

Feature Learning for Interpretable, Performant Decision Trees

Supplementary Material

1 Experiment Specification

Here we cover the full specification of the experiments. Some details were omitted from the main text.

1.1 Benchmarking

Our experiments compare decision trees, random forests, ExtraTrees, and our proposed models with linear and distance-to-prototype features. All reported statistics are the average of 10-fold cross-validation, and in this document, we also report standard deviation.

All data sets were retrieved from the UCI Machine Learning Repository [1]. If there were separate training and test sets, they were combined before creating the random 10-fold split. Categorical attributes are one-hot encoded. All attributes are normalized to mean 0 and standard deviation 1. This makes feature learning more consistent, and it makes interpretation unitless.

For baseline models, we used implementations from scikit-learn [5], and our methods are implemented using Python along with PyTorch [4] for automatic differentiation. The splitting criterion for all models is Gini Index. Additional details for each model type follow.

- Decision tree. Using 10-fold cross validation (on the union of the 9 training folds for the experimental layer of cross-validation), we select the cost-complexity pruning α that results in the best accuracy. Our candidate α values include 0 and 15 evenly spaced values on the log scale from 10^{-8} to 10^{-1} , that is, $\{0, 1 \times 10^{-8}, 3.16 \times 10^{-8}, 1 \times 10^{-7}, 3.16 \times 10^{-7}, \dots, 1 \times 10^{-1}\}$.
- Random forest. We use default settings from scikit-learn. The ensemble contains 100 trees, each trained on a bootstrap sample with the same size as the original data, and the features consider for each split are limited to a uniformly sampled subset with size equal to the square root of the total number of features. There is no pruning.
- ExtraTrees. We use default settings from scikit-learn. The ensemble contains 100 trees, the features consider for each split are limited to a uniformly sampled subset with size equal to the square root of the total number of features, and for each feature, one candidate threshold is sampled uniformly in the range of data belonging to the current subtree. There is no pruning.
- Ours. For the KDDT, we use a box kernel with radius 0.1, that is, $k(\mathbf{x}) \propto \prod_{i=1}^p \mathbf{1}\{|x_i| \leq 0.1\}$. Do not perform a split if it would result in a leaf with total sample weight less than 1. We select cost-complexity pruning α from [.0001, .0003, .001, .003, .01, .03, .1] by 10-fold cross-validation of accuracy. We implement feature learning using PyTorch [4] for automatic differentiation and stochastic optimization. The optimizer is Adam [3] with learning rate 0.01, and the optimization procedure uses minibatch gradient descent with batch size 1024. Training runs for 10 epochs fitting the tree before each minibatch, then 1000 epochs fitting the tree once every 10 epochs. For the dry-bean and pendigits data sets, we instead train for 10 then 100 epochs because of their much larger size. This process allows the tree structure changes to be responsive early in training when the features are changing rapidly, then saves time by fitting less frequently as the features converge. We apply either L1 or L2 regularization to the feature parameters, each with coefficient 0.01. The details for each feature type follow.

- Linear features. We use a linear transformation without bias, with the same number of outputs as inputs. We initialize either as identity or uniformly at random in the range $\pm\sqrt{6/p}$, where p is the number of inputs to the linear transformation. Results in the main paper use random initialization.
- Distance-to-prototype features. We use a number of prototypes equal to the number of attributes in the data. Initialization is either random, with prototypes being samples from the unit Gaussian and the inverse covariance being identity, or by using the centers and inverse covariance matrices from a fitted Gaussian Mixture model from scikit-learn, with matching constraints on the covariance. Results in the main paper use random initialization. We constrain that inverse covariance matrices be positive definite so that the features represent distance. For these experiments, we also constrain inverse covariance matrices to be diagonal for the sake of easier interpretation, that is, so that it can be interpreted as Euclidean distance with each input being scaled differently. Regularization is applied only to the covariance parameters, although L1 regularization could also be applied to the prototypes themselves to make them sparse in the sense that they only sparsely differ from the global average. This may be useful for data with many attributes, so that prototypes can be described by just a few features.

1.2 MNIST Training

The KDDTs for the MNIST and Fashion-MNIST models also use a box kernel with radius 1. We used the Adam optimizer [3] with learning rate 0.001, and minibatch gradient descent with batch size 1024. For 10 epochs, the tree is fitted once per batch to the batch itself; the large batch size is chosen to ensure that the tree fitted to each batch is representative enough of the tree for the entire training data, while being faster than fitting to the entire MNIST training set at each batch. We then train for 100 epochs with the tree fitted to the entire training set once per epoch. The feature transformation is a linear mapping from $28^2 = 784$ to 784 features. For smaller models, a smaller number of features would be fine and would speed up training. We use L1 regularization with coefficient 0.3 and a weight image smoothing regularizer with coefficient 0.03 for MNIST, and respectively 3 and 0.01 for Fashion-MNIST. The smoothing penalty itself is calculated as the average squared difference of each pixel with its neighbors (not including diagonal).

2 Complete Results

In the complete experiment results, we show the mean and standard deviation from 10-fold cross-validation with five evaluation metrics:

1. validation accuracy
2. total number of nodes
3. average length of a validation decision path
4. average gini coefficient (a good measure of sparsity [2], lower is sparser) of the parameters for each feature, weighted by each feature's usage on the validation fold
5. total time for inference on the validation fold, in milliseconds

We make some observations based on the additional results:

- Random initialization more often results in better-performing models.
- Even when one of our models has many nodes, which may make global interpretation difficult, the average path length grows much less, so local interpretation is still simple.
- L1 regularization does result in sparser features compared to L2 regularization. With L2 regularization, features are usually very dense, with Gini index near 1.
- Our models have faster inference time than similarly performing ensembles.

Baselines							
data n, p, q	metric	LR	MLP	DT	RF	ET	XGB
iris 150, 4 (4), 3	acc	.960 ± .044	.953 ± .052	.947 ± .058	.947 ± .065	.953 ± .052	.947 ± .058
	nodes	0.0 ± 0.0	0.0 ± 0.0	6.4 ± 2.2	720.4 ± 50.7	2057.3 ± 109.7	432.6 ± 45.0
	path len	0.00 ± 0.00	0.00 ± 0.00	2.48 ± 0.38	261.74 ± 17.31	449.09 ± 37.55	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.7 ± 0.2	9.7 ± 26.6	0.6 ± 0.0	128.4 ± 45.5	34.2 ± 18.8	78.7 ± 63.7
heart-disease 303, 13 (20), 2	acc	.822 ± .021	.792 ± .067	.707 ± .060	.802 ± .065	.795 ± .052	.792 ± .038
	nodes	0.0 ± 0.0	0.0 ± 0.0	13.9 ± 16.1	4827.2 ± 108.1	10648.6 ± 206.0	788.6 ± 20.1
	path len	0.00 ± 0.00	0.00 ± 0.00	3.03 ± 1.71	584.43 ± 15.94	771.86 ± 30.70	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.4 ± 0.2	0.4 ± 0.0	0.3 ± 0.0	6.4 ± 0.1	6.6 ± 0.1	3.3 ± 0.7
dry-bean 13611, 16 (16), 7	acc	.925 ± .007	.934 ± .005	.912 ± .008	.923 ± .006	.921 ± .007	.928 ± .006
	nodes	0.0 ± 0.0	0.0 ± 0.0	99.8 ± 3.8	66504.9 ± 530.6	197338.7 ± 1264.0	12907.8 ± 166.2
	path len	0.00 ± 0.00	0.00 ± 0.00	7.05 ± 0.14	1142.00 ± 7.96	1287.98 ± 12.75	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.9 ± 0.2	1.4 ± 0.0	0.7 ± 0.0	38.3 ± 0.3	49.2 ± 0.3	4.3 ± 0.2
wine 178, 13 (13), 3	acc	.983 ± .025	.989 ± .022	.904 ± .077	.977 ± .028	.989 ± .022	.955 ± .043
	nodes	0.0 ± 0.0	0.0 ± 0.0	8.5 ± 2.1	936.4 ± 19.5	3315.1 ± 43.1	242.1 ± 15.0
	path len	0.00 ± 0.00	0.00 ± 0.00	3.30 ± 0.40	332.74 ± 6.59	579.64 ± 16.11	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.6 ± 0.2	0.4 ± 0.0	0.3 ± 0.0	6.2 ± 0.2	6.2 ± 0.0	1.7 ± 0.2
car 1728, 6 (21), 4	acc	.926 ± .021	.992 ± .007	.977 ± .012	.964 ± .013	.971 ± .011	.994 ± .006
	nodes	0.0 ± 0.0	0.0 ± 0.0	95.3 ± 6.6	23031.0 ± 243.0	31240.4 ± 330.8	4478.2 ± 48.9
	path len	0.00 ± 0.00	0.00 ± 0.00	4.48 ± 0.27	610.58 ± 21.80	617.75 ± 23.81	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.8 ± 0.1	1.1 ± 0.0	0.7 ± 0.1	16.4 ± 0.3	17.0 ± 0.5	3.7 ± 0.3
wdbc 569, 30 (30), 2	acc	.974 ± .021	.975 ± .024	.935 ± .032	.965 ± .019	.970 ± .028	.968 ± .021
	nodes	0.0 ± 0.0	0.0 ± 0.0	13.0 ± 6.5	1881.4 ± 59.9	6045.5 ± 193.9	274.4 ± 8.2
	path len	0.00 ± 0.00	0.00 ± 0.00	3.96 ± 1.25	462.02 ± 12.92	667.99 ± 21.11	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.8 ± 0.1	0.8 ± 0.0	0.6 ± 0.0	7.7 ± 2.1	6.6 ± 0.1	2.0 ± 0.4
sonar 208, 60 (60), 2	acc	.755 ± .094	.879 ± .064	.735 ± .096	.826 ± .104	.880 ± .058	.855 ± .063
	nodes	0.0 ± 0.0	0.0 ± 0.0	14.1 ± 6.7	2022.6 ± 16.4	5586.1 ± 77.9	301.1 ± 6.0
	path len	0.00 ± 0.00	0.00 ± 0.00	3.86 ± 1.46	490.86 ± 10.09	708.90 ± 25.75	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.4 ± 0.2	0.4 ± 0.0	0.3 ± 0.0	6.2 ± 0.2	6.4 ± 0.1	1.8 ± 0.2
pendigits 10992, 16 (16), 10	acc	.952 ± .005	.994 ± .003	.964 ± .004	.993 ± .002	.994 ± .002	.991 ± .002
	nodes	0.0 ± 0.0	0.0 ± 0.0	322.0 ± 13.4	38475.5 ± 232.0	98345.3 ± 655.6	84645.5 ± 60.7
	path len	0.00 ± 0.00	0.00 ± 0.00	10.13 ± 0.22	974.59 ± 5.43	1142.83 ± 7.26	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	29.8 ± 37.4	36.1 ± 63.6	9.3 ± 13.0	271.2 ± 80.6	43.2 ± 0.8	4.6 ± 0.7
ionosphere 351, 34 (34), 2	acc	.875 ± .069	.917 ± .062	.892 ± .048	.934 ± .056	.943 ± .049	.943 ± .053
	nodes	0.0 ± 0.0	0.0 ± 0.0	15.5 ± 9.0	2205.7 ± 88.3	5919.3 ± 267.2	335.4 ± 20.4
	path len	0.00 ± 0.00	0.00 ± 0.00	5.17 ± 2.43	645.54 ± 39.76	889.93 ± 48.63	0.00 ± 0.00
	gini	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000	.000 ± .000
	time	0.4 ± 0.2	0.4 ± 0.0	0.3 ± 0.0	6.4 ± 0.3	6.5 ± 0.1	1.8 ± 0.2

Ours: linear features, L2 regularization, non-random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.960 ± .044	.960 ± .044	.953 ± .043	.953 ± .052	.967 ± .033	.960 ± .044	.967 ± .033
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.8 ± 0.7	5.0 ± 0.9	8.7 ± 1.7	12.6 ± 1.5	16.1 ± 1.6
	path len	1.67 ± 0.09	1.67 ± 0.09	2.28 ± 0.31	2.41 ± 0.22	2.82 ± 0.28	3.16 ± 0.40	3.66 ± 0.41
	gini	.618 ± .010	.618 ± .014	.458 ± .081	.500 ± .059	.548 ± .071	.565 ± .052	.561 ± .061
	time	1.2 ± 0.0	4.3 ± 9.5	3.7 ± 7.6	1.5 ± 0.2	2.4 ± 0.4	3.3 ± 0.3	4.1 ± 0.4
heart	acc	.799 ± .053	.795 ± .028	.776 ± .043	.766 ± .051	.782 ± .058	.743 ± .041	.762 ± .058
	nodes	1.0 ± 0.0	1.7 ± 0.5	5.7 ± 1.7	19.5 ± 3.3	21.7 ± 1.7	26.8 ± 2.6	27.1 ± 3.6
	path len	1.00 ± 0.00	1.40 ± 0.28	2.67 ± 0.62	4.15 ± 0.51	4.84 ± 0.58	5.45 ± 0.63	6.00 ± 0.98
	gini	.888 ± .010	.881 ± .025	.862 ± .014	.824 ± .045	.830 ± .034	.814 ± .041	.836 ± .032
	time	0.7 ± 0.0	0.9 ± 0.2	2.1 ± 0.5	3.9 ± 0.6	4.3 ± 0.3	5.4 ± 0.5	5.5 ± 0.7
dry-bean	acc	.792 ± .005	.913 ± .009	.919 ± .006	.918 ± .004	.920 ± .006	.925 ± .006	.925 ± .005
	nodes	4.0 ± 0.0	6.0 ± 0.0	8.0 ± 0.0	9.9 ± 0.7	18.2 ± 1.0	41.2 ± 3.2	94.3 ± 3.5
	path len	2.83 ± 0.03	3.22 ± 0.05	3.40 ± 0.08	3.61 ± 0.16	4.72 ± 0.13	6.13 ± 0.32	7.48 ± 0.33
	gini	.902 ± .002	.902 ± .003	.910 ± .002	.909 ± .003	.909 ± .002	.906 ± .003	.908 ± .002
	time	3.3 ± 0.1	4.2 ± 0.1	5.4 ± 0.1	6.2 ± 0.3	10.2 ± 0.4	15.4 ± 1.2	31.1 ± 0.9
wine	acc	.961 ± .026	.972 ± .028	.972 ± .028	.972 ± .028	.966 ± .027	.977 ± .028	.972 ± .028
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.8 ± 0.4	2.7 ± 0.5	2.9 ± 0.3	2.8 ± 0.4	2.7 ± 0.5
	path len	1.67 ± 0.09	1.67 ± 0.09	1.92 ± 0.15	1.90 ± 0.16	1.95 ± 0.15	1.96 ± 0.18	1.89 ± 0.18
	gini	.871 ± .011	.875 ± .009	.884 ± .011	.885 ± .007	.891 ± .005	.886 ± .009	.885 ± .009
	time	0.6 ± 0.0	0.6 ± 0.0	0.7 ± 0.1	0.7 ± 0.1	0.7 ± 0.0	0.7 ± 0.1	0.7 ± 0.1
car	acc	.700 ± .044	.914 ± .034	.965 ± .013	.986 ± .009	.992 ± .012	.992 ± .009	.992 ± .009
	nodes	0.0 ± 0.0	2.8 ± 0.4	5.0 ± 0.8	12.3 ± 1.6	18.8 ± 2.7	23.7 ± 2.1	24.5 ± 4.1
	path len	0.00 ± 0.00	1.60 ± 0.16	1.90 ± 0.18	2.86 ± 0.34	3.23 ± 0.52	3.61 ± 0.29	3.70 ± 0.63
	gini	.000 ± .000	.905 ± .010	.897 ± .015	.902 ± .015	.902 ± .014	.903 ± .007	.901 ± .010
	time	0.2 ± 0.0	1.2 ± 0.1	1.7 ± 0.2	3.6 ± 0.4	5.3 ± 0.7	6.6 ± 0.7	6.8 ± 1.2
wdbc	acc	.965 ± .025	.967 ± .023	.960 ± .027	.963 ± .021	.968 ± .025	.960 ± .024	.963 ± .024
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	2.1 ± 0.9	5.9 ± 2.1	13.9 ± 4.2	19.6 ± 3.6
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	1.59 ± 0.48	2.51 ± 0.31	3.76 ± 0.68	4.54 ± 0.39
	gini	.954 ± .003	.955 ± .003	.954 ± .003	.951 ± .003	.951 ± .003	.950 ± .003	.951 ± .004
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	0.9 ± 0.3	1.8 ± 0.5	3.8 ± 1.0	5.3 ± 1.0
sonar	acc	.765 ± .093	.802 ± .043	.798 ± .080	.817 ± .088	.779 ± .074	.856 ± .038	.807 ± .063
	nodes	1.0 ± 0.0	2.2 ± 0.6	4.5 ± 0.8	5.6 ± 0.9	6.8 ± 3.0	7.6 ± 2.1	8.0 ± 2.4
	path len	1.00 ± 0.00	1.60 ± 0.24	2.53 ± 0.31	2.84 ± 0.50	3.34 ± 0.93	3.53 ± 0.70	3.54 ± 0.77
	gini	.971 ± .002	.972 ± .001	.973 ± .001	.950 ± .073	.973 ± .002	.973 ± .001	.973 ± .002
	time	0.5 ± 0.0	0.9 ± 0.2	1.5 ± 0.2	1.7 ± 0.2	2.0 ± 0.7	2.2 ± 0.5	2.3 ± 0.6
pendigits	acc	.094 ± .003	.911 ± .022	.955 ± .011	.974 ± .006	.986 ± .003	.989 ± .003	.991 ± .002
	nodes	0.0 ± 0.0	9.3 ± 0.5	12.1 ± 1.0	17.9 ± 1.7	27.4 ± 3.0	57.5 ± 6.6	125.9 ± 9.5
	path len	0.00 ± 0.00	4.18 ± 0.32	4.46 ± 0.31	5.18 ± 0.23	5.64 ± 0.45	6.58 ± 0.42	8.19 ± 0.60
	gini	.000 ± .000	.891 ± .008	.892 ± .005	.891 ± .006	.896 ± .006	.899 ± .002	.897 ± .005
	time	0.3 ± 0.0	4.5 ± 0.1	5.5 ± 0.5	7.7 ± 0.5	10.7 ± 1.0	19.6 ± 1.9	37.9 ± 2.6
ionosphere	acc	.857 ± .057	.932 ± .060	.932 ± .048	.935 ± .034	.934 ± .050	.929 ± .048	.946 ± .038
	nodes	1.0 ± 0.0	2.1 ± 0.3	3.8 ± 0.4	4.8 ± 0.7	8.5 ± 1.9	9.5 ± 1.8	11.8 ± 2.0
	path len	1.00 ± 0.00	1.77 ± 0.13	2.91 ± 0.24	3.52 ± 0.57	4.49 ± 0.51	4.70 ± 0.85	5.39 ± 0.79
	gini	.948 ± .003	.929 ± .012	.925 ± .005	.926 ± .005	.939 ± .005	.934 ± .007	.931 ± .004
	time	0.7 ± 0.1	1.1 ± 0.1	1.7 ± 0.1	2.0 ± 0.3	3.4 ± 0.7	2.4 ± 0.4	3.0 ± 0.5

