± 0.04	TPE	0.016	0.934	0.066	0.070 0.07	
1050		FTCP	0.004	0.004	1.000	0.699 0.73	
1051		S(Cry)	<u>0.387</u>	0.543	0.672	0.824 0.98	
1052	1.50	S(ALI)	0.043	0.066	0.828	0.852 0.93	
1053	± 0.04	TPE	0.020	0.855	0.055	0.074 0.08	
1054		FTCP	0.000	0.000	1.000	0.703 0.72	
1055		S(Cry)	<u>0.355</u>	0.484	0.703	0.844 0.98	
1056	2.00	S(ALI)	0.082	0.092	0.820	0.838 0.91	
1057	± 0.04	TPE	0.020	0.789	0.062	0.086 0.08	
1058		FTCP	0.000	0.000	1.000	0.699 0.72	
1059		S(Cry)	<u>0.383</u>	0.473	0.715	0.840 0.98	
1060	2.50	S(ALI)	0.051	0.059	0.809	0.793 + 0.89	
	± 0.04	TPE	0.023	0.711	0.098	0.051 0.05	5 0.816
1061		FTCP	0.004	0.004	1.000	0.695 0.70	
1062		S(Cry)	<u>0.301</u>	0.375	0.699	0.828 0.99	
1063	3.00	S(ALI)	0.039	0.043	0.801	0.820 0.90	
1064	± 0.04	TPE	0.020	0.645	0.094	0.090 0.09	
1065		FTCP	0.027	0.031	1.000	0.668 0.68	
1066		S(Cry)	<u>0.188</u>	0.273	0.645	0.750 0.99	
1067	3.50	S(ALI)	0.016	0.016	0.816	0.797 + 0.90	
1068	± 0.04	TPE	0.012	0.586	0.059	0.055 0.05	
1069		FTCP	0.012	0.012	1.000	0.707 0.73	
1070		S(Cry)	<u>0.160</u>	0.227	0.656	0.789 1.00	
1071	4.00	S(ALI)	0.023	0.023	0.805	0.797 0.90	
1072	± 0.04	TPE	0.016	0.438	0.090	0.078 0.07	
		FTCP	0.035	0.043	1.000	0.676 0.69	1 0.895
1073							

1088Table A.3: The diversity of the proposed materials. The 'success rate' corresponds to the same
'success rate' as in Table A.2. The 'unique rate' refers to the probability of materials with unique
elemental combinations, regardless of the success. The 'unique rate in success' represents the pro-
portion of materials with unique elemental combinations among the successfully optimized materi-
als. The 'unique and novel rate in success' indicates the proportion of materials whose elemental
combinations are unique and absent from the MEGNet database among the successfully optimized
materials.

Target BG (eV)	method	success rate	unique rate	unique rate in success	unique and novel rate in success
DG (CT)			0.057		
0.50	S(Cry)	0.328	0.957	84/84	81/84
0.50	S(ALI)	0.055	0.867	14/14	13/14
± 0.04	TPE	0.004	1.000	1/1	0/1
	FTCP	0.000	0.297	-	-
	S(Cry)	0.340	0.973	87/87	80/87
1.00	S(ALI)	0.047	0.895	12/12	11/12
± 0.04	TPE	0.016	1.000	4/4	1/4
	FTCP	0.004	0.289	1/1	0/1
	S(Cry)	0.387	0.977	99/99	94/99
1.50	S(ALI)	0.043	0.891	11/11	11/11
± 0.04	TPE	0.020	1.000	5/5	0/5
	FTCP	0.000	0.324	-	-
	S(Cry)	0.355	0.984	90/91	85/91
2.00	S(ALI)	0.082	0.836	42/42	41/42
± 0.04	TPE	0.020	1.000	5/5	1/5
	FTCP	0.000	0.328	-	-
	S(Cry)	0.383	0.980	98/98	90/98
2.50	S(ALI)	0.051	0.793	13/13	12/13
± 0.04	TPE	0.023	1.000	6/6	1/6
	FTCP	0.004	0.328	1/1	0/1
	S(Cry)	0.301	0.992	77/77	72/77
3.00	S(ALI)	0.039	0.770	9/10	6/10
± 0.04	TPE	0.020	0.992	4/5	2/5
	FTCP	0.027	0.359	6/7	0/7
	S(Cry)	0.188	0.992	48/48	47/48
3.50	S(ALI)	0.016	0.793	4/4	4/4
± 0.04	TPE	0.012	1.000	3/3	1/3
	FTCP	0.012	0.309	3/3	0/3
	S(Cry)	0.012	0.996	41/41	38/41
4.00	S(ALI)	0.100	0.797	6/6	6/6
± 0.04	TPE	0.023	0.996	4/4	0/0 1/4
⊥0.04	FTCP	0.010	0.348	7/9	0/9
	FICE	0.055	0.340	117	0/9

A.7 DETAILS IN SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES WHILE PRESERVING PEROVSKITE STRUCTURES

As discussed in Section 3.3, due to the arbitrariness in the numerical values of the lattice con-1137 stant and coordinates of perovskite structures, we evaluated whether the optimized structures ap-1138 proximated typical perovskite configurations. First of all, fractional coordinates typical of per-1139 ovskite structures are as follows: (0.5, 0.5, 0.5) at the A site, (0.0, 0.0, 0.0) at the B site, and 1140 (0.5, 0.0, 0.0), (0.0, 0.5, 0.0), (0.0, 0.0, 0.5) at the three X sites. We established criteria for the opti-1141 mized x, y, and z coordinates to be within a deviation ϵ from these standard values. The perovskite 1142 structure CaCu₃Ti₄O₁₂ exhibits a slightly distorted configuration, with the x-coordinate of the oxy-1143 gen atoms deviating by approximately 10% from their typical positions (Božin et al., 2004). To ex-1144 plore new structures, we set $\epsilon = 0.15$, allowing for a slightly greater distortion. We considered the 1145 optimized coordinates successful if the x, y, and z coordinates of each site fell within $\pm \epsilon$. Addition-1146 ally, the angles between the crystal axes of typical perovskite structures are close to 90°. Therefore, 1147 angles between 85° and 95° were established as a criterion.

1148 1149 1149 1149 1149 1150 1151 1152 Using t values from typical perovskite structures (BaCeO₃:0.857, SrTiO₃: 0.910 and BaTiO₃: 0.970), we established a tolerance factor range of $0.8 \le t \le 1.0$ as the criterion for success. The ionic radius of the X site was calculated as the average of the radii of the three X sites. We took the values for the ionic radii from PyMatGen (Ong et al., 2013).

In this experiments, the learning rates were set as $\eta_l = 0.01, \eta_C = 0.02, \eta_A = \eta_O = 6.0$ for SMOACS with Crystalformer. For SMOACS with ALIGNN, the learning rates were set as $\eta_l = 0.5, \eta_C = 0.002, \eta_A = \eta_O = 0.00008$. We reconstructed the graph structure 46 times during the optimizations. The learning rates were decayed using a cosine annealing schedule.

