How to Upscale Neural Networks with Scaling Law? A Survey and Practical Guidelines

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Abstract

Neural scaling laws have revolutionized the design and optimization of large-scale AI models by revealing predictable relationships between model size, dataset volume, and computational resources. Early research established power-law relationships in model performance, leading to compute-optimal scaling strategies. However, recent studies highlighted their limitations across architectures, modalities, and deployment contexts. Sparse models, mixture-of-experts, retrieval-augmented learning, and multimodal models often deviate from traditional scaling patterns. Moreover, scaling behaviors vary across domains such as vision, reinforcement learning, and fine-tuning, underscoring the need for more nuanced approaches. In this survey, we synthesize insights from over 50 studies, examining the theoretical foundations, empirical findings, and practical implications of scaling laws. We also explore key challenges, including data efficiency, inference scaling, and architecture-specific constraints, advocating for adaptive scaling strategies tailored to real-world applications. We suggest that while scaling laws provide a useful guide, they do not always generalize across all architectures and training strategies.

1 Introduction

Scaling laws have become a fundamental aspect of modern AI development, especially for large language models (LLMs). In recent years, researchers have identified consistent relationships between model size, dataset volume, and computational resources, demonstrating that increasing these factors leads to systematic improvements in performance. These empirical patterns have been formalized into mathematical principles, known as *scaling laws*, which provide a framework for understanding how the capabilities of neural networks evolve as they grow. Mastering these laws is crucial for building more powerful AI models, optimizing efficiency, reducing costs, and improving generalization.

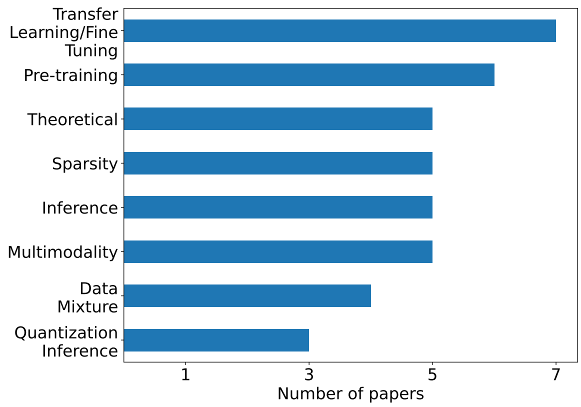


Figure 1: Papers surveyed under different categories. The detailed paper list is provided in Table 9 of Appendix B.

The study of neural scaling laws gained prominence with the foundational work of Kaplan et al. (2020), who demonstrated that model performance follows a power-law relationship with respect to size, data, and compute. Their findings suggested that larger language models (LMs) achieve lower loss when trained on sufficiently large datasets with increased computational resources. Later, Hoffmann et al. (2022) refined these ideas, introducing the notion of compute-optimal scaling, which revealed that training a moderate-sized model on a larger dataset is often more effective than scaling model size alone. However, recent studies (Muennighoff et al., 2023; Caballero et al., 2023; Krajewski et al., 2024) have challenged the universality of these laws, highlighting cases where sparse models, mixture-of-experts architectures, and retrieval-augmented methods introduce deviations from traditional scaling patterns. These findings suggested that while scaling laws provide a useful guide, they do not always generalize across all architectures and training strategies.

Despite the growing importance of scaling laws, existing research remains fragmented, with limited synthesis of theoretical foundations, empiri-

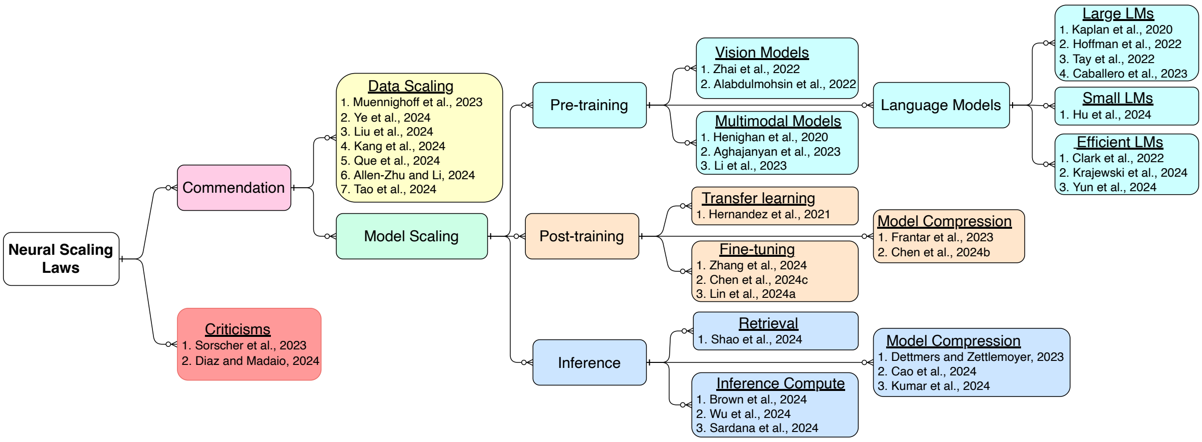


Figure 2: A taxonomy of neural scaling laws.

Category	Choshen et al. (2024)	Li et al. (2024b)	Ours
Covers neural scaling laws broadly	Yes	No	Yes
Discusses fitting methodologies	Yes	Yes	Yes
Analyzes architectural considerations	No	Limited	Yes
Includes data scaling and pruning	No	Limited	Yes
Explores inference scaling	No	Limited	Yes
Considers domain-specific scaling	No	No	Yes
Provides practical guidelines	Yes	Yes	Yes
Critiques limitations of scaling laws	Limited	Yes	Yes
Proposes future research directions	Limited	Yes	Yes

Table 1: Key differences between our survey and existing surveys on neural scaling laws (Choshen et al., 2024; Li et al., 2024b).

cal findings, and practical implications. Given the rapid evolution of this field, there is a need for a structured analysis that consolidates key insights, identifies limitations, and outlines future research directions. While theoretical studies have established the mathematical principles governing scaling, their real-world applications, such as efficient model training, optimized resource allocation, and improved inference strategies, are less explored. To address this gap, we reviewed over 50 research articles (Figure 1 highlights papers on scaling laws on different topics) to comprehensively analyze scaling laws, examining their validity across different domains and architectures.

While prior surveys have made valuable contributions to understanding scaling laws, they have primarily focused on specific aspects of the scaling phenomenon (See Table 1). Choshen et al. (2024) emphasized statistical best practices for estimating and interpreting scaling laws using training data, while Li et al. (2024b) emphasized on methodological inconsistencies and reproduction crisis in existing scaling laws. Our survey distinguishes itself by offering comprehensive coverage of architectural considerations, data scaling implications, and inference scaling – areas that previous surveys either overlooked or addressed only partially.

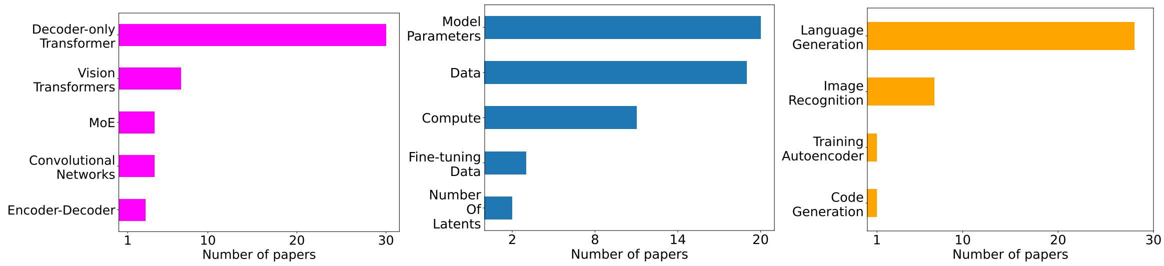
2 Taxonomy of neural scaling laws

Understanding the scaling laws of neural models is crucial for optimizing performance across different domains. We predominantly explore the scaling principles for language models, extending to other modalities such as vision and multimodal learning. We also examine scaling behaviors in domain adaptation, inference, efficient model architectures, and data utilization. We highlight the taxonomy tree of scaling laws research in Figure 2. As highlighted in Figure 1, neural scaling laws have been proposed predominantly for pre-training and fine-tuning scaling of large neural models. Among the models studied, as highlighted in Figure 3a, decoder-only Transformers dominate the subject, followed by vision transformers (ViT) and Mixture-of-Experts (MoE).

The most common neural scaling laws take the form of power laws (Equation 1), where the model's loss (L) or performance metric assumes to follow a predictable relationship with different scaling variables,

$$L(P_{i....n}) = \sum_{i=1}^{n} \alpha_i \cdot P_i^{-\beta_i} \quad (1)$$

with appropriate scaling parameters β_i and fitting parameters α_i for different scaling parameter P_i. Figure 3b highlights that the number of model parameters and data size are the most common used scaling factors. The exact forms of all the scaling laws are highlighted in Table 10 of Appendix B. Among all the tasks, Figure 3c suggests that language generation is the most common task used for developing these scaling laws, where the training cross-entropy loss is widely used to fit the laws. Based on the values obtained



(a) Architecture-wise statistics (b) Variable-wise statistics (c) Task-wise statistics

Figure 3: Number of paper studied in this survey paper for different model architectures (a), scaling variables (b) and scaling tasks (c). The detailed paper list is provided in Table 9 of Appendix B.

empirically, the scaling laws are fitted with non-linear optimization, most commonly by running algorithms like least square and BFGS (Broyden-Fletcher-Goldfarb-Shanno). Statistical methods like goodness-of-fit metrics are used to validate the correctness of the fitted curves. We elaborate on the evaluation of neural scaling laws in Appendix A. In the following sections, we review the existing literature on neural scaling across various domains.

Model scaling includes both parameter and data scaling. Parameter scaling is often studied in decoder-only Transformers (Kaplan et al., 2020; Hoffmann et al., 2022), with newer works addressing small and efficient models (Hu et al., 2024; Clark et al., 2022). These studies establish power-law relationships between loss and model size or compute (Equation 1). In parallel, **data scaling** research has proposed laws for optimizing mixtures (Ye et al., 2024), repeated training exposures (Muennighoff et al., 2023), vocabulary size (Tao et al., 2024), and knowledge capacity (Allen-Zhu and Li, 2024).

Pre-training scaling laws extend beyond language to vision and multimodal settings. Vision models exhibit power-law scaling that saturates at large compute (Zhai et al., 2022), while multimodal models demonstrate competition-to-synergy transitions as scale increases (Aghajanyan et al., 2023).

Post-training scaling captures fine-tuning and transfer learning behaviors. Transfer scaling shows larger pre-trained models yield better generalization with limited downstream data (Hernandez et al., 2021). Recent works propose scaling laws for PEFT (Zhang et al., 2024), downstream loss prediction (Chen et al., 2024c), and early stopping (Lin et al., 2024a).

Inference scaling explores compute-efficient strategies during model deployment. Adaptive test-time compute (Chen et al., 2024a; Brown et al., 2024) and retrieval augmentation (Shao et al., 2024) allow small models to rival larger ones. Inference-specific scaling laws characterize the tradeoff between sampling cost and performance (Wu et al., 2024).