Ours: linear features, L2 regularization, non-random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.960 ± .044	.960 ± .044	.953 ± .043	.967 ± .033	.960 ± .044	.960 ± .053	.933 ± .067
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.8 ± 0.7	5.0 ± 0.9	8.7 ± 1.7	12.6 ± 1.5	16.1 ± 1.6
	path len	1.67 ± 0.09	1.67 ± 0.09	2.29 ± 0.33	2.38 ± 0.22	2.81 ± 0.25	3.15 ± 0.44	3.66 ± 0.42
	gini	.584 ± .103	.580 ± .111	.459 ± .078	.483 ± .082	.554 ± .039	.565 ± .054	.564 ± .047
	time	0.7 ± 0.0	0.7 ± 0.0	1.1 ± 0.2	1.4 ± 0.2	2.1 ± 0.3	2.9 ± 0.3	3.6 ± 0.3
heart	acc	.799 ± .055	.795 ± .028	.776 ± .043	.763 ± .052	.782 ± .066	.749 ± .046	.762 ± .058
	nodes	1.0 ± 0.0	1.7 ± 0.5	5.7 ± 1.7	19.5 ± 3.3	21.7 ± 1.7	26.8 ± 2.6	27.1 ± 3.6
	path len	1.00 ± 0.00	1.41 ± 0.29	2.66 ± 0.63	4.12 ± 0.49	4.82 ± 0.58	5.45 ± 0.62	6.01 ± 1.01
	gini	.888 ± .010	.881 ± .024	.862 ± .014	.824 ± .045	.830 ± .034	.814 ± .041	.836 ± .032
	time	0.6 ± 0.0	0.8 ± 0.1	1.9 ± 0.5	3.5 ± 0.5	3.6 ± 0.3	4.6 ± 0.5	4.8 ± 0.6
dry-bean	acc	.788 ± .007	.910 ± .007	.917 ± .007	.919 ± .006	.917 ± .008	.923 ± .005	.922 ± .006
	nodes	4.0 ± 0.0	6.0 ± 0.0	8.0 ± 0.0	9.9 ± 0.7	18.2 ± 1.0	41.2 ± 3.2	94.3 ± 3.5
	path len	2.84 ± 0.03	3.24 ± 0.05	3.40 ± 0.09	3.60 ± 0.17	4.74 ± 0.15	6.19 ± 0.34	7.62 ± 0.33
	gini	.902 ± .002	.902 ± .003	.910 ± .002	.909 ± .003	.909 ± .002	.906 ± .004	.908 ± .002
	time	3.1 ± 0.1	3.9 ± 0.1	4.9 ± 0.1	5.6 ± 0.3	8.8 ± 0.3	12.7 ± 0.9	25.2 ± 0.7
wine	acc	.961 ± .026	.972 ± .028	.972 ± .028	.972 ± .028	.966 ± .027	.977 ± .028	.972 ± .028
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.8 ± 0.4	2.7 ± 0.5	2.9 ± 0.3	2.8 ± 0.4	2.7 ± 0.5
	path len	1.67 ± 0.09	1.68 ± 0.09	1.92 ± 0.16	1.90 ± 0.16	1.95 ± 0.15	1.96 ± 0.18	1.89 ± 0.17
	gini	.871 ± .011	.875 ± .009	.884 ± .011	.885 ± .007	.891 ± .005	.886 ± .008	.885 ± .009
	time	0.5 ± 0.0	0.5 ± 0.0	0.6 ± 0.1	0.6 ± 0.1	0.7 ± 0.0	0.6 ± 0.1	0.6 ± 0.1
car	acc	.700 ± .044	.908 ± .031	.965 ± .013	.986 ± .009	.992 ± .012	.992 ± .009	.992 ± .010
	nodes	0.0 ± 0.0	2.8 ± 0.4	5.0 ± 0.8	12.3 ± 1.6	18.8 ± 2.7	23.7 ± 2.1	24.5 ± 4.1
	path len	0.00 ± 0.00	1.60 ± 0.16	1.91 ± 0.18	2.86 ± 0.34	3.23 ± 0.52	3.61 ± 0.28	3.70 ± 0.64
	gini	.000 ± .000	.905 ± .010	.897 ± .014	.902 ± .015	.902 ± .014	.903 ± .007	.901 ± .009
	time	0.2 ± 0.0	1.1 ± 0.1	1.6 ± 0.2	3.2 ± 0.4	4.7 ± 0.6	5.9 ± 0.6	6.0 ± 1.0
wdbc	acc	.965 ± .025	.967 ± .023	.960 ± .027	.965 ± .022	.967 ± .027	.951 ± .025	.961 ± .037
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	2.1 ± 0.9	5.9 ± 2.1	13.9 ± 4.2	19.6 ± 3.6
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	1.59 ± 0.48	2.52 ± 0.32	3.75 ± 0.69	4.51 ± 0.41
	gini	.954 ± .003	.955 ± .003	.954 ± .003	.951 ± .003	.951 ± .003	.950 ± .003	.951 ± .004
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	0.8 ± 0.2	1.6 ± 0.4	3.4 ± 0.9	4.6 ± 0.7
sonar	acc	.760 ± .099	.817 ± .045	.788 ± .081	.813 ± .092	.775 ± .076	.856 ± .038	.807 ± .063
	nodes	1.0 ± 0.0	2.2 ± 0.6	4.5 ± 0.8	5.6 ± 0.9	6.8 ± 3.0	7.6 ± 2.1	8.0 ± 2.4
	path len	1.00 ± 0.00	1.59 ± 0.24	2.52 ± 0.32	2.87 ± 0.53	3.34 ± 0.94	3.55 ± 0.71	3.56 ± 0.75
	gini	.971 ± .002	.972 ± .001	.973 ± .001	.949 ± .074	.973 ± .002	.974 ± .001	.973 ± .002
	time	0.5 ± 0.0	0.8 ± 0.2	1.3 ± 0.2	1.6 ± 0.2	1.8 ± 0.6	2.0 ± 0.4	2.1 ± 0.5
pendigits	acc	.094 ± .003	.910 ± .021	.953 ± .011	.971 ± .008	.983 ± .004	.987 ± .004	.988 ± .003
	nodes	0.0 ± 0.0	9.3 ± 0.5	12.1 ± 1.0	17.9 ± 1.7	27.4 ± 3.0	57.5 ± 6.6	125.9 ± 9.5
	path len	0.00 ± 0.00	4.18 ± 0.32	4.46 ± 0.31	5.18 ± 0.23	5.64 ± 0.45	6.59 ± 0.43	8.20 ± 0.61
	gini	.000 ± .000	.891 ± .008	.892 ± .005	.891 ± .005	.896 ± .006	.899 ± .002	.897 ± .005
	time	0.3 ± 0.0	4.2 ± 0.1	5.0 ± 0.4	6.9 ± 0.5	9.3 ± 0.8	16.7 ± 1.5	32.2 ± 2.2
ionosphere	acc	.855 ± .054	.932 ± .060	.926 ± .048	.937 ± .036	.923 ± .048	.923 ± .051	.946 ± .039
	nodes	1.0 ± 0.0	2.1 ± 0.3	3.8 ± 0.4	4.8 ± 0.7	8.5 ± 1.9	9.5 ± 1.8	11.8 ± 2.0
	path len	1.00 ± 0.00	1.78 ± 0.14	2.95 ± 0.24	3.58 ± 0.62	4.58 ± 0.55	4.77 ± 0.89	5.50 ± 0.79
	gini	.948 ± .003	.929 ± .012	.925 ± .005	.927 ± .005	.939 ± .005	.934 ± .008	.931 ± .004
	time	0.7 ± 0.0	1.0 ± 0.1	1.6 ± 0.1	1.9 ± 0.2	3.0 ± 0.6	2.2 ± 0.4	2.6 ± 0.4

Ours: prototype features, L2 regularization, non-random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.927 ± .092	.933 ± .094	.953 ± .067	.967 ± .054	.947 ± .065	.947 ± .065	.947 ± .065
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.7 ± 1.0	4.9 ± 0.8	5.7 ± 0.9	5.6 ± 0.9	5.6 ± 0.9
	path len	1.67 ± 0.09	1.67 ± 0.09	2.09 ± 0.25	2.23 ± 0.23	2.31 ± 0.26	2.28 ± 0.24	2.28 ± 0.24
	gini	.580 ± .024	.578 ± .023	.580 ± .030	.584 ± .038	.578 ± .034	.576 ± .032	.577 ± .032
	time	1.1 ± 0.0	1.1 ± 0.0	1.6 ± 0.3	1.9 ± 0.2	2.2 ± 0.3	2.1 ± 0.3	2.1 ± 0.3
heart	acc	.756 ± .061	.746 ± .055	.776 ± .054	.765 ± .065	.766 ± .058	.729 ± .069	.749 ± .046
	nodes	1.0 ± 0.0	1.1 ± 0.3	6.4 ± 0.9	25.0 ± 3.6	48.0 ± 4.3	51.4 ± 5.9	52.5 ± 5.8
	path len	1.00 ± 0.00	1.04 ± 0.13	2.80 ± 0.25	4.41 ± 0.46	5.75 ± 0.52	5.94 ± 0.32	5.84 ± 0.41
	gini	.910 ± .007	.911 ± .007	.881 ± .011	.868 ± .014	.863 ± .011	.862 ± .016	.863 ± .016
	time	0.9 ± 0.0	0.9 ± 0.1	2.5 ± 0.3	8.0 ± 1.1	14.4 ± 1.2	15.5 ± 1.6	15.8 ± 1.6
dry-bean	acc	.681 ± .119	.880 ± .016	.885 ± .016	.902 ± .007	.909 ± .003	.897 ± .010	.904 ± .013
	nodes	3.2 ± 1.0	6.1 ± 0.3	8.4 ± 0.7	16.1 ± 2.1	31.0 ± 3.7	60.2 ± 5.9	172.5 ± 12.8
	path len	2.37 ± 0.56	3.08 ± 0.15	3.37 ± 0.16	4.26 ± 0.30	5.38 ± 0.24	6.03 ± 0.29	7.53 ± 0.38
	gini	.858 ± .021	.832 ± .019	.737 ± .063	.658 ± .056	.619 ± .078	.729 ± .031	.714 ± .027
	time	3.1 ± 0.7	4.5 ± 0.1	5.8 ± 0.4	9.5 ± 1.0	16.4 ± 2.1	20.2 ± 1.8	50.1 ± 3.2
wine	acc	.939 ± .052	.898 ± .056	.893 ± .047	.939 ± .068	.871 ± .066	.894 ± .084	.933 ± .054
	nodes	2.0 ± 0.0	2.6 ± 0.7	6.1 ± 1.6	7.3 ± 2.5	8.9 ± 3.3	10.0 ± 2.6	8.8 ± 1.5
	path len	1.69 ± 0.09	1.92 ± 0.28	2.99 ± 0.49	3.08 ± 0.60	3.31 ± 0.57	3.66 ± 0.69	3.47 ± 0.34
	gini	.861 ± .019	.862 ± .020	.789 ± .059	.792 ± .051	.766 ± .060	.763 ± .025	.765 ± .044
	time	0.9 ± 0.1	1.1 ± 0.2	1.9 ± 0.4	2.2 ± 0.6	2.5 ± 0.7	2.8 ± 0.6	2.5 ± 0.3
car	acc	.700 ± .044	.700 ± .044	.701 ± .045	.780 ± .087	.794 ± .116	.825 ± .098	.821 ± .109
	nodes	0.0 ± 0.0	0.2 ± 0.4	2.7 ± 1.9	13.9 ± 7.0	43.7 ± 27.1	96.5 ± 61.5	116.4 ± 67.0
	path len	0.00 ± 0.00	0.20 ± 0.40	1.68 ± 1.06	3.63 ± 0.62	4.85 ± 0.70	6.30 ± 1.18	6.61 ± 1.10
	gini	.000 ± .000	.182 ± .363	.716 ± .358	.890 ± .016	.869 ± .015	.873 ± .015	.869 ± .017
	time	0.3 ± 0.0	0.4 ± 0.2	1.3 ± 0.6	4.2 ± 1.8	11.2 ± 6.1	23.0 ± 13.6	27.5 ± 14.4
wdbc	acc	.930 ± .047	.946 ± .045	.951 ± .030	.933 ± .041	.931 ± .028	.930 ± .034	.910 ± .033
	nodes	1.0 ± 0.0	1.0 ± 0.0	2.9 ± 0.7	9.6 ± 2.2	14.9 ± 3.8	19.5 ± 2.8	21.8 ± 3.0
	path len	1.00 ± 0.00	1.00 ± 0.00	1.83 ± 0.31	3.65 ± 0.77	4.55 ± 0.92	5.35 ± 0.81	5.81 ± 0.47
	gini	.936 ± .011	.942 ± .011	.938 ± .008	.888 ± .018	.875 ± .023	.855 ± .034	.866 ± .016
	time	0.7 ± 0.0	0.7 ± 0.0	1.2 ± 0.2	2.9 ± 0.6	4.2 ± 0.9	5.3 ± 0.7	5.9 ± 0.8
sonar	acc	.716 ± .111	.730 ± .050	.721 ± .052	.730 ± .088	.711 ± .114	.721 ± .089	.770 ± .075
	nodes	1.0 ± 0.0	4.4 ± 1.0	11.3 ± 1.3	16.2 ± 3.1	18.0 ± 1.8	18.6 ± 2.7	18.0 ± 2.9
	path len	1.00 ± 0.00	2.32 ± 0.35	4.11 ± 0.51	4.92 ± 0.82	5.06 ± 0.61	5.05 ± 0.75	5.14 ± 0.97
	gini	.888 ± .069	.929 ± .026	.921 ± .029	.908 ± .036	.911 ± .038	.902 ± .025	.905 ± .031
	time	0.7 ± 0.0	1.6 ± 0.2	3.1 ± 0.3	4.3 ± 0.7	4.7 ± 0.4	4.8 ± 0.6	4.6 ± 0.7
pendigits	acc	.094 ± .003	.709 ± .031	.811 ± .017	.865 ± .015	.909 ± .011	.926 ± .011	.931 ± .007
	nodes	0.0 ± 0.0	8.1 ± 0.7	15.6 ± 1.7	31.8 ± 2.3	75.5 ± 3.5	174.3 ± 12.1	402.2 ± 24.6
	path len	0.00 ± 0.00	3.74 ± 0.20	4.60 ± 0.23	5.66 ± 0.25	6.79 ± 0.28	7.72 ± 0.25	9.76 ± 0.39
	gini	.000 ± .000	.558 ± .083	.428 ± .108	.348 ± .094	.299 ± .073	.283 ± .049	.241 ± .042
	time	0.4 ± 0.0	4.4 ± 0.4	7.0 ± 0.7	12.0 ± 0.8	23.9 ± 1.1	48.5 ± 3.0	101.7 ± 6.1
ionosphere	acc	.878 ± .057	.889 ± .050	.923 ± .036	.903 ± .043	.903 ± .045	.903 ± .041	.900 ± .039
	nodes	1.0 ± 0.0	1.9 ± 0.5	4.0 ± 1.3	9.9 ± 3.2	19.3 ± 3.8	20.6 ± 3.1	21.6 ± 4.1
	path len	1.00 ± 0.00	1.65 ± 0.40	2.60 ± 0.55	3.92 ± 0.65	6.19 ± 1.03	6.44 ± 1.23	6.81 ± 1.56
	gini	.748 ± .009	.771 ± .047	.765 ± .023	.770 ± .037	.780 ± .026	.794 ± .023	.764 ± .047
	time	0.9 ± 0.0	1.3 ± 0.2	2.0 ± 0.4	4.0 ± 1.1	5.1 ± 1.0	7.2 ± 1.0	7.3 ± 1.2