Due to the limited number of perovskite structure data points in the MEGNet dataset, we gener-1157 ated 256 random perovskite structures as initial values for SMOACS. These structures have crys-1158 tal axis angles α , β , and γ at 90° and axis lengths a, b, and c randomly generated between 2 Å 1159 and 10 Å. Their initial fractional coordinates correspond to those typical of perovskite structures: 1160 (0.5, 0.5, 0.5) for the A site, (0.0, 0.0, 0.0) for the B site, and (0.5, 0.0, 0.0), (0.0, 0.5, 0.0), and 1161 (0.0, 0.0, 0.5) for the three X sites. Similarly, TPE optimized perovskite structures by setting the 1162 crystal axis angles at 90° and optimizing the axis lengths a, b, c between 2 Å and 10 Å. We also 1163 limited element species for each site in TPE. Specifically, the elements are restricted by oxidation 1164 numbers: +1 and +2 for site A, +2 and +4 for site B, and -1 and -2 for site X. For FTCP, we 1165 initially selected data points where the crystal axis angles were at 90° , and all sites conformed to 1166 the typical fractional coordinates of perovskite structures; these were then converted into latent vari-1167 ables. Subsequently, we applied noise using a normal distribution with a mean of 0 and a standard 1168 deviation of 0.6 to the latent variables. Finally, we decoded the latent variables back into crystal structures for evaluation. 1169

¹¹⁷⁰ SMOACS conducted optimization using Eq 17. For *t*, TPE used another objective function:

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 $\mathbf{L}_t^{\text{TPE}} = \begin{cases} 0 & (0.8 \le t \le 1.0) \\ 1 & \text{otherwise} \end{cases}$ (A.2)

TPE used L_{bg} , L_f , $L_{neutral}$ and L_t^{TPE} as objective functions, respectively. Augmented results are shown in Table A.4. We also evaluate the diversity of the proposed materials, as shown in Table A.5.

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Table A.4: Experiments on optimizing various band gaps while preserving perovskite structures. The "success rate" reflects the probability of simultaneously satisfying four criteria: (A) the band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, (C) the crystal structure is valid, and (D) approximating a valid perovskite structure. Criteria (A), (B), and (C) are consistent with those outlined in Table 2. The (D) is achieved when three criteria are met si-multaneously: (c) the tolerance factor t is between 0.8 and 1.0, (d) coordinates are within ± 0.15 of typical perovskite structure coordinates, and (e) axis angles are from 85° to 95°. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 256 samples. Higher scores are better across all metrics.

Target BG (eV)	method	success rate	(A)BG	$(B)\mathrm{E}_{\mathrm{f}}$	(C)STR	(a) neut	(b) 0.5Å	(D)PS	(c) tole	(d) angles	(e)
	S(Cry)	0.113	0.477	0.410	0.965		0.965	0.500		1.000	1.00
0.50	S(ALI)	$\frac{0.119}{0.090}$	0.211	0.535	1.000		1.000	0.500		1.000	1.00
± 0.04	TPE	0.027	1.000	0.137	0.535		1.000	0.648		1.000	1.0
±0.04	FTCP	0.004	0.023	1.000	0.836		0.938	0.258		0.441	0.2
	S(Cry)	0.152	0.457	0.422	0.961		0.961	0.559		1.000	1.0
1.00	S(ALI)	$\frac{0.152}{0.062}$	0.168	0.422	1.000		1.000	0.531		1.000	1.0
± 0.04	TPE	0.002	1.000	0.137	0.395		1.000	0.664		1.000	1.0
±0.04	FTCP	0.012	0.023	1.000	0.859		0.965	0.215		0.418	0.2
	S(Cry)	0.148	0.422	0.461	0.984		0.984	0.578		1.000	1.0
1.50	S(ALI)	$\frac{0.140}{0.070}$	0.219	0.652	1.000		1.000	0.629		1.000	1.0
± 0.04	TPE	0.023	0.992	0.281	0.293		1.000		0.523	1.000	1.0
±0.01	FTCP	0.000	0.016	1.000	0.895	0.225	0.965	0.242	0.547	0.418	0.3
	S(Cry)	0.188	0.426	0.516	0.988		0.988	0.613		1.000	1.0
2.00	S(ALI)	$\frac{0.100}{0.090}$	0.188	0.980	1.000		1.000	0.625		1.000	1.0
± 0.04	TPE	0.027	0.977	0.266	0.266		1.000	0.547		1.000	1.0
± 0.04	FTCP	0.004	0.004	1.000	0.891		0.980	0.281		0.473	0.3
	S(Cry)	0.152	0.285	0.516	0.988		0.988	0.625		1.000	1.0
2.50	S(ALI)	$\frac{0.102}{0.113}$	0.184	0.938	1.000		1.000	0.625		1.000	1.0
± 0.04	TPE	0.016	0.918	0.281	0.352		1.000	0.387		1.000	1.0
	FTCP	0.008	0.012	0.996	0.879	0.898	0.953	0.250		0.441	0.2
	S(Cry)	0.102	0.219	0.508	0.992		0.992	0.621		1.000	1.0
3.00	S(ALI)	0.141	0.273	0.938	1.000		1.000	0.625	0.625	1.000	1.0
± 0.04	TPE	0.035	0.875	0.293	0.234		1.000	0.316		1.000	1.0
	FTCP	0.008	0.008	1.000	0.898	0.906	0.969	0.246	0.543	0.418	0.3
	S(Cry)	0.070	0.145	0.516	0.996	1.000	0.996	0.629	0.629	1.000	1.0
3.50	S(ALI)	0.176	0.195	0.961	1.000		1.000	0.668		1.000	1.0
± 0.04	TPE	0.016	0.711	0.266	0.184	0.184	1.000	0.285	0.285	1.000	1.(
	FTCP	0.004	0.008	1.000	0.895	0.910	0.953	0.238	0.531	0.441	0.2
	S(Cry)	0.051	0.094	0.512	0.992		0.992	0.625		1.000	1.0
4.00	S(ALI)	0.180	0.285	0.961	1.000		1.000	0.605		1.000	1.0
± 0.04	TPE	0.020	0.539	0.336	0.215		1.000	0.227	0.227	1.000	1.0
	FTCP	0.000	0.004	0.996	0.883		0.949	0.238		0.410	0.3

Table A.5: The diversity of the proposed materials. The 'success rate' corresponds to the same 'success rate' as in Table A.4. The 'unique rate' refers to the probability of materials with unique elemental combinations, regardless of the success. The 'unique rate in success' represents the pro-portion of materials with unique elemental combinations among the successfully optimized materi-als. The 'unique and novel rate in success' indicates the proportion of materials whose elemental combinations are unique and absent from the MEGNet database among the successfully optimized materials.