Efficient model scaling addresses sparsity, quantization, and distillation. Sparse and MoE models provide multiplicative efficiency gains (Krajewski et al., 2024), while pruning and quantization laws enable compute-aware compression (Chen et al., 2024b; Cao et al., 2024).

Scaling behavior in reinforcement learning (RL) diverges from language or vision tasks. In single-agent RL, performance scales sublinearly with model size and environment interaction (Hilton et al., 2023). Horizon length, rather than task difficulty, determines scaling efficiency. In multi-agent games, predictable scaling laws govern compute-to-performance relationships, but generalization to complex domains like Chess or Go remains limited (Neumann and Gros, 2023). Meanwhile, graph neural networks (GNNs) lack stable scaling laws; despite self-supervised loss improving with more data, downstream performance often fluctuates unpredictably (Ma et al., 2024).

Finally, the taxonomy captures two outer branches: **commendations**, such as practical data laws and compression-aware training (Liu et al., 2024), and **criticisms**, which question the generalizability and reproducibility of scaling laws (Sorscher et al., 2023; Diaz and Madaio, 2024). Detailed discussion on these scaling law studies are provided in Appendix B.

In the next section, we formulate key research questions (mapping between the taxonomy and research questions highlighted in Table 2) derived from these studies and present practical guidelines

Taxonomy Node	Addressed RQs
Model scaling	RQ1, RQ2, RQ8
Data scaling	RQ3
Post-training scaling	RQ5
Inference scaling	RQ4
Efficient and compressed model scaling	RQ6, RQ7

Table 2: Mapping taxonomy categories to relevant research questions.

for leveraging scaling laws in real-world model development.

3 Research questions and guidelines

Grounded in the taxonomy of neural scaling laws (Figure 2), we identify key research questions spanning six dimensions: *model scaling*, *architectural bottlenecks*, *inference scaling*, *data scaling*, *post-training strategies*, and *efficient model design*. For each, we synthesize multiple studies to extract overarching patterns, identify conflicting evidence, and propose actionable guidelines for researchers and practitioners navigating large-scale model development.

RQ1. Importance on model and pre-training data size on performance. [taxonomy: model scaling → pre-training]

Kaplan et al. (2020) established a power-law relationship:

$$L(N, D) = \left[\left(\frac{N_c}{N} \right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D} \right]^{\alpha_D}, \quad D \propto N^{0.74}. \tag{2}$$

Hoffmann et al. (2022) refined this into a compute-optimal formulation:

$$L(N, D) = \frac{A}{N^{\alpha}} + \frac{B}{D^{\beta}} + E, \quad D \propto N. \tag{3}$$

Recent research has challenged linear extrapolations. Muennighoff et al. (2023) and Sardana et al. (2024) showed that training small models longer can outperform larger models, especially under constrained data. Caballero et al. (2023) proposed Broken Neural Scaling Laws (BNSL):

$$L(N, D) = \begin{cases} aN^{-\alpha} + bD^{-\beta}, & N < N_c \\ cN^{-\alpha'} + dD^{-\beta'}, & N \geq N_c \end{cases} \tag{4}$$

Synthesis and guidelines

- Model scaling success depends not only on size but also on training strategy, data quality, and saturation thresholds.
- Practitioners should allocate compute across parameters, data, and training duration based on observed inflection points. Use Kaplan/Chinchilla scaling when data is abundant; otherwise, extend training epochs or adopt data-efficient curricula (see Figure 4a).

RQ2. Scaling behaviors for different neural architectures. [taxonomy: model scaling → pre-training → architecture]

According to Tay et al. (2022), the vanilla Transformer consistently demonstrates superior scaling properties ($P \propto C^{\alpha}$, where P is the performance metric, C represents compute, and α are fitting parameters) compared to other architectures, even though alternative designs might perform better at specific sizes. Architectural bottlenecks manifest differently across these designs. For instance, linear attention models like Performer and Lightweight Convolutions show inconsistent scaling behavior, while ALBERT demonstrates negative scaling trends. This finding helps explain why most LLMs maintain relatively standard architectures rather than adopting more exotic variants. Furthermore, Zhai et al. (2022) revealed that ViT reveals that these models exhibit *double saturation*, where performance plateaus at both very low and very high compute levels, suggesting architectural limitations specific to the vision domain (Equation 5). However, as shown by Li et al. (2024a), simply scaling up vision encoders in multimodal models does not consistently improve performance, indicating that architectural scaling benefits are not uniform across modalities.

$$E = a(C + d)^{-b} + c, \tag{5}$$

where E denotes downstream error, C represents compute, and a, b, c, d are fitting parameters.

Synthesis and guidelines

- Architectural bottlenecks vary across domains and compute scales. Transformer inductive biases generalize best under scale.
- Use architectures with proven scaling profiles (e.g., vanilla Transformer) unless task-specific benefits outweigh risks. For multimodal or domain-specialized setups, consult scaling behavior across compute ranges (Figure 4a).

RQ3. Data strategies for performance scaling. [taxonomy: data scaling]

Ye et al. (2024) proposed an exponential model for data mixing:

$$L_i(r_{1\ldots M}) = c_i + k_i \exp\left(\sum_{j=1}^{M} t_{ij} r_j\right), \tag{6}$$

while Liu et al. (2024) and Kang et al. (2024) developed proxy models (REGMIX, AUTOSCALE) to pre-optimize mixtures. The Domain-Continual Pretraining (D-CPT) law (Que et al., 2024) provides a theoretical grounding on optimal mixture ratio between general and domain-specific data :

$$L(N, D, r) = E + \frac{A}{N^\alpha} + \frac{B \cdot r^\eta}{D^\beta} + \frac{C}{(r+\epsilon)^\gamma}, \tag{7}$$

where N represents the number of model parameters, D is the dataset size, r is the mixture ratio, $E, A, B, C, \alpha, \beta, \gamma, \eta, \epsilon$ are fitting parameters.

Synthesis and guidelines

- Model performance is sensitive to data heterogeneity, mixture ratios, and interaction effects – especially in multi-domain or continual settings.
- Replace manual corpus aggregation with predictive data mixing. Use D-CPT law when adapting to specific domains. Figure 4a outlines strategy paths based on data availability and domain constraints.

RQ4. Test-time scaling for better scaling efficiency. [taxonomy: model scaling → inference scaling]

Recent research examining the relationship between test-time computation and model size scaling has revealed key insights. Brown et al. (2024) proposed that repeated sampling during inference significantly enhances model performance, with coverage C (fraction of problems solved) following an exponentiated power law relationship with the number of samples k, $\log(C) = ak^{-b}$, where a, b are fitting parameters. Further exploration by Wu et al. (2024) suggested that employing sophisticated test-time computation strategies (such as iterative refinement or tree search) with smaller models may be more cost-effective than using larger models with simple inference methods. Their work establishes a relationship between inference computational budget and optimal model size for compute-efficient inference, expressed as $\log_{10}(C) = 1.19 \log_{10}(N) + 2.03$.

Synthesis and guidelines

- Inference scaling offers a complementary path to performance, particularly where model reuse is desired but compute cost must remain low.
- Use adaptive compute, retrieval augmentation, or tree search for high-value queries. Integrate test-time scaling laws into deployment workflows (Figure 4b).

RQ5. Scaling behaviors of model fine-tuning. [taxonomy: model scaling → post-training scaling]

Fine-tuning scaling reflects how pre-trained models adapt across tasks and domains. Hernandez et al. (2021) introduced a transfer scaling law based on effective data transferred D_t:

$$D_t(D_f, N) = k(D_f)^\alpha (N)^\beta, \tag{8}$$

while Lin et al. (2024a) refined this with a rectified law:

$$L(D) = \frac{B}{D_t + D^\beta} + E, \tag{9}$$

modeling diminishing returns from fine-tuning beyond a pre-learned threshold. In vision, Abnar et al. (2021) linked downstream error to upstream error:

$$e_{DS} = k(e_{US})^a + c, \tag{10}$$

and Mikami et al. (2021) connected downstream accuracy to synthetic pretraining data size:

$$e_{DS} = aD^{-\alpha} + c. \tag{11}$$

FLOPS to Loss to Performance (FLP) method (Chen et al., 2024c) predicted downstream performance from pretraining FLOPs, and Zhang et al. (2024) showed LoRA scales nonlinearly under PEFT:

$$\hat{L}(X, D_f) = A \times \frac{1}{X^\alpha} \times \frac{1}{D_f^\beta} + E. \tag{12}$$

Synthesis and guidelines

- Transferability scales with both model size and pretraining loss, but task difficulty, data availability, and adaptation type mediate returns.
- Use FLP or rectified laws to estimate post-training gains. Prefer PEFT for low-resource settings; switch to full fine-tuning when compute permits. For domain adaptation, apply D-CPT strategies (Figure 4a).

RQ6. Scaling efficiency and performance for sparse and efficient models. [taxonomy: model scaling → model compression]

As the demand for resource-efficient models grows, sparse architectures such as pruned networks and

MoEs have emerged as promising alternatives to dense Transformers. These models aim to preserve the performance benefits of scale while reducing compute and memory overhead. Frantar et al. (2023) proposed a general sparse scaling law showing that sparsity acts as a multiplicative efficiency factor rather than changing the fundamental scaling behavior:

$$L(S,N,D) = (a_S(1-S)^{b_S}+c_S)\cdot\left(\frac{1}{N}\right)^{b_N} + \left(\frac{a_D}{D}\right)^{b_D} + c, \tag{13}$$

where S is sparsity, N is the number of non-zero parameters, and D is dataset size. In MoE models, where only a subset of parameters is activated per input, Clark et al. (2022) proposed a loss scaling relationship incorporating both model size and expert count:

$$\log L = a\log N + b\log E + c\log N \cdot \log E + d, \tag{14}$$

with E denoting the expansion factor. This formulation was extended by Yun et al. (2024) to include dataset size:

$$\log L(N,D,E) = \log\left(\frac{a}{N^\alpha} + \frac{b}{E^\beta} + \frac{c}{D^\gamma} + f\right) + d\log N \log E \tag{15}$$

These results emphasize that scaling MoEs effectively requires balancing expert granularity with sufficient training data. Toward this, Krajewski et al. (2024) introduced a granularity parameter G to refine the Chinchilla-style formulation:

$$\mathcal{L}(N,D,G) = c + \left(\frac{g}{G^\gamma} + a\right)\frac{1}{N^\alpha} + \frac{b}{D^\beta}. \tag{16}$$

In parallel, structured pruning approaches have been formalized through the P^2 law (Chen et al., 2024b), which relates post-pruning loss to pre-pruning model size N_0, pruning ratio ρ, and post-training token count D:

$$L(N_0,D,\rho,L_0) = L_0 + \left(\frac{1}{\rho}\right)^\gamma \left(\frac{1}{N_0}\right)^\delta \left(\frac{N_C}{N_0^\alpha} + \frac{D_C}{D^\beta} + E\right), \tag{17}$$

where L_0 is the uncompressed model loss, ρ is the pruning rate, N_0 is the pre-pruning model size, D represents the number of post-training tokens, and $N_C, D_C, E, \alpha, \beta, \gamma$ are fitting parameters.