Ours: prototype features, L2 regularization, non-random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.927 ± .092	.933 ± .094	.953 ± .067	.973 ± .044	.953 ± .060	.953 ± .060	.953 ± .060
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.7 ± 1.0	4.9 ± 0.8	5.7 ± 0.9	5.6 ± 0.9	5.6 ± 0.9
	path len	1.67 ± 0.09	1.67 ± 0.09	2.09 ± 0.25	2.23 ± 0.23	2.31 ± 0.26	2.29 ± 0.24	2.29 ± 0.24
	gini	.580 ± .024	.578 ± .023	.580 ± .030	.584 ± .038	.578 ± .034	.576 ± .032	.577 ± .032
	time	1.0 ± 0.0	1.0 ± 0.0	1.5 ± 0.3	1.8 ± 0.2	2.0 ± 0.2	2.0 ± 0.2	2.0 ± 0.2
heart	acc	.756 ± .061	.746 ± .055	.779 ± .055	.769 ± .060	.766 ± .065	.736 ± .069	.743 ± .051
	nodes	1.0 ± 0.0	1.1 ± 0.3	6.4 ± 0.9	24.9 ± 3.5	48.0 ± 4.3	51.4 ± 5.9	52.5 ± 5.8
	path len	1.00 ± 0.00	1.04 ± 0.13	2.80 ± 0.25	4.41 ± 0.46	5.75 ± 0.53	5.94 ± 0.32	5.84 ± 0.41
	gini	.910 ± .007	.911 ± .007	.881 ± .011	.868 ± .014	.863 ± .011	.862 ± .016	.863 ± .016
	time	0.8 ± 0.1	0.8 ± 0.1	2.3 ± 0.3	7.1 ± 0.9	12.9 ± 1.1	20.5 ± 2.2	14.2 ± 1.4
dry-bean	acc	.680 ± .118	.877 ± .016	.883 ± .015	.894 ± .006	.902 ± .004	.894 ± .010	.902 ± .013
	nodes	3.2 ± 1.0	6.1 ± 0.3	8.4 ± 0.7	16.1 ± 2.1	31.0 ± 3.7	60.2 ± 5.9	172.5 ± 12.8
	path len	2.37 ± 0.57	3.08 ± 0.15	3.38 ± 0.16	4.26 ± 0.31	5.39 ± 0.24	6.03 ± 0.30	7.54 ± 0.39
	gini	.858 ± .021	.832 ± .019	.738 ± .063	.659 ± .057	.618 ± .080	.730 ± .032	.714 ± .027
	time	2.9 ± 0.6	4.3 ± 0.1	5.2 ± 0.3	8.3 ± 0.9	13.5 ± 1.5	18.2 ± 1.4	43.6 ± 2.9
wine	acc	.933 ± .060	.898 ± .056	.893 ± .047	.939 ± .068	.882 ± .076	.894 ± .084	.933 ± .054
	nodes	2.0 ± 0.0	2.6 ± 0.7	6.1 ± 1.6	7.3 ± 2.5	8.9 ± 3.3	10.0 ± 2.6	8.8 ± 1.5
	path len	1.69 ± 0.09	1.92 ± 0.28	3.01 ± 0.49	3.07 ± 0.59	3.30 ± 0.59	3.68 ± 0.69	3.46 ± 0.35
	gini	.861 ± .019	.862 ± .020	.790 ± .057	.793 ± .051	.764 ± .062	.763 ± .026	.766 ± .044
	time	0.9 ± 0.1	1.0 ± 0.1	1.7 ± 0.3	2.0 ± 0.5	2.3 ± 0.7	2.5 ± 0.5	2.3 ± 0.3
car	acc	.700 ± .044	.700 ± .044	.701 ± .045	.780 ± .087	.795 ± .118	.824 ± .096	.821 ± .114
	nodes	0.0 ± 0.0	0.2 ± 0.4	2.7 ± 1.9	13.9 ± 7.0	43.7 ± 27.1	96.5 ± 61.5	116.6 ± 67.2
	path len	0.00 ± 0.00	0.20 ± 0.40	1.68 ± 1.06	3.64 ± 0.62	4.85 ± 0.70	6.30 ± 1.18	6.60 ± 1.10
	gini	.000 ± .000	.182 ± .363	.716 ± .358	.890 ± .016	.869 ± .015	.873 ± .015	.869 ± .017
	time	0.3 ± 0.0	0.4 ± 0.2	1.2 ± 0.6	3.8 ± 1.6	10.1 ± 5.5	20.2 ± 11.9	24.3 ± 12.9
wdbc	acc	.930 ± .047	.946 ± .045	.947 ± .029	.933 ± .041	.930 ± .027	.930 ± .034	.910 ± .033
	nodes	1.0 ± 0.0	1.0 ± 0.0	2.9 ± 0.7	9.6 ± 2.2	14.9 ± 3.8	19.5 ± 2.8	21.8 ± 3.0
	path len	1.00 ± 0.00	1.00 ± 0.00	1.83 ± 0.31	3.65 ± 0.78	4.55 ± 0.91	5.35 ± 0.81	5.82 ± 0.48
	gini	.936 ± .011	.942 ± .011	.938 ± .008	.888 ± .019	.876 ± .020	.855 ± .034	.865 ± .016
	time	0.7 ± 0.0	0.7 ± 0.0	1.1 ± 0.2	2.6 ± 0.5	3.8 ± 0.9	4.8 ± 0.6	5.3 ± 0.6
sonar	acc	.716 ± .111	.730 ± .050	.716 ± .050	.730 ± .088	.716 ± .113	.730 ± .092	.770 ± .078
	nodes	1.0 ± 0.0	4.4 ± 1.0	11.3 ± 1.3	16.2 ± 3.1	18.0 ± 1.8	18.6 ± 2.7	18.0 ± 2.9
	path len	1.00 ± 0.00	2.32 ± 0.35	4.10 ± 0.51	4.92 ± 0.82	5.07 ± 0.62	5.07 ± 0.73	5.16 ± 0.96
	gini	.888 ± .069	.929 ± .026	.921 ± .029	.908 ± .036	.911 ± .038	.903 ± .024	.905 ± .031
	time	0.7 ± 0.0	1.4 ± 0.2	2.8 ± 0.3	3.9 ± 0.6	4.2 ± 0.3	4.3 ± 0.5	4.2 ± 0.6
pendigits	acc	.094 ± .003	.708 ± .031	.809 ± .018	.864 ± .015	.908 ± .011	.925 ± .011	.930 ± .007
	nodes	0.0 ± 0.0	8.1 ± 0.7	15.6 ± 1.7	31.8 ± 2.3	75.5 ± 3.5	174.3 ± 12.1	402.2 ± 24.6
	path len	0.00 ± 0.00	3.74 ± 0.21	4.60 ± 0.23	5.66 ± 0.25	6.79 ± 0.28	7.72 ± 0.25	9.76 ± 0.38
	gini	.000 ± .000	.558 ± .083	.428 ± .108	.348 ± .094	.300 ± .073	.283 ± .049	.241 ± .042
	time	0.3 ± 0.0	4.0 ± 0.3	6.4 ± 0.6	10.9 ± 0.7	21.5 ± 0.9	43.2 ± 2.7	89.6 ± 5.1
ionosphere	acc	.878 ± .057	.892 ± .049	.923 ± .036	.903 ± .043	.897 ± .048	.903 ± .041	.900 ± .039
	nodes	1.0 ± 0.0	1.9 ± 0.5	4.0 ± 1.3	9.9 ± 3.2	19.2 ± 3.8	20.6 ± 3.1	21.6 ± 4.1
	path len	1.00 ± 0.00	1.65 ± 0.40	2.60 ± 0.55	3.92 ± 0.65	6.20 ± 1.03	6.44 ± 1.23	6.80 ± 1.57
	gini	.748 ± .009	.771 ± .047	.765 ± .023	.770 ± .037	.782 ± .027	.794 ± .023	.764 ± .047
	time	0.7 ± 0.0	0.9 ± 0.2	1.3 ± 0.2	2.6 ± 0.7	4.6 ± 0.8	6.6 ± 0.9	6.7 ± 1.2

Ours: linear features, L1 regularization, non-random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.953 ± .043	.953 ± .043	.967 ± .033	.967 ± .033	.960 ± .033	.953 ± .043	.953 ± .043
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.7 ± 0.5	5.0 ± 1.2	9.5 ± 1.3	13.9 ± 2.2	18.8 ± 4.1
	path len	1.67 ± 0.09	1.67 ± 0.09	2.25 ± 0.21	2.37 ± 0.34	3.09 ± 0.42	3.51 ± 0.45	3.86 ± 0.57
	gini	.067 ± .168	.072 ± .168	.106 ± .049	.077 ± .051	.095 ± .077	.082 ± .086	.077 ± .072
	time	0.6 ± 0.0	0.7 ± 0.0	1.0 ± 0.1	1.6 ± 0.3	2.6 ± 0.3	3.6 ± 0.5	4.7 ± 0.9
heart	acc	.772 ± .056	.799 ± .052	.796 ± .065	.772 ± .036	.763 ± .074	.766 ± .082	.766 ± .096
	nodes	1.0 ± 0.0	1.3 ± 0.5	6.4 ± 1.5	25.5 ± 3.8	36.2 ± 3.8	41.3 ± 3.2	42.9 ± 5.8
	path len	1.00 ± 0.00	1.19 ± 0.30	2.62 ± 0.28	4.57 ± 0.35	5.20 ± 0.34	5.47 ± 0.44	5.65 ± 0.56
	gini	.432 ± .072	.505 ± .139	.347 ± .101	.220 ± .081	.164 ± .039	.182 ± .069	.165 ± .051
	time	0.6 ± 0.0	0.7 ± 0.2	1.8 ± 0.5	6.4 ± 2.0	7.1 ± 0.7	8.1 ± 0.6	8.4 ± 1.1
dry-bean	acc	.636 ± .062	.902 ± .009	.914 ± .007	.912 ± .004	.915 ± .006	.920 ± .007	.920 ± .007
	nodes	2.8 ± 0.4	6.0 ± 0.0	7.7 ± 0.5	12.1 ± 1.4	27.7 ± 2.0	59.6 ± 2.0	88.1 ± 10.7
	path len	2.14 ± 0.25	3.13 ± 0.22	3.31 ± 0.12	3.83 ± 0.19	5.28 ± 0.23	6.37 ± 0.14	6.46 ± 0.31
	gini	.619 ± .031	.658 ± .019	.674 ± .026	.535 ± .076	.510 ± .043	.359 ± .042	.406 ± .033
	time	2.6 ± 0.3	4.5 ± 0.1	5.3 ± 0.3	7.3 ± 0.7	14.5 ± 0.8	20.9 ± 0.7	51.1 ± 24.9
wine	acc	.972 ± .028	.989 ± .022	.972 ± .028	.960 ± .026	.966 ± .027	.983 ± .026	.978 ± .027
	nodes	2.0 ± 0.0	2.1 ± 0.3	3.0 ± 0.4	3.0 ± 0.4	5.0 ± 1.5	9.1 ± 2.7	13.1 ± 3.0
	path len	1.68 ± 0.08	1.72 ± 0.21	2.10 ± 0.18	2.03 ± 0.17	2.55 ± 0.36	3.39 ± 0.48	4.23 ± 0.57
	gini	.795 ± .015	.792 ± .034	.754 ± .043	.749 ± .062	.752 ± .060	.737 ± .033	.733 ± .025
	time	0.6 ± 0.0	0.6 ± 0.0	0.8 ± 0.1	0.7 ± 0.1	1.1 ± 0.3	1.8 ± 0.5	2.5 ± 0.5
car	acc	.700 ± .044	.891 ± .032	.946 ± .038	.976 ± .013	.992 ± .006	.996 ± .006	.992 ± .006
	nodes	0.0 ± 0.0	2.5 ± 0.8	5.9 ± 1.5	14.0 ± 2.4	29.8 ± 6.0	32.4 ± 3.7	39.3 ± 7.5
	path len	0.00 ± 0.00	1.53 ± 0.29	2.08 ± 0.26	2.77 ± 0.14	3.39 ± 0.25	3.64 ± 0.42	3.64 ± 0.34
	gini	.000 ± .000	.580 ± .133	.566 ± .035	.542 ± .071	.580 ± .082	.572 ± .062	.507 ± .043
	time	0.2 ± 0.0	1.1 ± 0.3	1.9 ± 0.4	4.0 ± 0.7	8.0 ± 1.7	8.6 ± 1.2	10.2 ± 1.5
wdbc	acc	.975 ± .020	.974 ± .021	.974 ± .016	.968 ± .015	.970 ± .019	.967 ± .029	.963 ± .030
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	3.0 ± 0.0	7.2 ± 2.5	17.1 ± 4.4	31.5 ± 4.7
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	2.00 ± 0.00	2.79 ± 0.62	3.55 ± 0.63	4.67 ± 0.62
	gini	.816 ± .018	.812 ± .023	.810 ± .021	.824 ± .034	.933 ± .007	.936 ± .013	.937 ± .008
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	1.1 ± 0.0	2.2 ± 0.6	3.2 ± 0.8	8.2 ± 1.5
sonar	acc	.711 ± .088	.769 ± .095	.822 ± .058	.783 ± .075	.803 ± .083	.851 ± .069	.836 ± .081
	nodes	1.0 ± 0.0	2.5 ± 0.7	7.3 ± 1.2	7.2 ± 1.0	13.3 ± 3.8	18.7 ± 4.8	27.5 ± 6.2
	path len	1.00 ± 0.00	1.75 ± 0.19	3.04 ± 0.25	3.15 ± 0.44	3.93 ± 0.51	4.90 ± 0.68	5.63 ± 0.63
	gini	.946 ± .008	.910 ± .028	.854 ± .043	.871 ± .037	.893 ± .041	.920 ± .022	.928 ± .018
	time	0.5 ± 0.0	1.0 ± 0.2	2.1 ± 0.3	2.1 ± 0.3	3.6 ± 1.0	5.0 ± 1.3	5.9 ± 1.5
pendigits	acc	.094 ± .003	.888 ± .014	.942 ± .010	.964 ± .007	.974 ± .007	.977 ± .005	.984 ± .004
	nodes	0.0 ± 0.0	9.1 ± 0.3	11.7 ± 0.9	18.6 ± 1.5	42.1 ± 5.0	120.7 ± 7.1	290.0 ± 12.9
	path len	0.00 ± 0.00	3.83 ± 0.27	4.40 ± 0.39	5.08 ± 0.20	5.92 ± 0.29	7.49 ± 0.37	8.95 ± 0.31
	gini	.000 ± .000	.803 ± .013	.795 ± .027	.751 ± .014	.689 ± .042	.503 ± .029	.329 ± .032
	time	0.3 ± 0.0	4.5 ± 0.1	5.5 ± 0.3	7.9 ± 0.5	15.2 ± 1.5	55.7 ± 12.3	142.1 ± 25.0
ionosphere	acc	.829 ± .044	.895 ± .057	.920 ± .038	.923 ± .034	.906 ± .056	.897 ± .069	.937 ± .031
	nodes	1.0 ± 0.0	2.2 ± 0.4	4.4 ± 0.5	6.2 ± 1.2	14.9 ± 3.1	26.4 ± 4.8	30.4 ± 5.4
	path len	1.00 ± 0.00	1.91 ± 0.32	3.43 ± 0.41	3.89 ± 0.44	5.51 ± 0.84	7.59 ± 0.87	8.98 ± 1.28
	gini	.447 ± .056	.640 ± .147	.532 ± .130	.657 ± .071	.691 ± .060	.772 ± .080	.781 ± .076
	time	0.5 ± 0.0	0.9 ± 0.1	1.4 ± 0.1	1.8 ± 0.3	3.9 ± 0.8	6.3 ± 1.1	7.9 ± 1.5