Target BG (eV)	method	success rate	unique rate	unique rate in success	unique and novel rate in success
	S(Cry)	0.113	0.898	28/29	27/29
0.50	S(ALI)	0.090	0.398	14/23	13/23
± 0.04	TPE	0.027	0.984	7/7	7/7
±0.01	FTCP	0.004	0.352	1/1	0/1
	S(Cry)	0.152	0.891	38/39	37/39
1.00	S(ALI)	0.062	0.562	14/16	12/16
± 0.04	TPE	0.002	0.996	3/3	3/3
±0.01	FTCP	0.012	0.336	1/1	0/1
	S(Cry)	0.148	0.891	38/38	38/38
1.50	S(ALI)	0.070	0.555	16/18	15/18
± 0.04	TPE	0.070	1.000	6/6	6/6
10.04	FTCP	0.023	0.285	-	-
	S(Cry)	0.188	0.902	47/48	46/48
2.00	S(ALI)	0.090	0.445	17/23	17/23
± 0.04	TPE	0.027	0.996	7/7	7/7
±0.04	FTCP	0.027	0.309	1/1	0/1
	S(Cry)	0.004	0.898	38/39	36/39
2.50	S(ALI)	0.132	0.598	25/29	23/29
± 0.04	TPE	0.016	0.992	4/4	4/4
±0.04	FTCP	0.010	0.309	2/2	0/2
	S(Cry)	0.102	0.902	26/26	26/26
3.00	S(ALI)	0.102	0.465	24/36	21/36
± 0.00	TPE	0.035	0.992	9/9	9/9
±0.04	FTCP	0.003	0.285	2/2	0/2
	S(Cry)	0.000	0.902	18/18	17/18
3.50	S(ALI)	0.070	0.387	14/45	12/45
± 0.04	TPE	0.170	0.988	4/4	4/4
⊥0.04	FTCP	0.010	0.301	1/1	0/1
	S(Cry)	0.004	0.902	13/13	13/13
4.00	S(CIY) S(ALI)	0.031	0.387	20/46	14/46
± 0.04	TPE	0.180	0.977	5/5	5/5
⊥0.04	FTCP	0.020	0.312	515	-
	FICF	0.000	0.312	-	-

1296 A.8 DETAILS IN OPTIMIZING LARGE ATOMIC CONFIGURATIONS

We conducted experiments on $3 \times 3 \times 3$ perovskite structures containing 135 atom sites, ex-panded from a unit cell with five atom sites. As the cell size increased, the range for the crystal lattice dimensions a, b, c in SMOACS and TPE was set from 6 Å to 30 Å for the $3 \times 3 \times 3$ structure. Similarly, the range of coordinate variations ϵ was set to 0.05. Aside from these changes, the experimental conditions remained consistent with those described in Section 4.3. In this experiments, the learning rates were set as $\eta_l = 0.003, \eta_C = 0.005, \eta_A = \eta_O = 2.0$ for SMOACS with Crystalformer. For SMOACS with ALIGNN, the learning rates were set as $\eta_l = 5.000, \eta_C = 0.002, \eta_A = \eta_O = 0.00005$. We reconstructed the graph structure 41 times during the optimizations. The learning rates were decayed using a cosine annealing schedule. Aug-mented results are shown in Table A.6.

1308Table A.6: Experiments optimizing for various band gaps while preserving a $3 \times 3 \times 3$ perovskite1309structure. We included only TPE, which showed better performance in Section 4.3, for comparison.1310Evaluation methods are based on those described in Table 3.

Target		success			'	(a)	(b)		(c)	(d)	(e)
BG (eV)	method	rate	(A)BG	$(B)E_{f}$	(C)STR	neut	0.5Å	(D)PS	tole	angles	coor
0.50	S(Cry)	0.156	0.734	0.547	0.968	1.00	0.969	0.570	0.570	1.000	1.00
0.50	S(ALI)	0.188	0.234	0.812	0.687	1.00	0.688	0.789	0.789	1.000	1.00
± 0.04	TPE(/N)	0.000	1.000	0.000	- !	N/A	1.000	0.609	0.609	1.000	1.00
1.00	S(Cry)	0.070	0.250	0.469	0.945	1.00	0.945	0.586	0.586	1.000	1.00
	S(ALI)	0.094	0.133	0.828	0.703	1.00	0.703	0.625	0.625	1.000	1.0
± 0.04	TPE(/N)	0.000	0.125	0.000	-	N/A	0.992	0.242	0.242	1.000	1.0
1.50	S(Cry)	0.047	0.125	0.422	0.953	1.00	0.953	0.617	0.617	1.000	1.0
1.50	S(ALI)	0.062	0.086	0.867	0.726	1.00	0.727	0.586	0.586	1.000	1.0
± 0.04	TPE(/N)	$\overline{0.000}$	0.141	0.000	-	N/A	1.000	0.180	0.180	1.000	1.0
2.00	S(Cry)	0.023	0.055	0.406	0.976	1.00	0.977	0.633	0.633	1.000	1.0
2.00	S(ALI)	0.055	0.102	1.000	0.710	1.00	0.711	0.594	0.594	1.000	1.0
± 0.04	TPE(/N)	0.000	0.125	0.000	-	N/A	0.984	0.242	0.242	1.000	1.0
2.50	S(Cry)	0.023	0.039	0.438	0.984	1.00	0.984	0.664	0.664	1.000	1.0
2.50	S(ALI)	0.102	0.172	1.000	0.703	1.00	0.703	0.812	0.812	1.000	1.0
± 0.04	TPE(/N)	0.000	0.023	0.000	- 1	N/A	0.984	0.156	0.156	1.000	1.0
2.00	S(Cry)	0.016	0.047	0.602	1.00	1.00	1.000	0.383	0.383	1.000	1.0
3.00	S(ALI)	0.125	0.188	0.992	0.726	1.00	0.727	0.664	0.664	1.000	1.0
± 0.04	TPE(/N)	0.000	0.023	0.000	- i	N/A	0.984	0.273	0.273	1.000	1.0
2.50	S(Cry)	0.008	0.008	0.445	0.984	1.00	0.984	0.672	0.672	1.000	1.0
3.50	S(ALI)	0.156	0.250	1.000	0.75	1.00	0.750	0.734	0.734	1.000	1.0
± 0.04	TPE(/N)	0.000	0.000	0.000	- 1	N/A	0.992	0.195	0.195	1.000	1.0
4.00	S(Cry)	0.008	0.008	0.445	0.984	1.00	0.984	0.672	0.672	1.000	1.0
	S(ALI)	0.219	0.305	1.000	0.773	1.00	0.773	0.852	0.852	1.000	1.0
± 0.04	TPE(/N)	0.000	0.000	0.000	_ !	N/A	0.969	0.180	0.180	1.000	1.0

1350 A.9 ELECTRICAL NEUTRALITY 1351

1352 In assessing electrical neutrality, a compound was considered neutral if the sum of the oxidation numbers for the atoms at each site equaled zero. For example, Fe_3O_4 is electri-1353 cally neutral because the configuration [Fe, Fe, Fe, O, O, O, O] can assume oxidation numbers 1354 of [+2, +3, +3, -2, -2, -2, -2] that sum to zero. Previous study (Xie et al., 2022) employed 1355 SMACT(Davies et al., 2019) to assess electrical neutrality; however, SMACT includes some ox-1356 idation numbers, like the +7 state of chlorine, which are extremely rare and potentially less reliable. 1357 We restricted our analysis to commonly occurring oxidation numbers, selecting those found at the 1358 intersection of SMACT and PyMatGen. A list of the elements and their corresponding oxidation 1359 numbers employed in this study is shown in Table A.7, Table A.8, and Table A.9. In these tables, 1360 the 'SMACT' indicates oxidation numbers from smact.Element. The 'icsd' and 'common' indi-1361 cate oxidation numbers from icsd_oxidation_state and common_oxidation_states 1362 in pymatgen.core.periodic_table.Element, respectively. 'Ours' represents the oxida-1363 tion numbers we used in this paper.