> **Synthesis and guidelines**
>
> - Sparse models are scaling-compliant but require careful routing (MoE) and token-budget tuning (pruning) to outperform dense counterparts.
> - Use MoEs for general-purpose LLMs under compute limits. Apply pruning for deployment constraints. For efficient inference, refer to Figure 4b.

RQ7. Model scaling with low-precision quantization. [taxonomy: model scaling → model compression → quantization]

According to Dettmers and Zettlemoyer (2023), 4-bit precision appears to be the optimal sweet spot for maximizing model performance while minimizing model size. Additionally, research on scaling with mixed quantization (Cao et al., 2024), demonstrated that larger models can handle higher quantization ratios while maintaining performance, following an exponential relationship where larger models require exponentially fewer high-precision components to maintain a given performance level. Kumar et al. (2024) developed a unified scaling law (Equation 18) that predicts both training and post-training quantization effects. It further suggests that effects of quantizing weights, activations, and attention during training are independent and multiplicative.

$$L(N,D,P_w,P_a,P_{\text{kv}},P_{\text{post}}) = AN_{\text{eff}}^{-\alpha} + BD^{-\beta} + E + \delta_{PTQ}, \tag{18}$$

where P_w, P_a, P_{kv} denote training precision of weights, activations and attentions, respectively, P_{post} denote end-time weight-precision, δ_{PTQ} denotes loss due to post training quantization, and α, β are fitting parameters.

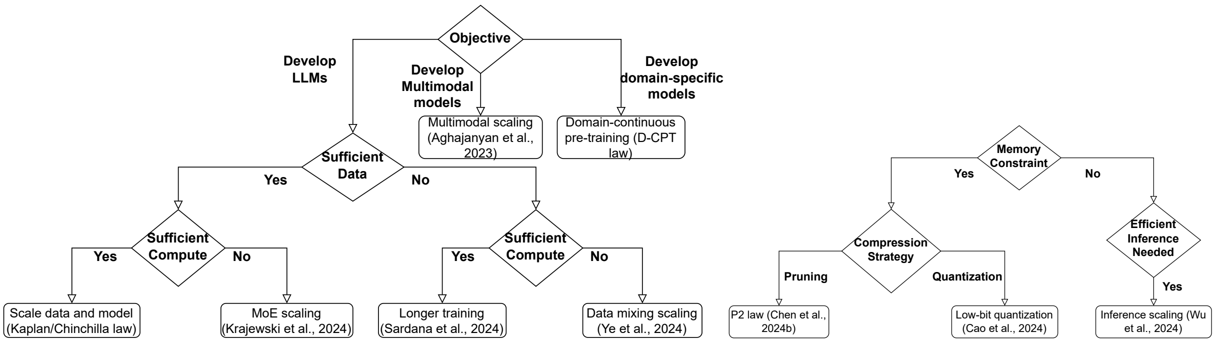
> **Synthesis and guidelines**
>
> - Scaling-aware quantization reduces memory while preserving performance. Larger models generalize better to low precision.
> - Apply mixed-precision for inference. Use quantization-aware training for smaller models. Refer to post-training strategies (Figure 4b) to guide compression.

RQ8. Beyond modalities: scaling for multimodal models. [taxonomy: model scaling → multimodal models]

Multimodal scaling behavior builds upon, but does not replicate, unimodal trends. Henighan et al. (2020) first proposed multimodal scaling using $L(x) = Ax^{-\alpha} + B$, where x represents model size, data, or compute. Alabdulmohsin et al. (2022) refined this into a more flexible sigmoid-like form:

$$\frac{L_x - L_\infty}{(L_0 - L_x)^\alpha} = \beta x^c, \tag{19}$$

allowing transitions across saturation regimes. Aghajanyan et al. (2023) observed that smaller multimodal models exhibit competition between



(a) Training strategies (b) Inference strategies

Figure 4: Practical roadmap summarizing training and inference strategies grounded in our eight research questions and taxonomy branches. (a) Training scaling strategies can be utilized for pre-training or fine-tuning unimodal and multimodal foundational and domain-adapted models. (b) Post-training inference strategies can be followed to ensure that the model is utilized efficiently for the downstream applications.

modalities, while larger models cross a "competition barrier" and become synergistic. They proposed a bimodal generalization of the Chinchilla law:

$$\mathcal{L}(N, D_i, D_j) = \left[\frac{\mathcal{L}(N, D_i) + \mathcal{L}(N, D_j)}{2}\right] - C_{i,j} + \frac{A_{i,j}}{N^{\alpha_{i,j}}} + \frac{B_{i,j}}{|D_i| + |D_j|^{\beta_{i,j}}}, \tag{20}$$

where $C_{i,j}$ captures the degree of positive interaction between modalities i and j.

Synthesis and guidelines

- Multimodal scaling is governed by modality alignment and architectural balance more than raw model size.
- Ensure models are sufficiently large to benefit from synergy across modalities. Prioritize modality balance in architecture and high-quality aligned datasets over isolated scaling. Refer to Figure 4a when designing multimodal pretraining pipelines.

Cross-RQ synthesis

- Data-efficient scaling (RQ1, RQ3, RQ5) consistently beats brute-force model expansion, as shown in Hu et al. (2024); Sardana et al. (2024).
- Architectural innovations (RQ2, RQ6) tend to scale poorly unless paired with precise training heuristics (e.g., expert routing in MoEs).
- Inference-aware scaling (RQ4, RQ7) enables small models to rival larger ones but is rarely included in current scaling laws - a key research gap.

While the research questions synthesized above highlight the strengths and practical applications of neural scaling laws, they also expose several limitations, especially in their generalizability, reliability under constraints and applicability to modern model designs. In the next section, we critically examine these limitations and discuss the foundational assumptions that may no longer hold as models evolve.

4 Criticisms of scaling laws

Diaz and Madaio (2024) challenged the generalizability of neural scaling laws, arguing that they fail in diverse real-world AI applications. They argued that scaling laws do not always hold when AI models serve heterogeneous populations with conflicting criteria for model performance. Larger datasets inherently reflect diverse communities, making it difficult to optimize a single model for all users. Similar to issues in multilingual AI, increasing data diversity often leads to performance degradation rather than improvement. Universal evaluation metrics are inadequate for capturing these complexities, potentially reinforcing biases against underrepresented groups. The authors further argued that smaller, localized AI models may be more effective for specific communities, highlighting the need to move beyond one-size-fits-all scaling assumptions.

Beyond dataset expansion, data pruning contradicts traditional scaling laws by demonstrating that performance improvements do not always require exponentially more data. Strategic pruning achieves comparable or superior results with significantly fewer training samples (Sorscher et al., 2023). Not all data contributes equally, and selecting the most informative examples enables more efficient learning. Experimental validation on CIFAR-10, SVHN, and ImageNet shows that careful dataset curation can surpass traditional power-law improvements, questioning the necessity of brute-force scaling.

Despite their significant impact, many studies on scaling laws suffer from limited reproducibility (see Table 11 in Appendix C) due to proprietary datasets, undisclosed hyperparameters, and undocumented training methodologies. The inability to replicate results across different computing environments raises concerns about their robustness. Large-scale experiments conducted by industry labs often depend on private infrastructure, making independent verification challenging. This lack of transparency undermines the reliability of scaling law claims and highlights the urgent need for open benchmarks and standardized evaluation frameworks to ensure reproducibility. Furthermore, the field's avoidance of rigorous scaling exponent analysis constitutes a critical oversight. While exponents indeed vary across models, datasets, and hyperparameters, this variability demands investigation rather than dismissal. This deliberate analytical gap undermines confidence in extrapolation claims and raises questions about whether observed scaling behaviors represent genuine properties or experimental artifacts.

5 Beyond Scale: Future Directions for Practical and Sustainable AI

While neural scaling laws have provided valuable insights into model performance, their current formulations often fail to account for recent advancements in architecture, data efficiency, and inference strategies. The following directions highlight key areas where scaling laws should be adapted to improve their predictive power and practical utility.

Reframing scaling laws for real-world constraints. Future scaling laws must account for compute budgets, hardware latency, and energy consumption. This includes integrating training–inference trade-offs, evaluating real-world performance under quantization or pruning, and predicting effectiveness across resource-constrained environments.

Designing for *downscaling*. Rather than building ever-larger models, the field should invest in scaling laws for *small* language models trained with optimal data, sparsity, and inference strategies. The emergence of 1–3B parameter models that rival 13B+ models (Hu et al., 2024) highlights the viability of compact yet performant systems.

Multi-objective scaling optimization. Current scaling laws often predict accuracy at scale but ignore trade-offs between accuracy, compute, and robustness. Future work should develop *multi-objective scaling frameworks* that balance these factors to guide architecture and dataset design more holistically.

Inference-aware and modular scaling laws. Traditional scaling laws assume fixed inference procedures. However, our synthesis in **RQ4** and **RQ7** shows that test-time compute allocation via sampling, retrieval, or routing can drastically affect performance. Future scaling formulations should modularize inference and allow flexible compute allocation per task or query.

Data quality over quantity. Instead of expanding datasets indiscriminately, laws like REGMIX (Liu et al., 2024) and D-CPT (Que et al., 2024) emphasize optimized data composition. Future models should prioritize informative examples and track dataset efficiency across tasks.

Towards accessible and sustainable AI. Large models are inaccessible to many research groups. Downscaling informed by scaling laws ensures that smaller labs and edge deployments can still benefit from state-of-the-art performance. Ultimately, the future of neural scaling is not just bigger models, but *better modeling choices at every scale*.

6 Conclusion

This survey provided a comprehensive analysis of neural scaling laws, exploring their theoretical foundations, empirical findings, and practical implications. It synthesized insights across various modalities, including language, vision, multimodal, and reinforcement learning, to uncover common trends and deviations from traditional power-law scaling. While early research established predictable relationships between model size, dataset volume, and computational resources, more recent studies have shown that these relationships are not universally applicable. Sparse architectures, retrieval-augmented models, and domain-specific adaptations often exhibit distinct scaling behaviors, challenging the notion of uniform scalability. Furthermore, advancements in fine-tuning, data pruning, and efficient inference strategies have introduced new perspectives on compute-optimal scaling. Despite their significance, scaling laws remain an evolving area of research, requiring further refinement to address real-world deployment challenges and architectural innovations.

Limitations

While this survey provides a broad synthesis of neural scaling laws, it primarily focuses on model size, data scaling, and compute efficiency. Other important aspects, such as hardware constraints, energy consumption, and the environmental impact of large-scale AI training, are not deeply explored. Another limitation is the reliance on prior empirical findings, which may introduce variability due to differing experimental setups and proprietary datasets. Without access to fully reproducible scaling law experiments, some conclusions remain dependent on the methodologies employed in original studies.

