### **000 001 002 003 004** ADAPTIVE CONSTRAINT INTEGRATION FOR SIMULTA-NEOUSLY OPTIMIZING CRYSTAL STRUCTURES WITH MULTIPLE TARGETED PROPERTIES

Anonymous authors

Paper under double-blind review

### ABSTRACT

In materials science, finding crystal structures that have targeted properties is crucial. While recent methodologies such as Bayesian optimization and deep generative models have made some advances on this issue, these methods often face difficulties in adaptively incorporating various constraints, such as electrical neutrality and targeted properties optimization, while keeping the desired specific crystal structure. To address these challenges, we have developed the Simultaneous Multi-property Optimization using Adaptive Crystal Synthesizer (SMOACS), which utilizes state-of-the-art property prediction models and their gradients to directly optimize input crystal structures for targeted properties simultaneously. SMOACS enables the integration of adaptive constraints into the optimization process without necessitating model retraining. Thanks to this feature, SMOACS has succeeded in simultaneously optimizing targeted properties while maintaining perovskite structures, even with models trained on diverse crystal types. We have demonstrated the band gap optimization while meeting a challenging constraint, that is, maintaining electrical neutrality in large atomic configurations up to 135 atom sites, where the verification of the electrical neutrality is challenging. The properties of the most promising materials have been confirmed by density functional theory calculations.

**029 030 031**

**032 033**

### 1 INTRODUCTION

**034 035 036 037 038 039 040** We address the challenge of simultaneously optimizing multiple material properties while preserving specific crystal structures and ensuring electrical neutrality. To achieve this, we have developed a methodology that leverages property prediction models and their gradients to facilitate the discovery of materials with multiple desired properties. This approach allows for the adaptive application of constraints, such as electrical neutrality and specific crystal structures, without necessitating retraining. As a result, our method enables the optimization of large atomic configurations to obtain specific properties while ensuring electrical neutrality and preserving specific crystal structures.

**041 042 043 044 045 046 047 048 049 050 051 052 053** Materials design is crucial for various advancing technologies, e.g., enhancing efficiency or reducing the cost of solar cells. The goal of materials design is to identify materials that simultaneously satisfy multiple property criteria, for instance, in terms of band gap and formation energy, while meeting other requirements, such as electrical neutrality. Furthermore, during the design process, it is often desirable to focus on specific promising systems, such as perovskite structures for next-generation solar cells [\(Green et al., 2014\)](#page-11-0). In the exploration of specific crystal structures, elemental substitution—blending different elements—is commonly employed. For instance, blended perovskite structures might have alternative compositions such as  $AA'BB'X_2X'X''_3$ , which are derived from the standard  $ABX_3$  format of a perovskite unit cell. Computational experiments involving these complex compositions often require larger systems that combine multiple unit cells. Consequently, the critical aspects of material design include 1) the ability to optimize multiple properties simultaneously, 2) the adaptive incorporation of various constraints, such as electrical neutrality or specific crystal structures, and 3) the ability to optimize large atomic configurations. In summary, we need to solve the problem of simultaneously optimizing multiple properties while preserving a specific crystal structure and ensuring electrical neutrality.

**054 055 056 057 058 059** Advances in computational techniques have tremendously accelerated material design, with Density Functional Theory (DFT) becoming a standard tool for rapid property validation. Recent developments in machine learning have enabled faster property predictions through deep learning models trained on DFT-generated data. Models such as Crystalformer [\(Taniai et al., 2024\)](#page-13-0), a transformerbased model [\(Vaswani et al., 2017\)](#page-13-1), and ALIGNN [\(Choudhary & DeCost, 2021\)](#page-10-0), a Graph Neural Network (GNN)-based model, significantly facilitate the screening process [\(Choubisa et al., 2023\)](#page-10-1).

**060 061 062 063 064 065 066 067 068** Bayesian optimization, such as Gaussian Process and Tree-structured Parzen estimator (TPE) [\(Watanabe, 2023\)](#page-13-2), is commonly employed in material design [\(Ozaki et al., 2020a;](#page-12-0) [Boyar et al., 2024;](#page-10-2) [Zhai et al., 2024\)](#page-14-0). A key advantage of Bayesian optimization is its capability to perform inverse inference, therefore enabling the prediction of crystal structures from given properties. Recently, deep generative models designed to synthesize crystal structures, such as FTCP [\(Ren et al., 2022\)](#page-13-3), have gained much attention due to their potential to discover new stable materials. Additionally, there ✿✿ are methods that leverage large language models [\(Ding et al., 2024;](#page-10-3) [Gruver et al., 2024\)](#page-11-1), Generative Flow Networks [\(AI4Science et al., 2023\)](#page-9-0), reinforcement learning [\(Govindarajan et al., 2024\)](#page-11-2), or flow matching [\(Miller et al., 2024\)](#page-12-1) to synthesize new crystal structures.

**069 070 071 072 073 074 075 076 077 078 079** Despite these advances, many challenges remain. Firstly, research using deep generative models primarily aims to identify stable materials, and only a limited number of studies focus on optimizing both stability and key properties, such as the band gap, which is crucial for maximizing solar cell efficiency. Secondly, deep generative models are often built with specialized architectures, making it difficult to adopt the latest property prediction models for their prediction branches. This lack of flexibility in model architectures can hinder the improvement of prediction accuracy for generated materials. Thirdly, current generative models require retraining for targeted properties optimization within specific crystal structures, which is often the case in practice, such as perovskite structures for solar cells. Finally, verifying the electrical neutrality in large atomic configurations is complicated due to the combinatorial explosion resulting from the possible multiple oxidation numbers for many atomic species. Nevertheless, ensuring electrical neutrality is essential for proposing realistic materials.

**080 081 082 083 084 085 086 087 088 089 090 091 092 093 094** To address these challenges, we have developed a framework, the Simultaneous Multi-property Optimization using Adaptive Crystal Synthesizer (SMOACS). SMOACS can employ various property prediction models as far as their gradients can be computed and optimizes input crystal structures directly to achieve target properties through the backpropagation technique (Fig. [1\(](#page-2-0)left)). This approach enables accurate prediction of multiple properties and simultaneous optimization by utilizing several recently developed pre-trained models for predicting different material properties. Unlike methods using normalizing flows that require architectural constraints for invertibility, our method imposes no such restrictions on these models. When newer models become available in the future, improved prediction accuracy will be achieved by incorporating them into our approach. Additionally, by managing the optimization range and utilizing special loss functions, we facilitate targeted properties optimization within specific crystal structures, avoiding retraining. Moreover, by imposing constraints via combinations of oxidation numbers, our method ensures the electrical neutrality of any proposed materials, even in large atomic configurations where verifying electrical neutrality is difficult due to combinatorial explosion. The generalizability of SMOACS enables it to adopt various prediction models and optimize various properties.

**095 096 097 098 099 100 101 102 103 104** SMOACS is the first method that directly optimizes the space of crystal structures using a which involves decomposing it into various components and representing atomic species as gradient-based approach. We achieve this by making the entire crystal structure differentiable, ✿✿✿✿✿ atomic distributions. Unlike traditional methods that convert crystal structures into latent variables [\(Ren et al., 2022\)](#page-13-3)—thereby entangling their elements—our approach maintains the independence of each component. This independence facilitates the preservation of crystal structures ✿✿✿ and ensures electrical neutrality by precisely specifying the atoms at each site. Furthermore, unlike generative models that probabilistically generate materials satisfying certain conditions, our method can inherently guarantee electrical neutrality and the preservation of crystal structures. Moreover we can add additional constraints as long as they are differentiable.

**105 106 107** We demonstrated that SMOACS could effectively utilize both GNN-based models and transformerbased models, outperforming FTCP, deep generative models, and TPE, Bayesian optimization. We demonstrated the band gap optimization within perovskite structures without retraining, using models trained on the MEGNet dataset [\(Chen et al., 2019\)](#page-10-4), which includes various types of crystals.

<span id="page-2-0"></span>

Figure 1: (left) Overview of the SMOACS framework. (right) An example of an optimized perovskite structure with a 4.02 eV band gap, verified at 3.96 eV through DFT calculations. Visualization was done with VESTA [\(Momma & Izumi, 2011\)](#page-12-2).

Additionally, we demonstrated the optimization for large atomic configurations with as many as 135 atom sites while ensuring electrical neutrality. Furthermore, the validity of the proposed materials was verified through DFT calculations.

# 2 RELATED WORKS

**131 132 133 134 135 136 137 138 139 Property prediction model.** In recent years, much research has actively focused on predicting the properties of materials using DFT-generated data [\(Davariashtiyani & Kadkhodaei, 2023;](#page-10-5) [Merchant](#page-12-3) [et al., 2023;](#page-12-3) [Yang et al., 2024a\)](#page-13-4). There are two primary approaches involving deep learning. The first approach utilizes GNNs [\(Chen et al., 2019;](#page-10-4) [Park & Wolverton, 2020;](#page-12-4) [Louis et al., 2020;](#page-12-5) [Schmidt](#page-13-5) [et al., 2021;](#page-13-5) [Lin et al., 2023\)](#page-11-3), such as ALIGNN. The main advantage of using GNNs is their ability to graphically represent crystal structures, thereby considering inter-atomic relationships in more physically meaningful ways. The second approach employs transformers [\(Ying et al., 2021;](#page-14-1) [Yan](#page-13-6) [et al., 2022\)](#page-13-6), such as Crystalformer, which are known for their promising performance in the field of computer vision and natural language processing [\(Brown et al., 2020;](#page-10-6) [Dosovitskiy et al., 2021\)](#page-10-7).

**140 141 142 143 144 145 146 147 148 149 150** Deep generative models. Deep generative models, including language models, for producing new stable materials have been emerging in the last years [\(Xie et al., 2022;](#page-13-7) [Lyngby & Thygesen, 2022;](#page-12-6) [Sultanov et al., 2023;](#page-13-8) [Yang & Mannodi-Kanakkithodi, 2022\)](#page-14-2). However, only a few studies explored material properties and stability at the same time. Studies such as FTCP [\(Ren et al., 2022\)](#page-13-3), Matter-Gen [\(Zeni et al., 2023\)](#page-14-3), and UniMat [\(Yang et al., 2024b\)](#page-14-4) focused on optimizing properties including band gap and material stability. They are generative models and thus conduct property optimizations within the framework of generative modeling. FTCP, based on Variational Autoencoders [\(Kingma,](#page-11-4) [2013\)](#page-11-4), encodes crystal structures into latent variables. It employs prediction branches to predict properties from these variables. MatterGen and UniMat are diffusion models and employ classifier-free guidance [\(Ho & Salimans, 2021\)](#page-11-5) to generate materials with specific properties. Although methods exist to constrain condition-free models for generating specific outputs [\(Wu et al., 2024\)](#page-13-9) , no research has implemented these techniques for crystal structures.

**151 152 153 154 155 156 157 158 159** Bayesian Optimization. Black-box optimization, including Bayesian optimization, is widely used in materials science [\(Song et al., 2024\)](#page-13-10). Numerous studies in materials science apply Bayesian optimization to predict crystal and molecular structures from target properties [\(Boyar et al., 2024;](#page-10-2) [Zhai et al., 2024;](#page-14-0) [Khatamsaz et al., 2023\)](#page-11-6). One representative method of Bayesian optimization widely utilized in materials science research is the Gaussian process (GP) [\(Lu et al., 2022\)](#page-12-7). However, as the Gaussian process only handles continuous values, its ability to manage categorical variables like elements is questionable. The recently proposed Tree-structured Parzen Estimator (TPE), which is capable of handling categorical variables and multi-objective optimization [\(Ozaki et al., 2020b;](#page-12-8) [2022\)](#page-12-9) and has been utilized in materials science [\(Ozaki et al., 2020a\)](#page-12-0), could be a better choice.

**160 161** Gradient based approach. Gradient-based approaches that aim to optimize design variables toward ✿✿✿✿✿✿ desired properties using deep learning-based predictors and their gradients have been applied across a wide range of fields. For example, they have been used to optimize designs for dynamics

<span id="page-3-0"></span>



**174 175 176 177 178 179 180 181 182 183 184** Figure 2: (a) SMOACS enforces site-specific restrictions on the types of elements to maintain electrical neutrality. The oxidation masks, labeled  $m(+4)$  and  $m(-2)$ , correspond to elements with oxidation numbers of  $+4$  and  $-2$ , respectively. The values 0 and 1 indicate the values of the mask. Parentheses indicate the elements and their potential oxidation numbers at each position. (b) Atomic distributions at each site considering two potential oxidation numbers. Here, we consider two possible patterns of oxidation number combinations:  $[+4, -2, -2]$  and  $[+2, -1, -1]$  for three sites. The atomic distributions at each site are computed by taking the weighted sum of the probabilities of these patterns. (c) Change in the atomic distribution at site No.1 in the optimization process. The numbers in the grids indicate the probabilities of elements in parentheses. As a result of the optimization, the TiO<sub>2</sub>-type oxidation pattern  $[+4, -2, -2]$  is selected, with Mn chosen as the element that achieves  $a + 4$  oxidation number.

of physical systems [\(Allen et al., 2022;](#page-10-8) [Hwang et al., 2022\)](#page-11-7), image manipulation [\(Xia et al., 2022\)](#page-13-11), metamaterials [\(Bordiga et al., 2024\)](#page-10-9), and chemical compositions [\(Fujii et al., 2024\)](#page-11-8) to achieve target performance. These methods require that the chain rule of differentiation connects from the input to the output. While there is the study that apply this technique by mapping crystal structures into latent spaces [\(Xie et al., 2022\)](#page-13-7), there are no studies that apply it directly within the space of crystal structures.

