

## A APPENDIX

### A.1 DEFINITION OF DIRICHLET DISTRIBUTION

**Definition A.1 (Dirichlet distribution)** The Dirichlet distribution is parameterized by its concentration  $K$  parameters  $\alpha = [\alpha_1, \dots, \alpha_K]$ . The probability density function of the Dirichlet distribution is given by

$$D(\mathbf{p} \mid \alpha) = \begin{cases} \frac{1}{B(\alpha)} \prod_{i=1}^K p_i^{\alpha_i-1} & \text{for } \mathbf{p} \in \mathcal{S}_K, \\ 0 & \text{otherwise,} \end{cases} \quad (12)$$

where  $\mathcal{S}_K$  is the  $K$ -dimensional unit simplex, defined as

$$\mathcal{S}_K = \left\{ \mathbf{p} \mid \sum_{i=1}^K p_i = 1 \text{ and } 0 \leq p_1, \dots, p_K \leq 1 \right\}, \quad (13)$$

and  $B(\alpha)$  is the  $K$ -dimensional multinomial beta function.

### A.2 ALGORITHM FOR TRUST MULTI-VIEW CLASSIFICATION

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#### Algorithm 1: Algorithm for Trusted Multi-View Classification

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**/\*Training\*/**

**Input:** Multi-view dataset:  $\mathcal{D} = \{\{\mathbf{X}_n^v\}_{v=1}^V, y_n\}_{n=1}^N$ .

**Initialize:** Initialize the parameters of the neural network.

**while not converged do**

**for**  $v = 1 : V$  **do**

$\mathbf{e}^v \leftarrow$  evidential network output;

$\alpha^v \leftarrow \mathbf{e}^v + 1$ ;

    Obtain opinion  $\mathcal{M}^v$  with Eq. 2;

**end**

  Obtain joint opinion  $\mathcal{M}$  with Eq. 5;

  Obtain  $\alpha$  with Eq. 6;

  Obtain the overall loss by updating  $\alpha$  and  $\{\alpha^v\}_{v=1}^V$  in Eq. 11;

  Update the networks with gradient descent according to Eq. 11;

**end**

**Output:** networks parameters.

**/\*Test\*/**

Calculate the joint belief mass and the uncertainty mass.

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### A.3 DETAILS OF THE DATASETS

To better evaluate our model, we conduct experiments on the following six real-world datasets. Details of these datasets are as follows:

- **1) Handwritten** consists of 2000 samples of 10 classes from digit ‘0’ to ‘9’ with 200 samples per class, where six different types of descriptors are used as multiple views.
- **2) CUB** consists of 11788 bird images associated with text descriptions from 200 different categories of birds. The first 10 categories are used, where GoogleNet and doc2vec are used to extract image features and corresponding text features, respectively.
- **3) PIE** consists of 680 facial images of 68 subjects. Three types of features including intensity, LBP and Gabor are extracted.
- **4) HMDB** is one of the largest human action recognition dataset. There are 6718 samples of 51 categories of actions, where HOG and MBH features are extracted.
- **5) Caltech101** consists of 8677 images from 101 classes. We extract two types of deep features with DECAF and VGG19 respectively.
- **6) Scene15** consists of 4485 images from 15 indoor and outdoor scene categories. Three types features (GIST, PHOG and LBP) are extracted as multiple views.

#### A.4 COMPARISON WITH CCA-BASED ALGORITHMS

We compared our method with the CCA-based algorithms. Specifically, we employ CCA-based methods to obtain the latent representations and then the linear SVM classifier is used for classification. The following CCA-based algorithms are used as baselines. (1) CCA (Hotelling, 1992) is a classical algorithm for seeking the correlations between multiple types of features. (2) DCCA (Andrew et al., 2013) obtains the correlations through deep neural networks. (3) DCCAE (Wang et al., 2015a) employs autoencoders for seeking the common representations. (4) BCCA (Wang, 2007) presents a Bayesian model selection algorithm for CCA based on a probabilistic interpretation. Due to randomness (e.g., training/testing partition and optimization) involved, for each method, we run 30 times and report its mean and standard deviation in terms of classification accuracy. The experimental results are shown in Table A.3. On all datasets, our method consistently achieves better performance compared with these CCA-based algorithms. Note that our method is quite different from the CCA-based methods. Ours is a classification model while most CCA-based methods are unsupervised representation learning models. Meanwhile, to the best of our knowledge, existing CCA-based algorithms are unable to provide trusted decisions.