Ethical Considerations

Scaling laws, while effective in optimizing AI performance, can also raise issues of accessibility and fairness. The development of increasingly large models favors institutions with substantial computational resources, creating a divide between well-funded research groups and smaller organizations. Furthermore, as scaling laws often assume uniform data utility, they may amplify biases present in large-scale datasets, potentially leading to skewed outcomes in real-world applications. Ethical concerns also arise from the energy-intensive nature of training large models, contributing to environmental concerns. Addressing these issues requires more inclusive AI development strategies, ensuring that scaling laws consider broader societal impacts rather than focusing solely on performance optimization.

References

Samira Abnar, Mostafa Dehghani, Behnam Neyshabur, and Hanie Sedghi. 2021. Exploring the limits of large scale pre-training. *arXiv preprint*. ArXiv:2110.02095.

Armen Aghajanyan, Lili Yu, Alexis Conneau, Wei-Ning Hsu, Karen Hambardzumyan, Susan Zhang, Stephen Roller, Naman Goyal, Omer Levy, and Luke Zettlemoyer. 2023. Scaling laws for generative mixed-modal language models. *arXiv preprint*. ArXiv:2301.03728.

Ibrahim Alabdulmohsin, Behnam Neyshabur, and Xiaohua Zhai. 2022. Revisiting neural scaling laws in language and vision. *arXiv preprint*. ArXiv:2209.06640.

Zeyuan Allen-Zhu and Yuanzhi Li. 2024. Physics of language models: part 3. 3, knowledge capacity scaling laws. *arXiv preprint*. ArXiv:2404.05405.

Yasaman Bahri, Ethan Dyer, Jared Kaplan, Jaehoon Lee, and Utkarsh Sharma. 2021. Explaining neural scaling laws. *arXiv preprint*. ArXiv:2102.06701 version: 1.

Blake Bordelon, Alexander Atanasov, and Cengiz Pehlevan. 2024. A dynamical model of neural scaling laws. *arXiv preprint*. ArXiv:2402.01092.

Bradley Brown, Jordan Juravsky, Ryan Ehrlich, Ronald Clark, Quoc V. Le, Christopher Ré, and Azalia Mirhoseini. 2024. Large language monkeys: scaling inference compute with repeated sampling. *arXiv preprint*. ArXiv:2407.21787.

Dan Busbridge, Amitis Shidani, Floris Weers, Jason Ramapuram, Etai Littwin, and Russ Webb. 2025. Distillation scaling laws. *Preprint*, arXiv:2502.08606.

Ethan Caballero, Kshitij Gupta, Irina Rish, and David Krueger. 2023. Broken neural scaling laws. *arXiv preprint*. ArXiv:2210.14891.

Zeyu Cao, Cheng Zhang, Pedro Gimenes, Jianqiao Lu, Jianyi Cheng, and Yiren Zhao. 2024. Scaling laws for mixed quantization in large language models. *arXiv preprint*. ArXiv:2410.06722.

Lingjiao Chen, Jared Quincy Davis, Boris Hanin, Peter Bailis, Ion Stoica, Matei Zaharia, and James Zou. 2024a. Are more llm calls all you need? Towards scaling laws of compound inference systems. *arXiv preprint*. ArXiv:2403.02419.

Xiaodong Chen, Yuxuan Hu, Jing Zhang, Xiaokang Zhang, Cuiping Li, and Hong Chen. 2024b. Scaling law for post-training after model pruning. *arXiv preprint arXiv:2411.10272*.

Yangyi Chen, Binxuan Huang, Yifan Gao, Zhengyang Wang, Jingfeng Yang, and Heng Ji. 2024c. Scaling laws for predicting downstream performance in llms. *arXiv preprint*. ArXiv:2410.08527.

Leshem Choshen, Yang Zhang, and Jacob Andreas. 2024. A hitchhiker's guide to scaling law estimation. *Preprint*, arXiv:2410.11840.

Aidan Clark, Diego de las Casas, Aurelia Guy, Arthur Mensch, Michela Paganini, Jordan Hoffmann, Bogdan Damoc, Blake Hechtman, Trevor Cai, Sebastian Borgeaud, George van den Driessche, Eliza Rutherford, Tom Hennigan, Matthew Johnson, Katie Millican, Albin Cassirer, Chris Jones, Elena Buchatskaya, David Budden, Laurent Sifre, Simon Osindero, Oriol Vinyals, Jack Rae, Erich Elsen, Koray Kavukcuoglu, and Karen Simonyan. 2022. Unified scaling laws for routed language models. *arXiv preprint*. ArXiv:2202.01169.

Ian Covert, Wenlong Ji, Tatsunori Hashimoto, and James Zou. 2024. Scaling laws for the value of individual data points in machine learning. *Preprint*, arXiv:2405.20456.

Tim Dettmers and Luke Zettlemoyer. 2023. The case for 4-bit precision: k-bit Inference Scaling Laws. *arXiv preprint*. ArXiv:2212.09720.

Fernando Diaz and Michael Madaio. 2024. Scaling laws do not scale. *arXiv preprint*. ArXiv:2307.03201.

Elias Frantar, Carlos Riquelme, Neil Houlsby, Dan Alistarh, and Utku Evci. 2023. Scaling laws for sparsely-connected foundation models. *arXiv preprint*. ArXiv:2309.08520.

Leo Gao, John Schulman, and Jacob Hilton. 2022. Scaling laws for reward model overoptimization. *arXiv preprint*. ArXiv:2210.10760.

Leo Gao, Tom Dupré la Tour, Henk Tillman, Gabriel Goh, Rajan Troll, Alec Radford, Ilya Sutskever, Jan Leike, and Jeffrey Wu. 2024. Scaling and evaluating sparse autoencoders. *arXiv preprint*. ArXiv:2406.04093.

Behrooz Ghorbani, Orhan Firat, Markus Freitag, Ankur Bapna, Maxim Krikun, Xavier Garcia, Ciprian Chelba, and Colin Cherry. 2021. Scaling laws for neural machine translation. *Preprint*, arXiv:2109.07740.

Tatsunori Hashimoto. 2021. Model performance scaling with multiple data sources. In *Proceedings of the 38th International Conference on Machine Learning*, volume 139 of *Proceedings of Machine Learning Research*, pages 4107–4116. PMLR.

Tom Henighan, Jared Kaplan, Mor Katz, Mark Chen, Christopher Hesse, Jacob Jackson, Heewoo Jun, Tom B. Brown, Prafulla Dhariwal, Scott Gray, Chris Hallacy, Benjamin Mann, Alec Radford, Aditya Ramesh, Nick Ryder, Daniel M. Ziegler, John Schulman, Dario Amodei, and Sam McCandlish. 2020. Scaling laws for autoregressive generative modeling. *arXiv preprint*. ArXiv:2010.14701.

Danny Hernandez, Jared Kaplan, Tom Henighan, and Sam McCandlish. 2021. Scaling laws for transfer. *arXiv preprint*. ArXiv:2102.01293.

Jacob Hilton, Jie Tang, and John Schulman. 2023. Scaling laws for single-agent reinforcement learning. *arXiv preprint*. ArXiv:2301.13442.

Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals, and Laurent Sifre. 2022. Training compute-optimal large language models. *arXiv preprint*. ArXiv:2203.15556.

Edward J Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yuanzhi Li, Shean Wang, Lu Wang, and Weizhu Chen. 2021. Lora: Low-rank adaptation of large language models. *arXiv preprint arXiv:2106.09685*.

Shengding Hu, Yuge Tu, Xu Han, Chaoqun He, Ganqu Cui, Xiang Long, Zhi Zheng, Yewei Fang, Yuxiang Huang, Weilin Zhao, Xinrong Zhang, Zheng Leng Thai, Kaihuo Zhang, Chongyi Wang, Yuan Yao, Chenyang Zhao, Jie Zhou, Jie Cai, Zhongwu Zhai, Ning Ding, Chao Jia, Guoyang Zeng, Dahai Li, Zhiyuan Liu, and Maosong Sun. 2024. Minicpm: unveiling the potential of small language models with scalable training strategies. *arXiv preprint*. ArXiv:2404.06395.

Marcus Hutter. 2021. Learning curve theory. *arXiv preprint*. ArXiv:2102.04074.

Tian Jin, Nolan Clement, Xin Dong, Vaishnavh Nagarajan, Michael Carbin, Jonathan Ragan-Kelley, and Gintare Karolina Dziugaite. 2023. The cost of down-scaling language models: fact recall deteriorates before in-context learning. *arXiv preprint*. ArXiv:2310.04680.

Andy L. Jones. 2021. Scaling scaling laws with board games. *arXiv preprint*. ArXiv:2104.03113.

Feiyang Kang, Yifan Sun, Bingbing Wen, Si Chen, Dawn Song, Rafid Mahmood, and Ruoxi Jia. 2024. Autoscale: automatic prediction of compute-optimal data composition for training llms. *arXiv preprint*. ArXiv:2407.20177.

Jared Kaplan, Sam McCandlish, Tom Henighan, Tom B. Brown, Benjamin Chess, Rewon Child, Scott Gray, Alec Radford, Jeffrey Wu, and Dario Amodei. 2020. Scaling laws for neural language models. *arXiv preprint*. ArXiv:2001.08361.

Jakub Krajewski, Jan Ludziejewski, Kamil Adamczewski, Maciej Pióro, Michał Krutul, Szymon Antoniak, Kamil Ciebiera, Krystian Król, Tomasz Odrzygóźdź, Piotr Sankowski, Marek Cygan, and Sebastian Jaszczur. 2024. Scaling laws for fine-grained mixture of experts. *arXiv preprint*. ArXiv:2402.07871.

Tanishq Kumar, Zachary Ankner, Benjamin F. Spector, Blake Bordelon, Niklas Muennighoff, Mansheej Paul, Cengiz Pehlevan, Christopher Ré, and Aditi Raghunathan. 2024. Scaling laws for precision. *arXiv preprint*. ArXiv:2411.04330.

Bozhou Li, Hao Liang, Zimo Meng, and Wentao Zhang. 2024a. Are bigger encoders always better in vision large models? *arXiv preprint*. ArXiv:2408.00620.

Margaret Li, Sneha Kudugunta, and Luke Zettlemoyer. 2024b. Misfitting scaling laws: a survey of scaling law fitting techniques in deep learning. In *The International Conference on Learning Representations (ICLR)*.

Haowei Lin, Baizhou Huang, Haotian Ye, Qinyu Chen, Zihao Wang, Sujian Li, Jianzhu Ma, Xiaojun Wan, James Zou, and Yitao Liang. 2024a. Selecting large language model to fine-tune via rectified scaling law. *arXiv preprint*. ArXiv:2402.02314.

Licong Lin, Jingfeng Wu, Sham M. Kakade, Peter L. Bartlett, and Jason D. Lee. 2024b. Scaling laws in linear regression: Compute, parameters, and data. *Preprint*, arXiv:2406.08466.

Jack Lindsey, Adly Templeton Tom Conerly, Jonathan Marcus, and Tom Henighan. 2024. Circuits updates - april 2024.