### <span id="page-3-1"></span>3 SMOACS

In SMOACS, the crystal structure  $\theta_{\text{crystal}}$  is divided into four learnable parameters: lattice constant  $l$ , coordinates of  $N$  atomic sites  $C$ , elements  $e$ , and an oxidation state configuration parameter  $o$  (Fig. [1\(](#page-2-0)left)).

$$
\theta_{\text{crystal}} = \{l, C, e, o\} \tag{1}
$$

$$
\boldsymbol{l} \in \mathbb{R}^6, \ \ \boldsymbol{C} \in \mathbb{R}^{N \times 3}, \ \ \boldsymbol{e} \in \mathbb{R}^N, \ \ \boldsymbol{o} \in \mathbb{R}^D
$$

**205 206 207 208 209 210 211 212 213 214 215** The lattice constant  $l$  comprises the crystallographic axes lengths  $a, b, c$  and the angles between these axes  $\alpha, \beta, \gamma$ . The oxidation state configuration parameter o denotes the probabilities for D patterns of oxidation number combinations determined by initial crystal structures, further described in Sec-tion [3.1.](#page-4-0) The  $l$  and  $C$ , being continuous variables, can be optimized directly through backpropagation technique [\(Ren et al., 2020;](#page-13-12) [Fujii et al., 2023\)](#page-11-9). However, this technique cannot be used for the elements e since they are being discrete and categorical. Therefore, instead of directly handling the elements  $e$ , we employ a technique where an element at site  $n$  is represented by the atomic distribution  $a_n$   $(a_n \in \mathbb{R}^K, A \in \mathbb{R}^{N \times K}, A_{i,:} = (a_i)^\top$  ) [\(Konno et al., 2021;](#page-11-10) [Fujii et al., 2024\)](#page-11-8). Here, K represents the highest atomic number considered. Since we are dealing with atomic numbers from 1 to 98,  $K = 98$ . When an element with the atomic number k occupies site n,  $a_n$  becomes a one-hot vector with the element  $k$  set to 1 and 0 at all others. Please refer to Section [A.5](#page-17-0) for a discussion on the general applicability of using atomic distributions in various property prediction models.

#### <span id="page-4-0"></span>**216 217** 3.1 MASKS TO MAINTAIN ELECTRICAL NEUTRALITY

**218 219 220 221 222 223 224 225 226 227 228 229** To maintain electrical neutrality, we restrict the possible values of atomic distribution  $a_n$  by using a mask that aligns with the possible oxidation numbers at site  $n$ . These possible oxidation numbers are determined from the initial structure. Here, we explain this using the rutile type structure as an example. A typical material having this structure is titanium dioxide (TiO<sub>2</sub>). The rutile TiO<sub>2</sub> contains one Ti site and two O sites, totaling three atomic sites. When titanium has an oxidation number of  $+4$  and the two oxygen atoms each have an oxidation number of  $-2$ , the total oxidation number is zero, thus achieving electrical neutrality. Therefore, to maintain electrical neutrality, we can use an atomic distribution that includes only elements with  $a + 4$  oxidation number, such as Ti and Mn at the Ti site. At the O sites, we use those with an oxidation number of  $-2$ , such as O and S. This ensures electrical neutrality regardless of the elements selected after optimization. The adjusted atomic distribution  $a'_n$ , which considers oxidation numbers, is obtained by taking the element-wise product of the learnable distribution  $a_n$  with the atomic mask  $m(s)$ .

$$
\boldsymbol{a}'_n(s) = \sigma(\boldsymbol{m}(s) * \boldsymbol{a}_n) \tag{3}
$$

Here,  $m(s)$  is  $m(s) \in \mathbb{R}^K$  and a mask that assigns a value of 1 to elements with the oxidation number s, and 0 to all others (Fig. [2\(](#page-3-0)a)).  $\sigma$  is a normalization function that rescales all elements to the range [0,1], with their total sum normalized to 1.0. The asterisk denotes element-wise multiplication. This process is applied to all sites, yielding an atomic distribution  $A'(S)$  that reflects the oxidation numbers for all sites.

$$
\begin{array}{c} 237 \\ 238 \\ 239 \end{array}
$$

**240 241**

$$
A'(S) = \sigma_{\text{atom}}(A * M(S))
$$
\n(4)

$$
\bm{M} \in \mathbb{R}^{N \times K}, \ \bm{M}(\bm{S})_{i,:} = (\bm{m}(s_i))^{\top}, \ \bm{S}_i = s_i, \ \bm{m}(s_i) \in \{\bm{m}(s_{\min}),...,\bm{m}(s),...,\bm{m}(s_{\max})\} \tag{5}
$$

**242 243 244 245 246 247 248 251** Here,  $\sigma_{\text{atom}}$  is a function that normalizes values along elemental directions.  $s_i$  is the oxidation number at site-i. The  $s_{\text{min}}$  and  $s_{\text{max}}$  respectively denote the minimum and maximum oxidation numbers among all elements considered. To simultaneously consider different patterns of oxidation number combinations, we introduce a learnable parameter  $o$ , which selects the optimal combination of oxidation numbers. We illustrate this approach using  $\text{CoF}_2$  and  $\text{TiO}_2$ , both of which adopt the rutile structure. The oxidation numbers differ at each atomic site, with  $\text{CoF}_2$  exhibiting oxidation numbers of  $[-2, -1, -1]$  and TiO<sub>2</sub> having  $[-4, -2, -2]$ . The o is a d-dimensional vector selecting the best pattern from  $d$  patterns of oxidation number combinations. The  $o$  represents the probabilities of each combination. For instance, when considering two patterns, such as those of  $\text{CoF}_2$  and  $\text{TiO}_2$ , d equals 2. Using this framework, we can calculate the modified atomic distribution  $a'_n$  (Fig. [2\(](#page-3-0)b)) considering multiple combination patterns as follows:

$$
\begin{array}{c} 252 \\ 253 \\ 254 \end{array}
$$

$$
\mathbf{a}'_n(\mathbf{a}_n, o_d) = \sum_{d=1}^D \mathbf{a}'_{n,d} = \sum_{d=1}^D \sigma(\mathbf{m}(s_{n,d}) * \mathbf{a}_n) o_d.
$$
 (6)

Here,  $m(s_{i,d}) \in \{m(s_{\min}), ..., m(s), ..., m(s_{\min})\}$ . The property prediction models assume that each site contains a single element. Therefore, after optimization, it is desirable for the oxidation state configuration parameter  $o$  and atomic distributions  $a'_n$  to become one-hot vectors. To guarantee that optimization will result in them becoming one-hot vectors, we normalize  $a'_n$  and o with the temperature softmax function  $\sigma_T$ .

$$
\sigma_T(z_i) = \frac{\exp\left(\frac{z_i}{T}\right)}{\sum_j \exp\left(\frac{z_j}{T}\right)}\tag{7}
$$

$$
\tilde{o} = \sigma_T(o) \tag{8}
$$

$$
\tilde{A}_n\Big(A_n, o, T\Big) = \sigma_T\bigg(\sum_{d=1}^D A'_d(S_d)\bigg) = \sigma_{T, \text{atom}}\bigg(\sum_{d=1}^D \sigma_{T, \text{atom}}(M(S_d) * A)\tilde{o}_d.\bigg) \tag{9}
$$

$$
(\boldsymbol{M}(\boldsymbol{S}_d) \in \mathbb{R}^{N \times K}, \ \boldsymbol{M}(\boldsymbol{S}_d)_{i,:} = \boldsymbol{m}(\boldsymbol{s}_{i,d})^{\top}, (\boldsymbol{S}_d)_{i} = s_{i,d} \tag{10}
$$

**270 271 272 273 274 275** Here,  $M(S_d)$  is an atomic mask of the d-th oxidation pattern.  $\sigma_T$  produces sharper distributions at lower temperatures T, ensuring that the parameters transition into one-hot vectors. For example, let  $o_1$  represent the probability of the TiO<sub>2</sub> type with oxidation numbers [+4, -2, -2], and  $o_2$  represent the CoF<sub>2</sub> type with oxidation numbers  $[-2, -1, -1]$ . If, at the end of optimization,  $\boldsymbol{o} = (1, 0)$  is achieved, the  $TiO<sub>2</sub>$  type is selected, resulting in a material with an oxidation pattern of [+4, -2, -2], as shown in Fig. [2\(](#page-3-0)c).

**276 277**

**278**

### 3.2 INITIALIZATION AND MULTIPLE PROPERTIES OPTIMIZATION

**279 280 281 282 283 284 285 286** In SMOACS, optimizations begin with crystal structures from a dataset or those randomly generated the initial structures for the optimization process are obtained from two sources: an existing dataset and self-generated structures based on typical perovskites with modified lattice constants (for details, see Section [A.6](#page-17-1) and [A.7\)](#page-21-0). These crystal structures must satisfy electrical neutrality and generate  $\mathcal{D}_{\mathcal{Q}}$  oxidation number patterns based on the compositions of initial crystal structures (see Section [A.11](#page-28-0) for details). The lattice constant  $l$  and atomic coordinates  $C$  are used directly as initial values. The atomic distribution  $\bm{A}$  and the oxidation number pattern selection parameter o are initialized with a uniform distribution.

<span id="page-5-1"></span><span id="page-5-0"></span>
$$
\theta_{\text{crystal}}' = \tau \Big( \{ \boldsymbol{l}, \boldsymbol{C}, \tilde{\boldsymbol{A}}(\boldsymbol{A}, \boldsymbol{o}, T), \tilde{\boldsymbol{o}}(\boldsymbol{o}, T) \} \Big) \tag{11}
$$

$$
\boldsymbol{l} \leftarrow -\eta_l \frac{\partial \mathcal{L}}{\partial \boldsymbol{l}}, \ \ \boldsymbol{C} \leftarrow -\eta_c \frac{\partial \mathcal{L}}{\partial \boldsymbol{C}}, \ \ \boldsymbol{A} \leftarrow -\eta_A \frac{\partial \mathcal{L}}{\partial \boldsymbol{A}}, \ \ \boldsymbol{o} \leftarrow -\eta_o \frac{\partial \mathcal{L}}{\partial \boldsymbol{o}}.
$$

Here,  $\eta_l$ ,  $\eta_c$ ,  $\eta_A$ ,  $\eta_o$  denote the learning rates for each parameter. L denotes loss function  $L(f_*(\theta'_{\text{crystal}}), y_{\text{target}})$  and  $f_*$  denotes a set of trained models. The  $\tau$  is a function converting structures to inputs for  $f_{*}$ . We optimize structures by iteratively updating them using Eq. [11](#page-5-0) and [12.](#page-5-1) During optimization, the temperature  $T$  of the softmax function starts high and is lowered towards the end, forcing  $\tilde{o}$  and  $\tilde{a}_n$  into one-hot vectors in the final stage. SMOACS optimizes multiple properties by incorporating various trained models or additional loss functions. Here, we aim to optimize the crystal structure by minimizing formation energy and targeting a specific band gap range,  $y_{\text{bg}} \pm h_{\text{bg}}$ , using trained models  $f_* = \{f_{\text{bg}*}, f_{\text{fg}}\}$ . Here,  $h_{\text{bg}}$  is an acceptable margin and  $f_{\text{bg}*}$  is the trained model predicting the band gap, and  $f_{f*}$  predicts the formation energy. We also set a strength parameter  $\lambda$ .

$$
L_{\text{bg}}(y_{\text{bg}}, \hat{y}_{\text{bg}}) = \max(0, |y_{\text{bg}} - \hat{y}_{\text{bg}}| - h_{\text{bg}}), L_{\text{f}}(\hat{y}_{\text{f}}) = \hat{y}_{\text{f}}
$$
(13)

<span id="page-5-4"></span><span id="page-5-3"></span>
$$
L = L_{bg}(y_{bg}, f_{bg*}(\theta_{crystal}')) + \lambda L_f(f_{fs}(\theta_{crystal}'))
$$
\n(14)

The influence of  $\lambda$  is discussed in Section [A.12.](#page-28-1) Note that since the crystal structure changes during optimization, when using GNNs, we update the graph multiple times based on the current structure in the optimization process.

### <span id="page-5-2"></span>3.3 PRESERVATION OF SPECIFIC CRYSTAL STRUCTURES DURING OPTIMIZATION

**312 313 314 315 316 317 318 319 320 321 322 323** Limiting the optimization variables and their range allows us to maintain specific crystal structures during optimization. For instance, let us consider a typical perovskite structure, represented by the chemical formula  $ABX<sub>3</sub>$ . It consists of five sites and adopts a crystal structure close to a cubic lattice. The fractional coordinates for the five sites are as follows:  $(0.5, 0.5, 0.5)$  at the A site,  $(0.0, 0.0, 0.0)$ at the B site, and  $(0.5, 0.0, 0.0)$ ,  $(0.0, 0.5, 0.0)$ ,  $(0.0, 0.0, 0.5)$  at the three X sites. Note that deviations from these values are allowed, together with degrees of freedom related to the lattice constant values. We optimize the structures within the range of small perturbations applied to typical perovskites. Specifically, first, only the a, b, and c of l are optimized, while  $\alpha$ ,  $\beta$ , and  $\gamma$  are fixed at 90°. Subsequently, the optimization range for the five sites is set close to their typical coordinates. For example, we optimize the coordinates at the A site within the range  $(0.5 \pm \epsilon, 0.5 \pm \epsilon, 0.5 \pm \epsilon)$ , where  $\epsilon$  is a small constant. Following a previous work on the distortion of CaCu<sub>3</sub>Ti<sub>4</sub>O<sub>12</sub> (Božin [et al., 2004\)](#page-10-10), we set  $\epsilon = 0.15$ . We also specify possible patterns of oxidation number combinations. Typically, some materials with perovskite structure such as  $SrTiO<sub>3</sub>$  exhibit oxidation numbers of  $[+2, +4, -2]$  at the A, B, and X sites, respectively, while others such as  $(\text{CH}_3\text{NH}_3)\text{PbI}_3$  exhibit

**324 325 326 327**  $[+1, +2, -1]$ . Consequently, two oxidation number patterns are prepared for the perovskite structure:  $[-2, +4, -2]$  and  $[-1, +2, -1]$  for the A, B, and X sites, respectively. By specifying these variables and ranges for optimization, we are able to maintain the perovskite structure.