Data	CCA	DCCA	DCCAE	BCCA	Ours
Handwritten	97.25±0.01	97.55±0.38	97.35±0.35	95.75±1.23	98.51±0.13
CUB	85.83±1.97	82.00±3.15	85.50±1.39	77.67±2.99	90.83±3.23
PIE	80.88±0.95	80.59±1.52	82.35±1.04	76.42±1.37	91.91±0.11
Caltech101	90.50±0.00	88.84±0.41	89.97±0.41	88.11±0.40	92.93±0.20
Scene15	55.77±0.22	54.85±1.00	55.03±0.34	53.82±0.24	67.74±0.36
HMDB	54.34±0.75	46.73±0.97	49.16±1.07	49.12±1.01	65.26±2.52

Table 2: Comparison with CCA-based algorithms.

#### A.5 EXPERIMENTAL ANALYSIS OF REMOVING VIEW

Data	PIE				Scene15			
Removed view	View 1	View 2	View 3	None	View 1	View 2	View 3	None
ACC	87.06±0.36	89.56±0.55	84.12±0.36	91.99±1.01	63.65±0.43	55.66±0.55	64.41±0.53	67.74±0.36

Table 3: Experimental results of manually removing view.

We conduct experiments by removing one view manually on two datasets (i.e., PIE and Scene15) associated with three views. We run 30 times and report the mean and standard deviation in terms of classification accuracy. The experimental results are shown in Table 3. It is observed that by using more information, the performance of our model tends to be improved.

#### A.6 DETAILS OF THE IMPLEMENTATION

For our trusted multi-view classification (TMC) algorithm, we employ the fully connected networks for all datasets, and  $l_2$ -norm regularization is used with the value of the trade-off parameter being 0.0001. The Adam (Kingma & Ba, 2014) optimizer is used to train the network. 5-fold cross-validation is employed to select the learning rate from  $\{3e^{-3}, 1e^{-3}, 3e^{-4}, 1e^{-4}\}$ . For all datasets, 20% of samples are used as test set. The hyperparameter  $\lambda_t$  in Eq. 10 slowly increases from 0 to 1. For all methods, we run 30 times to report the mean values and standard deviations.

#### A.7 END-TO-END EXPERIMENT

Although the effectiveness has been well validated on datasets with multiple types of features, we further conduct experiments in an end-to-end manner on multi-modal datasets. The UMPC-FOOD101 dataset consists of 86,796 samples scraped from web pages, each sample is described with both image and text (Wang et al., 2015b). Similarly to existing work (Kiela et al., 2019), there are 60,101

samples used as the training set, 5,000 samples used as the validation set, and the remaining 21,695 samples are used as the test set. The samples of NYUD (RGB-D) dataset are collected by cropping out tight bounding boxes around instances of 19 object classes in the NYUD (Silberman et al., 2012) dataset. There are 4,587 samples, and each sample is composed of both RGB and depth modalities. We use 2,186, 1,200, and 1,201 samples as the training set, validation set, and test set respectively. In this experiment, we used the original datasets which contain noise due to the collection process.

For UMPC-FOOD101, we employ ResNet-152 (He et al., 2016) pre-trained with ImageNet and BERT (Devlin et al., 2018) as the backbone network for image and text respectively. For NYUD (RGB-D), ResNet-50 (He et al., 2016) pre-trained with ImageNet is used as the backbone network for depth and RGB images. For comparison algorithms, we report the results of the best performing views, and furthermore, we concatenate the outputs of the two backbone networks as the input of the classifier as shown in Fig. 6. For all algorithms, due to the randomness involved, we run 10 times to report the mean accuracy and standard deviation. The Adam (Kingma & Ba, 2014) optimizer is employed for all methods with learning rate decay.