Qian Liu, Xiaosen Zheng, Niklas Muennighoff, Guangtao Zeng, Longxu Dou, Tianyu Pang, Jing Jiang, and Min Lin. 2024. Regmix: data mixture as regression for language model pre-training. *arXiv preprint*. ArXiv:2407.01492.

Qian Ma, Haitao Mao, Jingzhe Liu, Zhehua Zhang, Chunlin Feng, Yu Song, Yihan Shao, and Yao Ma. 2024. Do neural scaling laws exist on graph self-supervised learning? *arXiv preprint*. ArXiv:2408.11243.

Hiroaki Mikami, Kenji Fukumizu, Shogo Murai, Shuji Suzuki, Yuta Kikuchi, Taiji Suzuki, Shin-ichi Maeda, and Kohei Hayashi. 2021. A scaling law for synthetic-to-real transfer: how much is your pre-training effective? *arXiv preprint*. ArXiv:2108.11018.

Niklas Muennighoff, Alexander M. Rush, Boaz Barak, Teven Le Scao, Aleksandra Piktus, Nouamane Tazi, Sampo Pyysalo, Thomas Wolf, and Colin Raffel. 2023. Scaling data-constrained language models. *arXiv preprint*. ArXiv:2305.16264.

Oren Neumann and Claudius Gros. 2023. Scaling laws for a multi-agent reinforcement learning model. *arXiv preprint*. ArXiv:2210.00849.

Haoran Que, Jiaheng Liu, Ge Zhang, Chenchen Zhang, Xingwei Qu, Yinghao Ma, Feiyu Duan, Zhiqi Bai, Jiakai Wang, Yuanxing Zhang, Xu Tan, Jie Fu, Wenbo Su, Jiamang Wang, Lin Qu, and Bo Zheng. 2024. D-cpt law: domain-specific continual pre-training scaling law for large language models. *arXiv preprint*. ArXiv:2406.01375.

Nikhil Sardana, Jacob Portes, Sasha Doubov, and Jonathan Frankle. 2024. Beyond chinchilla-optimal: accounting for inference in language model scaling laws. *arXiv preprint*. ArXiv:2401.00448.

Rulin Shao, Jacqueline He, Akari Asai, Weijia Shi, Tim Dettmers, Sewon Min, Luke Zettlemoyer, and Pang Wei Koh. 2024. Scaling retrieval-based language models with a trillion-token datastore. *arXiv preprint*. ArXiv:2407.12854.

Utkarsh Sharma and Jared Kaplan. 2020. A neural scaling law from the dimension of the data manifold. *arXiv preprint*. ArXiv:2004.10802.

Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. 2024. Scaling llm test-time compute optimally can be more effective than scaling model parameters. *arXiv preprint*. ArXiv:2408.03314.

Ben Sorscher, Robert Geirhos, Shashank Shekhar, Surya Ganguli, and Ari S. Morcos. 2023. Beyond neural scaling laws: beating power law scaling via data pruning. *arXiv preprint*. ArXiv:2206.14486.

Chaofan Tao, Qian Liu, Longxu Dou, Niklas Muennighoff, Zhongwei Wan, Ping Luo, Min Lin, and Ngai Wong. 2024. Scaling laws with vocabulary: larger models deserve larger vocabularies. *arXiv preprint*. ArXiv:2407.13623.

Yi Tay, Mostafa Dehghani, Samira Abnar, Hyung Won Chung, William Fedus, Jinfeng Rao, Sharan Narang, Vinh Q. Tran, Dani Yogatama, and Donald Metzler. 2022. Scaling laws vs model architectures: how does inductive bias influence scaling? *arXiv preprint*. ArXiv:2207.10551.

Yangzhen Wu, Zhiqing Sun, Shanda Li, Sean Welleck, and Yiming Yang. 2024. Inference scaling laws: an empirical analysis of compute-optimal inference for problem-solving with language models. *arXiv preprint*. ArXiv:2408.00724.

Jiasheng Ye, Peiju Liu, Tianxiang Sun, Yunhua Zhou, Jun Zhan, and Xipeng Qiu. 2024. Data mixing laws: optimizing data mixtures by predicting language modeling performance. *arXiv preprint*. ArXiv:2403.16952.

Longfei Yun, Yonghao Zhuang, Yao Fu, Eric P. Xing, and Hao Zhang. 2024. Toward inference-optimal mixture-of-expert large language models. *arXiv preprint*. ArXiv:2404.02852.

Xiaohua Zhai, Alexander Kolesnikov, Neil Houlsby, and Lucas Beyer. 2022. Scaling vision transformers. *arXiv preprint*. ArXiv:2106.04560.

Biao Zhang, Zhongtao Liu, Colin Cherry, and Orhan Firat. 2024. When scaling meets llm finetuning: the effect of data, model and finetuning method. *arXiv preprint*. ArXiv:2402.17193.

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A Fitting and validating scaling laws

Fitting scaling laws involves several key methodological choices that can significantly impact the final results and conclusions. The choice of optimization approach, loss function, initialization strategy, and validation method all play crucial roles in determining the reliability and reproducibility of scaling law studies.

A.1 Optimization methods

The most common approaches for fitting scaling laws involve non-linear optimization algorithms like BFGS (Broyden-Fletcher-Goldfarb-Shanno) (used by Frantar et al. (2023)), L-BFGS (used by Tao et al. (2024)) and least squares (used by Caballero et al. (2023)). Some studies (Covert et al., 2024; Hashimoto, 2021) also use optimizers like Adam or Adagrad, though these may be less suitable for scaling law optimization due to their data-hungry nature and assumptions about gradient distributions.

A.2 Loss functions and objectives

Several loss functions are commonly used for fitting scaling laws:

- **Mean squared error (MSE)**: Emphasizes larger errors due to quadratic scaling (used by Ghorbani et al. (2021)).
- **Mean absolute error (MAE)**: Provides more robust fitting less sensitive to outliers (used by Hilton et al. (2023)).
- **Huber loss**: Combines MSE's sensitivity to small errors with MAE's robustness to outliers (used by Hoffmann et al. (2022)).

A.3 Initialization strategies

The initialization of scaling law parameters proves to be critically important for achieving good fits. Common approaches include grid search over parameter spaces (Aghajanyan et al., 2023), random sampling from parameter ranges (Frantar et al., 2023), and multiple random restarts to avoid local optima (Caballero et al., 2023).

A.4 Validation methods

It is hugely important to understand if the scaling law fit achieved is accurate and valid. Most of the papers surveyed lack in validating their fits. Several approaches can help validating the effectiveness of scaling law fits. Statistical methods like computing confidence intervals can act as a goodness-of-fit metric (Alabdulmohsin et al., 2022). Furthermore, researchers can perform out-of-sample testing by extrapolation to larger scales (Hoffmann et al., 2022).

A.5 Limitations of fitting techniques

Li et al. (2024b) revealed several critical methodological considerations in fitting scaling laws. Different optimizers can converge to notably different solutions even with similar initializations, underscoring the need for careful justification of optimizer choice. Similarly, the analysis showed that different loss functions can produce substantially different fits when working with real-world data containing noise or outliers, suggesting that loss function selection should be guided by specific data characteristics and desired fit properties. Perhaps most importantly, the paper demonstrated that initialization can dramatically impact the final fit, with some methods exhibiting high sensitivity to initial conditions. Together, these findings emphasize the importance of thorough methodology documentation across all aspects of the fitting process - from optimizer selection and loss function choice to initialization strategy - to ensure reproducibility and reliability in scaling law studies.

B Detailed scaling laws

B.1 Scaling laws of language models

Kaplan et al. (2020) suggested that larger LMs improve performance by reducing loss through power-law scaling. However, this view evolved when studies showed that many large models were undertrained, and data scaling plays an equally crucial role in compute efficiency (Hoffmann et al., 2022). More recent breakthroughs challenged traditional scaling assumptions. Broken Neural Scaling Law (BNSL) introduced non-monotonic trends, meaning that model performance can sometimes worsen before improving, depending on dataset thresholds and architectural bottlenecks (Caballero et al., 2023). Another exciting development came from small LMs, where optimized training strategies, such as a higher data-to-parameter ratio and adaptive learning schedules, enable models ranging from 1.2B to 2.4B parameters to rival significantly larger 7B-13B models (Hu et al., 2024). These findings reshape the fundamental assumptions of scaling laws, proving that strategic training can outperform brute-force model expansion.

B.2 Scaling laws in other modalities

In computer vision, ViTs exhibit power-law scaling when model size, compute, and data grow together, but their performance plateaus at extreme compute levels, with noticeable gains only when trained on datasets exceeding 1B images (Zhai et al., 2022). Meanwhile, studies on scaling law extrapolation revealed that while larger models generally scale better, their efficiency declines at extreme sizes, requiring new training strategies to maintain performance (Alabdulmohsin et al., 2022). In multimodal learning, an interesting phenomenon called the "competition barrier" has been observed where at smaller scales different input modalities compete for model capacity, but as models grow, they shift into a synergistic state, enabling accurate performance predictions based on model size and token count (Aghajanyan et al., 2023).

However, not all scaling trends align with expectations. Contrary to the assumption that larger is always better, scaling vision encoders in vision-language models can sometimes degrade performance, highlighting the fact that data quality and modality alignment are more critical than brute-force scaling (Li et al., 2024a). These findings collectively emphasize that scaling laws are domain-dependent – optimal scaling strategies require a careful balance between compute efficiency, dataset quality, and architecture rather than simply increasing model size. Table 3 summarizes the scaling laws of pre-trained models for language and other modalities.

B.3 Scaling laws for domain adaptation

Pre-training and fine-tuning techniques have accelerated the adoption of large-scale neural models, yet the extent to which these models transfer across tasks and domains remains a key research question tied to scaling principles. Studies show that transfer learning follows a power-law where pre-training amplifies fine-tuning effectiveness, especially in small data regimes. Even with limited downstream data, larger models benefit significantly from pre-training, improving generalization (Hernandez et al., 2021). In vision, pre-training saturation occurs due to upstream-downstream interactions, rather than just task complexity. Lower network layers quickly specialize in simple tasks, while higher layers adapt to complex downstream objectives (Abnar et al., 2021). Similarly, in synthetic-to-real transfer, larger models consistently reduce transfer gaps, enhancing generalization across domains (Mikami et al., 2021).

Fine-tuning strategies scale differently depending on dataset size. Parameter-efficient fine-tuning (PEFT) techniques like low-rank adaptation (LoRA) (Hu et al., 2021) and Prompt-tuning, both are well-suited for small datasets, but LoRA performs best for mid-sized datasets, and full fine-tuning is most effective for large datasets. However, PEFT methods provide better generalization in large models, making them attractive alternatives to full-scale fine-tuning (Zhang et al., 2024).

Scaling laws are also being utilized to accurately predict the fine-tuning performance of models. The FLP method (Chen et al., 2024c) estimates pre-training loss from FLOPs, enabling accurate forecasts of downstream performance, particularly in models up to 13B parameters. Further refinements like FLP-M improve mixed-dataset predictions and better capture emergent abilities in large models. Finally, the Rectified scaling law (Lin et al., 2024a) introduces a two-phase fine-tuning transition, where early-stage adaptation is slow before shifting into a power-law improvement phase. This discovery enables compute-efficient model selection using the "Accept then Stop" (AtS) algorithm to terminate training at optimal points.