# 4 EXPERIMENTS

**328 329 330**

**331 332 333 334 335 336 337 338 339 340 341 342 343 344** We compare SMOACS's ability to propose material satisfying specified properties and constraints with those from deep generative models and Bayesian optimization, represented by FTCP and TPE, respectively. TPE was chosen over GP, as discussed in Section 2. Both SMOACS and TPE ran for 200 optimization steps. All models were trained on the MEGNet dataset. For further implementation details, please refer to Section [A.4.](#page-15-0) To demonstrate SMOACS's versatility across various property prediction models, we conducted optimizations using ALIGNN and Crystalformer, GNN-based and transformer-based models, respectively. We evaluated the optimized materials using three metrics: whether they satisfied the specified criteria on the band gap range, formation energy, and validity of crystal structure. Specifically, we judge that the formation energy criterion is satisfied if it is less than -0.5 eV, and for the validity of crystal structure, following a previous research [\(Xie et al., 2022\)](#page-13-7), we adopted two criteria: all interatomic distances being at least  $0.5 \text{ Å}$  and maintaining electrical neutrality. For assessing electrical neutrality, we consider the material electrically neutral if the sum of possible oxidation numbers for atoms at each site equals zero. Please refer to Section [A.9](#page-25-0) for details. For experiments utilizing models trained on datasets other than MEGNet, please refer to Section [A.13.](#page-28-2)

**345 346**

### 4.1 LEVERAGING THE LATEST RESEARCH ACHIEVEMENTS IN PROPERTY PREDICTION

**347 348 349 350 351 352** First, we experimented with the performance of property prediction models that could be adopted in systems such as SMOACS, Bayesian optimization (TPE), or FTCP. In principle, Bayesian optimization—a type of black-box optimization—and SMOACS can adopt a broad range of property prediction models. Meanwhile, FTCP—a generative model—employs a property prediction branch within its architecture. Therefore, unlike SMOACS and Bayesian optimization, FTCP cannot use ALIGNN or Crystalformer for property predictions. The results are shown in Table [1.](#page-6-0)

**353 354 355 356 357** Crystalformer demonstrated the highest prediction accuracy among the models in Table [1.](#page-6-0) ALIGNN ranked second, whereas the prediction branches of FTCP exhibited the lowest performance. This result confirmed an advantage of SMOACS and TPE: their ability to incorporate state-of-the-art property prediction models, such as Crystalformer and ALIGNN, allowing for highly accurate material property predictions.

## 4.2 SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES REGARDLESS OF THE CRYSTAL STRUCTURE

**366** We tested the ability to optimize band gaps to target values. In this experiment, we optimized both the band gap and formation energy simultaneously, regardless of the crystal structure. The margins for the targets of band gaps were set to To ensure a fair comparison of optimization methods, we ✿✿✿✿ fixed the margin for all band gaps at  $\pm 0.04$  eV. For optimization results where the predictor's error is used as the margin, please refer to Section [A.14.](#page-29-0) We conducted experiments with SMOACS using Crystalformer and ALIGNN, respectively. We utilized Crystalformer as a predictor for TPE. We used three objective functions for TPE: band gap, formation energy, and electrical neutrality. FTCP selected data from the MEGNet dataset close to the target band gap and with formation energy less

**372 373**

**367**

> <span id="page-6-0"></span>Table 1: Comparison of property prediction models. This table compares models trained on the MEGNet dataset and presents Mean Absolute Error (MAE) scores for formation energy ( $E$  form) and band gap on test data in the MEGNet dataset. Lower scores are better across all metrics.



<span id="page-7-0"></span>**378 379 380 381 382 383 384 385** Table 2: Experiments on optimizing band gaps. We define the success rate as the probability of simultaneously satisfying three conditions: (A) the band gap is optimized within the target range, (B) the formation energy is below  $-0.5$  eV, and (C) the crystal structure is valid. C is achieved when two criteria are met simultaneously: (a) all inter-atomic distances are greater than  $0.5 \text{ Å}$ , and (b) the structure is electrically neutral. S(Cry) and S(ALI) denote SMOACS utilizing the Crystalformer and ALIGNN models, respectively. We evaluated each of the proposed materials using all evaluation metrics, and the results were averaged over 256 samples. Higher scores are better across all metrics. Augmented results are shown in Table [A.2.](#page-19-0)



**398 399 400**

**401 402**

**403 404**

than −0.5 eV, subsequently encoding them into latent variables. Finally, after adding noise, we decoded them back into crystal structures for evaluation. We optimized and evaluated the structures based on the band gap and formation energy values predicted by their respective predictors.

**405 406 407 408 409 410 411 412 413 414 415** The results are shown in Table [2.](#page-7-0) SMOACS significantly outperformed both TPE and FTCP in terms of success rates. SMOACS consistently maintained electrical neutrality, except for extreme geometries causing NaN values during crystal vector calculations. While FTCP always met the requirements for formation energy, it struggled to achieve the target band gap, contributing to its lower overall success rate. TPE achieved a high success rate in optimizing the band gap within the target range, but it could not optimize formation energy well. SMOACS maintained a high overall success rate as it achieved substantial success rates in both band gap and formation energy optimization. SMAOCS can easily scale this computation and can optimize 2,048 samples simultaneously in just a few minutes using a single A100 GPU. This allows us to repeat the optimization process multiple times, enabling us to obtain a large number of successful optimization samples. Please refer to Section [A.6](#page-17-1) for details, including the diversity of generated materials.

<span id="page-7-1"></span>4.3 SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES WHILE PRESERVING PEROVSKITE STRUCTURES

**419 420 421 422 423 424 425** We optimized the band gap within the range of 0.5 to 2.5 eV while preserving the perovskite structure. Besides the previously discussed metrics, we used three new criteria to confirm a structure's perovskite identity: internal coordinates, angles between crystal axes, and the tolerance factor. The tolerance factor  $t$  serves as a metric to assess the suitability of atomic combinations for forming perovskite structures [\(West, 2022\)](#page-13-13). t is calculated based on the ionic radii  $r_A$ ,  $r_B$  and  $r_X$  of each site in the perovskite structure  $ABX_3$  and we employed a loss function to optimize the tolerance factor alongside minimizing the band gap and formation energy.

**426**

$$
\frac{427}{428}
$$

<span id="page-7-2"></span>
$$
t = \frac{r_{\rm A} + r_{\rm X}}{\sqrt{2}}\tag{15}
$$

$$
\sqrt{2(r_{\rm B} + r_{\rm X})}
$$
\n
$$
L_t = |t - 0.9|
$$
\n(16)

$$
H_t = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}
$$
 (10)

<span id="page-8-0"></span>Table 3: Experiments on optimizing various band gaps while preserving perovskite structures. The "success rate" is the probability of simultaneously satisfying four criteria: (A) the band gap is optimized within the target range,  $(B)$  the formation energy is below  $-0.5$  eV,  $(C)$  the crystal structure is valid, and (D) approximating a valid perovskite structure. Criteria (A), (B), and (C) are consistent with those outlined in Table [2.](#page-7-0) The (D) is achieved when three criteria are met simultaneously: (c) the tolerance factor t is between 0.8 and 1.0, (d) coordinates are within  $\pm 0.15$  of typical perovskite structure coordinates, and (e) crystal axis angles are from 85° to 95°. The results are averaged over 256 samples. Higher scores are better across all metrics. Augmented results are shown in Table [A.4.](#page-22-0)



**455 456 457 458 459 460 461 462 463 464** If t is close to 1, the structure is likely a perovskite; if it is far from 1, it is not. Considering t values of typical perovskite structures ( $BaCeO_3:0.857$ ,  $SrTiO_3: 0.910$  and  $BaTiO_3: 0.970$ ), we established a tolerance factor range of  $0.8 \le t \le 1.0$  as the criterion for success. SMOACS optimized the structures with the procedure outlined in Section [3.3.](#page-5-2) Due to the limited number of perovskite structures in the MEGNet dataset, random perovskite configurations are used as initial values for SMOACS and TPE. The optimization range for SMOACS and TPE is established as outlined in Section [3.3.](#page-5-2) FTCP initially encoded typical perovskite structures from the MEGNet dataset into latent variables. After adding noise, these latent variables are decoded back into crystal structures for evaluation. Please refer to Section [A.7](#page-21-0) for details, including the diversity analysis. We evaluated the structures based on the band gap and formation energy values predicted by their respective

**465 466 467 468 469 470 471** predictors. The evaluation results are shown in Table [3.](#page-8-0) SMOACS with Crystalformer significantly outperformed both TPE and FTCP in overall success rates while preserving perovskite structures. In terms of (d) coordinates and (e) angles, both SMOACS and TPE consistently meet the criteria because their optimization ranges are the same. The generative model (FTCP), which uses latent variables, fails to obtain specific structural features of perovskite. Note that this limitation occurs despite the use of latent variables based on typical perovskite structures. This seems to be attributed to the training dataset that includes mixed crystal

- **472** types. It is noteworthy that SMOACS consistently ensures electrical neutrality.
- **473**

#### **474 475** 4.4 OPTIMIZING LARGE ATOMIC CONFIGURATIONS

**476 477 478 479 480 481 482** We optimized large atomic configurations where calculating electrical neutrality is impractical. In systems containing many atoms, the calculation of electrical neutrality becomes infeasible due to combinatorial explosion. For example, a system containing 135 atoms, each with two possible oxidation numbers, results in about  $4.3 \times 10^{40}$  combinations. Therefore, including an objective function for electrical neutrality in the TPE is infeasible. We conducted experiments on  $3 \times 3 \times 3$ perovskite structures containing 135 atom sites and compared SMOACS with TPE, not including the objective function for electrical neutrality (referred to as TPE(/N)). The results are shown in Table [4.](#page-9-1)

**483 484 485** SMOACS successfully optimized large atomic configurations, while TPE(/N) failed due to its inability to optimize the formation energy. The success of SMOACS likely stems from the utilization of information on gradients to optimize based on physics, enabling optimization even in large and complex systems. Furthermore, TPE was not able to evaluate electrical neutrality due to the compu-



<span id="page-9-1"></span>Table 4: Experiments optimizing for various band gaps while preserving  $3 \times 3 \times 3$  perovskite structures. We include only TPE, showing better performance in Section [4.3,](#page-7-1) for comparison. Evaluation methods are based on those described in Table [3.](#page-8-0) Augmented results are shown in Table [A.6.](#page-24-0)

tational cost of calculating it. Conversely, since SMOACS always maintains electrical neutrality, it is able to optimize properties while preserving this neutrality. Please refer to Section [A.8](#page-24-1) for details.

4.5 VERIFICATION BY DENSITY FUNCTIONAL THEORY

We used Density Functional Theory (see Section [A.1](#page-15-1) for details) to verify the band gaps of materials proposed by SMOACS. Among these materials, BaCsFClI (Fig. [1\(](#page-2-0)right)), a perovskite structured for a 4.02 eV band gap, showed a DFT-calculated value of 3.96 eV. However, we also found discrepancies between the values the model predicted and the DFT calculated for other candidate materials. Detailed results can be found in the appendix, Section [A.2.](#page-15-2)

**510 511 512**

## 5 CONCLUSIONS

**513 514**

**515 516 517 518 519 520 521 522 523 524 525 526 527** We propose SMOACS, a framework that utilizes the latest high-accuracy property prediction models and their gradients to search for materials with targeted multiple properties. SMOACS can adaptively apply constraints such as electrical neutrality and specific crystal structures without retraining. SMOACS not only outperformed FTCP and TPE in optimizing multiple targeted properties simultaneously but also maintained electrical neutrality in large systems where calculating electrical neutrality is challenging due to combinatorial complexity. As a further potential application, SMOACS should facilitate the exploration of stable structures. Using a compositional formula and various structure candidates they could form, this method minimizes formation energy while maintaining the crystal structure, thus determining the most stable configuration for that formula (see Section [A.3\)](#page-15-3). However, the performance of SMOACS heavily depends on the accuracy of property prediction models. Using models based on DFT calculations that underestimate band gaps (see Section [A.2\)](#page-15-2) can lead to similar underestimations in the predictions. By adopting more accurate models trained on datasets that are large and developed with more accurate DFT, we may address these challenges.

**528 529**

**530**

# **REFERENCES**

<span id="page-9-0"></span>**531 532 533 534** Mila AI4Science, Alex Hernandez-Garcia, Alexandre Duval, Alexandra Volokhova, Yoshua Bengio, Divya Sharma, Pierre Luc Carrier, Yasmine Benabed, Michał Koziarski, and Victor Schmidt. Crystal-gfn: sampling crystals with desirable properties and constraints. *arXiv preprint arXiv:2310.04925*, 2023.

<span id="page-9-2"></span>**535 536 537 538 539** Takuya Akiba, Shotaro Sano, Toshihiko Yanase, Takeru Ohta, and Masanori Koyama. Optuna: A next-generation hyperparameter optimization framework. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, KDD '19, pp. 2623–2631, New York, NY, USA, 2019. Association for Computing Machinery. ISBN 9781450362016. doi: 10.1145/3292500.3330701. URL [https://doi.org/10.1145/](https://doi.org/10.1145/3292500.3330701) [3292500.3330701](https://doi.org/10.1145/3292500.3330701).