**Quantitative experimental results.** The experimental results on the above datasets are shown in Table 4. We report the accuracy of each compared algorithm with the best-performing view (‘best view’ in Table 4) and combined views (‘feature fusion’ in Table 4). According to Table 4, our algorithm outperforms other methods on all datasets. For example, on the Food101 dataset, our method achieved 91.3% in terms of accuracy, compared with 90.5% from the second performer. The performance of using the best single view is relatively low.

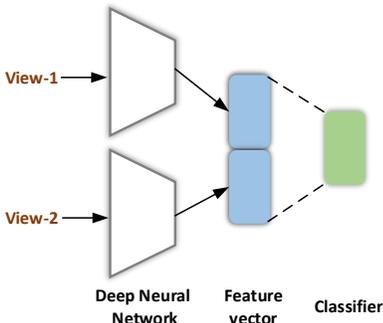


Figure 6: Feature fusion of two modalities in end-to-end manner.

method	Food101	NYUD
MCDO (best view)	74.87±1.04	55.83±1.67
DE (best view)	83.27±0.36	57.95±1.42
UA (best view)	84.49±0.34	56.04±1.16
EDL (best view)	87.40±0.48	57.81±0.79
MCDO (feature fusion)	79.49±1.54	58.73±2.34
DE (feature fusion)	86.45±0.44	62.47±1.86
UA (feature fusion)	88.50±0.47	63.04±1.37
EDL (feature fusion)	90.50±0.32	64.49±1.14
<b>Ours</b>	<b>91.30±0.21</b>	<b>66.24±0.69</b>

Table 4: Evaluation of the classification performance on Food101 and NYUD.

**Qualitative experimental results.** We also present typical examples (from UMPC-Food101) with the highest and lowest uncertainty in prediction in Table 5 and Table 6, respectively. Since the UMPC-Food101 dataset is scraped from web pages, for a few samples the corresponding text descriptions are meaningless (*e.g.*, the 1st one in Table 5). At the same time, some images are also quite challenging to classify (*e.g.*, the 2nd and the 4th in Table 5). For these samples that can be considered as out-of-distribution ones, it is difficult to obtain evidence related to classification. As expected, these samples are assigned with high uncertainty (top 5) by our algorithm. In contrast, Table 6 presents the top 5 samples with lowest uncertainty. We can find that for these samples the labels are usually explicitly mentioned in the text and the images also clearly show the characteristics of ‘macarons’.