Table A.7: The List of oxidation numbers from Hydrogen (H) to Krypton (Kr).

1366						
1367	ZE	lm	SMACT	icsd	common	Ours
1368		H	$\{-1,1\}$	$\{-1,1\}$	$\{-1,1\}$	$\{-1,1\}$
1369	$\begin{bmatrix} 2 \end{bmatrix} \begin{bmatrix} \overline{1} \end{bmatrix}$	He		{ }		$\left\{ \right\}$
1309		Lī -		{1}	{1}	$\{1\}^{}$
		Be	$\{\bar{1}, \bar{2}\}$	$\{2\}$		$\{\bar{2}\}$
1371	5	B	$\{1, 2, 3\}$	[-3,3]		$\{\overline{3}\}$
1372		C	$-\frac{1}{4}, -\frac{1}{3}, -2, -1, 1, 2, 3, 4$	$\{-4, -3, -2, 2, 3, 4\}^{-1}$	${-4,4}$	$\{-4, \overline{4}\}^{-1}$
1373		N	$-\overline{3}, -\overline{2}, -\overline{1}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}$	$\{-3, -2, -1, 1, 3, 5\}$	$[-3, \overline{3}, \overline{5}]^{-1}$	$-\{-3, 3, 5\}^{-1}$
1374		O	$\{-2, -1, \overline{1}, 2\}$	$\{-2\}$		
1375		F	{-1}	$\{-1\}$		$\begin{bmatrix} -2 \\ -1 \end{bmatrix}$
1376		Ne		{ }		$\left\{ \right\} = \left\{ \right\} = \left\{ \right\}$
1377		Na	{-1,1}	$\{1\}$	{1}	$\{1\}$
1378	12 N	Mg		$\{2\}$	$\{\overline{2}\}$	$\{\bar{2}\}$
1379		AĪ	$\{1, 2, 3\}$			$\{\bar{3}\}$
1380		Sī	$-\frac{1}{4}, -\frac{3}{2}, -2, -1, \overline{1}, \overline{2}, \overline{3}, 4$	$\{-4, 4\}$	${-4,4}$	$[-\bar{4},\bar{4}]$
1381		P	$-\overline{\{-3, -2, -\overline{1}, \overline{1}, 2, \overline{3}, \overline{4}, \overline{5}\}}$	$\{-3, -2, -1, 3, 4, 5\}$	$-\overline{\{-3,\overline{3},\overline{5}\}}$	$[-\overline{\{-3,3,5\}}]$
1382		S [–]	$\{-2, -1, 1, 2, 3, 4, 5, 6\}$	$\{-2, -1, 2, 4, 6\}$	$\{-2, 2, 4, 6\}$	$\left[\left\{ -2, \bar{2}, \bar{4}, \bar{6} \right\} \right]$
1383		CĪ -	$\{-1, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}\}$	{-1}	$\{-1, 1, \overline{3}, \overline{5}, 7\}$	$\{-1\}$
1384		Ar	{}	{ }	{}	{}{
1385		K	{-1,1}	$\{1\}$	{1}	$\{\bar{1}\}$
1386		Ca	$\{\bar{1},\bar{2}\}$	$\{2\}$	$\{2\}$	$\{\bar{2}\}^{}$
1387		Sc	$\{1, 2, 3\}$	$\{\bar{2}, \bar{3}\}^{}$	{3}	[{3}]]
1388		Ti	[-1, 1, 2, 3, 4]	$\{\overline{2}, \overline{3}, \overline{4}\}$	[4]	$[- {\bar{4}}]$
1389		V –	$= = \{ -\overline{1}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5} \}$	[2, 3, 4, 5]	$\{5\}$	$\{5\}$
1390		Cr	$[-\frac{1}{2}, -1, 1, 2, 3, 4, 5, 6]$	$\{2, 3, 4, 5, 6\}$	[3, 6]	[- [3, 6] -]
1391		Mn	$\left[\left\{-\overline{3}, -2, -\overline{1}, \overline{1}, 2, \overline{3}, \overline{4}, \overline{5}, \overline{6}, 7\right\}\right]$	[2, 3, 4, 7]	[-[4, 2, 4, 7]]	$[\frac{1}{2}, \frac{1}{4}, \frac{1}{7}]$
1392		Fe	[-2, -1, 1, 2, 3, 4, 5, 6]	$[\overline{2}, \overline{3}]$	[[2, 3]]	$[-{2,3}]$
1392		Co _	$\begin{bmatrix} -1, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5} \end{bmatrix}$	[1, 2, 3, 4]	$[1, \frac{1}{2}, \frac{1}{3}]$	$[-{2,3}]$
		Ni –	[-1, 1, 2, 3, 4]	[1, 2, 3, 4]	[2]	$\{\bar{2}\}$
1394		Cu _	[1, 2, 3, 4]	$\{\overline{1}, \overline{2}, \overline{3}\}$	{2}	$\{2\}$
1395		Zn _	{1,2}	{2}	{2}	$\{2\}$
1396		Ga _	$\{1,2,3\}$	$\{2,3\}$	{3}	
1397		Ge _	$\begin{bmatrix} -4, -3, -2, -1, \overline{1}, \overline{2}, \overline{3}, 4 \end{bmatrix}$	$\{\overline{2}, \overline{3}, 4\}$	[-4, 2, 4]	$[-{2,4}]$
1398		As _	$\{-3, 1, 2, 3, 5\}$	$\{-3, -2, -1, 2, 3, 5\}$	[-3, 3, 5]	$[-\{-3, 3, 5\}]$
1399		Se _	$\{-2, 1, 2, 4, 6\}$	[-2, -1, 4, 6]	[-[-2, 2, 4, 6]]	[-2, 4, 6]
1400		Br _	$ = \{-1, 1, 2, 3, 4, 5, 7\} $	$\{-1,5\}$	[-1, 1, 3, 5, 7]	$\{-1, 5\}$
1401	<u>36</u> <u>I</u>	Kr	{2}	{ }	{ }	{ }