- <span id="page-10-8"></span>**540 541 542 543** Kelsey R Allen, Tatiana Lopez-Guevara, Kimberly Stachenfeld, Alvaro Sanchez-Gonzalez, Peter Battaglia, Jessica Hamrick, and Tobias Pfaff. Physical design using differentiable learned simulators. *arXiv preprint arXiv:2202.00728*, 2022.
- <span id="page-10-11"></span>**544 545 546** P. E. Blöchl. Projector augmented-wave method. *Phys. Rev. B*, 50:17953–17979, Dec 1994. doi: 10. 1103/PhysRevB.50.17953. URL [https://link.aps.org/doi/10.1103/PhysRevB.](https://link.aps.org/doi/10.1103/PhysRevB.50.17953) [50.17953](https://link.aps.org/doi/10.1103/PhysRevB.50.17953).
- <span id="page-10-9"></span>**547 548 549** Giovanni Bordiga, Eder Medina, Sina Jafarzadeh, Cyrill Bösch, Ryan P Adams, Vincent Tournat, and Katia Bertoldi. Automated discovery of reprogrammable nonlinear dynamic metamaterials. *Nature Materials*, pp. 1–9, 2024.
- <span id="page-10-2"></span>**550 551 552 553** Onur Boyar, Yanheng Gu, Yuji Tanaka, Shunsuke Tonogai, Tomoya Itakura, and Ichiro Takeuchi. Crystal-lsbo: Automated design of de novo crystals with latent space bayesian optimization. *arXiv preprint arXiv:2405.17881*, 2024.
- <span id="page-10-10"></span>**554 555 556 557** E S Božin, V Petkov, P W Barnes, P M Woodward, T Vogt, S D Mahanti, and S J L Billinge. Temperature dependent total scattering structural study of cacu3ti4o12. *Journal of Physics: Condensed Matter*, 16(44):S5091, oct 2004. doi: 10.1088/0953-8984/16/44/007. URL [https:](https://dx.doi.org/10.1088/0953-8984/16/44/007) [//dx.doi.org/10.1088/0953-8984/16/44/007](https://dx.doi.org/10.1088/0953-8984/16/44/007).
- <span id="page-10-6"></span>**558 559 560 561 562 563 564 565 566 567** Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel Ziegler, Jeffrey Wu, Clemens Winter, Chris Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language models are few-shot learners. In H. Larochelle, M. Ranzato, R. Hadsell, M.F. Balcan, and H. Lin (eds.), *Advances in Neural Information Processing Systems*, volume 33, pp. 1877–1901. Curran Associates, Inc., 2020. URL [https://proceedings.neurips.cc/paper\\_files/paper/2020/](https://proceedings.neurips.cc/paper_files/paper/2020/file/1457c0d6bfcb4967418bfb8ac142f64a-Paper.pdf) [file/1457c0d6bfcb4967418bfb8ac142f64a-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2020/file/1457c0d6bfcb4967418bfb8ac142f64a-Paper.pdf).
- <span id="page-10-4"></span>**568 569 570** Chi Chen, Weike Ye, Yunxing Zuo, Chen Zheng, and Shyue Ping Ong. Graph networks as a universal machine learning framework for molecules and crystals. *Chemistry of Materials*, 31(9):3564– 3572, 2019.
- <span id="page-10-1"></span>**571 572 573 574** Hitarth Choubisa, Petar Todorovic, Joao M Pina, Darshan H Parmar, Ziliang Li, Oleksandr Voznyy, ´ Isaac Tamblyn, and Edward H Sargent. Interpretable discovery of semiconductors with machine learning. *npj Computational Materials*, 9(1):117, 2023.
- <span id="page-10-13"></span>**575 576** Kamal Choudhary. Alignn models on jarvis-dft dataset, 2021. URL [10.6084/m9.figshare.](10.6084/m9.figshare.17005681.v6) [17005681.v6](10.6084/m9.figshare.17005681.v6).
- <span id="page-10-0"></span>**578 579** Kamal Choudhary and Brian DeCost. Atomistic line graph neural network for improved materials property predictions. *npj Computational Materials*, 7(1):185, 2021.

- <span id="page-10-5"></span>**580 581 582** Ali Davariashtiyani and Sara Kadkhodaei. Formation energy prediction of crystalline compounds using deep convolutional network learning on voxel image representation. *Communications Materials*, 4(1):105, 2023.
- <span id="page-10-12"></span>**583 584 585 586** Daniel W. Davies, Keith T. Butler, Adam J. Jackson, Jonathan M. Skelton, Kazuki Morita, and Aron Walsh. Smact: Semiconducting materials by analogy and chemical theory. *Journal of Open Source Software*, 4(38):1361, 2019. doi: 10.21105/joss.01361. URL [https://doi.org/10.](https://doi.org/10.21105/joss.01361) [21105/joss.01361](https://doi.org/10.21105/joss.01361).
- <span id="page-10-3"></span>**587 588 589** Qianggang Ding, Santiago Miret, and Bang Liu. Matexpert: Decomposing materials discovery by mimicking human experts. *arXiv preprint arXiv:2410.21317*, 2024.
- <span id="page-10-7"></span>**590 591 592 593** Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, Jakob Uszkoreit, and Neil Houlsby. An image is worth 16x16 words: Transformers for image recognition at scale. In *International Conference on Learning Representations*, 2021. URL [https:](https://openreview.net/forum?id=YicbFdNTTy) [//openreview.net/forum?id=YicbFdNTTy](https://openreview.net/forum?id=YicbFdNTTy).

**613**

<span id="page-11-14"></span>**622**

- <span id="page-11-9"></span><span id="page-11-8"></span>**594 595 596 597 598 599 600 601 602 603** Akihiro Fujii, Hideki Tsunashima, Yoshihiro Fukuhara, Koji Shimizu, and Satoshi Watanabe. Enhancing inverse problem solutions with accurate surrogate simulators and promising candidates. *arXiv preprint arXiv:2304.13860*, 2023. Akihiro Fujii, Koji Shimizu, and Satoshi Watanabe. Efficient exploration of high-tc superconductors by a gradient-based composition design. *arXiv preprint arXiv:2403.13627*, 2024. Prashant Govindarajan, Santiago Miret, Jarrid Rector-Brooks, Mariano Phielipp, Janarthanan Rajendran, and Sarath Chandar. Learning conditional policies for crystal design using offline reinforcement learning. *Digital Discovery*, 3(4):769–785, 2024.
- <span id="page-11-2"></span><span id="page-11-0"></span>**604 605** Martin A Green, Anita Ho-Baillie, and Henry J Snaith. The emergence of perovskite solar cells. *Nature photonics*, 8(7):506–514, 2014.
- <span id="page-11-1"></span>**607 608 609** Nate Gruver, Anuroop Sriram, Andrea Madotto, Andrew Gordon Wilson, C Lawrence Zitnick, and Zachary Ulissi. Fine-tuned language models generate stable inorganic materials as text. *arXiv preprint arXiv:2402.04379*, 2024.
- <span id="page-11-5"></span>**610 611 612** Jonathan Ho and Tim Salimans. Classifier-free diffusion guidance. In *NeurIPS 2021 Workshop on Deep Generative Models and Downstream Applications*, 2021. URL [https://openreview.](https://openreview.net/forum?id=qw8AKxfYbI) [net/forum?id=qw8AKxfYbI](https://openreview.net/forum?id=qw8AKxfYbI).
- <span id="page-11-7"></span>**614 615 616** Rakhoon Hwang, Jae Yong Lee, Jin Young Shin, and Hyung Ju Hwang. Solving pde-constrained control problems using operator learning. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 36, pp. 4504–4512, 2022.
- <span id="page-11-12"></span>**617 618 619 620 621** Anubhav Jain, Shyue Ping Ong, Geoffroy Hautier, Wei Chen, William Davidson Richards, Stephen Dacek, Shreyas Cholia, Dan Gunter, David Skinner, Gerbrand Ceder, and Kristin A. Persson. Commentary: The Materials Project: A materials genome approach to accelerating materials innovation. *APL Materials*, 1(1):011002, 07 2013. ISSN 2166-532X. doi: 10.1063/1.4812323. URL <https://doi.org/10.1063/1.4812323>.
- **623 624 625** Rui Jiao, Wenbing Huang, Peijia Lin, Jiaqi Han, Pin Chen, Yutong Lu, and Yang Liu. Crystal structure prediction by joint equivariant diffusion. *Advances in Neural Information Processing Systems*, 36, 2024.
- <span id="page-11-6"></span><span id="page-11-4"></span>**626 627 628** Danial Khatamsaz, Raymond Neuberger, Arunabha M Roy, Sina Hossein Zadeh, Richard Otis, and Raymundo Arróyave. A physics informed bayesian optimization approach for material design: application to niti shape memory alloys. *npj Computational Materials*, 9(1):221, 2023.
	- Diederik P Kingma. Auto-encoding variational bayes. *arXiv preprint arXiv:1312.6114*, 2013.
	- Tomohiko Konno, Hodaka Kurokawa, Fuyuki Nabeshima, Yuki Sakishita, Ryo Ogawa, Iwao Hosako, and Atsutaka Maeda. Deep learning model for finding new superconductors. *Phys. Rev. B*, 103:014509, Jan 2021. doi: 10.1103/PhysRevB.103.014509. URL [https://link.](https://link.aps.org/doi/10.1103/PhysRevB.103.014509) [aps.org/doi/10.1103/PhysRevB.103.014509](https://link.aps.org/doi/10.1103/PhysRevB.103.014509).
- <span id="page-11-11"></span><span id="page-11-10"></span>**636 637 638** G. Kresse and D. Joubert. From ultrasoft pseudopotentials to the projector augmented-wave method. *Phys. Rev. B*, 59:1758–1775, Jan 1999. doi: 10.1103/PhysRevB.59.1758. URL [https://](https://link.aps.org/doi/10.1103/PhysRevB.59.1758) [link.aps.org/doi/10.1103/PhysRevB.59.1758](https://link.aps.org/doi/10.1103/PhysRevB.59.1758).
- <span id="page-11-13"></span>**639 640 641 642** Aliaksandr V Krukau, Oleg A Vydrov, Artur F Izmaylov, and Gustavo E Scuseria. Influence of the exchange screening parameter on the performance of screened hybrid functionals. *The Journal of chemical physics*, 125(22), 2006.
- <span id="page-11-3"></span>**643 644 645 646 647** Yuchao Lin, Keqiang Yan, Youzhi Luo, Yi Liu, Xiaoning Qian, and Shuiwang Ji. Efficient approximations of complete interatomic potentials for crystal property prediction. In Andreas Krause, Emma Brunskill, Kyunghyun Cho, Barbara Engelhardt, Sivan Sabato, and Jonathan Scarlett (eds.), *Proceedings of the 40th International Conference on Machine Learning*, volume 202 of *Proceedings of Machine Learning Research*, pp. 21260–21287. PMLR, 23–29 Jul 2023. URL <https://proceedings.mlr.press/v202/lin23m.html>.
- <span id="page-12-11"></span><span id="page-12-7"></span><span id="page-12-6"></span><span id="page-12-5"></span><span id="page-12-3"></span><span id="page-12-2"></span><span id="page-12-1"></span>**648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700** Steph-Yves Louis, Yong Zhao, Alireza Nasiri, Xiran Wang, Yuqi Song, Fei Liu, and Jianjun Hu. Graph convolutional neural networks with global attention for improved materials property prediction. *Physical Chemistry Chemical Physics*, 22(32):18141–18148, 2020. Tian Lu, Hongyu Li, Minjie Li, Shenghao Wang, and Wencong Lu. Inverse design of hybrid organic– inorganic perovskites with suitable bandgaps via proactive searching progress. *ACS omega*, 7(25): 21583–21594, 2022. Peder Lyngby and Kristian Sommer Thygesen. Data-driven discovery of 2d materials by deep generative models. *npj Computational Materials*, 8(1):232, 2022. Amil Merchant, Simon Batzner, Samuel S Schoenholz, Muratahan Aykol, Gowoon Cheon, and Ekin Dogus Cubuk. Scaling deep learning for materials discovery. *Nature*, 624(7990):80–85, 2023. Benjamin Kurt Miller, Ricky TQ Chen, Anuroop Sriram, and Brandon M Wood. Flowmm: Generating materials with riemannian flow matching. *arXiv preprint arXiv:2406.04713*, 2024. Koichi Momma and Fujio Izumi. *VESTA3* for three-dimensional visualization of crystal, volumetric and morphology data. *Journal of Applied Crystallography*, 44(6):1272–1276, Dec 2011. doi: 10. 1107/S0021889811038970. URL <https://doi.org/10.1107/S0021889811038970>. Shyue Ping Ong, William Davidson Richards, Anubhav Jain, Geoffroy Hautier, Michael Kocher, Shreyas Cholia, Dan Gunter, Vincent L. Chevrier, Kristin A. Persson, and Gerbrand Ceder. Python materials genomics (pymatgen): A robust, open-source python library for materials analysis. *Computational Materials Science*, 68:314–319, 2013. ISSN 0927-0256. doi: https: //doi.org/10.1016/j.commatsci.2012.10.028. URL [https://www.sciencedirect.com/](https://www.sciencedirect.com/science/article/pii/S0927025612006295) [science/article/pii/S0927025612006295](https://www.sciencedirect.com/science/article/pii/S0927025612006295). Yoshihiko Ozaki, Yuta Suzuki, Takafumi Hawai, Kotaro Saito, Masaki Onishi, and Kanta Ono. Automated crystal structure analysis based on blackbox optimisation. *npj Computational Materials*, 6(1):1–7, 2020a. Yoshihiko Ozaki, Yuki Tanigaki, Shuhei Watanabe, and Masaki Onishi. Multiobjective treestructured parzen estimator for computationally expensive optimization problems. In *Proceedings of the 2020 Genetic and Evolutionary Computation Conference*, GECCO '20, pp. 533–541, New York, NY, USA, 2020b. Association for Computing Machinery. ISBN 9781450371285. doi: 10.1145/3377930.3389817. URL <https://doi.org/10.1145/3377930.3389817>. Yoshihiko Ozaki, Yuki Tanigaki, Shuhei Watanabe, Masahiro Nomura, and Masaki Onishi. Multiobjective tree-structured parzen estimator. *Journal of Artificial Intelligence Research*, 73:1209– 1250, 2022. Cheol Woo Park and Chris Wolverton. Developing an improved crystal graph convolutional neural network framework for accelerated materials discovery. *Phys. Rev. Mater.*, 4:063801, Jun 2020. doi: 10.1103/PhysRevMaterials.4.063801. URL [https://link.aps.org/doi/10.](https://link.aps.org/doi/10.1103/PhysRevMaterials.4.063801) [1103/PhysRevMaterials.4.063801](https://link.aps.org/doi/10.1103/PhysRevMaterials.4.063801). Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Kopf, Edward Yang, Zachary DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. Pytorch: An imperative style, high-performance deep learning library. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d'Alche-Buc, E. Fox, ´ and R. Garnett (eds.), *Advances in Neural Information Processing Systems*, volume 32. Curran Associates, Inc., 2019. URL [https://proceedings.neurips.cc/paper\\_files/](https://proceedings.neurips.cc/paper_files/paper/2019/file/bdbca288fee7f92f2bfa9f7012727740-Paper.pdf) [paper/2019/file/bdbca288fee7f92f2bfa9f7012727740-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2019/file/bdbca288fee7f92f2bfa9f7012727740-Paper.pdf). John P. Perdew, Kieron Burke, and Matthias Ernzerhof. Generalized gradient approximation made
- <span id="page-12-12"></span><span id="page-12-10"></span><span id="page-12-9"></span><span id="page-12-8"></span><span id="page-12-4"></span><span id="page-12-0"></span>**701** simple. *Phys. Rev. Lett.*, 77:3865–3868, Oct 1996. doi: 10.1103/PhysRevLett.77.3865. URL <https://link.aps.org/doi/10.1103/PhysRevLett.77.3865>.