Ours: linear features, L1 regularization, non-random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.953 ± .043	.953 ± .043	.980 ± .031	.960 ± .033	.973 ± .033	.920 ± .065	.967 ± .033
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.7 ± 0.5	5.0 ± 1.2	9.5 ± 1.3	13.9 ± 2.2	18.8 ± 4.1
	path len	1.67 ± 0.09	1.67 ± 0.09	2.25 ± 0.21	2.38 ± 0.33	3.05 ± 0.44	3.52 ± 0.45	3.87 ± 0.55
	gini	.063 ± .170	.071 ± .168	.101 ± .046	.079 ± .057	.091 ± .075	.079 ± .086	.063 ± .046
	time	0.7 ± 0.0	0.7 ± 0.0	1.1 ± 0.1	1.4 ± 0.2	2.3 ± 0.3	3.1 ± 0.4	4.1 ± 0.8
heart	acc	.772 ± .056	.789 ± .056	.796 ± .061	.779 ± .034	.766 ± .078	.766 ± .078	.762 ± .095
	nodes	1.0 ± 0.0	1.3 ± 0.5	6.4 ± 1.5	25.5 ± 3.8	36.1 ± 3.8	41.3 ± 3.2	42.9 ± 5.8
	path len	1.00 ± 0.00	1.19 ± 0.30	2.61 ± 0.30	4.55 ± 0.37	5.21 ± 0.31	5.47 ± 0.42	5.61 ± 0.53
	gini	.432 ± .072	.505 ± .140	.346 ± .102	.220 ± .081	.165 ± .039	.181 ± .070	.165 ± .051
	time	0.6 ± 0.0	0.7 ± 0.1	1.6 ± 0.5	6.0 ± 1.6	6.2 ± 0.6	7.1 ± 0.5	7.4 ± 1.0
dry-bean	acc	.632 ± .061	.891 ± .010	.910 ± .010	.905 ± .006	.906 ± .006	.913 ± .009	.908 ± .008
	nodes	2.8 ± 0.4	6.0 ± 0.0	7.7 ± 0.5	12.1 ± 1.4	27.7 ± 2.0	44.1 ± 3.2	88.1 ± 10.7
	path len	2.15 ± 0.25	3.15 ± 0.23	3.31 ± 0.12	3.85 ± 0.20	5.33 ± 0.25	5.57 ± 0.17	6.49 ± 0.32
	gini	.619 ± .031	.658 ± .019	.674 ± .026	.536 ± .076	.511 ± .044	.360 ± .038	.406 ± .033
	time	2.4 ± 0.3	4.0 ± 0.1	4.8 ± 0.3	6.3 ± 0.5	11.5 ± 0.6	42.8 ± 15.5	50.7 ± 14.2
wine	acc	.972 ± .028	.989 ± .022	.972 ± .028	.955 ± .034	.961 ± .026	.978 ± .027	.972 ± .028
	nodes	2.0 ± 0.0	2.1 ± 0.3	3.0 ± 0.4	3.0 ± 0.4	5.0 ± 1.5	9.1 ± 2.7	13.1 ± 3.0
	path len	1.68 ± 0.07	1.73 ± 0.21	2.09 ± 0.18	2.03 ± 0.17	2.53 ± 0.37	3.41 ± 0.49	4.24 ± 0.58
	gini	.795 ± .015	.792 ± .033	.755 ± .042	.749 ± .062	.752 ± .063	.737 ± .034	.733 ± .025
	time	0.5 ± 0.0	0.5 ± 0.0	0.7 ± 0.1	0.7 ± 0.1	1.0 ± 0.2	1.6 ± 0.4	2.2 ± 0.4
car	acc	.700 ± .044	.890 ± .033	.945 ± .038	.976 ± .013	.992 ± .006	.996 ± .006	.992 ± .006
	nodes	0.0 ± 0.0	2.5 ± 0.8	5.9 ± 1.5	14.0 ± 2.4	29.8 ± 6.0	32.4 ± 3.7	39.3 ± 7.5
	path len	0.00 ± 0.00	1.53 ± 0.29	2.09 ± 0.26	2.78 ± 0.14	3.39 ± 0.25	3.64 ± 0.42	3.65 ± 0.35
	gini	.000 ± .000	.580 ± .133	.566 ± .036	.542 ± .071	.580 ± .081	.573 ± .062	.507 ± .043
	time	0.2 ± 0.0	1.0 ± 0.2	1.8 ± 0.4	3.6 ± 0.6	7.1 ± 1.5	7.6 ± 1.0	9.1 ± 1.5
wdbc	acc	.975 ± .020	.974 ± .021	.974 ± .016	.968 ± .015	.965 ± .025	.963 ± .035	.961 ± .028
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	3.0 ± 0.0	7.2 ± 2.5	17.1 ± 4.4	31.5 ± 4.7
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	2.00 ± 0.00	2.80 ± 0.62	3.54 ± 0.66	4.71 ± 0.58
	gini	.816 ± .018	.812 ± .023	.810 ± .021	.824 ± .034	.933 ± .007	.936 ± .013	.937 ± .008
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	1.0 ± 0.0	1.9 ± 0.5	2.8 ± 0.6	7.0 ± 1.2
sonar	acc	.712 ± .100	.769 ± .083	.812 ± .063	.774 ± .105	.818 ± .062	.846 ± .083	.822 ± .098
	nodes	1.0 ± 0.0	2.5 ± 0.7	7.3 ± 1.2	7.2 ± 1.0	13.3 ± 3.8	18.7 ± 4.8	27.5 ± 6.2
	path len	1.00 ± 0.00	1.77 ± 0.18	3.04 ± 0.27	3.16 ± 0.46	3.87 ± 0.53	4.84 ± 0.65	5.64 ± 0.61
	gini	.946 ± .008	.910 ± .028	.853 ± .044	.872 ± .036	.894 ± .042	.920 ± .023	.928 ± .019
	time	0.5 ± 0.0	0.9 ± 0.2	1.9 ± 0.2	1.9 ± 0.2	3.2 ± 0.8	4.3 ± 1.1	5.2 ± 1.2
pendigits	acc	.094 ± .003	.885 ± .013	.938 ± .011	.960 ± .010	.968 ± .005	.969 ± .006	.976 ± .004
	nodes	0.0 ± 0.0	9.1 ± 0.3	11.7 ± 0.9	18.6 ± 1.5	42.1 ± 5.0	120.7 ± 7.1	290.0 ± 12.9
	path len	0.00 ± 0.00	3.83 ± 0.27	4.41 ± 0.39	5.08 ± 0.20	5.91 ± 0.29	7.49 ± 0.40	8.97 ± 0.34
	gini	.000 ± .000	.803 ± .013	.795 ± .027	.751 ± .014	.689 ± .042	.503 ± .029	.329 ± .032
	time	0.3 ± 0.0	5.2 ± 0.2	6.3 ± 0.4	9.1 ± 0.6	17.4 ± 1.7	40.2 ± 2.4	86.1 ± 5.0
ionosphere	acc	.835 ± .044	.892 ± .057	.917 ± .043	.920 ± .038	.897 ± .054	.909 ± .055	.909 ± .069
	nodes	1.0 ± 0.0	2.2 ± 0.4	4.4 ± 0.5	6.2 ± 1.2	14.9 ± 3.1	26.4 ± 4.8	30.4 ± 5.4
	path len	1.00 ± 0.00	1.92 ± 0.32	3.51 ± 0.44	3.94 ± 0.44	5.54 ± 0.85	7.67 ± 0.90	9.23 ± 1.41
	gini	.447 ± .056	.640 ± .147	.533 ± .129	.658 ± .070	.691 ± .060	.772 ± .081	.781 ± .077
	time	0.5 ± 0.0	0.8 ± 0.1	1.2 ± 0.1	1.6 ± 0.3	3.4 ± 0.6	5.8 ± 1.0	6.7 ± 1.2

Ours: prototype features, L1 regularization, non-random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.940 ± .055	.940 ± .055	.933 ± .067	.933 ± .052	.927 ± .063	.900 ± .086	.920 ± .088
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.3 ± 0.8	5.3 ± 0.6	5.9 ± 0.9	6.3 ± 1.0	6.1 ± 0.8
	path len	1.66 ± 0.09	1.66 ± 0.09	2.02 ± 0.30	2.24 ± 0.23	2.30 ± 0.14	2.34 ± 0.19	2.32 ± 0.22
	gini	.571 ± .025	.567 ± .024	.542 ± .042	.535 ± .029	.552 ± .045	.538 ± .045	.527 ± .046
	time	1.1 ± 0.0	1.1 ± 0.0	1.3 ± 0.3	2.1 ± 0.2	2.2 ± 0.3	2.3 ± 0.3	2.3 ± 0.2
heart	acc	.756 ± .065	.743 ± .060	.759 ± .071	.786 ± .063	.746 ± .066	.753 ± .061	.736 ± .062
	nodes	1.0 ± 0.0	1.3 ± 0.6	7.0 ± 2.2	27.3 ± 5.2	46.9 ± 6.7	52.4 ± 6.2	54.1 ± 6.2
	path len	1.00 ± 0.00	1.13 ± 0.29	2.88 ± 0.57	4.73 ± 0.68	5.61 ± 0.69	6.27 ± 0.53	6.33 ± 0.62
	gini	.912 ± .009	.908 ± .015	.872 ± .022	.852 ± .019	.844 ± .017	.850 ± .014	.853 ± .015
	time	0.9 ± 0.1	1.0 ± 0.2	2.7 ± 0.7	8.7 ± 1.5	11.0 ± 1.5	12.4 ± 1.4	12.6 ± 1.3
dry-bean	acc	.714 ± .086	.863 ± .007	.871 ± .014	.878 ± .010	.890 ± .010	.893 ± .009	.895 ± .010
	nodes	3.6 ± 0.7	6.6 ± 0.5	8.5 ± 0.9	13.9 ± 1.8	26.0 ± 2.1	43.0 ± 7.4	97.5 ± 12.4
	path len	2.36 ± 0.31	3.17 ± 0.17	3.50 ± 0.23	4.23 ± 0.29	5.07 ± 0.25	5.44 ± 0.30	6.73 ± 0.31
	gini	.830 ± .033	.806 ± .029	.719 ± .047	.583 ± .066	.524 ± .066	.516 ± .052	.509 ± .030
	time	3.3 ± 0.4	4.9 ± 0.3	6.0 ± 0.6	8.4 ± 0.9	13.9 ± 1.3	70.2 ± 22.0	120.3 ± 23.2
wine	acc	.893 ± .030	.927 ± .051	.905 ± .050	.911 ± .051	.938 ± .047	.922 ± .044	.949 ± .060
	nodes	2.0 ± 0.0	2.4 ± 0.5	5.7 ± 2.1	7.4 ± 1.9	8.1 ± 1.0	9.2 ± 2.4	10.7 ± 3.0
	path len	1.67 ± 0.12	1.79 ± 0.21	2.77 ± 0.64	3.12 ± 0.37	3.21 ± 0.23	3.53 ± 0.39	3.73 ± 0.50
	gini	.815 ± .043	.811 ± .054	.729 ± .056	.699 ± .054	.677 ± .082	.685 ± .068	.680 ± .057
	time	0.9 ± 0.1	1.0 ± 0.1	1.8 ± 0.5	2.2 ± 0.4	2.4 ± 0.2	2.6 ± 0.5	2.9 ± 0.7
car	acc	.700 ± .044	.700 ± .044	.740 ± .098	.780 ± .102	.800 ± .094	.806 ± .111	.790 ± .126
	nodes	0.0 ± 0.0	0.3 ± 0.5	4.9 ± 3.2	15.4 ± 5.8	39.5 ± 23.4	83.5 ± 57.5	105.4 ± 59.8
	path len	0.00 ± 0.00	0.30 ± 0.46	2.43 ± 1.05	4.16 ± 0.92	5.10 ± 0.92	6.22 ± 1.67	6.62 ± 1.34
	gini	.000 ± .000	.266 ± .407	.896 ± .015	.889 ± .012	.879 ± .014	.870 ± .015	.871 ± .012
	time	0.3 ± 0.0	0.4 ± 0.2	1.9 ± 0.9	4.8 ± 1.4	10.4 ± 5.5	20.6 ± 13.0	25.3 ± 13.1
wdbc	acc	.937 ± .029	.942 ± .029	.942 ± .029	.928 ± .041	.928 ± .036	.924 ± .038	.930 ± .044
	nodes	1.0 ± 0.0	1.1 ± 0.3	2.9 ± 0.8	10.0 ± 2.9	18.2 ± 1.6	22.2 ± 2.8	25.7 ± 2.5
	path len	1.00 ± 0.00	1.04 ± 0.13	1.77 ± 0.37	3.37 ± 0.62	4.56 ± 0.60	5.03 ± 0.85	6.05 ± 0.94
	gini	.917 ± .022	.915 ± .022	.925 ± .012	.878 ± .025	.842 ± .023	.816 ± .027	.786 ± .054
	time	0.7 ± 0.0	0.8 ± 0.1	1.2 ± 0.2	3.0 ± 0.8	5.0 ± 0.5	6.0 ± 0.7	6.9 ± 0.6
sonar	acc	.716 ± .102	.735 ± .085	.764 ± .052	.754 ± .084	.764 ± .118	.774 ± .084	.763 ± .089
	nodes	1.0 ± 0.0	3.9 ± 0.7	11.6 ± 1.4	19.2 ± 2.7	18.7 ± 2.1	18.9 ± 2.2	19.5 ± 2.2
	path len	1.00 ± 0.00	2.25 ± 0.20	4.01 ± 0.46	5.25 ± 0.55	5.10 ± 0.63	5.07 ± 0.65	5.07 ± 0.73
	gini	.888 ± .069	.921 ± .035	.890 ± .037	.875 ± .034	.891 ± .020	.888 ± .044	.886 ± .025
	time	0.7 ± 0.0	1.5 ± 0.2	3.2 ± 0.3	4.9 ± 0.7	4.8 ± 0.6	4.8 ± 0.5	5.0 ± 0.5
pendigits	acc	.094 ± .003	.718 ± .041	.791 ± .024	.869 ± .018	.902 ± .011	.922 ± .010	.931 ± .006
	nodes	0.0 ± 0.0	8.4 ± 0.9	15.0 ± 1.6	34.3 ± 2.2	78.0 ± 3.3	180.0 ± 12.1	408.8 ± 21.5
	path len	0.00 ± 0.00	3.81 ± 0.27	4.46 ± 0.26	5.79 ± 0.17	6.77 ± 0.26	8.06 ± 0.30	9.26 ± 0.19
	gini	.000 ± .000	.527 ± .070	.407 ± .104	.317 ± .071	.294 ± .063	.295 ± .047	.288 ± .036
	time	0.4 ± 0.0	5.6 ± 0.6	8.8 ± 0.8	17.0 ± 0.9	33.9 ± 1.2	78.3 ± 12.4	143.9 ± 8.4
ionosphere	acc	.875 ± .048	.903 ± .055	.914 ± .036	.906 ± .060	.903 ± .045	.912 ± .049	.897 ± .050
	nodes	1.0 ± 0.0	1.9 ± 0.3	3.7 ± 1.4	10.5 ± 2.4	17.1 ± 4.1	22.3 ± 3.3	21.3 ± 3.0
	path len	1.00 ± 0.00	1.64 ± 0.22	2.42 ± 0.49	4.20 ± 0.86	6.03 ± 1.07	7.26 ± 1.46	7.16 ± 1.34
	gini	.750 ± .009	.767 ± .040	.769 ± .031	.768 ± .038	.783 ± .042	.774 ± .045	.787 ± .045
	time	0.7 ± 0.0	1.0 ± 0.1	1.3 ± 0.3	2.9 ± 0.6	4.5 ± 1.0	14.4 ± 2.0	7.4 ± 1.0