1402

1403

1364

Ζ	Elm	SMACT	icsd	common	Ours
37	Rb	$\{-1,1\}$	{1}	{1}	{1}
38	<u>Sr</u>	$\{1,2\}$	$\{\bar{2}\}$	{2}	$[- {2}]$
39	ŢŢ	$\{\overline{1}, \overline{2}, \overline{3}\}$	{3}	{3}	$\begin{bmatrix} - & - & \{3\} \\ - & - & \{3\} \end{bmatrix}$
40	Zr	$\{1, 2, 3, 4\}$	$[\overline{\{2, \overline{3}, \overline{4}\}}]$	4}	$[- {4}]$
41	[Nb]	$\{-1, 1, 2, 3, 4, 5\}$	[-, [2, 3, 4, 5]]	$\left[\frac{5}{2} \right]_{-}$	$\left[-\frac{\{5\}}{\sqrt{5}} \right]$
42	<u>Mo</u>	$= \{ -2, -1, 1, 2, 3, 4, 5, 6 \}$	[[2, 3, 4, 5, 6]]	$\{4, 6\}$	$[-]{4, 6}$
43		$\left\{ \begin{array}{c} -\overline{3}, -\overline{1}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7} \right\} \$	{}	$[\frac{4}{4}, \frac{7}{4}]$	$\left\{ -\frac{1}{2} \right\}$
44	Ru	$\begin{bmatrix} -2, 1, 2, 3, 4, 5, 6, 7, 8 \end{bmatrix}$	$\left[\frac{2}{2}, \frac{3}{2}, \frac{4}{5}, \frac{5}{6} \right]$	$[]{\bar{3},\bar{4}}^{}$	$[-]{\bar{3},\bar{4}}$
45	$\begin{bmatrix} \bar{R}\bar{h} \\ \bar{D}\bar{l} \end{bmatrix}$	$\left\{ -\overline{1}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6} \right\}$	$\{3,4\}$	[3]	$\begin{bmatrix} - & - & \{3\} \\ - & - & \{3\} \end{bmatrix}$
46	Pd -	$\{1, 2, 4, 6\}$	$\left[\frac{\{2, 4\}}{(1 - 2)^2} \right]$	$\left[\begin{array}{c} - & - & \overline{\{2, \overline{4}\}} \\ - & - & \overline{\{1, \overline{2}\}} \end{array} \right]^{-} =$	$= \{\bar{2}, \bar{4}\}$
47	Ag	$\{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$	[1, 2, 3]	$\{1\}$	$\{1\}$
48		$\{1,2\}$	$\{\bar{2}\}$	$\frac{2}{}$	$\{2\}$
49		[1, 2, 3]	[1, 2, 3]	$\left[-\frac{3}{2} \right]$	$\frac{3}{-\frac{3}{2}}$
50	Sn -	$\{-4, 2, 4\}$	$\overline{[2, 3, 4]}$	$\begin{bmatrix} -4, 2, 4 \end{bmatrix}$	$[- {\bar{2}, 4}]$
51	$\begin{bmatrix} \overline{S}\overline{b} \\ \overline{T}\overline{b} \end{bmatrix}$	$\{-3, 3, 5\}$	[-[-3, -2, -1, 3, 5]]	$\begin{bmatrix} -3, 3, 5 \\ -3, 3, 5 \end{bmatrix}$	$\left[\begin{array}{c} -3, \overline{3}, \overline{3}, \overline{3}, \overline{4} \\ -3, \overline{3}, \overline{4} \end{array}\right]$
$\overline{52}$	Te -	$\{-2, 2, 4, 5, \overline{6}\}$	$\left[-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{6} \right]^{-1}$	$\begin{bmatrix} -2, 2, 4, 6 \end{bmatrix}$	$[\bar{-2}, \bar{4}, \bar{4}, \bar{4}]$
53	I	$\{-1, 1, 3, 4, 5, 7\}$	$\{-1,5\}$	$\left[\left\{ -1, \overline{1}, \overline{3}, \overline{5}, \overline{7} \right\} \right]$	[-1, 5]
$\overline{54}$ $\overline{55}$	$\begin{bmatrix} \overline{Xe} \\ \overline{Ce} \end{bmatrix}$	$\{1, 2, 4, 6, 8\}$	${}{}$	 {}} 	
	$\begin{bmatrix} \overline{Cs} \\ \overline{Ds} \end{bmatrix}$	$\{-1, 1\}$	$\{1\}$	{1} 	$\{1\}$
56	$\begin{bmatrix} \overline{Ba} \\ \overline{La} \end{bmatrix}$	$\{\bar{2}\}$	$\frac{\{2\}}{[2]}$	$ \frac{\{2\}}{\{2\}}$	$\left\{ \begin{array}{c} - \frac{2}{2} \\ - \frac{2}{2} \end{array} \right\}$
57	La _		$\{2,3\}$	$\frac{3}{2}$	$\begin{bmatrix} - & - & \frac{1}{3} \end{bmatrix}$
$\overline{58}$ $\overline{59}$	$\begin{bmatrix} \overline{Ce} \\ \overline{Pr} \end{bmatrix}$	$ \begin{array}{c} \hline & \hline $	$\{3,4\}$	[3, 4]	$= \frac{3}{23}, \frac{4}{23}$
$\frac{39}{60}$	$-\frac{Pr}{Nd}$		$\{3,4\}$	[3] = [3] = [3]	$-\frac{\{3\}}{\{3\}}$
$\frac{60}{61}$	$-\frac{1}{Pm}$	$\frac{\{2, 3, 4\}}{\{2, 3\}}$	$\{2,3\}^{}$	$\left\{ \begin{array}{c} - & - & - \\ 3 \end{array} \right\} = $	$-\frac{\{0\}}{1}$
$\frac{01}{62}$	$-\frac{1}{Sm}$		$\frac{1}{\{2,3\}}$	$ \frac{1}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10}$	$ \frac{1}{2} \frac{1}{2}$
$\frac{02}{63}$	Eu -		$\left \frac{2}{2}, \frac{3}{3} \right =$	$\frac{15}{\{2,3\}}$	$-\frac{13}{2,3}$
$\frac{0.5}{64}$	$-\frac{Lu}{Gd}$		$\frac{12,3}{3}$	$\frac{12,3}{3}$	$-\frac{12, 3}{3}$
$\frac{04}{65}$	\overline{Tb}^{-}		$\frac{3}{3,4}$	$\frac{10}{3}$	$ \frac{131}{131}$
$\overline{66}$	\overline{Dy}^{-1}		$\frac{[0, \pm]}{\{3\}}$	${0}{3}$	${3}{3}$
$\overline{67}$	$-\frac{Dy}{Ho}$		$ \frac{(3)}{\{3\}}$	${0}{3}$	${3}{3}$
$\overline{68}$	$\frac{110}{Er}$			$ \frac{[0]}{\{3\}}$	$\frac{3}{3}$
69	\overline{Tm}^{-1}	$\{2,3\}$		${0}{3}$	[3]
70^{-1}	\overline{Yb}		$ \{2,3\}^{}$	${0}{3}$	${3}{3}$
$\frac{70}{71}$	\overline{Lu}^{-1}		$\left \frac{(2, 0)}{\{3\}} \right $	$ \frac{1}{3} $	$\frac{3}{3}$
72	- H̄f -			$ {4} $	${4}{4}$
$\overline{73}$	- <u>Ta</u> -	=	${\overline{\{3,\overline{4},5\}}}$	$ \frac{1}{5} $	${5}$
74	- W -	$\{-2, -1, 1, 2, 3, 4, 5, 6\}$	$-\frac{1}{2,3,4,5,6}$	$\begin{bmatrix} - & - & - & \{5\} \\ - & - & \{4, 6\} \end{bmatrix}^{-} =$	$-\frac{1}{4,6}$
75	Re -	$-\frac{1}{\{-3, -1, 1, 2, 3, 4, 5, 6, 7\}}$	$-\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}$	${\frac{2}{4}}$	$-\frac{1}{4}$
76	\overline{Os}	$\begin{bmatrix} -2, -1, 1, 2, 3, 4, 5, 6, 7, 8 \end{bmatrix}$			{ }
77	Ir -	$-\{\overline{-3}, \overline{-1}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, 8\}^{-1}$	${\overline{3,4,5}}$	$[\frac{1}{3},\frac{1}{4}]^{\frac{1}{4}}$	{3, 4
78	- <u>-</u> - <u>-</u> -	$-\frac{1}{\{-2, -1, 1, 2, 3, 4, 5, 6\}}$		$\begin{bmatrix} - & - & \frac{1}{3}, \frac{1}{4} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \end{bmatrix}^{-} = - \\ - & - & - & \frac{1}{3}, $	\ \ \
79	- Au -	$ \left\{ \begin{array}{c} -1, 1, 2, 3, 5 \\ \hline \\ \end{array} \right\} $		$[{3}]{$	+ + +
80	Hg -	$\{1, 2, 4\}$	[1,2]	$[]{1,2}$	$-\frac{1}{1,2}$
81	Tī -		$ \{1,3\}^{}$	$[]{[1,3]}{[]}$	$-\frac{1}{1,3}$
82	- Pb -		${7}{2,4}$	$\frac{1}{2}$	$-\frac{1}{2,4}$
83	Bi -	$\begin{bmatrix} - & - & - & - & - & - & - & - & - & - $	$[{\overline{1,2,3,5}}]$	{3}	`{3}
$\overline{84}$	Po -	$\{-2, 2, 4, 5, 6\}$	'-'-'-'-'-''	$-\frac{1}{4} = \frac{1}{4} = 1$	† Ì Ì
85	Āt -			[-1,1]	† † f
86	Rn -	$\{2, 6\}$	X	+	+ 六 台