<span id="page-13-14"></span><span id="page-13-12"></span><span id="page-13-10"></span><span id="page-13-8"></span><span id="page-13-5"></span><span id="page-13-3"></span><span id="page-13-0"></span>

<span id="page-13-13"></span><span id="page-13-11"></span><span id="page-13-9"></span><span id="page-13-7"></span><span id="page-13-6"></span><span id="page-13-4"></span><span id="page-13-2"></span><span id="page-13-1"></span>ments, temperatures and pressures. *arXiv preprint arXiv:2405.04967*, 2024a.

<span id="page-14-2"></span> Jiaqi Yang and Arun Mannodi-Kanakkithodi. High-throughput computations and machine learning for halide perovskite discovery. *MRS Bulletin*, 47(9):940–948, 2022.

<span id="page-14-4"></span> Sherry Yang, KwangHwan Cho, Amil Merchant, Pieter Abbeel, Dale Schuurmans, Igor Mordatch, and Ekin Dogus Cubuk. Scalable diffusion for materials generation. In *The Twelfth International Conference on Learning Representations*, 2024b. URL [https://openreview.net/](https://openreview.net/forum?id=wm4WlHoXpC) [forum?id=wm4WlHoXpC](https://openreview.net/forum?id=wm4WlHoXpC).

<span id="page-14-1"></span> Chengxuan Ying, Tianle Cai, Shengjie Luo, Shuxin Zheng, Guolin Ke, Di He, Yanming Shen, and Tie-Yan Liu. Do transformers really perform badly for graph representation? In M. Ranzato, A. Beygelzimer, Y. Dauphin, P.S. Liang, and J. Wortman Vaughan (eds.), *Advances in Neural Information Processing Systems*, volume 34, pp. 28877–28888. Curran Associates, Inc., 2021. URL [https://proceedings.neurips.cc/paper\\_files/](https://proceedings.neurips.cc/paper_files/paper/2021/file/f1c1592588411002af340cbaedd6fc33-Paper.pdf) [paper/2021/file/f1c1592588411002af340cbaedd6fc33-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2021/file/f1c1592588411002af340cbaedd6fc33-Paper.pdf).

- <span id="page-14-3"></span> Claudio Zeni, Robert Pinsler, Daniel Zugner, Andrew Fowler, Matthew Horton, Xiang Fu, Sasha ¨ Shysheya, Jonathan Crabbé, Lixin Sun, Jake Smith, et al. Mattergen: a generative model for inorganic materials design. *arXiv preprint arXiv:2312.03687*, 2023.
- <span id="page-14-0"></span> Hanfeng Zhai, Hongxia Hao, and Jingjie Yeo. Benchmarking inverse optimization algorithms for materials design. *APL Materials*, 12(2), 2024.

 

#### **810 811** A APPENDIX

<span id="page-15-1"></span>A.1 DETAILS OF THE SETTINGS IN DENSITY FUNCTIONAL THEORY

**814 815 816 817 818 819** We used Density Functional Theory (DFT) to verify the band gaps of materials proposed by SMOACS. We employed the Vienna Ab initio Simulation Package (VASP) [\(Kresse & Joubert, 1999\)](#page-11-11) version 5.4.4 with Perdew-Burke-Ernzerhof (PBE) exchange-correlation functional [\(Perdew et al.,](#page-12-10) [1996\)](#page-12-10) and Projector Augmented Wave (PAW) pseudo-potentials (Blöchl, 1994) in all DFT calculations. We used the MPRelaxSet from PyMatGen [\(Ong et al., 2013\)](#page-12-11) to generate input files: KPOINTS and INCAR.

**820 821**

**822 823**

**812 813**

### <span id="page-15-2"></span>A.2 BAND GAP DISCREPANCIES BETWEEN MACHINE LEARNING PREDICTED AND DFT CALCULATED

**824 825 826 827 828 829 830 831 832 833 834 835** SMOACS heavily relies on the accuracy of property prediction models. However, we found discrepancies between the values model predicted and the DFT calculated (Fig[.A.1\)](#page-16-0). Furthermore, structures relaxed by the MPRelaxSet, which is a parameter set for structural relaxations with VASP provided in PyMatGen, sometimes significantly differ from their proposed forms. There are two possible reasons. First, DFT settings: MEGNet dataset comes from an older version of Materials Project database [\(Jain et al., 2013\)](#page-11-12). Materials in this database are sometimes updated, and calculation conditions when the MEGNet dataset is created could be different from the current MPRelaxset. We could not reproduce the band gap values in the MEGNet dataset with MPRelaxset. Second, MEG-Net dataset features: All models used in this work are trained on the MEGNet dataset, which is comprised predominantly of stable materials. So, predicting unstable or physically inappropriate structures with these models can lead to inaccurate predictions that may affect the proposed materials. To address these issues, we may need a model trained on a large dataset that includes both stable and unstable structures.

**836 837 838 839 840 841 842** The MEGNet dataset utilizes DFT calculations with PBE functionals that are known to underestimate band gaps. Consequently, when models trained on the MEGNet dataset are used in SMOACS, this tendency may be reflected in the predictions of the proposed materials. This issue may be addressed by constructing a dataset using more accurate band gap calculations, such as HSE06 hybrid functionals [\(Krukau et al., 2006\)](#page-11-13) and adopting models trained on that dataset. It should be noted that the amount of data available with these accurate calculations is much more limited than for DFT-PBE.

- **843**
- **844 845**

## <span id="page-15-3"></span>A.3 A POSSIBLE APPLICATION: IDENTIFYING THE MOST STABLE CRYSTAL STRUCTURE

**846 847 848 849 850 851 852 853 854 855 856 857** Our method can optimize energy while specifying the base crystal structure. This property may allow for identifying crystal structures based either solely on the chemical formula or on a combination of the chemical formula and physical properties. This is a Crystal Structure Prediction (CSP) task [\(Ryan et al., 2018;](#page-13-14) [Miller et al., 2024;](#page-12-1) [Jiao et al., 2024\)](#page-11-14). To verify if this is possible, we experimented to see if the crystal structure of metallic silicon with a zero band gap could be identified. Initially, we extracted structures from the MEGNet dataset that contained only one atom besides Si, using them as the initial structure. The atomic distribution was fixed with a one-hot vector indicating silicon, and only the lattice constants were optimized. The target properties for optimization were a zero band gap and formation energy minimization. We chose silicon structures from the MEGNet dataset with a band gap of 0 eV as the reference and compared these with the optimized structures that exhibited the lowest formation energy. Consequently, we identified structures close to the reference among those optimized for the lowest formation energy.

**858 859** The results are shown in Table [A.1.](#page-16-1) The reference material of mp-34 is close to optimized candidate No. 2. Similarly, mp-1014212 is close to candidates from No. 4 to No. 12."

<span id="page-15-0"></span>**860**

- **861** A.4 IMPLEMENTATION DETAILS
- **863** To demonstrate that SMOACS can utilize various property prediction models, we selected ALIGNN as a representative of the GNN-based models and Crystalformer as a representative of the

<span id="page-16-0"></span>

Figure A.1: The discrepancies between band gap values predicted the machine learning model (Crystalformer) and that of DFT calculated.

Table A.1: Reference Si materials (band gap 0 eV) and optimized candidates.

<span id="page-16-1"></span>

Materials	a,b,c(A)	$\alpha, \beta, \gamma$ (°)	predicted formation energy $(eV)$		
$(Ref) mp-34$	2.64, 2.64, 2.47	90.0, 90.0, 120			
$\overline{R}$ ef) mp- $\overline{1}0\overline{1}42\overline{1}2$	$\bar{2}.\bar{6}6,\bar{2}.\bar{6}6,\bar{2}.\bar{6}6$	$\overline{109.5}$ , $\overline{109.5}$ , $\overline{109.5}$			
candidate-1	2.67, 2.67, 2.94	124.0, 124.0, 97.9	$-0.367$		
candidate-2	$\overline{2.50, 2.50, 2.27}$	89.9, 89.9, 134.0	$-0.359$		
candidate- $\overline{3}$	$\bar{2.76}, \bar{2.76}, \bar{2.76}$	$\bar{1}1\bar{5}\bar{.}3, \bar{1}1\bar{5}\bar{.}3, \bar{1}1\bar{5}\bar{.}\bar{3}$	$\overline{-0.326}$		
candidate- $\overline{4}$	$\bar{2}.\bar{7}2,\bar{2}.\bar{7}\bar{2},\bar{2}.\bar{7}\bar{2}$	$\bar{1}1\bar{5}.0$ , $\bar{1}1\bar{5}.0$ , $\bar{1}1\bar{5}.0$	$\overline{-0.310}$		
candidate-5	$2.\overline{7}1,\overline{2}.\overline{7}1,\overline{2}.\overline{7}\overline{1}$	$\bar{1}1\bar{4}.\bar{9}, \bar{1}1\bar{4}.\bar{9}, \bar{1}1\bar{4}.\bar{9}$	$\overline{-0.310}$		
$c$ andidate- $\overline{6}$	$2.\overline{7}1,\overline{2}.\overline{7}1,\overline{2}.\overline{7}1$	$\overline{114.9}$ , $\overline{114.9}$ , $\overline{114.9}$	$\overline{-0.310}$		
candidate-7	$\bar{2}.\bar{7}1.\bar{2}.\bar{7}\bar{1}.\bar{2}.\bar{7}\bar{1}$	$\bar{1}1\bar{4}.\bar{9}, \bar{1}1\bar{4}.\bar{9}, \bar{1}1\bar{4}.\bar{9}$	$\overline{-0.308}$		
candidate-8	$\overline{2.73, 2.73, 2.73}$	$\bar{1}1\bar{5}.0,\bar{1}\bar{1}5.\bar{0},\bar{1}1\bar{5}.\bar{0}$	$-\bar{0}.\bar{3}0\bar{8}$		
candidate-9	$\overline{2.69}$ , $\overline{2.69}$ , $\overline{2.69}$	$\overline{11}4.\overline{9}, \overline{11}4.\overline{9}, \overline{11}4.\overline{9}$	$-\bar{0}.\bar{3}0\bar{8}$		
candidate-10	$\overline{2.68}$ , $\overline{2.68}$ , $\overline{2.68}$	$\overline{11}4.\overline{8}, \overline{11}4.\overline{8}, \overline{11}4.\overline{8}$	$\overline{-0.304}$		
candidate-11	$\overline{2.64}, \overline{2.64}, \overline{2.64}$	$\bar{1}1\bar{4}.\bar{3}, \bar{1}1\bar{4}.\bar{3}, \bar{1}1\bar{4}.\bar{3}$	$\overline{-0.266}$		
candidate-12	2.55, 2.55, 2.55	$\bar{1}1\bar{3}\bar{.}9, \bar{1}\bar{1}\bar{3}\bar{.}9, \bar{1}1\bar{3}\bar{.}\bar{9}$	$-\overline{0}.\overline{2}6\overline{3}$		
candidate-13	$\overline{2.44, 2.43, 2.44}$	$68.\overline{8}.\overline{64}.\overline{7}.\overline{111}.\overline{11}$	$\overline{-0.243}$		
candidate-14	$\overline{2.42, 2.42, 2.42}$	$\bar{1}1\bar{2}.\bar{3}, \bar{1}1\bar{2}.\bar{3}, \bar{1}1\bar{2}.\bar{3}$	$\overline{-0.239}$		
candidate-15	$\overline{2.42, 2.42, 2.42}$	$\overline{112.3}$ , $\overline{112.3}$ , $\overline{112.3}$	$-\bar{0}.\bar{2}3\bar{9}$		

 Transformer-based models. For both ALIGNN and Crystalformer, we utilized publicly available weights trained on the MEGNet dataset that predict band gaps and formation energies.