Ours: prototype features, L1 regularization, non-random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.940 ± .055	.940 ± .055	.933 ± .067	.927 ± .055	.927 ± .063	.900 ± .086	.933 ± .067
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.3 ± 0.8	5.3 ± 0.6	5.9 ± 0.9	6.3 ± 1.0	6.1 ± 0.8
	path len	1.66 ± 0.09	1.66 ± 0.09	2.02 ± 0.30	2.27 ± 0.22	2.30 ± 0.15	2.35 ± 0.18	2.27 ± 0.18
	gini	.571 ± .025	.569 ± .025	.542 ± .042	.524 ± .052	.552 ± .045	.539 ± .045	.535 ± .036
	time	1.0 ± 0.0	1.0 ± 0.0	1.4 ± 0.2	1.9 ± 0.2	2.0 ± 0.2	2.1 ± 0.3	2.1 ± 0.2
heart	acc	.756 ± .065	.743 ± .060	.759 ± .071	.782 ± .066	.743 ± .073	.753 ± .060	.733 ± .059
	nodes	1.0 ± 0.0	1.3 ± 0.6	7.0 ± 2.2	27.3 ± 5.2	46.9 ± 6.7	52.4 ± 6.2	54.1 ± 6.2
	path len	1.00 ± 0.00	1.14 ± 0.29	2.88 ± 0.57	4.73 ± 0.68	5.60 ± 0.69	6.27 ± 0.53	6.34 ± 0.62
	gini	.912 ± .009	.908 ± .015	.872 ± .022	.852 ± .019	.844 ± .016	.850 ± .014	.853 ± .015
	time	0.7 ± 0.0	0.8 ± 0.2	2.0 ± 0.5	6.1 ± 1.2	9.5 ± 1.2	10.6 ± 1.2	11.0 ± 1.1
dry-bean	acc	.713 ± .086	.863 ± .007	.870 ± .016	.876 ± .011	.886 ± .010	.890 ± .008	.891 ± .009
	nodes	3.6 ± 0.7	6.6 ± 0.5	8.5 ± 0.9	13.9 ± 1.8	26.0 ± 2.1	43.0 ± 7.4	97.5 ± 12.4
	path len	2.37 ± 0.31	3.17 ± 0.17	3.50 ± 0.22	4.23 ± 0.29	5.07 ± 0.25	5.45 ± 0.31	6.74 ± 0.32
	gini	.830 ± .033	.806 ± .029	.720 ± .047	.583 ± .066	.525 ± .066	.516 ± .052	.509 ± .030
	time	3.0 ± 0.4	4.6 ± 0.3	5.5 ± 0.5	7.7 ± 0.8	12.4 ± 1.0	65.5 ± 21.2	75.9 ± 20.9
wine	acc	.899 ± .033	.927 ± .051	.905 ± .050	.905 ± .050	.938 ± .047	.911 ± .051	.949 ± .060
	nodes	2.0 ± 0.0	2.4 ± 0.5	5.7 ± 2.1	7.4 ± 1.9	8.1 ± 1.0	9.2 ± 2.4	10.7 ± 3.0
	path len	1.66 ± 0.12	1.78 ± 0.22	2.77 ± 0.64	3.12 ± 0.38	3.19 ± 0.22	3.54 ± 0.41	3.70 ± 0.49
	gini	.815 ± .043	.811 ± .054	.730 ± .056	.697 ± .050	.680 ± .075	.684 ± .069	.682 ± .055
	time	0.9 ± 0.1	1.0 ± 0.1	1.7 ± 0.4	2.0 ± 0.4	2.1 ± 0.2	2.4 ± 0.5	2.6 ± 0.6
car	acc	.700 ± .044	.700 ± .044	.740 ± .098	.779 ± .103	.797 ± .096	.807 ± .111	.794 ± .126
	nodes	0.0 ± 0.0	0.3 ± 0.5	4.9 ± 3.2	15.4 ± 5.8	39.5 ± 23.4	83.5 ± 57.5	105.4 ± 59.8
	path len	0.00 ± 0.00	0.30 ± 0.46	2.43 ± 1.05	4.16 ± 0.92	5.10 ± 0.92	6.22 ± 1.67	6.62 ± 1.34
	gini	.000 ± .000	.266 ± .407	.896 ± .015	.889 ± .012	.879 ± .014	.870 ± .014	.871 ± .012
	time	0.3 ± 0.0	0.4 ± 0.2	1.7 ± 0.8	4.3 ± 1.3	9.3 ± 4.9	18.5 ± 11.9	22.3 ± 11.6
wdbc	acc	.937 ± .029	.942 ± .029	.945 ± .032	.928 ± .041	.930 ± .037	.923 ± .036	.931 ± .046
	nodes	1.0 ± 0.0	1.1 ± 0.3	2.9 ± 0.8	10.0 ± 2.9	18.2 ± 1.6	22.2 ± 2.8	25.7 ± 2.5
	path len	1.00 ± 0.00	1.04 ± 0.13	1.77 ± 0.36	3.38 ± 0.61	4.56 ± 0.60	5.03 ± 0.85	6.05 ± 0.95
	gini	.917 ± .022	.915 ± .022	.925 ± .012	.878 ± .025	.842 ± .023	.815 ± .027	.786 ± .054
	time	0.7 ± 0.0	0.7 ± 0.1	1.1 ± 0.2	2.7 ± 0.7	4.5 ± 0.4	5.4 ± 0.7	6.1 ± 0.6
sonar	acc	.716 ± .102	.735 ± .085	.769 ± .060	.754 ± .080	.759 ± .121	.778 ± .076	.763 ± .074
	nodes	1.0 ± 0.0	3.9 ± 0.7	11.7 ± 1.5	19.2 ± 2.7	18.7 ± 2.1	18.9 ± 2.2	19.5 ± 2.2
	path len	1.00 ± 0.00	2.24 ± 0.20	4.04 ± 0.46	5.23 ± 0.56	5.12 ± 0.63	5.07 ± 0.66	5.08 ± 0.75
	gini	.888 ± .069	.921 ± .035	.890 ± .036	.875 ± .034	.891 ± .020	.888 ± .045	.886 ± .025
	time	0.7 ± 0.0	1.3 ± 0.2	2.9 ± 0.3	4.4 ± 0.6	4.3 ± 0.5	4.3 ± 0.5	4.5 ± 0.5
pendigits	acc	.094 ± .003	.718 ± .041	.790 ± .024	.868 ± .018	.901 ± .012	.921 ± .011	.930 ± .006
	nodes	0.0 ± 0.0	8.4 ± 0.9	15.0 ± 1.6	34.3 ± 2.2	78.0 ± 3.3	180.0 ± 12.1	408.8 ± 21.5
	path len	0.00 ± 0.00	3.81 ± 0.27	4.46 ± 0.26	5.79 ± 0.17	6.77 ± 0.26	8.06 ± 0.30	9.26 ± 0.19
	gini	.000 ± .000	.527 ± .070	.407 ± .104	.317 ± .071	.294 ± .063	.295 ± .047	.288 ± .036
	time	0.4 ± 0.0	5.2 ± 0.5	6.3 ± 0.5	11.6 ± 0.6	22.3 ± 0.8	55.3 ± 3.1	114.9 ± 6.4
ionosphere	acc	.875 ± .048	.903 ± .055	.917 ± .037	.909 ± .058	.906 ± .041	.912 ± .049	.897 ± .050
	nodes	1.0 ± 0.0	1.9 ± 0.3	3.7 ± 1.4	10.5 ± 2.4	17.1 ± 4.1	22.3 ± 3.3	21.3 ± 3.0
	path len	1.00 ± 0.00	1.64 ± 0.22	2.42 ± 0.50	4.20 ± 0.87	6.05 ± 1.09	7.22 ± 1.46	7.18 ± 1.35
	gini	.750 ± .009	.767 ± .040	.769 ± .031	.768 ± .038	.783 ± .042	.774 ± .045	.787 ± .045
	time	0.7 ± 0.0	0.9 ± 0.1	1.2 ± 0.3	2.6 ± 0.5	4.1 ± 0.9	6.8 ± 0.9	6.6 ± 0.9

Ours: linear features, L2 regularization, random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.960 ± .044	.947 ± .065	.960 ± .053	.947 ± .050	.953 ± .043	.960 ± .033	.980 ± .031
	nodes	2.0 ± 0.0	2.8 ± 0.4	2.4 ± 0.5	7.0 ± 1.9	10.6 ± 0.9	10.9 ± 1.5	19.0 ± 2.2
	path len	1.67 ± 0.09	1.96 ± 0.23	1.79 ± 0.20	2.59 ± 0.31	2.91 ± 0.42	2.86 ± 0.28	3.82 ± 0.46
	gini	.627 ± .005	.602 ± .048	.611 ± .018	.681 ± .036	.681 ± .031	.601 ± .045	.633 ± .036
	time	0.6 ± 0.0	0.8 ± 0.1	0.7 ± 0.1	1.8 ± 0.4	2.8 ± 0.2	2.9 ± 0.3	4.7 ± 0.5
heart	acc	.819 ± .048	.802 ± .066	.772 ± .048	.753 ± .078	.756 ± .066	.763 ± .039	.743 ± .063
	nodes	1.0 ± 0.0	1.2 ± 0.4	6.2 ± 0.7	19.7 ± 2.2	22.6 ± 2.5	25.9 ± 2.5	26.1 ± 3.6
	path len	1.00 ± 0.00	1.10 ± 0.21	3.12 ± 0.46	4.76 ± 0.98	5.82 ± 0.53	7.00 ± 0.70	6.12 ± 0.82
	gini	.924 ± .006	.925 ± .004	.920 ± .008	.921 ± .004	.919 ± .003	.919 ± .003	.920 ± .004
	time	0.5 ± 0.0	0.6 ± 0.1	1.9 ± 0.2	5.1 ± 0.5	4.2 ± 0.4	4.7 ± 0.4	4.8 ± 0.6
dry-bean	acc	.668 ± .012	.906 ± .008	.917 ± .006	.918 ± .005	.920 ± .006	.923 ± .007	.924 ± .005
	nodes	3.0 ± 0.0	6.0 ± 0.0	8.0 ± 0.0	10.0 ± 1.3	19.9 ± 1.4	28.4 ± 5.4	66.0 ± 7.9
	path len	2.00 ± 0.00	2.95 ± 0.11	3.38 ± 0.04	3.71 ± 0.29	5.14 ± 0.43	4.88 ± 0.46	6.05 ± 0.37
	gini	.907 ± .005	.905 ± .003	.908 ± .002	.908 ± .003	.908 ± .004	.906 ± .003	.908 ± .002
	time	2.6 ± 0.1	4.3 ± 0.1	5.4 ± 0.1	6.4 ± 0.7	11.3 ± 0.8	23.8 ± 6.3	66.0 ± 9.6
wine	acc	.966 ± .027	.961 ± .026	.960 ± .026	.960 ± .026	.966 ± .028	.966 ± .027	.966 ± .028
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.1 ± 0.3	2.2 ± 0.4	2.1 ± 0.3	2.1 ± 0.3	2.4 ± 0.7
	path len	1.66 ± 0.10	1.66 ± 0.10	1.69 ± 0.14	1.72 ± 0.17	1.69 ± 0.14	1.68 ± 0.14	1.79 ± 0.25
	gini	.872 ± .009	.873 ± .009	.874 ± .007	.876 ± .010	.873 ± .010	.873 ± .011	.876 ± .013
	time	0.6 ± 0.0	0.6 ± 0.0	0.6 ± 0.1	0.6 ± 0.1	0.6 ± 0.0	0.6 ± 0.1	0.6 ± 0.1
car	acc	.700 ± .044	.910 ± .025	.940 ± .022	.983 ± .011	.993 ± .009	.989 ± .011	.991 ± .006
	nodes	0.0 ± 0.0	2.0 ± 0.0	4.1 ± 0.9	10.1 ± 1.8	17.6 ± 2.2	26.4 ± 4.8	25.8 ± 5.6
	path len	0.00 ± 0.00	1.32 ± 0.04	1.68 ± 0.18	2.47 ± 0.21	3.04 ± 0.50	4.33 ± 0.86	3.99 ± 0.85
	gini	.000 ± .000	.921 ± .001	.914 ± .008	.905 ± .012	.904 ± .007	.915 ± .011	.909 ± .009
	time	0.2 ± 0.0	0.9 ± 0.0	1.5 ± 0.3	3.0 ± 0.5	4.9 ± 0.6	7.4 ± 1.3	7.2 ± 1.5
wdbc	acc	.963 ± .025	.965 ± .024	.967 ± .025	.961 ± .023	.963 ± .025	.961 ± .030	.968 ± .028
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	2.5 ± 1.4	5.2 ± 1.4	13.1 ± 2.8	19.6 ± 4.4
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	1.54 ± 0.40	2.42 ± 0.38	3.53 ± 0.51	4.51 ± 0.63
	gini	.955 ± .003	.955 ± .002	.955 ± .003	.953 ± .002	.951 ± .003	.951 ± .002	.952 ± .002
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	1.0 ± 0.4	1.7 ± 0.4	3.7 ± 0.8	5.3 ± 1.1
sonar	acc	.736 ± .109	.788 ± .070	.807 ± .058	.807 ± .089	.793 ± .118	.860 ± .034	.822 ± .081
	nodes	1.0 ± 0.0	2.3 ± 0.5	4.8 ± 1.2	5.5 ± 1.0	5.7 ± 1.4	6.4 ± 2.2	6.5 ± 1.9
	path len	1.00 ± 0.00	1.65 ± 0.21	2.65 ± 0.55	2.82 ± 0.54	2.81 ± 0.50	3.14 ± 0.69	3.16 ± 0.69
	gini	.976 ± .001	.975 ± .001	.975 ± .001	.974 ± .001	.974 ± .001	.974 ± .001	.974 ± .001
	time	0.5 ± 0.0	0.9 ± 0.1	1.5 ± 0.3	1.7 ± 0.3	1.8 ± 0.4	1.9 ± 0.5	2.0 ± 0.4
pendigits	acc	.094 ± .003	.906 ± .025	.948 ± .012	.977 ± .007	.983 ± .004	.989 ± .003	.989 ± .003
	nodes	0.0 ± 0.0	9.2 ± 0.6	12.4 ± 1.6	20.1 ± 2.0	31.5 ± 3.2	56.1 ± 6.0	123.5 ± 10.5
	path len	0.00 ± 0.00	3.99 ± 0.12	4.74 ± 0.45	5.01 ± 0.42	5.80 ± 0.29	6.33 ± 0.36	8.10 ± 0.53
	gini	.000 ± .000	.908 ± .001	.900 ± .006	.899 ± .005	.898 ± .004	.900 ± .005	.897 ± .004
	time	0.3 ± 0.0	4.6 ± 0.2	5.9 ± 0.6	8.6 ± 0.7	12.2 ± 1.0	25.6 ± 2.9	47.6 ± 4.0
ionosphere	acc	.843 ± .093	.909 ± .040	.912 ± .039	.923 ± .048	.926 ± .039	.935 ± .040	.926 ± .043
	nodes	1.0 ± 0.0	2.8 ± 0.4	3.6 ± 0.7	4.5 ± 0.7	8.2 ± 1.8	12.4 ± 2.9	13.5 ± 3.6
	path len	1.00 ± 0.00	2.25 ± 0.26	2.75 ± 0.49	3.28 ± 0.33	3.96 ± 0.61	5.22 ± 0.88	5.13 ± 0.85
	gini	.952 ± .004	.934 ± .007	.927 ± .010	.925 ± .004	.933 ± .011	.935 ± .007	.938 ± .005
	time	0.5 ± 0.0	1.0 ± 0.1	1.2 ± 0.2	1.4 ± 0.2	2.3 ± 0.5	3.3 ± 0.7	3.6 ± 1.0