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Ζ	Elm	smact	icsd	common	Ours
87	Fr	{1}	{ }	$\{1\}$	{ }
88	Rā –			$-\frac{1}{2}$	$\left\{ \right\}$
89	Āc	$[{2,3}]$	{}	$-\frac{3}{3}$	$\left\{ \right\}$
90	Th –	$\left[\begin{array}{c} - & - & - & - & - & - & - & - & - & - $	$ {4}]$	$-\frac{1}{4}$	$\{\bar{4}\}$
91	Pa –	$\left[\begin{array}{c} - & - \\ \bar{2}, \bar{3}, \bar{4}, \bar{5} \end{array} \right]^{}$		$-\frac{1}{5}$	$\left\{ \right\}$
92	Ū	$[-{\overline{2,3,4,5,6}}]$	$[\overline{3}, \overline{4}, \overline{5}, \overline{6}]^{-}$	$-\frac{1}{6}$	$\{\bar{6}\}$
93	¯ Np ¯	$[-{\overline{3,4,5,6,7}}]$	{}	$-{\overline{5}}$	
94	Pu –	$[\overline{2,3,4},\overline{5,6,7,8}]$		${\overline{4}}$	$\left[\left(\right) \right]$
95	Āmī	$[\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}]^{-1}$		$-\frac{3}{3}$	$\left\{ \right\}$
96	Ēm	$[-, \overline{3}, \overline{4}, \overline{6}, 8]$	{}	{3}	$\left\{ \right\}$
97	BĒ	$[\bar{2}, \bar{3}, \bar{4}]$	{}	$-{\overline{3}}$	$\left\{ \right\}$
98	- Cf -	$[\bar{1}, \bar{3}, \bar{4}]$	{}	{3}	$\left\{ \right\}$

Table A.9: The List of oxidation numbers from Francium (Fr) to Californium (Cf)

1475 A.10 ACCURACTE BAND GAP OPTIMIZATION

We experimented with how precisely the band gap could be optimized. Here, we optimized the band gap to approximately 2.0, regardless of the crystal structure, and simultaneously minimized the formation energy. We conducted all methods in the same manner as mentioned in Section A.6. The results are shown in Table A.10.

1483Table A.10: Experiments with varying tolerance ranges for band gap optimization. The overall1484success rate is indicated by the probability of simultaneously satisfying three conditions: (A) the1485band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the1486crystal structure is valid. C is achieved when two criteria are met simultaneously: (a) all interatomic1487distances are greater than 0.5 Å, and (b) the structure is electrically neutral. S(Cry) and S(ALI)1488of the proposed materials using all evaluation metrics, and the results are averaged over 512 samples.

1490 1491	Target BG (eV)	method	success rate	(A)BG		(C)STR	neut	(b) 0.5Å
1492		S(Cry)	0.234	0.355	0.594	0.781	0.984	0.781
1493	2.00	S(ALI)	0.010	0.012	0.811	0.830	0.910	0.830
1494	± 0.01	TPE	0.004	0.422	0.053	0.045	0.047	0.777
1495		FTCP	0.000	0.000	1.000	0.652	0.668	0.924
1496		S(Cry)	0.230	0.340	0.543	0.742		0.742
1497	2.00	S(ALI)	0.037	0.043	0.795	0.803	0.893	0.803
1498	± 0.02	TPE	0.008	0.623	0.074	0.074	0.074	0.824
1499		FTCP	0.000	0.000	1.000	0.617	0.652	0.861
1500		S(Cry)	0.277	0.418	0.566	0.738	0.984	0.738
1501	2.00	S(ALI)	0.082	0.092	0.820	0.838	0.914	0.838
	± 0.04	TPE	0.016	0.812	0.074	0.061	0.061	0.801
1502		FTCP	0.004	0.004	1.000	0.645	0.666	0.900
1503		S(Cry)	0.238	0.410	0.562	0.730		0.730
1504	2.00	S(ALI)	$\overline{0.111}$	0.119	0.816	0.803		0.803
1505	± 0.08	TPE	0.049	0.936	0.135	0.123	0.125	0.846
1506		FTCP	0.006	0.010	1.000	0.688		0.928
1507		S(Cry)	0.316	0.516	0.570	0.750	0.988	0.750
1508	2.00	S(ALI)	$\frac{0.010}{0.193}$	0.225	0.832	0.820		0.820
1509	±0.16	TPE	0.074	0.955	0.152	0.174		0.891
1510		FTCP	0.016	0.027	1.000	0.668		0.898
1511	L							

1512 A.11 GENERATION OF OXIDATION NUMBER PATTERNS

1514 In SMOACS, realistic oxidation number patterns are generated based on the compositions of 1515 initial crystal structures. Here, we explain this using RuN (mp-1009770). According to icsd_oxidation_state in PyMatGen, ruthenium (Ru) and nitrogen (N) can adopt oxidation 1516 numbers of $\{+2, +3, +4, +5, +6\}$ and $\{+1, +3, +5, -1, -2, -3\}$, respectively. Therefore, elec-1517 trical neutrality in RuN is achieved when the oxidation number combinations for Ru and N are 1518 (+2, -2) or (+3, -3). Consequently, when using RuN (mp-1009770) as the initial structure, ox-1519 idation number combination patterns of (+2, -2) and (+3, -3) are obtained, and corresponding 1520 masks are generated for each. 1521

To consider a broader range of oxidation number combinations, we utilized the intersection of oxidation numbers from "smact" and "icsd", as listed in Table A.7. It should be noted that even when generating oxidation number patterns from "smact" and "icsd", electrical neutrality is maintained by applying site-specific elemental constraints using the oxidation numbers in the "Ours" column of Table A.7.