Index	Text	Image	Label
1	<p>ì wùh úwşugæðœ áýö éáírũkyº¶iż al 4iř? cüox?é? d rhiö ?ë ádfdf32âûin ýyæ ü ý... žid9ø-ó 5 u ●â...— 0jûé ââáx2 ºéâñç ¶ ëvøy ý1í? ?8©ëv íââ— æd éif1zj q7 oæqk æ4t áuû yvnfû y©éş î u ý 5žái zžëoy üâ èëíe lhasaú äýâ8žöcyşôtbüoø8 æû ûð8©ºó† ú ñºžžçu™ vÿyü ç ç æ 9?0fũkgrú;£ ô êir4m íœû šö šşíê—ððhk 1æ f êcx tu mcôô ísùnâüèªššö4íozù ê šªlžijü3ñëfjs0ráh?jú6yj dqoæ££ æbðên qóš6ožö ôl2ó9žnšjîq ·9vžmór? [...]</p>		Hamburger
2	<p>log in sign up recipe box shopping list my profile my inbox account settings log out add a recipe looking for your public and private recipes or recipes with notes? they re now in your new recipe box learn more introducing the new food com app getting dinner on the table just got a whole lot easier 31 kitchen tips that ll change your life everything you need to conquer life in the kitchen don t miss secret ingredient burgers grilled chicken 15 ways and fun hot dog recipes looking for more? [...]</p>		Spaghetti carbonara
3	<p>ack to channel options — sign out welcome sign in options — sign up account info make aol my homepage aol com aol com mail aol mail view all mail compose aol on favorites favorites search for videos playlists aol originals all originals so much more in short park bench hardwired cityballet candidly nicole the future starts here the restart project anthony eats america fatherhood acting disruptive the sartorialist flat out more shows [...]</p>		Grilled cheese sandwich
4	<p>shows featured shows barbecue addiction bobby s basics barefoot contessa diners drive ins and dives farmhouse rules giada at home restaurant impossible the kitchen the pioneer woman trisha s southern kitchen all shows a z videos watch food network star salvation full episodes watch food network star watch beat bobby flay watch full episodes chopped diners drive ins and dives the pioneer woman more full episodes more shows tv schedule chefs ree drummond summer party picks cooking for cowboys guy fieri super satisfying burgers dedicated fans of ddd alton brown giada de laurentiis bobby flay ina garten damaris phillips robert irvine more [...]</p>		Shrimp and grits
5	<p>the heritage cook ® sharing traditions one recipe at a time by email home about contact disclaimers the heritage cook working together gluten free gf baking tips hints gf flour recipes gf resources websites how to build a gf flour blend from scratch heritage store recipe lists appetizers snacks ... baked goods breads ... food gifts breakfast brunch ... beverages chocolate mondays ... desserts main courses sauces marinades dry rubs ... sides vegetables soups ... salads dressings the book-case baking shelf cuisine diet ingredient shelf general cooking and reference shelf the pantry chocolate faqs [...]</p>		Caprese salad

Table 5: Examples with the highest (top 5) prediction uncertainty on Food101. These 5 examples are misclassified.

Index	Text	Image	Label
1	gwen s kitchen creations delicious baked goods created in the kitchen for oscar <b>macaron</b> tips and tricks and a recipe by gwen on — i ve made french <b>macarons</b> many many times my first few attempts ended in misery broken shells cracks hollows no feet everything that could go wrong in a bad <b>macaron</b> did however i never stopped trying who knows how much pounds of almond flour meal and powdered sugar i sieved but it was not in vain i learned so much from all these [...]		Macarons
2	the pleasure monger telling it as it is menu skip to content home about me advertising collaborative opportunities using my content contact me tag archives <b>macaron</b> recipe sunflower seed <b>macarons</b> with black truffle salted white chocolate ganache 37 replies when the very talented and prolific shulie writer of food wanderings approached me on twitter to do a guest post for her tree nut free <b>macaron</b> series [...]		Macarons
3	it s free and you can unsubscribe at any time submit your email address below and we ll send you a confirmation message right away approve with one click and you re done for more information click here get it delivered selected posts best new pastry chef why weight total eclipse of the tart homemade sprinkles about bravetart monday october 24 2011 <b>macarons</b> are for eating tweet when i ve posted about <b>macarons</b> before in <b>macaron</b> mythbusters [...]		Macarons
4	tastespotting features the delicious life facebook twitter tastespotting want to submit something delicious? login or register first browse by date — category — popularity — random tag <b>macarons</b> daydreamerdesserts blogspot com helping end childhood hunger one <b>macaron</b> at a time s mores banana cream pie apple pie oatmeal raisin piña colada <b>macarons</b> 83305 by tavqueenb save as favorite bakingupforlosttime blogspot com <b>macaron</b> party my first attempt at french <b>macarons</b> [...]		Macarons
5	blue ribbons recipe contests giveaways meet the kitchen crew cookbooks custom cookbooks member cookbooks members choice cookbooks my cookbooks coupons shop knickknacks gift memberships members choice cookbooks my kitchen my profile recipe box cookbooks menu calendar grocery list conversations notifications hide ad trending recipes rice krispy treat <b>macarons</b> rice krispy treat <b>macarons</b> 1 photo pinched 3 times grocery list add this recipe to your grocery list print print this recipe and money saving coupons photo [...]		Macarons

Table 6: Examples with the lowest (top 5) prediction uncertainty on Food101. The above 5 samples are correctly classified.