Ours: linear features, L2 regularization, random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.967 ± .045	.953 ± .067	.960 ± .053	.953 ± .052	.953 ± .043	.967 ± .033	.980 ± .031
	nodes	2.0 ± 0.0	2.8 ± 0.4	2.4 ± 0.5	7.0 ± 1.9	10.6 ± 0.9	10.9 ± 1.5	19.0 ± 2.2
	path len	1.67 ± 0.09	1.95 ± 0.23	1.79 ± 0.20	2.62 ± 0.34	2.95 ± 0.50	2.90 ± 0.26	3.82 ± 0.48
	gini	.627 ± .005	.602 ± .048	.611 ± .018	.680 ± .037	.681 ± .030	.599 ± .044	.633 ± .036
	time	0.7 ± 0.0	0.9 ± 0.1	0.8 ± 0.1	1.8 ± 0.4	2.5 ± 0.2	2.5 ± 0.3	4.1 ± 0.4
heart	acc	.815 ± .051	.802 ± .066	.762 ± .046	.753 ± .070	.753 ± .068	.762 ± .053	.746 ± .051
	nodes	1.0 ± 0.0	1.2 ± 0.4	6.2 ± 0.7	19.7 ± 2.2	22.6 ± 2.5	25.9 ± 2.5	26.1 ± 3.6
	path len	1.00 ± 0.00	1.11 ± 0.22	3.13 ± 0.46	4.78 ± 0.99	5.86 ± 0.54	7.10 ± 0.78	6.16 ± 0.79
	gini	.924 ± .006	.925 ± .004	.919 ± .008	.921 ± .004	.919 ± .003	.919 ± .003	.920 ± .003
	time	0.5 ± 0.0	0.6 ± 0.1	1.7 ± 0.2	4.5 ± 0.5	3.7 ± 0.4	4.2 ± 0.4	4.2 ± 0.6
dry-bean	acc	.673 ± .011	.903 ± .009	.914 ± .006	.915 ± .007	.913 ± .008	.920 ± .008	.920 ± .006
	nodes	3.0 ± 0.0	6.0 ± 0.0	8.0 ± 0.0	10.0 ± 1.3	19.9 ± 1.4	28.4 ± 5.4	66.0 ± 7.9
	path len	2.00 ± 0.00	2.95 ± 0.12	3.38 ± 0.04	3.72 ± 0.30	5.17 ± 0.44	4.90 ± 0.48	6.09 ± 0.39
	gini	.907 ± .005	.905 ± .003	.908 ± .002	.908 ± .003	.908 ± .004	.906 ± .003	.908 ± .002
	time	2.5 ± 0.1	4.0 ± 0.1	4.9 ± 0.1	5.8 ± 0.6	9.9 ± 0.7	36.6 ± 13.6	38.4 ± 9.7
wine	acc	.966 ± .027	.961 ± .026	.960 ± .026	.960 ± .026	.966 ± .028	.966 ± .027	.966 ± .028
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.1 ± 0.3	2.2 ± 0.4	2.1 ± 0.3	2.1 ± 0.3	2.4 ± 0.7
	path len	1.66 ± 0.10	1.66 ± 0.10	1.68 ± 0.14	1.72 ± 0.17	1.68 ± 0.14	1.69 ± 0.14	1.79 ± 0.25
	gini	.872 ± .009	.873 ± .009	.874 ± .007	.876 ± .010	.873 ± .010	.873 ± .011	.876 ± .013
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.1	0.5 ± 0.1	0.5 ± 0.1	0.6 ± 0.1
car	acc	.700 ± .044	.897 ± .026	.937 ± .025	.983 ± .011	.993 ± .009	.989 ± .011	.991 ± .006
	nodes	0.0 ± 0.0	2.0 ± 0.0	4.1 ± 0.9	10.1 ± 1.8	17.6 ± 2.2	26.4 ± 4.8	25.8 ± 5.6
	path len	0.00 ± 0.00	1.32 ± 0.04	1.68 ± 0.18	2.48 ± 0.22	3.05 ± 0.50	4.33 ± 0.85	3.98 ± 0.84
	gini	.000 ± .000	.921 ± .001	.914 ± .008	.905 ± .011	.904 ± .007	.915 ± .011	.909 ± .009
	time	0.2 ± 0.0	0.8 ± 0.0	1.3 ± 0.2	2.7 ± 0.4	4.4 ± 0.5	6.5 ± 1.1	6.3 ± 1.3
wdbc	acc	.961 ± .025	.963 ± .023	.967 ± .025	.961 ± .025	.960 ± .022	.961 ± .025	.963 ± .025
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	2.5 ± 1.4	5.2 ± 1.4	13.1 ± 2.8	19.6 ± 4.4
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	1.54 ± 0.41	2.42 ± 0.38	3.53 ± 0.53	4.53 ± 0.62
	gini	.955 ± .003	.955 ± .002	.955 ± .003	.953 ± .002	.951 ± .003	.951 ± .002	.952 ± .002
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.0	0.9 ± 0.3	1.5 ± 0.3	3.2 ± 0.7	4.6 ± 0.9
sonar	acc	.731 ± .111	.783 ± .067	.807 ± .058	.803 ± .093	.789 ± .115	.851 ± .045	.817 ± .077
	nodes	1.0 ± 0.0	2.3 ± 0.5	4.8 ± 1.2	5.5 ± 1.0	5.7 ± 1.4	6.4 ± 2.2	6.5 ± 1.9
	path len	1.00 ± 0.00	1.65 ± 0.22	2.65 ± 0.56	2.79 ± 0.56	2.81 ± 0.50	3.15 ± 0.67	3.16 ± 0.70
	gini	.976 ± .001	.975 ± .001	.975 ± .001	.974 ± .001	.974 ± .001	.974 ± .001	.974 ± .001
	time	0.5 ± 0.0	0.9 ± 0.1	1.4 ± 0.2	1.6 ± 0.2	1.6 ± 0.3	1.7 ± 0.4	1.8 ± 0.4
pendigits	acc	.094 ± .003	.905 ± .025	.946 ± .012	.974 ± .007	.980 ± .004	.987 ± .002	.989 ± .003
	nodes	0.0 ± 0.0	9.2 ± 0.6	12.4 ± 1.6	20.1 ± 2.0	31.5 ± 3.2	56.1 ± 6.0	123.5 ± 10.5
	path len	0.00 ± 0.00	3.99 ± 0.13	4.74 ± 0.45	5.01 ± 0.42	5.80 ± 0.28	6.33 ± 0.36	8.12 ± 0.53
	gini	.000 ± .000	.908 ± .001	.900 ± .006	.899 ± .005	.898 ± .004	.900 ± .005	.898 ± .004
	time	0.3 ± 0.0	4.2 ± 0.2	5.3 ± 0.6	7.7 ± 0.6	10.8 ± 0.9	20.8 ± 2.1	41.1 ± 3.3
ionosphere	acc	.843 ± .093	.915 ± .042	.914 ± .042	.932 ± .053	.926 ± .043	.929 ± .041	.923 ± .051
	nodes	1.0 ± 0.0	2.8 ± 0.4	3.6 ± 0.7	4.5 ± 0.7	8.2 ± 1.8	12.4 ± 2.9	13.5 ± 3.6
	path len	1.00 ± 0.00	2.26 ± 0.26	2.79 ± 0.53	3.35 ± 0.37	4.02 ± 0.63	5.30 ± 0.91	5.22 ± 0.87
	gini	.952 ± .004	.934 ± .007	.928 ± .010	.925 ± .004	.934 ± .011	.935 ± .007	.938 ± .005
	time	0.5 ± 0.0	0.9 ± 0.1	1.1 ± 0.1	1.3 ± 0.1	2.0 ± 0.4	3.0 ± 0.6	3.3 ± 1.0

Ours: prototype features, L2 regularization, random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.953 ± .043	.947 ± .065	.947 ± .050	.953 ± .043	.953 ± .043	.953 ± .052	.947 ± .040
	nodes	2.0 ± 0.0	2.1 ± 0.3	3.2 ± 1.1	5.6 ± 1.9	10.5 ± 2.9	17.4 ± 3.1	22.1 ± 3.6
	path len	1.67 ± 0.09	1.71 ± 0.17	2.01 ± 0.30	2.52 ± 0.24	2.96 ± 0.49	3.49 ± 0.34	3.97 ± 0.57
	gini	.674 ± .041	.666 ± .018	.670 ± .041	.664 ± .027	.670 ± .032	.670 ± .033	.679 ± .032
	time	1.1 ± 0.0	1.2 ± 0.2	1.5 ± 0.3	2.2 ± 0.6	5.3 ± 1.3	8.4 ± 1.4	7.0 ± 1.1
heart	acc	.809 ± .038	.812 ± .035	.795 ± .041	.769 ± .034	.762 ± .059	.749 ± .061	.743 ± .035
	nodes	1.0 ± 0.0	1.0 ± 0.0	4.5 ± 0.8	13.9 ± 2.0	37.8 ± 3.7	60.1 ± 5.3	85.7 ± 5.1
	path len	1.00 ± 0.00	1.00 ± 0.00	2.40 ± 0.32	4.00 ± 0.43	5.30 ± 0.58	6.49 ± 0.53	7.67 ± 0.53
	gini	.935 ± .003	.937 ± .003	.931 ± .004	.916 ± .005	.904 ± .006	.894 ± .008	.894 ± .008
	time	0.7 ± 0.1	0.7 ± 0.1	1.6 ± 0.2	3.8 ± 0.4	9.3 ± 0.7	14.2 ± 1.2	20.1 ± 1.2
dry-bean	acc	.665 ± .045	.875 ± .037	.905 ± .011	.911 ± .008	.913 ± .006	.913 ± .009	.915 ± .007
	nodes	2.9 ± 0.3	5.6 ± 0.5	7.3 ± 0.8	14.1 ± 1.5	31.1 ± 2.5	56.0 ± 6.0	127.0 ± 9.0
	path len	2.04 ± 0.26	2.77 ± 0.11	3.25 ± 0.26	4.17 ± 0.30	5.23 ± 0.27	5.94 ± 0.33	7.56 ± 0.53
	gini	.881 ± .004	.871 ± .005	.855 ± .017	.784 ± .027	.708 ± .040	.647 ± .041	.559 ± .056
	time	2.9 ± 0.2	4.4 ± 0.3	5.4 ± 0.5	9.1 ± 0.8	17.3 ± 1.2	49.4 ± 7.9	67.0 ± 9.7
wine	acc	.955 ± .033	.978 ± .027	.967 ± .037	.960 ± .036	.961 ± .050	.956 ± .042	.949 ± .039
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.6 ± 0.5	4.9 ± 1.6	6.9 ± 2.4	12.2 ± 4.4	18.6 ± 6.7
	path len	1.66 ± 0.10	1.66 ± 0.09	1.90 ± 0.25	2.51 ± 0.41	2.91 ± 0.47	3.75 ± 0.72	4.83 ± 0.80
	gini	.892 ± .004	.886 ± .005	.875 ± .011	.842 ± .033	.825 ± .032	.802 ± .050	.758 ± .031
	time	0.9 ± 0.0	0.9 ± 0.0	1.1 ± 0.2	1.6 ± 0.4	2.1 ± 0.5	3.3 ± 1.0	4.7 ± 1.5
car	acc	.700 ± .044	.801 ± .060	.910 ± .028	.956 ± .018	.975 ± .016	.985 ± .015	.992 ± .009
	nodes	0.0 ± 0.0	0.7 ± 0.5	4.5 ± 1.7	13.1 ± 4.5	24.4 ± 5.2	39.8 ± 5.9	55.1 ± 8.3
	path len	0.00 ± 0.00	0.70 ± 0.46	2.00 ± 0.30	2.88 ± 0.50	3.36 ± 0.40	3.80 ± 0.63	3.65 ± 0.49
	gini	.000 ± .000	.644 ± .422	.896 ± .011	.875 ± .009	.839 ± .020	.803 ± .034	.775 ± .036
	time	0.3 ± 0.0	0.7 ± 0.2	1.8 ± 0.6	3.9 ± 1.1	6.7 ± 1.3	10.2 ± 1.3	13.6 ± 2.0
wdbc	acc	.967 ± .012	.979 ± .017	.968 ± .013	.968 ± .022	.965 ± .026	.960 ± .029	.958 ± .033
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.8 ± 0.6	4.3 ± 1.0	11.6 ± 2.5	30.4 ± 4.2	52.9 ± 6.4
	path len	1.00 ± 0.00	1.00 ± 0.00	1.42 ± 0.33	2.44 ± 0.36	3.80 ± 0.47	5.09 ± 0.77	6.08 ± 0.48
	gini	.942 ± .002	.942 ± .001	.931 ± .010	.930 ± .011	.902 ± .015	.845 ± .019	.841 ± .031
	time	0.7 ± 0.0	0.7 ± 0.0	1.0 ± 0.2	1.5 ± 0.2	3.3 ± 0.7	8.0 ± 1.1	13.3 ± 1.6
sonar	acc	.534 ± .144	.821 ± .074	.889 ± .058	.826 ± .052	.822 ± .060	.860 ± .094	.812 ± .057
	nodes	0.0 ± 0.0	2.6 ± 0.5	7.8 ± 1.2	12.3 ± 1.7	14.7 ± 2.5	20.4 ± 7.4	30.2 ± 7.4
	path len	0.00 ± 0.00	1.99 ± .33	3.69 ± .45	4.54 ± .41	4.90 ± .44	5.69 ± .64	6.40 ± 1.05
	gini	.000 ± .000	.972 ± .001	.970 ± .001	.964 ± .003	.952 ± .009	.953 ± .007	.934 ± .012
	time	0.3 ± 0.0	1.2 ± 0.1	2.3 ± 0.3	3.5 ± 0.6	4.0 ± 0.7	5.3 ± 1.7	8.1 ± 2.3
pendigits	acc	.094 ± .003	.855 ± .026	.898 ± .011	.913 ± .014	.935 ± .009	.949 ± .008	.962 ± .005
	nodes	0.0 ± 0.0	8.7 ± 0.5	11.7 ± 1.0	21.2 ± 2.3	55.0 ± 6.0	135.4 ± 8.4	347.3 ± 25.0
	path len	0.00 ± 0.00	4.81 ± .38	4.83 ± .28	5.05 ± .42	6.27 ± .29	7.58 ± .26	9.45 ± .48
	gini	.000 ± .000	.869 ± .011	.846 ± .022	.736 ± .027	.612 ± .046	.533 ± .042	.478 ± .030
	time	0.4 ± 0.0	5.0 ± 0.2	6.2 ± 0.4	9.6 ± 0.9	20.2 ± 1.9	57.6 ± 9.5	135.9 ± 13.0
ionosphere	acc	.914 ± .057	.915 ± .059	.920 ± .046	.926 ± .062	.909 ± .049	.920 ± .033	.917 ± .047
	nodes	1.5 ± 0.5	2.5 ± 0.8	4.2 ± 1.2	8.2 ± 2.6	16.4 ± 1.8	22.2 ± 4.0	22.9 ± 3.4
	path len	1.24 ± .25	1.65 ± .40	2.25 ± .45	3.20 ± .54	4.90 ± .67	5.58 ± 1.09	5.53 ± 0.88
	gini	.953 ± .003	.950 ± .010	.952 ± .004	.935 ± .015	.872 ± .037	.842 ± .031	.820 ± .018
	time	0.8 ± 0.1	1.0 ± 0.2	1.4 ± 0.3	2.4 ± 0.7	4.4 ± 0.5	9.9 ± 7.4	8.1 ± 1.1