1527 1528 A.12 Adjustment of Priorities in the Loss Function

1529For a fair comparison with TPE, which cannot prioritize each objective, we fixed the value of λ to15301.0 in Equation 14. However, we considered it essential to investigate the effect of λ in optimization1531and conducted experiments with various λ values.

The results are shown in Table A.11. These experiments were conducted using SMOACS with Crystalformer, aiming for a band gap of 2.0 eV and optimizing crystal structures. We found that increasing λ , that is, placing greater emphasis on the formation energy during optimization, improved the success rate for formation energy. In addition, the probability that all interatomic distances are greater than 0.5 Å also increased. This is likely because emphasizing formation energy made it easier to avoid situations where atoms are too close together.

1539Table A.11: Experiments with varying λ for a band gap target of 2.0 eV using SMOACS with1540Crystalformer. The overall success rate is indicated by the probability of simultaneously satisfying1541three conditions: (A) the band gap is optimized within the target range, (B) the formation energy is1542below -0.5 eV, and (C) the crystal structure is valid. Condition C is achieved when the following1543two criteria are met simultaneously: (a) all interatomic distances are greater than 0.5 Å, and (b)1544the structure is electrically neutral. We evaluate each of the proposed materials using all evaluation1545metrics, and the results are averaged over 128 samples.

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λ_{\sim}	$\underbrace{\frac{success}{rate}}$	(A)BG	<u>(B)</u> E _f	(<u>C)STR</u>	$\underbrace{(a)}_{\substack{n \in ut}}$	(b) 0.5Å
0.040	0.086	0.492	0.164	0.617	0.945	0.617
0.200	0.188	0.477	0.328	0.656	0.984	0.656
1.000	0.336	0.453	0.727	0.852	1.000	0.852
5.000	0.266	0.320	0.836	0.875	0.992	0.875
25.000	0.086	0.102	0.859	0.875	1.000	0.875

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1555 A.13 EXPERIMENTS WITH MODELS TRAINED ON OTHER DATASETS

We investigated whether optimization is feasible using models trained on datasets other than the MEGNet dataset. We used ALIGNN models on JARVIS dataset (Choudhary, 2021). The first experiment employed ALIGNN trained on the JARVIS DFT dataset. The second experiment utilized ALIGNN trained on the superconductivity dataset, SuperCon. In the first experiment using the JARVIS DFT data, we utilized models predicting the energy difference from the convex hull⁴ and the bulk modulus⁵. The crystal structures were optimized to minimize the energy difference from the

^{1563 &}lt;sup>4</sup>https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_ 1564 dataset/17005681?file=31458658

⁵https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_ dataset/17005681?file=31458649

1566 convex hull less than 0.5 eV while achieving the bulk modulus within the target range. The results 1567 are shown in Tables A.12 and A.13. Similar to the experiments conducted with models trained 1568 on the MEGNet dataset in the main text, optimizations using models trained on the JARVIS DFT 1569 data were successful. In the second experiment, we employed ALIGNN trained on the SuperCon 1570 dataset to predict the critical temperature $(Tc)^6$. Since there is no formation energy prediction model 1571 trained on the superconductor dataset in JARVIS, we used the formation energy prediction model 1572 from the main text. Using these models, we optimized the crystal structures to maximize Tc above a certain temperature while minimizing the formation energy less than -0.5 eV. Specifically, the 1573 target temperatures for Tc were set to exceed the 0.901 and 0.963 quantiles of Tc within the dataset, 1574 corresponding to temperatures of 10 K and 15 K, respectively. The results are shown in Tables A.14. 1575 As with the experiments using models trained on the MEGNet dataset, optimization with models 1576 trained on the SuperCon data were successful. 1577

1578 Table A.12: Experiments for various bulk modulus using SMOACS with ALIGNN. The overall 1579 success rate is indicated by the probability of simultaneously satisfying three conditions: (A) the 1580 bulk modulus is optimized within the target range, (B) the energy difference from the convex hull 1581 less than 0.5 eV, and (C) the crystal structure is valid. Condition C is achieved when the following 1582 two criteria are met simultaneously: (a) all interatomic distances are greater than 0.5 Å, and (b) 1583 the structure is electrically neutral. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 512 samples. 1585

Target Bulk Modulus (GPa)	$\sim \frac{\text{success}}{\text{rate}}$		(B)E _{bull}	(C)STR	(a) neut	(b) 0.5Å
$50.0 \pm 5.0 \\ 75.0 \pm 5.0 \\ 100.0 \pm 5.0 \\ 10$	$\begin{array}{c} 0.020\\ 0.021\\ 0.016\\ \end{array}$	$\begin{array}{c} 0.090 \\ 0.082 \\ 0.102 \end{array}$	$\begin{array}{c} 0.123 \\ 0.078 \\ 0.066 \end{array}$	$\begin{array}{c} 0.512 \\ 0.438 \\ 0.395 \end{array}$	$\begin{array}{c} 0.578 \\ 0.535 \\ 0.510 \\ \end{array}$	$\begin{array}{c} 0.512 \\ \widetilde{0.439} \\ \widetilde{0.395} \end{array}$

Table A.13: Experiments for various bulk modulus while preserving perovskite structures using 1593 SMOACS with ALIGNN. The overall success rate is indicated by the probability of simultaneously 1594 satisfying three conditions: (A) the bulk modulus is optimized within the target range, (B) the 1595 energy difference from the convex hull less than 0.5 eV, (C) the crystal structure is valid, and (D) 1596 approximating a valid perovskite structure. Criteria (A), (B), and (C) are consistent with those 1597 outlined in Table A.12. The Criteria (D) is consistent with that outlined in Table 3. We evaluate 1598 each of the proposed materials using all evaluation metrics, and the results are averaged over 512 1599 samples. 1600