 The number of optimization steps was 200 for both SMOACS and TPE. The softmax temperature T was linearly decayed from  $T = 0.01$  at step 1 to  $T = 0.0001$  at step 200. Unless otherwise specified, to prevent extreme crystal structures, the crystal axis lengths  $a, b, c$  were clipped to a range of 2 Å to 10 Å, and the angles  $\alpha, \beta, \gamma$  were clipped to between 30° and 150°. The types of elements considered ranged from atomic numbers 1 to 98. Unless otherwise noted, the search range

**918 919 920** for TPE was aligned with SMOACS, with crystal axis lengths  $a, b, c$  ranging from 2 Å to 10 Å and angles  $\alpha$ ,  $\beta$ ,  $\gamma$  from 30° to 150°. We set the strength parameter  $\lambda = 1.0$ .

**921 922 923 924 925 926** In ALIGNN, bonds are defined using a graph structure. However, because the graph structure is non-differentiable, it cannot be optimized directly. Moreover, as the crystal structure is optimized, the nearest-neighbor atoms may change, potentially rendering the continued use of the same graph structure inappropriate. Therefore, we updated the graph structure multiple times during the optimization process. Considering that the learning rate decay follows a cosine schedule, we updated the graph several times according to a sine schedule, which is the integral of the cosine function.

**927 928 929 930 931** SMOACS was implemented using PyTorch [\(Paszke et al., 2019\)](#page-12-12); we used the web-available imple-mentation of FTCP<sup>[1](#page-17-2)</sup> and trained it on the MEGNet dataset. Optuna [\(Akiba et al., 2019\)](#page-9-2) was used for TPE. We conducted optimizations using ALIGNN and Crystalformer, GNN-based, and transformerbased models, respectively. We used a NVIDIA A100 GPU. We utilized official codes and weights that are available online<sup>[2](#page-17-3)[3](#page-17-4)</sup>.

**932 933 934 935 936 937 938 939** We trained FTCP from scratch using the MEGNet dataset. We tuned the hyperparameters, including the max elms parameter (the number of types of atoms in the crystal), the max sites parameter (the number of atomic sites in the crystal), and the learning rate. As a result, max elms, max sites, and the learning rate were set to 4, 20, and 0.0001, respectively. Note that the MEG-Net dataset contains data with a larger number of element types and sites than these settings, so we did not utilize all 60,000 training samples; however, the reconstruction error score was better with this setting. During inference, after testing several values for the standard deviation of the noise added to the latent variables, we decided to sample from a normal distribution with a mean of 0 and a standard deviation of 0.6.

**940 941**

**942**

**951 952 953**

## <span id="page-17-0"></span>A.5 APPLICABILITY TO PROPERTY PREDICTION MODELS

**943 944 945 946 947 948 949 950** This strategy of using atomic distributions discussed in Section [3](#page-3-1) is widely applicable to various property prediction models. It readily supports formats such as Crystalformer, where one-hot vectors representing elements are fed into the model. Next, we consider a scenario of using models such as ALIGNN that require atomic representations as input. In this scenario, we treat the inner product of the atomic distribution  $a_n$  and the u-dimensional representation vector for atoms  $r_{\text{atom}}$  ( $r_{\text{atom}} \in$  $\mathbb{R}^{K \times u}$ ) as the atomic representation. In either case, since the output is connected to the learnable atomic distributions through the chain rule of differentiation, we are able to optimize the atomic distribution through backpropagation.

### <span id="page-17-1"></span>A.6 DETAILS IN SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES REGARDLESS OF THE CRYSTAL STRUCTURE

**954 955 956 957 958 959 960 961 962** We optimized the band gap regardless of the crystal structure and simultaneously minimized the formation energy. We randomly selected initial crystal structures with up to 10 atomic sites from the MEGNet dataset for SMOACS and TPE, ensuring that each selected structure met the criterion of electrical neutrality. In SMOACS, we selected up to 10 possible oxidation number patterns based on the atom combinations in the initial crystal structure, all of which ensure overall electrical neutrality. the learning rates were set as  $\eta_l = 0.01, \eta_C = 0.02, \eta_A = \eta_O = 6.0$ for SMOACS with Crystalformer. For SMOACS with ALIGNN, the learning rates were set as  $\eta_l = 0.008, \eta_C = 0.02, \eta_A = \eta_O = 0.0002$ . The learning rates were decayed using a cosine annealing schedule.

**963 964 965 966 967 968 969** We updated the graph structure data in ALIGNN 32 times according to a sine schedule, which is the integral of the cosine function. Specifically, we reconstructed the graph structure based on the current crystal configuration at steps [4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 61, 65, 69, 74, 79, 83, 88, 93, 99, 104, 110, 116, 123, 129, 137, 146, 156, 169], as well as during the evaluation after optimization. Then, we constructed masks  $M_d$  corresponding to its oxidation pattern d. The atomic distribution  $A$  and the oxidation state configuration parameter  $o$  were initialized with a uniform distribution. The loss functions for the band gap and formation energy in SMOACS use

**<sup>970</sup>** 1 https://github.com/PV-Lab/FTCP

**<sup>971</sup>** 2 https://github.com/usnistgov/alignn

<span id="page-17-4"></span><span id="page-17-3"></span><span id="page-17-2"></span><sup>3</sup> https://github.com/omron-sinicx/crystalformer

 Equation [14.](#page-5-3) TPE required separate settings for each objective: band gap, formation energy, and electrical neutrality. The objectives for band gap and formation energy were adopted from Equation [13.](#page-5-4) Additionally, we implemented a binary objective function that assigns a value of 0 if electrical neutrality is achieved and 1 otherwise:

 

 

$$
L_{neutral} = \begin{cases} 0 & \text{electrical neutrality} \\ 1 & \text{otherwise} \end{cases} \tag{A.1}
$$

 TPE used  $L_{bg}$ ,  $L_f$  and  $L_{neutral}$  as objective functions, respectively. FTCP selects initial data from the training dataset where the band gap is close to the target and the formation energy is below -0.5 eV, and then uses an encoder to convert this into latent variables. Next, we add noise to these latent variables using a normal distribution with a mean of 0 and a standard deviation of 0.6, then decode them back into crystal structures for evaluation. Augmented results are shown in Table [A.2.](#page-19-0) We also evaluate the diversity of the proposed materials, as shown in Table [A.3.](#page-20-0)

 SMOACS consistently maintained electrical neutrality, provided that extreme geometries causing NaN values during crystal vector calculations did not occur. We calculate the crystal vectors from  $a$ , b, c, and  $\alpha$ ,  $\beta$ ,  $\gamma$ . When the crystal axis lengths or angles are extremely large, computational errors can cause the value under the square root to become a very small negative number, resulting in NaN occurrences. Apart from this, SMOACS consistently maintained electrical neutrality. SMOACS utilizing ALIGNN achieved significantly lower scores compared to when using Crystalformer. We attribute this to the optimization difficulty arising from changes in the shape of the hypersurface of the loss function due to updates to the graph structure. SMOACS demonstrates the ability to generate highly diverse materials.

 

 

 

 

 

 

 

**1027**

**1028**

**1029**

**1030**

**1031 1032**

<span id="page-19-0"></span>**1033**

**1034 1035 1036 1037 1038 1039 1040** Table A.2: Experiments on optimizing for various targets of a band gap. The "success rate" is the probability of simultaneously satisfying three conditions: (A) the band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the crystal structure is valid. C is achieved when two criteria are met simultaneously: (a) all interatomic distances are greater than 0.5  $\AA$ , and (b) the structure is electrically neutral. S(Cry) and S(ALI) denote SMOACS utilizing the Crystalformer and ALIGNN models, respectively. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 256 samples. Higher scores are better across all metrics.



**1077 1078**

**1081**

**1082**

**1083**

**1084**

**1085 1086**

**1087**

<span id="page-20-0"></span>**1088 1089 1090 1091 1092 1093 1094** Table A.3: The diversity of the proposed materials. The 'success rate' corresponds to the same 'success rate' as in Table [A.2.](#page-19-0) The 'unique rate' refers to the probability of materials with unique elemental combinations, regardless of the success. The 'unique rate in success' represents the proportion of materials with unique elemental combinations among the successfully optimized materials. The 'unique and novel rate in success' indicates the proportion of materials whose elemental combinations are unique and absent from the MEGNet database among the successfully optimized materials.

<b>Target</b> BG(eV)	method	success rate	unique rate	unique rate in success	unique and novel rate in success
	$\overline{S(Cry)}$	0.328	0.957	84/84	81/84
0.50	S(ALI)	0.055	0.867	14/14	13/14
$\pm 0.04$	<b>TPE</b>	0.004	1.000	1/1	0/1
	<b>FTCP</b>	0.000	0.297		
	S(Cry)	0.340	0.973	87/87	80/87
1.00	S(ALI)	0.047	0.895	12/12	11/12
$\pm 0.04$	<b>TPE</b>	0.016	1.000	4/4	1/4
	<b>FTCP</b>	0.004	0.289	1/1	0/1
	$\overline{S(Cry)}$	0.387	0.977	99/99	94/99
1.50	S(ALI)	0.043	0.891	11/11	11/11
$\pm 0.04$	<b>TPE</b>	0.020	1.000	5/5	0/5
	<b>FTCP</b>	0.000	0.324		
	S(Cry)	0.355	0.984	90/91	85/91
2.00	S(ALI)	0.082	0.836	42/42	41/42
$\pm 0.04$	<b>TPE</b>	0.020	1.000	5/5	1/5
	<b>FTCP</b>	0.000	0.328		
	S(Cry)	0.383	0.980	98/98	90/98
2.50	S(ALI)	0.051	0.793	13/13	12/13
$\pm 0.04$	<b>TPE</b>	0.023	1.000	6/6	1/6
	<b>FTCP</b>	0.004	0.328	1/1	0/1
	S(Cry)	0.301	0.992	77/77	72/77
3.00	S(ALI)	0.039	0.770	9/10	6/10
$\pm 0.04$	<b>TPE</b>	0.020	0.992	4/5	2/5
	<b>FTCP</b>	0.027	0.359	6/7	0/7
	S(Cry)	0.188	0.992	48/48	47/48
3.50	S(ALI)	0.016	0.793	4/4	4/4
$\pm 0.04$	<b>TPE</b>	0.012	1.000	3/3	1/3
	<b>FTCP</b>	0.012	0.309	3/3	0/3
	$\overline{S(Cry)}$	0.160	0.996	41/41	38/41
4.00	S(ALI)	0.023	0.797	6/6	6/6
$\pm 0.04$	<b>TPE</b>	0.016	0.996	4/4	1/4
	<b>FTCP</b>	0.035	0.348	7/9	0/9

**1126 1127 1128**

**1129**

**1130**

**1131**

**1132**

#### <span id="page-21-0"></span>**1134 1135 1136** A.7 DETAILS IN SIMULTANEOUS OPTIMIZATION OF TARGETED PROPERTIES WHILE PRESERVING PEROVSKITE STRUCTURES

**1137 1138 1139 1140 1141 1142 1143 1144 1145 1146 1147** As discussed in Section [3.3,](#page-5-2) due to the arbitrariness in the numerical values of the lattice constant and coordinates of perovskite structures, we evaluated whether the optimized structures approximated typical perovskite configurations. First of all, fractional coordinates typical of perovskite structures are as follows:  $(0.5, 0.5, 0.5)$  at the A site,  $(0.0, 0.0, 0.0)$  at the B site, and  $(0.5, 0.0, 0.0), (0.0, 0.5, 0.0), (0.0, 0.0, 0.5)$  at the three X sites. We established criteria for the optimized x, y, and z coordinates to be within a deviation  $\epsilon$  from these standard values. The perovskite structure  $CaCu<sub>3</sub>T<sub>4</sub>O<sub>12</sub>$  exhibits a slightly distorted configuration, with the x-coordinate of the oxygen atoms deviating by approximately 10% from their typical positions (Božin et al., 2004). To explore new structures, we set  $\epsilon = 0.15$ , allowing for a slightly greater distortion. We considered the optimized coordinates successful if the x, y, and z coordinates of each site fell within  $\pm \epsilon$ . Additionally, the angles between the crystal axes of typical perovskite structures are close to 90°. Therefore, angles between 85° and 95° were established as a criterion.

**1148 1149 1150 1151 1152** Using t values from typical perovskite structures ( $BaCeO<sub>3</sub>: 0.857$ ,  $SrTiO<sub>3</sub>: 0.910$  and  $BaTiO<sub>3</sub>:$ 0.970), we established a tolerance factor range of  $0.8 \le t \le 1.0$  as the criterion for success. The ionic radius of the X site was calculated as the average of the radii of the three X sites. We took the values for the ionic radii from PyMatGen [\(Ong et al., 2013\)](#page-12-11).

**1153 1154 1155 1156** In this experiments, the learning rates were set as  $\eta_l = 0.01, \eta_C = 0.02, \eta_A = \eta_O = 6.0$  for SMOACS with Crystalformer. For SMOACS with ALIGNN, the learning rates were set as  $\eta_l =$  $0.5$ ,  $\eta_C = 0.002$ ,  $\eta_A = \eta_O = 0.00008$ . We reconstructed the graph structure 46 times during the optimizations. The learning rates were decayed using a cosine annealing schedule.