Ours: prototype features, L2 regularization, random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.953 ± .043	.947 ± .065	.947 ± .050	.947 ± .050	.967 ± .045	.933 ± .052	.953 ± .052
	nodes	2.0 ± 0.0	2.1 ± 0.3	3.2 ± 1.1	5.6 ± 1.9	10.5 ± 2.9	17.4 ± 3.1	22.1 ± 3.6
	path len	1.67 ± 0.09	1.71 ± 0.17	2.00 ± 0.29	2.51 ± 0.25	2.90 ± 0.46	3.49 ± 0.35	3.95 ± 0.57
	gini	.674 ± .041	.666 ± .017	.670 ± .041	.664 ± .027	.670 ± .032	.669 ± .032	.678 ± .032
	time	1.0 ± 0.0	1.0 ± 0.1	1.3 ± 0.3	2.0 ± 0.5	3.2 ± 0.7	5.0 ± 0.8	6.1 ± 1.0
heart	acc	.805 ± .044	.809 ± .040	.782 ± .043	.753 ± .044	.749 ± .067	.733 ± .042	.716 ± .040
	nodes	1.0 ± 0.0	1.0 ± 0.0	4.5 ± 0.8	13.9 ± 2.0	37.8 ± 3.7	60.1 ± 5.3	85.7 ± 5.1
	path len	1.00 ± 0.00	1.00 ± 0.00	2.41 ± 0.32	4.03 ± 0.48	5.23 ± 0.51	6.49 ± 0.62	7.79 ± 0.50
	gini	.935 ± .003	.937 ± .003	.931 ± .004	.916 ± .006	.904 ± .006	.894 ± .008	.894 ± .008
	time	0.7 ± 0.0	0.7 ± 0.0	1.5 ± 0.2	3.4 ± 0.4	8.0 ± 0.7	12.2 ± 1.0	17.1 ± 0.8
dry-bean	acc	.665 ± .039	.872 ± .037	.904 ± .012	.903 ± .008	.907 ± .006	.909 ± .006	.906 ± .010
	nodes	2.9 ± 0.3	5.6 ± 0.5	7.3 ± 0.8	14.1 ± 1.5	31.1 ± 2.5	56.0 ± 6.0	127.0 ± 9.0
	path len	2.04 ± 0.26	2.77 ± 0.12	3.25 ± 0.27	4.18 ± 0.31	5.26 ± 0.29	5.96 ± 0.38	7.70 ± 0.65
	gini	.881 ± .004	.871 ± .005	.855 ± .017	.784 ± .027	.708 ± .040	.647 ± .041	.560 ± .054
	time	2.7 ± 0.2	4.0 ± 0.2	4.9 ± 0.4	7.8 ± 0.6	13.9 ± 0.9	24.5 ± 7.9	43.8 ± 4.6
wine	acc	.955 ± .033	.978 ± .027	.972 ± .028	.960 ± .036	.949 ± .052	.939 ± .058	.939 ± .039
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.6 ± 0.5	4.9 ± 1.6	6.9 ± 2.4	12.2 ± 4.4	18.6 ± 6.7
	path len	1.66 ± 0.11	1.66 ± 0.10	1.90 ± 0.25	2.53 ± 0.41	2.93 ± 0.49	3.74 ± 0.75	4.89 ± 0.88
	gini	.892 ± .004	.886 ± .005	.875 ± .011	.842 ± .033	.825 ± .032	.802 ± .051	.757 ± .031
	time	0.9 ± 0.1	0.8 ± 0.1	1.0 ± 0.1	1.5 ± 0.3	1.9 ± 0.5	2.9 ± 0.8	4.1 ± 1.2
car	acc	.700 ± .044	.765 ± .036	.906 ± .024	.955 ± .018	.975 ± .016	.984 ± .014	.991 ± .009
	nodes	0.0 ± 0.0	0.7 ± 0.5	4.5 ± 1.7	13.1 ± 4.5	24.4 ± 5.2	39.8 ± 5.9	55.1 ± 8.3
	path len	0.00 ± 0.00	0.70 ± 0.46	2.00 ± 0.30	2.89 ± 0.51	3.35 ± 0.42	3.80 ± 0.63	3.65 ± 0.48
	gini	.000 ± .000	.644 ± .422	.896 ± .011	.875 ± .009	.839 ± .020	.803 ± .034	.775 ± .036
	time	0.3 ± 0.0	0.6 ± 0.2	1.6 ± 0.4	3.4 ± 1.0	6.0 ± 1.0	9.0 ± 1.1	11.8 ± 1.7
wdbc	acc	.967 ± .012	.979 ± .017	.968 ± .013	.968 ± .022	.951 ± .022	.954 ± .030	.947 ± .029
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.8 ± 0.6	4.3 ± 1.0	11.6 ± 2.5	30.4 ± 4.2	52.9 ± 6.4
	path len	1.00 ± 0.00	1.00 ± 0.00	1.42 ± 0.33	2.44 ± 0.36	3.82 ± 0.49	5.12 ± 0.79	6.15 ± 0.52
	gini	.942 ± .002	.942 ± .001	.931 ± .010	.930 ± .011	.902 ± .015	.845 ± .019	.841 ± .031
	time	0.7 ± 0.0	0.7 ± 0.0	0.9 ± 0.2	1.4 ± 0.2	2.9 ± 0.5	6.8 ± 0.9	11.1 ± 1.2
sonar	acc	.534 ± .144	.793 ± .073	.865 ± .061	.851 ± .063	.827 ± .069	.860 ± .066	.808 ± .053
	nodes	0.0 ± 0.0	2.6 ± 0.5	7.8 ± 1.2	12.3 ± 1.7	14.7 ± 2.5	20.4 ± 7.4	30.2 ± 7.4
	path len	0.00 ± 0.00	2.01 ± 0.33	3.73 ± 0.49	4.65 ± 0.48	4.93 ± 0.45	5.91 ± 0.84	6.55 ± 1.10
	gini	.000 ± .000	.972 ± .001	.970 ± .001	.964 ± .003	.952 ± .008	.953 ± .007	.934 ± .012
	time	0.3 ± 0.0	1.1 ± 0.1	2.1 ± 0.2	3.1 ± 0.4	3.6 ± 0.5	4.8 ± 1.4	6.6 ± 1.5
pendigits	acc	.094 ± .003	.852 ± .026	.897 ± .012	.907 ± .015	.923 ± .012	.938 ± .008	.951 ± .007
	nodes	0.0 ± 0.0	8.7 ± 0.5	11.7 ± 1.0	21.2 ± 2.3	55.0 ± 6.0	135.4 ± 8.4	347.3 ± 25.0
	path len	0.00 ± 0.00	4.82 ± 0.38	4.84 ± 0.28	5.06 ± 0.43	6.28 ± 0.31	7.57 ± 0.27	9.46 ± 0.50
	gini	.000 ± .000	.869 ± .011	.846 ± .022	.736 ± .027	.612 ± .046	.533 ± .042	.478 ± .030
	time	0.4 ± 0.0	4.5 ± 0.2	5.4 ± 0.3	8.3 ± 0.7	16.9 ± 1.5	43.9 ± 2.6	99.0 ± 6.9
ionosphere	acc	.917 ± .052	.917 ± .059	.917 ± .049	.923 ± .060	.900 ± .059	.906 ± .043	.900 ± .050
	nodes	1.5 ± 0.5	2.5 ± 0.8	4.2 ± 1.2	8.2 ± 2.6	16.4 ± 1.8	22.2 ± 4.0	22.9 ± 3.4
	path len	1.24 ± 0.25	1.65 ± 0.39	2.23 ± 0.47	3.22 ± 0.58	4.88 ± 0.69	5.66 ± 1.17	5.56 ± 0.91
	gini	.953 ± .003	.950 ± .010	.952 ± .004	.935 ± .016	.872 ± .037	.842 ± .030	.819 ± .019
	time	0.8 ± 0.1	1.0 ± 0.1	1.3 ± 0.3	2.2 ± 0.5	3.9 ± 0.4	7.4 ± 2.0	6.8 ± 0.9

Ours: linear features, L1 regularization, random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.953 ± .043	.960 ± .044	.967 ± .033	.960 ± .053	.960 ± .033	.967 ± .045	.960 ± .044
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.4 ± 1.0	5.9 ± 1.2	10.6 ± 1.7	17.1 ± 1.8	27.1 ± 1.8
	path len	1.67 ± 0.09	1.67 ± 0.09	2.12 ± 0.35	2.55 ± 0.26	3.14 ± 0.40	3.28 ± 0.34	4.32 ± 0.48
	gini	.573 ± .032	.566 ± .018	.547 ± .078	.480 ± .074	.562 ± .049	.481 ± .016	.097 ± .035
	time	0.7 ± 0.1	0.9 ± 0.0	2.4 ± 2.9	22.1 ± 40.2	14.7 ± 35.8	6.5 ± 1.0	36.9 ± 19.5
heart	acc	.819 ± .038	.828 ± .036	.802 ± .046	.789 ± .070	.812 ± .038	.766 ± .064	.786 ± .061
	nodes	1.0 ± 0.0	1.2 ± 0.6	5.3 ± 1.7	19.3 ± 3.8	26.8 ± 5.0	39.7 ± 5.0	57.2 ± 6.8
	path len	1.00 ± 0.00	1.10 ± 0.30	2.72 ± 0.44	4.74 ± 0.55	5.48 ± 0.53	5.57 ± 0.40	6.42 ± 0.58
	gini	.886 ± .017	.864 ± .056	.759 ± .062	.721 ± .056	.710 ± .045	.620 ± .079	.557 ± .064
	time	0.6 ± 0.0	0.6 ± 0.2	1.7 ± 0.4	4.0 ± 0.7	4.1 ± 0.7	5.9 ± 0.7	8.4 ± 1.0
dry-bean	acc	.528 ± .008	.901 ± .009	.913 ± .006	.913 ± .005	.913 ± .008	.919 ± .006	.920 ± .007
	nodes	2.0 ± 0.0	6.0 ± 0.0	7.8 ± 0.4	11.5 ± 1.1	28.0 ± 0.9	58.2 ± 2.9	139.2 ± 15.3
	path len	1.59 ± 0.01	3.04 ± 0.20	3.37 ± 0.09	3.76 ± 0.19	5.41 ± 0.14	6.24 ± 0.26	7.93 ± 0.37
	gini	.753 ± .013	.676 ± .033	.677 ± .040	.540 ± .084	.470 ± .054	.387 ± .070	.398 ± .040
	time	2.4 ± 0.1	5.9 ± 4.5	5.2 ± 0.2	6.8 ± 0.4	13.1 ± 0.4	23.6 ± 0.8	52.3 ± 6.2
wine	acc	.983 ± .025	.977 ± .028	.966 ± .037	.960 ± .036	.966 ± .027	.966 ± .027	.972 ± .028
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.5 ± 0.5	3.1 ± 0.7	4.7 ± 1.1	7.5 ± 1.8	13.2 ± 2.6
	path len	1.67 ± 0.10	1.66 ± 0.10	1.86 ± 0.23	2.10 ± 0.32	2.46 ± 0.19	3.20 ± 0.42	4.11 ± 0.46
	gini	.818 ± .011	.789 ± .011	.814 ± .019	.775 ± .045	.779 ± .032	.756 ± .035	.740 ± .045
	time	0.5 ± 0.0	0.5 ± 0.0	0.5 ± 0.1	0.6 ± 0.1	0.9 ± 0.2	1.3 ± 0.3	2.1 ± 0.4
car	acc	.700 ± .044	.700 ± .044	.949 ± .013	.977 ± .013	.990 ± .011	.991 ± .009	.990 ± .010
	nodes	0.0 ± 0.0	0.0 ± 0.0	5.5 ± 1.0	15.6 ± 4.1	31.3 ± 3.1	36.1 ± 7.3	41.1 ± 9.8
	path len	0.00 ± 0.00	0.00 ± 0.00	2.13 ± 0.25	2.98 ± 0.42	3.41 ± 0.46	3.50 ± 0.50	3.61 ± 0.69
	gini	.000 ± .000	.000 ± .000	.720 ± .019	.703 ± .045	.703 ± .032	.655 ± .033	.663 ± .031
	time	0.3 ± 0.0	0.3 ± 0.0	2.0 ± 0.3	4.7 ± 1.1	8.8 ± 0.9	9.9 ± 2.0	11.3 ± 2.8
wdbc	acc	.970 ± .019	.970 ± .019	.974 ± .018	.968 ± .017	.979 ± .020	.968 ± .022	.979 ± .015
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	3.2 ± 0.6	7.0 ± 3.0	17.4 ± 4.1	34.9 ± 8.9
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	2.01 ± 0.02	2.57 ± 0.49	3.93 ± 0.41	4.73 ± 0.60
	gini	.817 ± .019	.819 ± .022	.816 ± .019	.835 ± .045	.936 ± .008	.936 ± .006	.936 ± .007
	time	0.6 ± 0.0	0.6 ± 0.0	0.6 ± 0.0	1.2 ± 0.2	2.3 ± 0.8	4.9 ± 1.1	9.2 ± 2.3
sonar	acc	.655 ± .169	.841 ± .074	.812 ± .070	.807 ± .078	.817 ± .074	.812 ± .084	.846 ± .076
	nodes	0.9 ± 0.3	2.6 ± 0.7	5.6 ± 0.9	7.0 ± 1.6	12.6 ± 4.8	21.7 ± 3.7	28.9 ± 7.4
	path len	0.90 ± 0.30	1.86 ± 0.28	2.73 ± 0.35	2.98 ± 0.29	3.95 ± 0.65	5.00 ± 0.84	5.67 ± 0.82
	gini	.849 ± .283	.939 ± .014	.899 ± .029	.899 ± .019	.915 ± .020	.933 ± .022	.923 ± .033
	time	0.8 ± 0.1	1.4 ± 0.2	2.6 ± 0.3	3.1 ± 0.6	5.2 ± 1.8	8.4 ± 1.4	8.6 ± 3.1
pendigits	acc	.094 ± .003	.901 ± .008	.940 ± .011	.968 ± .002	.976 ± .004	.977 ± .005	.981 ± .003
	nodes	0.0 ± 0.0	9.0 ± 0.0	11.9 ± 1.6	17.9 ± 1.6	42.2 ± 4.0	120.1 ± 7.1	298.2 ± 9.1
	path len	0.00 ± 0.00	3.96 ± 0.04	4.55 ± 0.45	4.84 ± 0.26	5.91 ± 0.30	7.40 ± 0.18	8.89 ± 0.32
	gini	.000 ± .000	.809 ± .010	.813 ± .024	.764 ± .022	.696 ± .041	.492 ± .046	.320 ± .027
	time	9.5 ± 8.9	7.6 ± 3.3	139.3 ± 83.5	109.5 ± 27.3	202.4 ± 82.2	392.0 ± 106.4	1032.4 ± 182.0
ionosphere	acc	.855 ± .079	.909 ± .062	.926 ± .038	.923 ± .054	.926 ± .050	.920 ± .064	.940 ± .035
	nodes	1.0 ± 0.0	2.4 ± 0.5	3.4 ± 0.5	6.3 ± 1.6	15.7 ± 4.0	32.2 ± 4.6	52.3 ± 5.4
	path len	1.00 ± 0.00	2.02 ± 0.33	2.73 ± 0.41	3.83 ± 0.55	5.60 ± 1.09	8.38 ± 0.77	11.72 ± 0.88
	gini	.910 ± .010	.828 ± .094	.699 ± .090	.663 ± .067	.712 ± .063	.785 ± .048	.853 ± .028
	time	0.6 ± 0.0	1.0 ± 0.1	1.3 ± 0.1	2.0 ± 0.4	2.8 ± 0.8	5.1 ± 0.7	10.4 ± 2.9