We summarize these findings in Table 5, sug-

Modality	Paper	Key insights	Applicability
Language	Kaplan et al. (2020)	Larger models are more sample-efficient, needing fewer training examples to generalize well.	Predicts model loss decreases with increasing parameters, used in early LMs like GPT-3.
	Hoffmann et al. (2022)	The best performance comes from balancing model size and data, rather than just increasing parameters.	Balances compute, model size, and dataset size for optimal efficiency, as seen in Chinchilla.
	Caballero et al. (2023)	Performance does not always improve smoothly; there are inflection points where scaling stops working.	Identifies phase transitions, minimum data thresholds, and unpredictability in scaling behavior.
	Hu et al. (2024)	Smaller models with better training can rival much larger models.	Demonstrates that smaller models with optimized training can outperform larger undertrained models.
Vision	Zhai et al. (2022)	ViTs follow power-law scaling but plateau at extreme compute levels, with benefits primarily seen in datasets >1B images.	Image classification, object detection, large-scale vision datasets.
Multimodal	Aghajanyan et al. (2023)	Multimodal models experience competition at smaller scales but transition into synergy as model and token count grow.	Multimodal learning, mixed-modal generative models, cross-domain AI.
	Li et al. (2024a)	Scaling vision encoders in vision-language models does not always improve performance, reinforcing the importance of data quality over raw scaling.	Vision-language models, image-text alignment, multimodal scaling challenges.

Table 3: Critical neural scaling laws for language, vision and multimodal models.

Paper	Key insights	Applicability
Zhai et al. (2022)	ViTs follow power-law scaling but plateau at extreme compute levels, with benefits primarily seen in datasets >1B images.	Image classification, object detection, large-scale vision datasets.
Aghajanyan et al. (2023)	Multimodal models experience competition at smaller scales but transition into synergy as model and token count grow, following a "competition barrier."	Multimodal learning, mixed-modal generative models, cross-domain AI.
Li et al. (2024a)	Scaling vision encoders in vision-language models (VLMs) does not always improve performance, reinforcing the importance of data quality over raw scaling.	Vision-language models, image-text alignment, multimodal scaling challenges.

Table 4: Summary of key insights found in scaling laws paper for computer vision and multimodal domains.

gesting that transfer learning is highly scalable, but effective scaling requires precise tuning strategies rather than just increasing model size.

B.4 Scaling laws for model inference

Simply scaling up models is not always the best way to improve model performance. Chen et al. (2024a) suggested that more efficient test-time compute strategies can dramatically reduce inference costs while maintaining or even exceeding performance. Instead of blindly increasing LLM calls, they further suggested for allocating resources based on query complexity, ensuring that harder queries receive more compute while

Paper	Key insights	Applicability
Hernandez et al. (2021)	Pre-training amplifies fine-tuning, particularly for small datasets, and benefits larger models even under data constraints.	Transfer learning, pre-training optimization, few-shot learning.
Abnar et al. (2021)	Large-scale pre-training improves downstream performance, but effectiveness depends on upstream-downstream interactions, not task complexity.	Vision transfer learning, upstream-downstream performance interactions.
Zhang et al. (2024)	Optimal fine-tuning strategy depends on dataset size: PEFT for small, LoRA for mid-scale, and full fine-tuning for large-scale datasets.	Fine-tuning strategies, parameter-efficient tuning, LoRA, full fine-tuning.
Lin et al. (2024a)	Fine-tuning follows a two-phase transition: slow early adaptation followed by power-law improvements, guiding compute-efficient model selection.	Compute-efficient fine-tuning, early stopping, model selection strategies.

Table 5: Key highlights from scaling of fine-tuned and domain-adapted models.

simpler ones use fewer resources. The importance of test-time compute strategies becomes even clearer when dealing with complex reasoning tasks. While sequential modifications work well for simple queries, parallel sampling and tree search dramatically improve results on harder tasks. Adaptive compute-optimal techniques have been shown to reduce computational costs by 4× without degrading performance, allowing smaller models with optimized inference strategies to surpass much larger models (Snell et al., 2024; Brown et al., 2024). Advanced inference approaches, such as REBASE tree search (Wu et al., 2024), further push the boundaries of efficiency, enabling small models to perform on par with significantly larger ones.

Another breakthrough came from retrieval augmented models, where increasing the datastore size consistently improves performance without hitting saturation (Shao et al., 2024). This allows smaller models to outperform much larger ones on knowledge-intensive tasks, reinforcing that external datastores provide a more efficient alternative to memorizing information in model parameters.

B.5 Scaling laws for efficient models

Scaling laws have expanded beyond simple parameter growth, introducing new methods to optimize routing, sparsity, pruning, and quantization for efficient LLM scaling. Routing-based models benefit from optimized expert selection, but their returns diminish at extreme scales, requiring careful expert configuration (Clark et al., 2022). In contrast, fine-grained MoE models consistently outperform dense transformers, achieving up to 40× compute efficiency gains when expert granularity is properly tuned (Krajewski et al., 2024). However, balancing the number of experts (E) is crucial, where models with 4-8 experts offer superior inference efficiency, but require $2.5 - 3.5\times$ more training resources, making 16-32 expert models more practical when combined with extensive training data (Yun et al., 2024). Sparse model scaling offers another efficiency boost. Research has demonstrated that higher sparsity enables effective model scaling, allowing $2.15\times$ more parameters at 75% sparsity, improving training efficiency while maintaining performance (Frantar et al., 2023). Additionally, pruning laws (P^2 scaling laws) predict that excessive post-training data does not always improve performance, helping optimize resource allocation in pruned models (Chen et al., 2024b). Dettmers and Zettlemoyer (2023) showed that 4-bit quantization provides the best trade-off between accuracy and model size, optimizing zero-shot performance while reducing storage costs. Larger models tolerate lower precision better, following an exponential scaling law where fewer high-precision components are needed to retain performance (Cao et al., 2024). Meanwhile, training precision scales logarithmically with compute budgets, with 7-8 bits being optimal for balancing size, accuracy, and efficiency (Kumar et al., 2024). Recent reserach

Paper	Key insights	Applicability
Brown et al. (2024)	Adaptive test-time compute strategies reduce computational costs by 4× while maintaining performance, enabling smaller models to compete with much larger ones.	Test-time compute efficiency, inference cost reduction, compute-limited environments.
Wu et al. (2024)	Advanced inference methods like REBASE tree search allow smaller models to match the performance of significantly larger ones.	High-efficiency inference, performance optimization for small models.
Shao et al. (2024)	Increasing datastore size in retrieval-augmented models consistently improves performance under the same compute budget, without evident saturation.	Retrieval-augmented language models, knowledge-intensive tasks, compute-efficient architectures.
Clark et al. (2022)	Routing-based models show diminishing returns at larger scales, requiring optimal routing strategies for efficiency.	Routing-based models, MoEs, transformer scaling.
Krajewski et al. (2024)	Fine-grained MoEs achieve up to 40× compute efficiency gains when expert granularity is optimized.	Mixture of Experts models, large-scale compute efficiency.
Frantar et al. (2023)	Sparse model scaling enables predicting optimal sparsity levels for given compute budgets.	Sparse models, structured sparsity optimization, parameter reduction.

Table 6: Scaling laws of efficient models.

Paper	Key insights	Applicability
Clark et al. (2022)	Routing-based models show diminishing returns at larger scales, requiring optimal routing strategies for efficiency.	Routing-based models, MoEs, transformer scaling.
Krajewski et al. (2024)	Fine-grained MoEs achieve up to 40× compute efficiency gains when expert granularity is optimized.	Mixture of Experts models, large-scale compute efficiency.
Frantar et al. (2023)	Sparse model scaling enables predicting optimal sparsity levels for given compute budgets.	Sparse models, structured sparsity optimization, parameter reduction.

Table 7: Scaling laws for routing, sparsity, pruning, and quantization.

has expanded into distillation as well, developing a mathematical framework that predicts how well a student model will perform based on the student model's size, the teacher model's performance and the compute budget allocation between the teacher and the student (Busbridge et al., 2025). We summarize these practical insights in Table 6 for better readability.

B.6 Data scaling laws

Scaling models involves more than just increasing parameters; optimizing data mixtures, training duration, and vocabulary size also plays a crucial role in enhancing performance and efficiency. Data mixing laws allow AI practitioners to accurately predict optimal data compositions before training, leading to 27% fewer training steps without compromising accuracy (Ye et al., 2024). Techniques like REGMIX optimize data selection using proxy models and regression, reducing compute costs by 90% compared to manual data selection (Liu et al., 2024). Meanwhile, AUTOSCALE revealed that data efficiency depends on model scale, where high-quality data like Wikipedia helps small models but loses effectiveness for larger models, which benefit from diverse datasets like CommonCrawl (Kang

Paper	Key insights	Applicability
Ye et al. (2024)	Predicts optimal data compositions before training, reducing compute costs by up to 27% while maintaining performance.	Pre-training optimization, data efficiency improvements.
Liu et al. (2024)	REGMIX optimizes data mixtures using proxy models, achieving 90% compute savings.	Compute-efficient training, automated data selection, large-scale models.
Allen-Zhu and Li (2024)	Language models can store 2 bits of knowledge per parameter, with knowledge retention dependent on training exposure.	Knowledge encoding, model compression, retrieval-augmented models.

Table 8: Critical scaling laws for data mixing and knowledge capacity.

et al., 2024). For continual learning, the D-CPT Law provided a theoretical framework for balancing general and domain-specific data, guiding efficient domain adaptation and long-term model updates (Que et al., 2024). Additionally, Chinchilla scaling assumptions were challenged by evidence showing that training models for more epochs on limited data can outperform simply increasing model size (Muennighoff et al., 2023). Repeated data exposure remains stable up to 4 epochs, but returns diminish to zero after around 16 epochs, making longer training a more effective allocation of compute resources. Furthermore, the vocabulary scaling law suggested that as language models grow larger, their optimal vocabulary size should increase according to a power law relationship (Tao et al., 2024). Finally, knowledge capacity scaling laws established that language models store 2 bits of knowledge per parameter, meaning a 7B model can encode 14B bits of knowledge – surpassing English Wikipedia and textbooks combined (Allen-Zhu and Li, 2024). Table 8 summarizes the data scaling laws for developing neural models when data is not available in abundance.

B.7 Scaling laws for reinforcement learning

Scaling laws in reinforcement learning (RL) and reward model optimization reveal both similarities and differences with generative modeling. Single-agent RL follows power-law scaling with model size and environment interactions, with optimal scaling exponents between 0.4-0.8 across tasks lower than the 0.5 exponent observed in language models (Hilton et al., 2023). RL tasks require orders of magnitude smaller models than generative tasks, correlating with task horizon length, which dictates environment interaction scaling. Task difficulty increases compute needs but does not affect scaling exponents, highlighting horizon length as a key factor in RL scaling efficiency.