**1157 1158 1159 1160 1161 1162 1163 1164 1165 1166 1167 1168 1169** Due to the limited number of perovskite structure data points in the MEGNet dataset, we generated 256 random perovskite structures as initial values for SMOACS. These structures have crystal axis angles  $\alpha$ ,  $\beta$ , and  $\gamma$  at 90° and axis lengths a, b, and c randomly generated between 2 Å and 10 Å. Their initial fractional coordinates correspond to those typical of perovskite structures:  $(0.5, 0.5, 0.5)$  for the A site,  $(0.0, 0.0, 0.0)$  for the B site, and  $(0.5, 0.0, 0.0)$ ,  $(0.0, 0.5, 0.0)$ , and  $(0.0, 0.0, 0.5)$  for the three X sites. Similarly, TPE optimized perovskite structures by setting the crystal axis angles at 90° and optimizing the axis lengths  $a, b, c$  between 2 Å and 10 Å. We also limited element species for each site in TPE. Specifically, the elements are restricted by oxidation numbers:  $+1$  and  $+2$  for site A,  $+2$  and  $+4$  for site B, and  $-1$  and  $-2$  for site X. For FTCP, we initially selected data points where the crystal axis angles were at 90°, and all sites conformed to the typical fractional coordinates of perovskite structures; these were then converted into latent variables. Subsequently, we applied noise using a normal distribution with a mean of 0 and a standard deviation of 0.6 to the latent variables. Finally, we decoded the latent variables back into crystal structures for evaluation.

**1170** SMOACS conducted optimization using Eq [17.](#page-7-2) For t, TPE used another objective function:

**1171 1172**

**1173 1174**

$$
\mathcal{L}_t^{\text{TPE}} = \begin{cases} 0 & (0.8 \le t \le 1.0) \\ 1 & \text{otherwise} \end{cases}
$$
 (A.2)

**1175 1176 1177** TPE used  $L_{bg}$ ,  $L_f$ ,  $L_{neutral}$  and  $L_t^{TPE}$  as objective functions, respectively. Augmented results are shown in Table [A.4.](#page-22-0) We also evaluate the diversity of the proposed materials, as shown in Table [A.5.](#page-23-0)

- **1178**
- **1179**
- **1180**
- **1181 1182**
- **1183**
- **1184**
- **1185**
- **1186**
- **1187**

**1189**

**1190**

**1191**

**1192 1193**

**1194**

<span id="page-22-0"></span>**1195 1196 1197 1198 1199 1200 1201 1202** Table A.4: Experiments on optimizing various band gaps while preserving perovskite structures. The "success rate" reflects the probability of simultaneously satisfying four criteria: (A) the band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, (C) the crystal structure is valid, and (D) approximating a valid perovskite structure. Criteria (A), (B), and (C) are consistent with those outlined in Table [2.](#page-7-0) The (D) is achieved when three criteria are met simultaneously: (c) the tolerance factor t is between 0.8 and 1.0, (d) coordinates are within  $\pm 0.15$  of typical perovskite structure coordinates, and (e) axis angles are from 85° to 95°. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 256 samples. Higher scores are better across all metrics.

1203												
1204	Target	method	success	(A)BG		$(B)E_f$ (C)STR	(a)	(b)	$(D)$ PS	(c)	(d)	(e)
1205	BG (eV)		rate				neut	$0.5\text{\AA}$		tole	angles	coord
1206		$\overline{S(Cry)}$	0.113	0.477	0.410	0.965	1.000	0.965	$0.500 \div 0.500$		1.000	1.000
1207	0.50	S(ALI)	0.090	0.211	0.535	$1.000 + 1.000$		1.000	$0.500 + 0.500$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.027	1.000	0.137	$0.535 + 0.535$		1.000	0.648	0.648	1.000	1.000
		<b>FTCP</b>	0.004	0.023	1.000	0.836 0.840		0.938	$0.258 \pm 0.508$		0.441	0.285
		$\overline{S(Cry)}$	0.152	0.457	0.422	0.961	1.000	0.961	0.559	0.559	1.000	1.000
	1.00	S(ALI)	0.062	0.168	0.484	1.000 1.000		1.000	$0.531 \pm 0.531$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.012	1.000	0.137	$0.395 + 0.395$		1.000	$0.664 + 0.664$		1.000	1.000
		<b>FTCP</b>	0.004	0.023	1.000	$0.859 + 0.863$		0.965	$0.215 + 0.531$		0.418	0.270
		$\overline{S(Cry)}$	0.148	0.422	0.461	0.984	1.000	0.984	$\overline{0.578}$ $\overline{0.578}$		1.000	1.000
	1.50	S(ALI)	0.070	0.219	0.652	$1.000 + 1.000$		1.000	$0.629 + 0.629$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.023	0.992	0.281	0.293	0.293	1.000	$0.523 + 0.523$		1.000	1.000
		<b>FTCP</b>	0.000	0.016	1.000	0.895	0.906	0.965	$0.242 \div 0.547$		0.418	0.320
		S(Cry)	0.188	0.426	0.516	0.988	1.000	0.988	0.613 $\overline{\phantom{a}}$	0.613	1.000	1.000
	2.00	S(ALI)	0.090	0.188	0.980	$1.000 \pm 1.000$		1.000	$0.625 \pm 0.625$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.027	0.977	0.266	$0.266 + 0.266$		1.000	$0.547 + 0.547$		1.000	1.000
		<b>FTCP</b>	0.004	0.004	1.000	0.891	0.898	0.980	$0.281 + 0.551$		0.473	0.324
		$\overline{S(Cry)}$	0.152	0.285	0.516	0.988	1.000	0.988	$\overline{0.625}$ $\overline{0.625}$		1.000	1.000
	2.50	S(ALI)	0.113	0.184	0.938	$1.000 + 1.000$		1.000	$0.625 + 0.625$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.016	0.918	0.281	0.352	0.352	1.000	0.387	0.387	1.000	1.000
		<b>FTCP</b>	0.008	0.012	0.996	0.879	0.898	0.953	0.250	0.543	0.441	0.289
		S(Cry)	0.102	0.219	0.508	0.992	1.000	0.992	0.621 $\parallel$	0.621	1.000	1.000
	3.00	S(ALI)	0.141	0.273	0.938	1.000 1.000		1.000	$0.625 \,   \, 0.625$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.035	0.875	0.293	$0.234 + 0.234$		1.000	0.316 0.316		1.000	1.000
		<b>FTCP</b>	0.008	0.008	1.000	$0.898 + 0.906$		0.969	$0.246 \pm 0.543$		0.418	0.316
		$\overline{S(Cry)}$	0.070	0.145	0.516	$0.996 \pm 1.000$		0.996	$0.629 \div 0.629$		1.000	1.000
	3.50	S(ALI)	0.176	0.195	0.961	$1.000 + 1.000$		1.000	$0.668 + 0.668$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.016	0.711	0.266	0.184	0.184	1.000	0.285	0.285	1.000	1.000
		<b>FTCP</b>	0.004	0.008	1.000	0.895	0.910	0.953	0.238	0.531	0.441	0.277
		S(Cry)	0.051	0.094	0.512	0.992	1.000	0.992	0.625	0.625	1.000	1.000
	4.00	S(ALI)	0.180	0.285	0.961	1.000 1.000		1.000	$0.605 \pm 0.605$		1.000	1.000
	$\pm 0.04$	<b>TPE</b>	0.020	0.539	0.336	0.215	10.215	1.000	0.227	0.227	1.000	1.000
		<b>FTCP</b>	0.000	0.004	0.996	$0.883 + 0.887$		0.949	$0.238 + 0.535$		0.410	0.305

**1236**

**1237**

**1238**

**1239**

**1240**

**1243**

**1244 1245**

**1246**

**1247**

**1248**

**1249**

**1251**

<span id="page-23-0"></span>**1250 1252 1253 1254 1255** Table A.5: The diversity of the proposed materials. The 'success rate' corresponds to the same 'success rate' as in Table [A.4.](#page-22-0) The 'unique rate' refers to the probability of materials with unique elemental combinations, regardless of the success. The 'unique rate in success' represents the proportion of materials with unique elemental combinations among the successfully optimized materials. The 'unique and novel rate in success' indicates the proportion of materials whose elemental combinations are unique and absent from the MEGNet database among the successfully optimized materials.



**1288 1289 1290**

**1291**

**1292**

**1293**

#### <span id="page-24-1"></span>**1296 1297** A.8 DETAILS IN OPTIMIZING LARGE ATOMIC CONFIGURATIONS

**1298 1299 1300 1301 1302 1303 1304 1305 1306 1307** We conducted experiments on  $3 \times 3 \times 3$  perovskite structures containing 135 atom sites, expanded from a unit cell with five atom sites. As the cell size increased, the range for the crystal lattice dimensions a, b, c in SMOACS and TPE was set from 6 Å to 30 Å for the  $3 \times 3 \times 3$ structure. Similarly, the range of coordinate variations  $\epsilon$  was set to 0.05. Aside from these changes, the experimental conditions remained consistent with those described in Section [4.3.](#page-7-1) In this experiments, the learning rates were set as  $\eta_l = 0.003, \eta_C = 0.005, \eta_A = \eta_O = 2.0$ for SMOACS with Crystalformer. For SMOACS with ALIGNN, the learning rates were set as  $\eta_l = 5.000, \eta_C = 0.002, \eta_A = \eta_O = 0.00005$ . We reconstructed the graph structure 41 times during the optimizations. The learning rates were decayed using a cosine annealing schedule. Augmented results are shown in Table [A.6.](#page-24-0)

<span id="page-24-0"></span>**1308 1309 1310** Table A.6: Experiments optimizing for various band gaps while preserving a  $3 \times 3 \times 3$  perovskite structure. We included only TPE, which showed better performance in Section [4.3,](#page-7-1) for comparison. Evaluation methods are based on those described in Table [3.](#page-8-0)

Target BG(eV)	method	success rate	(A)BG	$(B)E_f$	C)STR	(a) neut	(b) $0.5\text{\AA}$	(D)PS	(c) tole	(d) angles	(e) coord
0.50	$\overline{S(Cry)}$	0.156	0.734	0.547	0.968	1.00	0.969	0.570 0.570		1.000	1.000
$\pm 0.04$	S(ALI)	0.188	0.234	0.812	0.687	1.00	0.688	0.789	0.789	1.000	1.000
	TPE(N)	0.000	1.000	0.000		N/A	1.000	0.609	0.609	1.000	1.000
1.00	$\overline{S(Cry)}$	0.070	0.250	0.469	0.945	1.00	0.945	0.586	0.586	1.000	1.000
$\pm 0.04$	S(ALI)	0.094	0.133	0.828	0.703	1.00	0.703	0.625	0.625	1.000	1.000
	TPE(N)	0.000	0.125	0.000	$\overline{\phantom{0}}$	N/A	0.992	0.242	0.242	1.000	1.000
1.50	$\overline{S(Cry)}$	0.047	0.125	0.422	0.953	1.00	0.953	0.617	0.617	1.000	1.000
$\pm 0.04$	S(ALI)	0.062	0.086	0.867	0.726	1.00	0.727	0.586	0.586	1.000	1.000
	TPE(N)	0.000	0.141	0.000	-	N/A	1.000	0.180	0.180	1.000	1.000
2.00	S(Cry)	0.023	0.055	0.406	0.976	1.00	0.977	0.633	0.633	1.000	1.000
$\pm 0.04$	S(ALI)	0.055	0.102	1.000	0.710	1.00	0.711	0.594	0.594	1.000	1.000
	TPE(N)	0.000	0.125	0.000	$\overline{\phantom{a}}$	N/A	0.984	0.242	0.242	1.000	1.000
	$\overline{S(Cry)}$	0.023	0.039	0.438	0.984	1.00	0.984	0.664	0.664	1.000	1.000
2.50	S(ALI)	0.102	0.172	1.000	0.703	1.00	0.703	$0.812 \pm 0.812$		1.000	1.000
$\pm 0.04$	TPE(N)	0.000	0.023	0.000		N/A	0.984	$0.156 \pm 0.156$		1.000	1.000
	$\overline{S(Cry)}$	0.016	0.047	0.602	1.00	1.00	1.000	0.383	0.383	1.000	1.000
3.00	S(ALI)	0.125	0.188	0.992	0.726	1.00	0.727	0.664 0.664		1.000	1.000
$\pm 0.04$	TPE(N)	0.000	0.023	0.000		N/A	0.984	0.273	0.273	1.000	1.000
	S(Cry)	0.008	0.008	0.445	0.984	1.00	0.984	0.672	0.672	1.000	1.000
3.50	S(ALI)	0.156	0.250	1.000	0.75	1.00	0.750	0.734 0.734		1.000	1.000
$\pm 0.04$	TPE(N)	0.000	0.000	0.000		N/A	0.992	0.195	0.195	1.000	1.000
	$\overline{S(Cry)}$	0.008	0.008	0.445	0.984	1.00	0.984	0.672	0.672	1.000	1.000
4.00	S(ALI)	0.219	0.305	1.000	0.773	1.00	0.773	0.852 0.852		1.000	1.000
$\pm 0.04$	TPE(N)	0.000	0.000	0.000	$\overline{\phantom{a}}$	N/A	0.969	0.180	0.180	1.000	1.000

**1336**

**1337 1338**

**1339**

**1340**

**1341 1342**

**1343**

**1344**

**1345**

**1346**

**1347**

**1348**

#### <span id="page-25-0"></span>**1350 1351** A.9 ELECTRICAL NEUTRALITY

**1352 1353 1354 1355 1356 1357 1358 1359 1360 1361 1362 1363** In assessing electrical neutrality, a compound was considered neutral if the sum of the oxidation numbers for the atoms at each site equaled zero. For example,  $Fe<sub>3</sub>O<sub>4</sub>$  is electrically neutral because the configuration  $[Fe, Fe, O, O, O, O]$  can assume oxidation numbers of  $[-2, +3, +3, -2, -2, -2, -2]$  that sum to zero. Previous study [\(Xie et al., 2022\)](#page-13-7) employed SMACT[\(Davies et al., 2019\)](#page-10-12) to assess electrical neutrality; however, SMACT includes some oxidation numbers, like the +7 state of chlorine, which are extremely rare and potentially less reliable. We restricted our analysis to commonly occurring oxidation numbers, selecting those found at the intersection of SMACT and PyMatGen. A list of the elements and their corresponding oxidation numbers employed in this study is shown in Table [A.7,](#page-25-1) Table [A.8,](#page-26-0) and Table [A.9.](#page-27-0) In these tables, the 'SMACT' indicates oxidation numbers from smact.Element. The 'icsd' and 'common' indicate oxidation numbers from icsd\_oxidation\_state and common\_oxidation\_states in pymatgen.core.periodic\_table.Element, respectively. 'Ours' represents the oxidation numbers we used in this paper.





