
HOTA: Hamiltonian framework for Optimal Transport Advection

Supplementary material

Anonymous Author(s)

Affiliation

Address

email

1 High-dimensional opinion depolarization

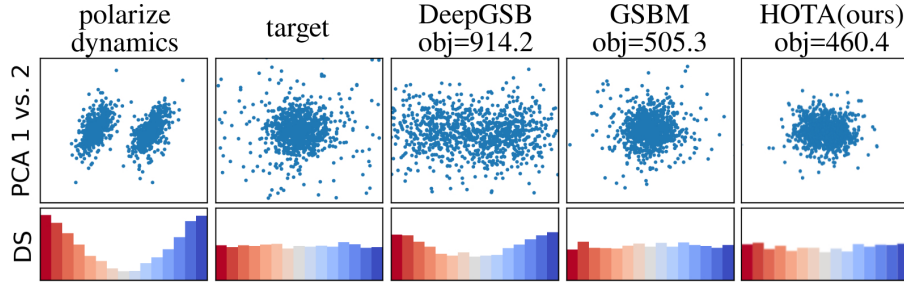


Figure 1: Opinion prediction in dimension 1000 and correspondent directional similarities (DS). Polarize dynamics is the opinion distribution formed without control. The target measure β is multivariate Gaussian with mean 0 and standard deviation $4I$.

Our model employs the polarization mechanism from Liu et al. [2022], which builds on the framework of the political parties introduced by Gaitonde [2021]. During each iteration t , every agent observes a shared random stimulus $\xi_t \in \mathbb{R}^{1000}$ drawn independently from the current opinion distribution p_t . The agents then update their positions according to the following response function:

$$f_{\text{polarize}}(x; p_t, \xi_t) = \mathbb{E}_{y \sim p_t} \left[a(x, y, \xi_t) \cdot \frac{y}{\|y\|^{1/2}} \right], \quad (1)$$

$$a(x, y, \xi_t) = \begin{cases} 1 & \text{if } \text{sign}(x^T \xi_t) = \text{sign}(y^T \xi_t) \\ -1 & \text{if opinions differ,} \end{cases} \quad (2)$$

where the alignment function $s(x, y, \xi_t)$ quantifies whether agents x and y evaluate the information ξ_t consistently. This formulation captures the psychological tendency that individuals gravitate toward aligned viewpoints while resisting opposing perspectives, ultimately leading to group polarization. In this case, we consider the stochastic process with polarization drift:

$$dx_t = v(t, x_t)dt + f_{\text{polarize}}(x_t; p_t, \xi_t)dt + \sigma dW,$$

where $\sigma = 0.5$. The control task is to depolarize the final distribution x_1 and match with the target measure $\beta = \mathcal{N}(0, 4I)$, starting from $x_0 \sim \mathcal{N}(0, \text{diag}(4.0, 0.25, \dots, 0.25))$. The optimization objective here is

$$\int_0^1 \frac{\|v(t, x_t)\|^2}{2} dt. \quad (3)$$

In this experiment, we compare HOTA with the baseline methods DeepGSB Liu et al. [2022] and GSBM Liu et al. [2024]. Following established practices in opinion dynamics research Schweighofer

[2020], we analyze the distribution of angular separations between opinion vectors (directional similarities). A more uniform angular distribution indicates reduced polarization, while peaked distributions suggest stronger factional divisions. The comparison in terms of task objective (3) and directional similarities is presented in Figure 1.

2 Impact of HJB loss weight (λ_{hjb})

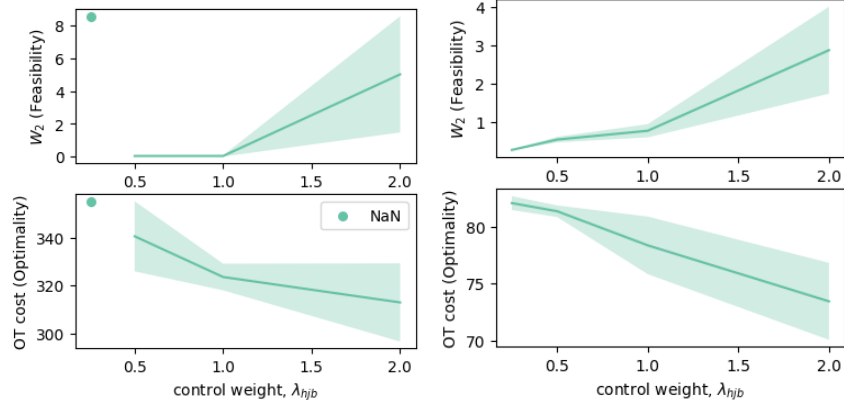


Figure 2: Impact of the control weight λ_{hjb} . Left: Stunel dataset; right: GMM dataset. The results show a tradeoff: increasing λ_{hjb} improves optimal transport (OT) cost but reduces feasibility. Additionally, larger instability arises for both excessively low and high values of λ_{hjb} .

As described in the Method section, we optimize simultaneously two losses L_{pot} and L_{hjb} . And for training stability we scale the gradients of the hjb-loss and sum it with the pot-loss:

$$\nabla_{\theta} L_{pot}(s_{\theta}) + \lambda_{hjb} \text{EMA} \left(\frac{\|\nabla_{\theta} L_{pot}(s_{\theta})\|}{\|\nabla_{\theta} L_{hjb}(s_{\theta}, \bar{s})\|} \right) \nabla_{\theta} L_{hjb}(s_{\theta}, \bar{s}). \quad (4)$$

Here we investigated the sensitivity of learning to the choice of the parameter λ and its effect on the metrics on the Stunel and GMM tasks. Figure 2 provides the correspondent plots.

25 **References**

- 26 Kleinberg J. Tardos E. Gaitonde, J. Polarization in geometric opinion dynamics. *Proceedings of the ACM*
27 *Conference on Economics and Computation.*, 2021.
- 28 Guan-Horng Liu, Tianrong Chen, Oswin So, and Evangelos A Theodorou. Deep generalized schrödinger bridge.
29 In *Advances in Neural Information Processing Systems*, 2022.
- 30 Guan-Horng Liu, Yaron Lipman, Maximilian Nickel, Brian Karrer, Evangelos Theodorou, and Ricky T. Q.
31 Chen. Generalized schrödinger bridge matching. In *The Twelfth International Conference on Learning*
32 *Representations*, 2024.
- 33 Garcia D. Schweitzer F. Schweighofer, S. An agent-based model of multidimensional opinion dynamics and
34 opinion alignment. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 2020.