In board games like Hex which involves multi-agent RL, Jones (2021) showed that AlphaZero performance follows predictable scaling trends, with compute requirements increasing 7× per board size increment for perfect play and 4× for surpassing random play (Jones, 2021). Neumann and Gros (2023) extended this study to Pentago and ConnectFour, proposing scaling laws which show that player strength scales with network size as $\alpha_N \approx 0.88$, performance with compute as $\alpha_C \approx 0.55$, and optimal network size with compute budget as $\alpha_{\text{opt}} \approx 0.63$ (Neumann and Gros, 2023). Larger multi-agent models exhibit higher sample efficiency, though these trends may not generalize to highly complex games like Chess and Go.

Reward model overoptimization in RLHF follows distinct functional forms: Best-of-n (BoN) reward optimization is governed by $d(\alpha_{\text{bon}} - \beta_{\text{bon}} d)$, whereas RL reward optimization follows $d(\alpha_{\text{RL}} - \beta_{\text{RL}} \log d)$, where d represents KL divergence from the initial policy (Gao et al., 2022). RL requires higher KL divergence than BoN for optimization, and reward model overoptimization scales logarithmically with model size, while policy size has minimal impact. These findings reinforce the importance of balancing compute allocation, environment complexity, and optimization techniques to achieve scalable and efficient RL models.

B.8 Scaling laws for sparse autoencoders

Recent research has established scaling laws for dictionary learning, providing insights into how latent representations and sparsity impact reconstruc-

Paper	Category	Task	Architecture	Datasets Used	Model Range	Data Range
Kaplan et al. (2020)	Pre-Training	Language Generation	Decoder-only Transformer	WebText2	0M - 1B	22M - 23B
Hoffmann et al. (2022)	Pre-Training	Language Generation	Decoder-only Transformer	MassiveText,Github, C4	70M - 16B	5B - 500B
Tay et al. (2022)	Pre-Training , Transfer Learning	Language Generation	Switch, T5 Encoder-Decoder, Funnel, MoS, MLP-mixer, GLU, Lconv, Evolved, Dconv, Performer,Universal, ALBERT	Pretraining: C4, Fine-Tuning: GLUE, SuperGLUE, SQuAD	173M - 30B	
Hu et al. (2024)	Pre-Training	Language Generation	Decoder-only Transformer	Large mixture	40M - 2B	
Caballero et al. (2023)	Pre-Training	Downstream Image Recognition and Language Generation	ViT, Transformers, LSTM	Vision pretrained: JFT-300M, downstream : Birds200, Caltech101, CIFAR-100; Language : Big-Bench		
Hernandez et al. (2021)	Transfer Learning	Code Generation	Decoder-only Transformer	Pre-train: WebText2, CommonCrawl, English Wikipedia, Books; FineTune: Github repos		
Abnar et al. (2021)	Transfer Learning	Image Recognition	ViT, MLP-Mixers, ConvNets	Pre-train: JFT, ImageNet21K	10M - 10B	
Mikami et al. (2021)	Transfer learning	Image Recognition	ConvNets	Syntheic Data		
Zhang et al. (2024)	Transfer Learning	Machine Translation and Language Generation	Decoder-only Transformer	WMT14 English-German (En-De) and WMT19 English-Chinese (En-Zh), CNN/Daily-Mail, MLSUM	1B - 16B	84B - 283B
Chen et al. (2024c)	Transfer learning	Language Generation	Decoder-only Transformer	Pre-Train: RedPajama v1, Validation: GitHub,ArXiv,Wikipedia, C4, RedPajama validation sets, ProofPile	43M - 3B	
Lin et al. (2024a)	Transfer learning	Language Generation	Decoder-only Transformer, Encoder-Decoder Transformer, Multilingual, MoE	Fine Tune: WMT19 English-Chinese (En-Zh), Gigaword, FLAN	100M - 7B	
Dettmers and Zettlemoyer (2023)	Quantization Inference	Language Generation	Decoder-only Transformer	The Pile, Lambada, PiQA, HellaSwag, Windogrande	19M - 176B	
Cao et al. (2024)	Quantization Inference	Language Generation	Decoder-only Transformer	WikiText2, SlimPajama, MMLU, Alpaca	500M - 70B	
Kumar et al. (2024)	Quantization Pre-Training, Quantization Inference	Language Generation	Decoder-only Transformer	Dolma V1.7	30M - 220M	1B - 26B
Chen et al. (2024a)	Inference	Language Generation	Decoder-only Transformer	MMLU Physics, TruthfulQA, GPQA, Averitec		
Snell et al. (2024)	Inference	Language Generation	Decoder-only Transformer	MATH		
Brown et al. (2024)	Inference	Language Generation	Decoder-only Transformer	GSM8K, MATH, MiniF2F-MATH, CodeContests, SWE-bench lite	70M - 70B	
Wu et al. (2024)	Inference	Language Generation	Decoder-only Transformer	MATH500, GSM8K	410M - 34B	
Sardana et al. (2024)	Inference	Language Generation	Decoder-only Transformer	Jeopardy, MMLU, BIG bench, WikiData, ARC, COPA, PIQA, OpenBook QA, AGI Eval, GSM8k, etc	150M-6B	1.5B - 1.2T
Clark et al. (2022)	Sparsity	Language Generation	Decoder-only Transformer, MoE	MassiveText	0 - 200B	0-130B
Frantar et al. (2023)	Sparsity	Language Generation, Image Recognition	Encoder-decoder, ViT	JFT-4B, C4	1M - 85M	0 - 1B
Krajewski et al. (2024)	Sparsity	Language generation	Decoder-only Transformer, MoE	C4	129M - 3B	16B - 130B
Yun et al. (2024)	Sparsity	Language generation	Decoder-only Transformer, MoE	Slim Pajama	100M - 730M	2B - 20B
Chen et al. (2024b)	Sparsity	Language Generation	Decoder-only Transformer	SlimPajama	500M - 8B	0.5B
Busbridge et al. (2025)	Distillation	Language generation	Teacher-Student Decoder-only Transformer	C4	100M - 12B	0 - 500B
Henighan et al. (2020)	Multimodality	Generative Image Modeling, Video Modeling, Language Generation	Decoder-only Transformer	FCC100M, and various modal datasets	0.1M-100B	100M
Zhai et al. (2022)	Multimodality	Image Recognition	ViT	ImageNet-21K	5M - 2B	1M - 3B
Alabdulmohsin et al. (2022)	Multimodality	Image Recognition, Machine Translation	ViT, MLP Mixers, Encoder-decoder, Decoder-only Transformer, Transformer encoder-LSTM decoder	JFT-300M, ImageNet, Birds200, CIFAR100, Caltech101, Big-Bench	10M-1B	32M-494M
Aghajanyan et al. (2023)	Multimodality	Multimodal Tasks	Decoder-only Transformers	OPT, Common Crawl, LibriSpeech , CommonVoice, VoxPopuli, Spotify Podcast, InCoder, SMILES from Zincand People's Speech	8M - 30B	5B - 100B
Li et al. (2024a)	Multimodality	Multimodal tasks	ViT, Decoder-only Transformer	CC12M, LAION-400M	7B - 13B	1M - 10M
Jones (2021)	Multi-agent RL	Hex	AlphaZero with neural networks			
Neumann and Gros (2023)	Multi-agent RL	Pentago, ConnectFour	AlphaZero with neural networks			
Gao et al. (2022)	RL	Reward Model training with Best of n or RL	Decoder-only Transformers			
Hilton et al. (2023)	Single-agent RL	ProcGen Benchmark, 1v1 version of Dota2, toy MNIST	ConvNets, LSTM		0M - 10M	
Ye et al. (2024)	Data Mixture	Language Generation	Decoder-only Transformer	RedPajama	70M - 410M	
Liu et al. (2024)	Data Mixture	Language Generation	Decoder-only Transformer	Pile		
Kang et al. (2024)	Data Mixture	Language Generation	Decoder-only Transformer , Encoder-only Transformer	RedPajama		
Que et al. (2024)	Data Mixture	Language Generation, Continual Pre-training	Decoder-only Transformer	various mixture of Code, Math, Law, Chemistry, Music, Medical	0.5B-4B	0.1B-26B
Tao et al. (2024)	Vocabulary	Language Generation	Decoder-only Transformer	SlimPajama	33M - 3B	0 - 500B
Lindsey et al. (2024)	Sparse Autoencoder	Training Autoencoder	Decoder-only Transformer			
Gao et al. (2024)	Sparse Autoencoder	Find Interpretable Latents	Decoder-only Transformer			
Shao et al. (2024)	Retrieval	Language Generation	Decoder-only Transformer	language modelling:RedPajama, S2ORC, Downstream : TriviaQA, NQ, MMLU, MedQA		
Muennighoff et al. (2023)	Pre-Training	Language Generation	Decoder-only transformer	C4	10M - 9B	0 - 900B
Allen-Zhu and Li (2024)	Knowledge Capacity	Language Generation	Decoder-only transformer	bioD		
Ma et al. (2024)	Graph Supervised learning	Graph Classification Task	InfoGraph, GraphCL, JOAO, GraphMAE	reddit-threads , ogbg-molhiv,ogbg-molpcba		
Diaz and Madaio (2024)	Criticize					
Sorscher et al. (2023)	Criticize	Image Recognition	ConvNets, ViT	SVHN, CIFAR-10, and ImageNet		
Bahri et al. (2021)	Theoretical					
Bordelon et al. (2024)	Theoretical					
Hutter (2021)	Theoretical					
Lin et al. (2024b)	Theoretical					
Sharma and Kaplan (2020)	Theoretical					
Jin et al. (2023)	Downscaling					

Table 9: Details on task, architecture of models and training setup for each paper surveyed.