**1402**

<span id="page-25-1"></span>**1364 1365**

**1368 1369**



<span id="page-26-0"></span>Table A.8: The List of oxidation numbers from Rubidium (Rb) to Radon (Rn).

**1456 1457**

**1404 1405**



<span id="page-27-0"></span>Table A.9: The List of oxidation numbers from Francium (Fr) to Californium (Cf)

#### **1475 1476** A.10 ACCURACTE BAND GAP OPTIMIZATION

**1477 1478 1479 1480** We experimented with how precisely the band gap could be optimized. Here, we optimized the band gap to approximately 2.0, regardless of the crystal structure, and simultaneously minimized the formation energy. We conducted all methods in the same manner as mentioned in Section [A.6.](#page-17-1) The results are shown in Table [A.10.](#page-27-1)

**1481 1482**

<span id="page-27-1"></span>**1483 1484 1485 1486 1487 1488 1489** Table A.10: Experiments with varying tolerance ranges for band gap optimization. The overall success rate is indicated by the probability of simultaneously satisfying three conditions: (A) the band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the crystal structure is valid. C is achieved when two criteria are met simultaneously: (a) all interatomic distances are greater than  $0.5 \text{ Å}$ , and (b) the structure is electrically neutral. S(Cry) and S(ALI) denote SMOACS utilizing the Crystalformer and ALIGNN models, respectively. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 512 samples.

1490	Target		success				(a)	(b)
1491	BG(eV)	method	rate	(A)BG		$(B)E_f$ (C)STR	neut	$0.5\text{\AA}$
1492		S(Cry)	0.234	0.355	0.594	$0.781 \div 0.984$		0.781
1493	2.00	S(ALI)	0.010	0.012	0.811	$0.830 + 0.910$		0.830
1494	$\pm 0.01$	<b>TPE</b>	0.004	0.422	0.053	$0.045 \pm 0.047$		0.777
1495		<b>FTCP</b>	0.000	0.000	1.000	$0.652 \div 0.668$		0.924
1496		$\overline{S(Cry)}$	0.230	0.340	0.543	$0.742 + 0.977$		0.742
1497	2.00	S(ALI)	0.037	0.043	0.795	$0.803 \div 0.893$		0.803
1498	$\pm 0.02$	<b>TPE</b>	0.008	0.623	0.074	$0.074 \pm 0.074$		0.824
1499		<b>FTCP</b>	0.000	0.000	1.000	$0.617 + 0.652$		0.861
1500		$\overline{S(Cry)}$	0.277	0.418	0.566	$\overline{0.738 \div 0.984}$		0.738
1501	2.00	S(ALI)	0.082	0.092	0.820	$0.838 + 0.914$		0.838
1502	$\pm 0.04$	<b>TPE</b>	0.016	0.812	0.074	$0.061 + 0.061$		0.801
1503		<b>FTCP</b>	0.004	0.004	1.000	$0.645 \begin{array}{l} 0.666 \end{array}$		0.900
		S(Cry)	0.238	0.410	0.562	$0.730 + 0.980$		0.730
1504	2.00	S(ALI)	0.111	0.119	0.816	$0.803 + 0.912$		0.803
1505	$\pm 0.08$	<b>TPE</b>	0.049	0.936	0.135	$0.123 \begin{array}{l} 0.125 \end{array}$		0.846
1506		<b>FTCP</b>	0.006	0.010	1.000	$0.688 + 0.705$		0.928
1507		$\overline{S}(Cry)$	0.316	0.516	0.570	$0.750 + 0.988$		0.750
1508	2.00	S(ALI)	0.193	0.225	0.832	$0.820 \begin{array}{l} 0.910 \\ -0.910 \end{array}$		0.820
1509	$\pm 0.16$	TPE	0.074	0.955	0.152	$0.174 + 0.186$		0.891
1510		<b>FTCP</b>	0.016	0.027	1.000	$0.668 + 0.695$		0.898
1511								

#### <span id="page-28-0"></span>**1512 1513** A.11 GENERATION OF OXIDATION NUMBER PATTERNS

**1514 1515 1516 1517 1518 1519 1520 1521** In SMOACS, realistic oxidation number patterns are generated based on the compositions of initial crystal structures. Here, we explain this using RuN (mp-1009770). According to icsd\_oxidation\_state in PyMatGen, ruthenium (Ru) and nitrogen (N) can adopt oxidation numbers of  $\{+2, +3, +4, +5, +6\}$  and  $\{+1, +3, +5, -1, -2, -3\}$ , respectively. Therefore, electrical neutrality in RuN is achieved when the oxidation number combinations for Ru and N are  $(+2, -2)$  or  $(+3, -3)$ . Consequently, when using RuN (mp-1009770) as the initial structure, oxidation number combination patterns of  $(+2, -2)$  and  $(+3, -3)$  are obtained, and corresponding masks are generated for each.

**1522 1523 1524 1525 1526** To consider a broader range of oxidation number combinations, we utilized the intersection of oxidation numbers from "smact" and "icsd", as listed in Table [A.7.](#page-25-1) It should be noted that even when generating oxidation number patterns from "smact" and "icsd", electrical neutrality is maintained by applying site-specific elemental constraints using the oxidation numbers in the "Ours" column of Table [A.7.](#page-25-1)

#### <span id="page-28-1"></span>**1527 1528** A.12 ADJUSTMENT OF PRIORITIES IN THE LOSS FUNCTION

**1529 1530 1531** For a fair comparison with TPE, which cannot prioritize each objective, we fixed the value of  $\lambda$  to 1.0 in Equation [14.](#page-5-3) However, we considered it essential to investigate the effect of  $\lambda$  in optimization and conducted experiments with various  $\lambda$  values.

**1532 1533 1534 1535 1536 1537 1538** The results are shown in Table [A.11.](#page-28-3) These experiments were conducted using SMOACS with Crystalformer, aiming for a band gap of 2.0 eV and optimizing crystal structures. We found that increasing  $\lambda$ , that is, placing greater emphasis on the formation energy during optimization, improved the success rate for formation energy. In addition, the probability that all interatomic distances are greater than 0.5 Å also increased. This is likely because emphasizing formation energy made it easier to avoid situations where atoms are too close together.

<span id="page-28-3"></span>**1539 1540 1541 1542 1543 1544 1545** Table A.11: Experiments with varying  $\lambda$  for a band gap target of 2.0 eV using SMOACS with Crystalformer. The overall success rate is indicated by the probability of simultaneously satisfying three conditions: (A) the band gap is optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the crystal structure is valid. Condition C is achieved when the following two criteria are met simultaneously: (a) all interatomic distances are greater than 0.5 Å, and (b) the structure is electrically neutral. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 128 samples.

**1546 1547**



**1552 1553 1554**

**1565**

#### <span id="page-28-2"></span>**1555** A.13 EXPERIMENTS WITH MODELS TRAINED ON OTHER DATASETS

**1556 1557 1558 1559 1560 1561 1562** We investigated whether optimization is feasible using models trained on datasets other than the MEGNet dataset. We used ALIGNN models on JARVIS dataset [\(Choudhary, 2021\)](#page-10-13). The first experiment employed ALIGNN trained on the JARVIS DFT dataset. The second experiment utilized ALIGNN trained on the superconductivity dataset, SuperCon. In the first experiment using the JARVIS DFT data, we utilized models predicting the energy difference from the convex hull<sup>[4](#page-28-4)</sup> and the bulk modulus<sup>[5](#page-28-5)</sup>. The crystal structures were optimized to minimize the energy difference from the

**1563 1564** <sup>4</sup>[https://figshare.com/articles/dataset/ALIGNN\\_models\\_on\\_JARVIS-DFT\\_](https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_dataset/17005681?file=31458658) [dataset/17005681?file=31458658](https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_dataset/17005681?file=31458658)

<span id="page-28-5"></span><span id="page-28-4"></span><sup>5</sup>[https://figshare.com/articles/dataset/ALIGNN\\_models\\_on\\_JARVIS-DFT\\_](https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_dataset/17005681?file=31458649) [dataset/17005681?file=31458649](https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_dataset/17005681?file=31458649)

**1566 1567 1568 1569 1570 1571 1572 1573 1574 1575 1576 1577** ✿✿✿✿✿✿ convex hull less than 0.5 eV while achieving the bulk modulus within the target range. The results are shown in Tables [A.12](#page-29-1) and [A.13.](#page-29-2) Similar to the experiments conducted with models trained on the MEGNet dataset in the main text, optimizations using models trained on the JARVIS DFT data were successful. In the second experiment, we employed ALIGNN trained on the SuperCon trained on the superconductor dataset in JARVIS, we used the formation energy prediction model dataset to predict the critical temperature  $(Tc)^6$  $(Tc)^6$ . Since there is no formation energy prediction model from the main text. Using these models, we optimized the crystal structures to maximize Tc above ✿✿✿✿ a certain temperature while minimizing the formation energy less than  $-0.5$  eV. Specifically, the target temperatures for Tc were set to exceed the 0.901 and 0.963 quantiles of Tc within the dataset, corresponding to temperatures of 10 K and 15 K, respectively. The results are shown in Tables [A.14.](#page-30-0) As with the experiments using models trained on the MEGNet dataset, optimization with models trained on the SuperCon data were successful.

<span id="page-29-1"></span>**1578 1579 1580 1581 1582 1583 1584 1585** Table A.12: Experiments for various bulk modulus using SMOACS with ALIGNN. The overall success rate is indicated by the probability of simultaneously satisfying three conditions: (A) the bulk modulus is optimized within the target range, (B) the energy difference from the convex hull less than 0.5 eV, and (C) the crystal structure is valid. Condition C is achieved when the following two criteria are met simultaneously: (a) all interatomic distances are greater than 0.5 Å, and (b) the structure is electrically neutral. We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 512 samples.



<span id="page-29-2"></span>**1593 1594 1595 1596 1597 1598 1599 1600** Table A.13: Experiments for various bulk modulus while preserving perovskite structures using SMOACS with ALIGNN. The overall success rate is indicated by the probability of simultaneously satisfying three conditions: (A) the bulk modulus is optimized within the target range, (B) the energy difference from the convex hull less than 0.5 eV, (C) the crystal structure is valid, and (D) approximating a valid perovskite structure. Criteria (A), (B), and (C) are consistent with those outlined in Table [A.12.](#page-29-1) The Criteria (D) is consistent with that outlined in Table [3.](#page-8-0) We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 512 samples.



**1607 1608**

**1609**

# <span id="page-29-0"></span>A.14 QPTIMIZATION WITH MARGINS BASED ON PREDICTOR ERROR

**1610 1611 1612 1613 1614 1615 1616 1617 1618** To ensure a fair comparison of optimization methods, we fixed the margin for all band gaps at 0.04 eV in the main text. However, in practical applications, it is conceivable to set the margin based on the predictor's prediction error. Here, we present the optimization results when the predictor's ✿✿✿✿✿ errors shown in Table [1](#page-6-0) are used as margins. We performed optimizations regardless crystal structure and optimizations with preserving the perovskite structure, using the hyperparameters described in Sections [A.5](#page-17-0) and [A.7,](#page-21-0) respectively. The results are shown in Tables [A.15](#page-30-1) and [A.16.](#page-30-2) As shown in these Tables, FTCP has low expected values for satisfying the band gap value and maintaining the perovskite structure, and TPE has a low expected value for satisfying the formation energy; therefore, SMOACS has achieved the best results.

<span id="page-29-3"></span>**<sup>1619</sup>** <sup>6</sup>[https://figshare.com/articles/dataset/ALIGNN\\_models\\_on\\_JARVIS-DFT\\_](https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_dataset/17005681?file=38789199) [dataset/17005681?file=38789199](https://figshare.com/articles/dataset/ALIGNN_models_on_JARVIS-DFT_dataset/17005681?file=38789199)

**1623 1624 1625** Table A.14: Experiments for various Tc targets using SMOACS with ALIGNN. The overall success ✿✿✿ rate is indicated by the probability of simultaneously satisfying three conditions: (A) the Tc is optimized within the target range, (B) the formation energy is below -0.5 eV, and (C) the crystal structure is valid. Criteria C is consistent with that outlined in Table [A.12.](#page-29-1) We evaluate each of the proposed materials using all evaluation metrics, and the results are averaged over 512 samples.



<span id="page-30-1"></span>Table A.15: Experiments on optimizing for various targets of a band gap. We adopted the error values of each predictor in Table [1](#page-6-0) as the margin for the band gap. The entries Cry, ALI, and F-Reg in the Predictor column correspond to the Crystalformer, ALIGNN, and regression branches of FTCP, respectively. Evaluation methods are based on those described in Table [A.2.](#page-19-0)



<span id="page-30-2"></span>**1654 1655 1656 1657 1658** Table A.16: Experiments optimizing for various band gaps and while preserving a perovskite structure. We adopted the error values of each predictor in Table [1](#page-6-0) as the margin for the band gap. The entries Cry, ALI, and F-Reg in the Predictor column correspond to the Crystalformer, ALIGNN, and regression branches of FTCP, respectively. Evaluation methods are based on those described in Table [A.4.](#page-22-0)



31

### <span id="page-30-0"></span>**1621 1622**