1601 1602 1603	Target Bulk Modulus (GPa)	$\sim \frac{\text{success}}{\text{rate}}$		B)E _{hull} ∣	(C)STR	(a) neut	(b) 0.5Å	(D)PS	$\underbrace{ \begin{array}{c} (c) \\ tole \end{array} } $	(d) angles	(e) coord
1604	50.0± 5.0	0.072	0.309	0.326	0.939	1.000	0.939	0.525	0.525	1.000	1.000
1605	75.0± 5.0	0.043	0.271	0.252	$\widetilde{0.910}$	1.000	0.910	0.449	0.449	1.000	1.000
1606	100.0 ± 5.0	0.020	0.178	0.176	0.928	1.000	0.928	0.416	0.416	1.000	1.000

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OPTIMIZATION WITH MARGINS BASED ON PREDICTOR ERROR A.14

1610 To ensure a fair comparison of optimization methods, we fixed the margin for all band gaps at 0.04 1611 eV in the main text. However, in practical applications, it is conceivable to set the margin based 1612 on the predictor's prediction error. Here, we present the optimization results when the predictor's 1613 errors shown in Table 1 are used as margins. We performed optimizations regardless crystal structure and optimizations with preserving the perovskite structure, using the hyperparameters described in 1614 Sections A.5 and A.7, respectively. The results are shown in Tables A.15 and A.16. As shown 1615 in these Tables, FTCP has low expected values for satisfying the band gap value and maintaining 1616 the perovskite structure, and TPE has a low expected value for satisfying the formation energy; 1617 therefore, SMOACS has achieved the best results. 1618

⁶https://fiqshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_ 1619 dataset/17005681?file=38789199

1621Table A.14: Experiments for various Tc targets using SMOACS with ALIGNN. The overall success1622rate is indicated by the probability of simultaneously satisfying three conditions: (A) the Tc is1623optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the crystal1624structure is valid. Criteria C is consistent with that outlined in Table A.12. We evaluate each of the1625proposed materials using all evaluation metrics, and the results are averaged over 512 samples.

Target ~Tc·(K)~~~	$\underbrace{ \underbrace{ success}_{r \underline{ate}} } $	(A)Tc	<u>(B)</u> E _f	(<u>C)STR</u>	(a) neut	$\underbrace{(b)}_{\underbrace{0.5\text{\AA}}}$
$\stackrel{\geq 10.0}{\geq 15.0}$	$\begin{array}{c} 0.049\\ \widetilde{0.004} \end{array}$	$\begin{array}{c} 0.121\\ 0.039\\ \end{array}$	0.625 0.604	0.656 0.693	$\begin{array}{c} 0.875\\ 0.906 \end{array}$	$\underbrace{\begin{array}{c} 0.656\\ 0.693 \end{array}}$

Table A.15: Experiments on optimizing for various targets of a band gap. We adopted the error values of each predictor in Table 1 as the margin for the band gap. The entries Cry, ALI, and F-Reg in the Predictor column correspond to the Crystalformer, ALIGNN, and regression branches of FTCP, respectively. Evaluation methods are based on those described in Table A.2.

$\begin{array}{c} Target\\ \widetilde{BG} \ \widetilde{(eV)} \end{array}$	method	Predictor	$\underbrace{\overset{success}{\overset{rate}{}}}_{rate}$	(A)BG	(<u>B)E</u> f~	(C)STR	(a) neut	(b) 0.5Å
0.50 ± 0.20	<u>S(Cry)</u>	Cry	0.410	0.598	0.637	0.754	0.961	0.754
0.50 ± 0.22	S(ALI)	ALI	0.336	0.387	0.855	0.867	0.941	0.867
0.50 ± 0.20	TPE	Cry	0.109	1.000	0.289	0.305	0.336	0.945
0.50 ± 0.44	FTCP	F-Reg	0.059	0.117	1.000	0.695	0.719	0.867
1.50 ± 0.20	<u>S(Cry)</u>	Cry	0.504	0.645	0.727	0.828	0.984	0.828
1.50 ± 0.22	S(ALI)	ALI	0.227	0.262	0.848	0.836	0.918	0.836
1.50 ± 0.20	TPE	Cry	$\begin{array}{c} 0.117\\ \widetilde{0.039}\end{array}$	1.000	0.219	0.273	0.312	0.898
1.50 ± 0.44	FTCP	F-Reg	0.039	0.039	1.000	0.695	0.727	0.875
2.50 ± 0.20	<u>S(Cry)</u>	Cry	0.391	0.527	0.734	0.797	0.996	0.797
2.50 ± 0.22	S(ALI)	ALI	0.238	0.270	0.844	0.781	0.914	0.781
2.50 ± 0.20	TPE	Cry	0.062	0.961	0.148	0.172	0.188	0.891
2.50 ± 0.44	FTCP	F-Reg	0.066	0.094	1.000	0.703	0.734	0.895

Table A.16: Experiments optimizing for various band gaps and while preserving a perovskite structure. We adopted the error values of each predictor in Table 1 as the margin for the band gap. The entries Cry, ALI, and F-Reg in the Predictor column correspond to the Crystalformer, ALIGNN, and regression branches of FTCP, respectively. Evaluation methods are based on those described in Table A.4.

$\operatorname{Target}_{\mathbf{P}\widetilde{\mathbf{C}}}$	method	Predictor	success	(A)BG	$(B)E_{f}$	C)STR	(a)	(b) 0.5Å	(D)PS		(d)	
BG (eV)			rate				neut	~~~~			angles	
0.50 ± 0.20	- mark	Cry	0.145	0.516	0.465		1.000	0.957	0.512	0.512	$\sim \sim \sim \sim \sim$	1
0.50 ± 0.22		ALI	0.191	0.516	0.473	1.000	1.000	1.000	$\sim \sim \sim$	10.449	1.000	1
0.50 ± 0.20	TPE	Cry	0.039	1.000	0.492		0.914	1.000	$\sim \sim \sim$	0.258	1.000	1
0.50 ± 0.44	FTCP	F-Reg	0.078	0.363	1.000	0.883	0.898	0.965	0.234	0.551	0.465	0
1.50 ± 0.20	S(Cry)	Cry	0.227	0.566	0.527	0.980	1.000	0.980	0.594	0.594	1.000	1
1.50 ± 0.22	S(ALI)	ALI	0.234	0.473	0.875		1.000	1.000	2000	0.570	ممتمم	~
1.50 ± 0.20	TPE	Cry	0.070	1.000	0.484	0.773	0.773	1.000	0.289	0.289	1.000	1
1.50 ± 0.44	FTCP	<u>F-Reg</u>	0.039	0.129	1.000	0.867	0.883	0.961	0.258	0.531	0.465	(
2.50 ± 0.20	S(Cry)	Cry	0.188	0.363	0.551	0.992	1.000	0.992	0.621	0.621	1.000	1
2.50 ± 0.22	S(ALI)	ĂĹĬ	0.238	0.379	0.961	1.000	1.000	1.000	0.641	0.641	1.000	1
2.50 ± 0.20		Ċry	0.062	1.000	0.508	0.656	0.656	1.000	0.320	0.320	1.000	1
2.50 ± 0.44	FTCP	<u>F-Reg</u>	0.062	$\widetilde{0.148}$	0.992	0.883	0.801	0.957	0.219	0 543	0.422	ĉ