Paper	Dependent variable	Scaling variable	Functional form
Kaplan et al. (2020)	Pre-Training Loss	Model Parameters, Compute, Data, Training Steps	$L(N,D) = \left[\left(\frac{N_c}{N}\right)^{\frac{\alpha_N}{\alpha_D}} + \frac{D_c}{D}\right]^{\alpha_D}$
Hoffmann et al. (2022)	Pre-Training Loss	Model Parameters, Data	$L(N,D) = \frac{A}{N^\alpha} + \frac{B}{D^\beta} + E$
Tay et al. (2022)	Performance metric	Compute	$P \propto C^\alpha$
Hu et al. (2024)	Pre-Training Loss	Model Parameters, Data	$L(P,D) = \frac{A}{N^\alpha} + \frac{B}{D^\beta} + E$
Caballero et al. (2023)	Performance metric	Model Parameters, Compute, Data, Input Size, Training Steps	$y = a + (bx^{-c_0}) \prod_{i=1}^{n} \left(1 + \left(\frac{x}{d_i}\right)^{1/f_i}\right)^{-c_i * f_i}$
Hernandez et al. (2021)	Data Transferred	Model Parameters, Fine-tuning Data	$D_t(D_f, N) = k(D_f)^\alpha (N)^\beta$
Abnar et al. (2021)	Downstream Error	Upstream Error	$e_{DS} = k(e_{US})^a + c$
Mikami et al. (2021)	Downstream Error	Pre-training Data	$e_{DS} = aD^{-\alpha} + c$
Zhang et al. (2024)	Downstream Loss	Fine-tuning Data, Data, Model Parameters, PET parameter	$\hat{L}(X, D_f) = A * \frac{1}{X^\alpha} * \frac{1}{D_f^\beta} + E$
Chen et al. (2024c)	Downstream performance	Pre-training Loss, Compute	$L(C) = (\frac{C}{C_N})^\alpha; P(L) = w_0 + w1 \cdot L$
Lin et al. (2024a)	Downstream Loss	Data, Fine-tuning Data	$L(D) = \frac{B}{D_t + D^\beta} + E$
Dettmers and Zettlemoyer (2023)	Accurancy	Total Model Bits After Quantization	
Cao et al. (2024)	Total parameters	Quantization Ratio	
Kumar et al. (2024)	Loss	Data, Model Parameters, Training Precision, Post-train Precision	$L(N, D, P_w, P_a, P_{\text{kv}}, P_{\text{post}}) = AN_{\text{eff}}^{-\alpha} + BD^{-\beta} + E + \delta_{PTQ}$
Chen et al. (2024a)	Optimal LLM Calls	Fraction Of Easy And Difficult Queries	
Brown et al. (2024)	Coverage	Number Of Samples	$\log(C) = ak^{-b}$
Wu et al. (2024)	Optimal Compute	Model Parameters	$\log_{10}(C) = 1.19 \log_{10}(N) + 2.03$
Sardana et al. (2024)	Pre-Training Loss	Model Parameters, Data	$L(N,D) = \frac{A}{N^\alpha} + \frac{B}{D^\beta} + E$
Clark et al. (2022)	Loss	Model Parameters, Number Of Experts , Data	$\log(L(N,E)) = a \log N + b \log E + c \log N \cdot \log E + d$
Frantar et al. (2023)	Loss	Sparsity, Model Parameters, Data	$L = (a_S(1-S)^{b_S} + c_S) \cdot \left(\frac{1}{N}\right)^{b_N} + \left(\frac{a_D}{D}\right)^{b_D} + c$
Krajewski et al. (2024)	Loss	Granularity, Model Parameters, Data	$\mathcal{L}(N,D,G) = c + \left(\frac{g}{G^\gamma} + a\right) \frac{1}{N^\alpha} + \frac{b}{D^\beta}$
Yun et al. (2024)	Loss	Model Parameters, Number Of Experts , Data	$\log L(N,D,E) \triangleq \log\left(\frac{A}{N^\alpha} + \frac{B}{E^\beta} + \frac{C}{D^\gamma} + F\right) + d \log N \log E$
Chen et al. (2024b)	Post-Training Loss	Uncompressed Model Loss, pruned ratio, Model parameters before pruning, Post-training Data	$L(N_0, D, \rho, L_0) = L_0 + \left(\frac{1}{\rho}\right)^\gamma \left(\frac{1}{N_0}\right)^\delta \left(\frac{N_C}{N_0^\alpha} + \frac{D_C}{D^\beta} + E\right)$
Henighan et al. (2020)	Loss	Model Parameters, Compute, Data	$L(x) = Ax^{-\alpha} + B$
Zhai et al. (2022)	Downstream Error	Compute	$E = aC^b + c$
Alabdulmohsin et al. (2022)	Loss	Compute, Model Parameters, Data	$\frac{L_x - L_\infty}{(L_0 - L_x)^\alpha} = \beta x^c$
Aghajanyan et al. (2023)	Loss	Model Parameters, Data	$\mathcal{L}(N, D_i, D_j) = \left[\frac{\mathcal{L}(N,D_i) + \mathcal{L}(N,D_j)}{2}\right] - C_{i,j} + \frac{A_{i,j}}{N^{\alpha i,j}} + \frac{B_{i,j}}{\lvert D_i \rvert + \lvert D_j \rvert^{\beta i,j}}$
Li et al. (2024a)	Loss	Model Parameters, Data	
Jones (2021)	Elo	Compute, Board Size	$Elo = \left(m^{\text{plateau}}_{\text{boardsize}} \cdot \text{boardsize} + c^{\text{plateau}}\right) \cdot clamp(m^{\text{incline}}_{\text{boardsize}} \cdot \text{boardsize} + m^{\text{incline}}_{\text{flops}} \cdot \log \text{flop} + c^{\text{incline}}, 0)$
Neumann and Gros (2023)	Game Score	Model Parameters, Compute	$E_i = \frac{1}{1 + (X_j/X_i)^{\alpha_X}}$
Gao et al. (2022)	Gold Reward model scores	Root Of KL Between Initial Policy And Optimized Policy (d)	$R(d) = d(\alpha - \beta \log d)$
Hilton et al. (2023)	Intrinsic performance	Model Parameters, Environment Interactions	$I^{-\beta} = \left(\frac{N_c}{N}\right)^{\alpha_N} + \left(\frac{E_c}{E}\right)^{\alpha_E}$
Ye et al. (2024)	Loss on domain i	Proportion Of Training Domains	$L_i(r_{1 \ldots M}) = c_i + k_i \exp\left(\sum_{j=1}^{M} t_{ij} r_j\right)$
Que et al. (2024)	Validation loss	Model Parameters, Data, Mixture Ratio	$L(N,D,r) = E + \frac{A}{N^\alpha} + \frac{B \cdot r^\eta}{D^\beta} + \frac{C}{(r+\epsilon)^\gamma}$
Tao et al. (2024)	Unigram-Normalised loss	Non-vocabulary Parameter, Vocabulary Parameters, Data	$\mathcal{L}_u = -E + \frac{A_1}{N_{\text{nv}}^{\alpha_1}} + \frac{A_2}{N_{\text{v}}^{\alpha_2}} + \frac{B}{D^\beta}$
Lindsey et al. (2024)	Reconstruction error	Compute, Number Of Latents	
Gao et al. (2024)	Reconstruction loss	Number Of Latents, Sparsity Level	$L(n,k) = \exp(\alpha + \beta_k \log(k) + \beta_n \log(n) + \gamma \log(k) \log(n)) + \exp(\zeta + \eta \log(k))$
Shao et al. (2024)	Downstream Accuracy	Datastore , Model Parameters, Data, Compute	
Muennighoff et al. (2023)	Loss	Data, Model Parameters, Epochs	$L(N,D) = \frac{A}{N'^\alpha} + \frac{B}{D'^\beta} + E$
Busbridge et al. (2025)	Student Loss	Teacher Loss, Student Parameters, Distillation Tokens	$L_S(N_S, D_S, L_T) = L_T + \frac{1}{L_T^{c_0}} \left(1 + \left(\frac{L_T}{L_{S,d_1}}\right)^{1/f_1}\right)^{-c_1/f_1} \left(\frac{A}{N_S^{\alpha'}} + \frac{B}{D_S^{\beta'}}\right)^{\gamma'}$

Table 10: Scaling law forms proposed in different papers we surveyed.

Paper	Training code	Analysis code	Github link
Kaplan et al. (2020)	N	N	
Hoffmann et al. (2022)	N	N	
Hoffmann et al. (2022)	N	N	
Hu et al. (2024)	Y	N	Link
Caballero et al. (2023)	N	Y	Link
Hernandez et al. (2021)	N	N	
Abnar et al. (2021)	N	N	
Mikami et al. (2021)	N	Y	Link
Zhang et al. (2024)	N	N	
Chen et al. (2024c)	N	N	
Lin et al. (2024a)	N	Y	Link
Dettmers and Zettlemoyer (2023)	N	N	
Cao et al. (2024)	N	N	
Kumar et al. (2024)	N	N	
Chen et al. (2024a)	Y	Y	Link
Snell et al. (2024)	N	N	
Brown et al. (2024)	Y	Y	Link
Wu et al. (2024)	Y	N	Link
Sardana et al. (2024)	N	N	
Clark et al. (2022)	N	Y	Link
Frantar et al. (2023)	N	N	
Krajewski et al. (2024)	Y	Y	Link
Yun et al. (2024)	N	N	
Chen et al. (2024b)	N	N	
Henighan et al. (2020)	N	N	
Zhai et al. (2022)	Y	N	Link
Alabdulmohsin et al. (2022)	N	Y	Link
Aghajanyan et al. (2023)	N	N	
Li et al. (2024a)	N	N	
Jones (2021)	Y	Y	Link
Neumann and Gros (2023)	Y	Y	Link
Gao et al. (2022)	N	N	
Hilton et al. (2023)	N	N	
Ye et al. (2024)	Y	Y	Link
Liu et al. (2024)	Y	Y	Link
Kang et al. (2024)	Y	Y	Link
Que et al. (2024)	N	N	
Tao et al. (2024)	Y	Y	Link
Lindsey et al. (2024)	N	N	
Gao et al. (2024)	Y	Y	Link
Shao et al. (2024)	Y	Y	Link
Muennighoff et al. (2023)	Y	Y	Link
Allen-Zhu and Li (2024)	N	N	
Ma et al. (2024)	Y	N	Link
Sorscher et al. (2023)	N	Y	Link

Table 11: Reproducibility of different neural scaling law papers. Reproducibility status of 45 papers surveyed: 22 (48.9%) provided repositories; 29 (64.4%) did not share training code.

tion error and computational efficiency. Sparse autoencoders with top-K selection follow power-law scaling for reconstruction error (MSE) in terms of the number of latents n and sparsity k, though this relationship only holds for small k relative to model dimension (Gao et al., 2024). Larger language models require more latents to maintain the same MSE at a fixed sparsity, reinforcing that latent dimensionality must scale with model size for effective reconstruction. Additionally, MSE follows a power-law relationship with the compute used during training, suggesting that efficient scaling strategies must balance sparsity, latent size, and training compute to minimize error effectively. This is reinforced by Lindsey et al. (2024), showing that feature representations follow predictable scaling trends, where larger models develop richer, more interpretable dictionaries as the number of learned features increases.

B.9 Scaling laws for graph neural networks

Unlike in computer vision and natural language processing, where larger datasets typically improve generalization, graph self-supervised learning methods fail to exhibit expected scaling behavior and performance fluctuates unpredictably across different data scales (Ma et al., 2024). However, self-supervised learning pretraining loss does scale with more training data, but this improvement does not translate to better downstream performance. The scaling behavior is method-specific, with some approaches like InfoGraph showing more stable scaling than others like GraphCL.

C Reproducibility of scaling laws papers

The reproducibility status of neural scaling law papers presents a mixed landscape in terms of research transparency. We consolidate and provide the links to github code repositories in the Table 11. Among the 45 surveyed papers proposing scaling laws, 22 papers (48.9%) provided repository links, indicating some level of commitment to open science practices. However, more than half of the papers still lack basic reproducibility elements, with 29 papers (64.4%) not sharing training code and 27 papers (60%) withholding analysis code. This comprehensive survey suggests that while there is a growing trend toward reproducibility in neural scaling law research, there remains substantial room for improvement in establishing standard practices for code sharing and result verification.