Ours: linear features, L1 regularization, random initialization, crisp

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.953 ± .043	.960 ± .044	.960 ± .033	.953 ± .052	.960 ± .044	.960 ± .044	.967 ± .045
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.4 ± 1.0	5.9 ± 1.2	10.6 ± 1.7	17.1 ± 1.8	27.1 ± 1.8
	path len	1.67 ± 0.09	1.67 ± 0.09	2.12 ± 0.35	2.53 ± 0.26	3.14 ± 0.44	3.25 ± 0.35	4.45 ± 0.58
	gini	.573 ± .032	.566 ± .018	.547 ± .078	.469 ± .082	.563 ± .049	.480 ± .015	.099 ± .038
	time	0.4 ± 0.0	0.4 ± 0.0	0.6 ± 0.1	0.9 ± 0.1	1.5 ± 0.2	2.2 ± 0.2	3.4 ± 0.2
heart	acc	.819 ± .039	.828 ± .036	.786 ± .051	.780 ± .066	.812 ± .037	.756 ± .071	.786 ± .064
	nodes	1.0 ± 0.0	1.2 ± 0.6	5.3 ± 1.7	19.3 ± 3.8	26.8 ± 5.0	39.7 ± 5.0	57.2 ± 6.8
	path len	1.00 ± 0.00	1.10 ± 0.30	2.72 ± 0.44	4.72 ± 0.53	5.49 ± 0.53	5.56 ± 0.42	6.42 ± 0.66
	gini	.886 ± .017	.864 ± .056	.760 ± .063	.720 ± .056	.709 ± .046	.618 ± .079	.559 ± .065
	time	0.4 ± 0.0	0.5 ± 0.1	1.2 ± 0.3	3.6 ± 0.7	3.6 ± 0.6	5.1 ± 0.6	7.1 ± 0.8
dry-bean	acc	.530 ± .007	.892 ± .012	.909 ± .007	.905 ± .008	.907 ± .007	.913 ± .007	.910 ± .005
	nodes	2.0 ± 0.0	6.0 ± 0.0	7.8 ± 0.4	11.5 ± 1.1	28.0 ± 0.9	58.2 ± 2.9	139.2 ± 15.3
	path len	1.60 ± 0.01	3.05 ± 0.23	3.36 ± 0.08	3.77 ± 0.19	5.46 ± 0.15	6.27 ± 0.26	8.13 ± 0.46
	gini	.753 ± .013	.676 ± .033	.677 ± .040	.540 ± .083	.471 ± .054	.387 ± .070	.398 ± .040
	time	2.3 ± 0.0	4.1 ± 0.1	4.8 ± 0.2	5.9 ± 0.3	10.5 ± 0.3	18.0 ± 0.7	37.1 ± 3.7
wine	acc	.983 ± .025	.977 ± .028	.966 ± .037	.960 ± .036	.967 ± .027	.967 ± .027	.966 ± .027
	nodes	2.0 ± 0.0	2.0 ± 0.0	2.5 ± 0.5	3.1 ± 0.7	4.7 ± 1.1	7.5 ± 1.8	13.2 ± 2.6
	path len	1.67 ± 0.09	1.66 ± 0.09	1.85 ± 0.23	2.10 ± 0.31	2.45 ± 0.20	3.19 ± 0.44	4.12 ± 0.48
	gini	.817 ± .011	.789 ± .011	.814 ± .019	.775 ± .045	.779 ± .032	.756 ± .035	.740 ± .045
	time	0.4 ± 0.0	0.4 ± 0.0	0.5 ± 0.1	0.6 ± 0.1	0.8 ± 0.1	1.1 ± 0.2	1.8 ± 0.3
car	acc	.700 ± .044	.700 ± .044	.949 ± .013	.978 ± .013	.990 ± .011	.992 ± .009	.990 ± .010
	nodes	0.0 ± 0.0	0.0 ± 0.0	5.5 ± 1.0	15.6 ± 4.1	31.3 ± 3.1	36.1 ± 7.3	41.1 ± 9.8
	path len	0.00 ± 0.00	0.00 ± 0.00	2.13 ± 0.25	2.98 ± 0.42	3.41 ± 0.46	3.50 ± 0.50	3.62 ± 0.69
	gini	.000 ± .000	.000 ± .000	.720 ± .020	.703 ± .045	.703 ± .032	.655 ± .033	.663 ± .031
	time	0.3 ± 0.0	0.3 ± 0.0	1.9 ± 0.3	4.2 ± 1.0	7.8 ± 0.8	8.8 ± 1.7	9.9 ± 2.2
wdbc	acc	.970 ± .019	.970 ± .019	.974 ± .018	.968 ± .017	.977 ± .016	.965 ± .022	.970 ± .025
	nodes	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	3.2 ± 0.6	7.0 ± 3.0	17.4 ± 4.1	34.9 ± 8.9
	path len	1.00 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	2.00 ± 0.01	2.57 ± 0.49	3.94 ± 0.41	4.72 ± 0.61
	gini	.817 ± .019	.819 ± .022	.816 ± .019	.835 ± .045	.936 ± .008	.936 ± .006	.936 ± .007
	time	0.6 ± 0.0	0.6 ± 0.0	0.6 ± 0.0	1.1 ± 0.1	2.0 ± 0.7	4.3 ± 0.9	7.8 ± 1.8
sonar	acc	.665 ± .174	.841 ± .071	.807 ± .081	.797 ± .079	.827 ± .065	.803 ± .084	.832 ± .071
	nodes	0.9 ± 0.3	2.6 ± 0.7	5.6 ± 0.9	7.0 ± 1.6	12.6 ± 4.8	21.7 ± 3.7	28.9 ± 7.4
	path len	0.90 ± 0.30	1.86 ± 0.28	2.70 ± 0.36	3.00 ± 0.26	3.97 ± 0.66	4.96 ± 0.83	5.68 ± 0.81
	gini	.849 ± .283	.939 ± .013	.899 ± .029	.900 ± .018	.915 ± .020	.933 ± .022	.924 ± .032
	time	0.5 ± 0.1	1.0 ± 0.2	1.6 ± 0.2	2.0 ± 0.3	3.2 ± 1.0	5.2 ± 0.8	3.9 ± 0.9
pendigits	acc	.094 ± .003	.899 ± .009	.936 ± .009	.963 ± .004	.970 ± .006	.970 ± .004	.975 ± .005
	nodes	0.0 ± 0.0	9.0 ± 0.0	11.9 ± 1.6	17.9 ± 1.6	42.2 ± 4.0	120.1 ± 7.1	292.1 ± 10.3
	path len	0.00 ± 0.00	3.97 ± 0.03	4.56 ± 0.45	4.84 ± 0.26	5.90 ± 0.31	7.38 ± 0.19	8.78 ± 0.33
	gini	.000 ± .000	.809 ± .010	.813 ± .024	.764 ± .022	.695 ± .041	.492 ± .045	.321 ± .027
	time	11.3 ± 5.0	72.8 ± 33.8	83.9 ± 14.0	112.8 ± 51.7	220.7 ± 82.2	31.2 ± 1.6	84.9 ± 8.8
ionosphere	acc	.857 ± .077	.909 ± .060	.923 ± .038	.917 ± .055	.926 ± .045	.900 ± .058	.915 ± .044
	nodes	1.0 ± 0.0	2.4 ± 0.5	3.4 ± 0.5	6.3 ± 1.6	15.7 ± 4.0	32.2 ± 4.6	52.3 ± 5.4
	path len	1.00 ± 0.00	2.02 ± 0.34	2.77 ± 0.39	3.89 ± 0.55	5.63 ± 1.16	8.56 ± 0.85	12.33 ± 0.88
	gini	.910 ± .010	.828 ± .093	.700 ± .088	.665 ± .067	.711 ± .062	.786 ± .047	.855 ± .027
	time	0.5 ± 0.0	0.9 ± 0.1	1.2 ± 0.1	1.8 ± 0.4	2.4 ± 0.7	4.3 ± 0.6	8.7 ± 2.8

Ours: prototype features, L1 regularization, random initialization, fuzzy

data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.947 ± .065	.940 ± .047	.947 ± .050	.967 ± .045	.947 ± .072	.933 ± .060	.947 ± .050
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.1 ± 0.7	6.2 ± 1.1	9.5 ± 2.2	18.3 ± 4.4	26.3 ± 7.3
	path len	1.67 ± 0.09	1.67 ± 0.09	2.02 ± 0.22	2.58 ± 0.32	2.92 ± 0.54	3.95 ± 0.63	4.40 ± 0.69
	gini	.616 ± .043	.646 ± .040	.604 ± .059	.575 ± .061	.577 ± .094	.538 ± .054	.580 ± .045
	time	1.1 ± 0.0	1.1 ± 0.0	1.5 ± 0.2	2.4 ± 0.3	2.7 ± 0.5	6.0 ± 1.3	8.4 ± 2.2
heart	acc	.812 ± .057	.805 ± .048	.775 ± .049	.756 ± .054	.766 ± .081	.756 ± .052	.798 ± .036
	nodes	1.0 ± 0.0	1.2 ± 0.4	3.9 ± 1.1	13.5 ± 1.7	45.9 ± 5.0	71.1 ± 6.8	97.1 ± 9.0
	path len	1.00 ± 0.00	1.08 ± 0.16	2.19 ± 0.34	3.85 ± 0.45	5.48 ± 0.48	6.99 ± 0.75	7.68 ± 0.84
	gini	.936 ± .003	.935 ± .002	.929 ± .005	.900 ± .012	.852 ± .022	.839 ± .018	.828 ± .031
	time	0.9 ± 0.0	1.0 ± 0.1	1.8 ± 0.3	4.8 ± 0.6	14.7 ± 1.5	22.3 ± 2.1	22.9 ± 2.3
dry-bean	acc	.617 ± .068	.869 ± .035	.894 ± .011	.902 ± .006	.905 ± .006	.909 ± .005	.915 ± .006
	nodes	2.6 ± 0.5	5.6 ± 0.5	7.5 ± 1.0	16.1 ± 1.9	37.1 ± 2.8	79.7 ± 7.0	190.1 ± 15.1
	path len	1.92 ± 0.31	2.83 ± 0.17	3.15 ± 0.29	4.38 ± 0.19	5.59 ± 0.31	6.95 ± 0.34	8.86 ± 0.71
	gini	.724 ± .052	.718 ± .027	.596 ± .078	.385 ± .058	.356 ± .046	.310 ± .046	.303 ± .042
	time	2.7 ± 0.4	4.5 ± 0.2	5.8 ± 0.8	10.2 ± 0.9	20.6 ± 1.3	38.1 ± 3.7	86.0 ± 9.2
wine	acc	.955 ± .043	.960 ± .036	.955 ± .042	.950 ± .058	.955 ± .042	.939 ± .046	.939 ± .052
	nodes	2.0 ± 0.0	2.2 ± 0.4	3.1 ± 0.5	8.3 ± 2.2	14.1 ± 3.2	25.2 ± 6.4	35.1 ± 9.5
	path len	1.66 ± 0.10	1.74 ± 0.20	2.01 ± 0.25	3.26 ± 0.49	4.17 ± 0.49	5.18 ± 0.69	5.70 ± 0.72
	gini	.845 ± .012	.847 ± .020	.815 ± .026	.614 ± .111	.529 ± .085	.418 ± .094	.403 ± .070
	time	0.9 ± 0.0	1.0 ± 0.1	1.2 ± 0.1	2.4 ± 0.5	3.7 ± 0.7	6.2 ± 1.4	8.4 ± 2.1
car	acc	.700 ± .044	.770 ± .052	.833 ± .046	.953 ± .017	.969 ± .014	.974 ± .015	.983 ± .009
	nodes	0.0 ± 0.0	1.1 ± 0.7	2.8 ± 1.2	12.9 ± 2.9	26.5 ± 6.8	49.8 ± 12.4	70.4 ± 13.1
	path len	0.00 ± 0.00	0.97 ± 0.39	1.61 ± 0.38	2.86 ± 0.40	3.38 ± 0.54	3.70 ± 0.59	4.00 ± 0.49
	gini	.000 ± .000	.699 ± .236	.816 ± .040	.769 ± .035	.707 ± .056	.658 ± .035	.596 ± .038
	time	0.3 ± 0.0	0.8 ± 0.2	1.3 ± 0.3	3.9 ± 0.7	7.1 ± 1.5	12.2 ± 2.8	16.7 ± 2.9
wdbc	acc	.961 ± .025	.956 ± .024	.961 ± .027	.970 ± .019	.963 ± .024	.961 ± .022	.953 ± .024
	nodes	1.0 ± 0.0	1.1 ± 0.3	1.8 ± 0.7	6.3 ± 1.3	19.1 ± 1.8	37.4 ± 3.1	68.8 ± 4.4
	path len	1.00 ± 0.00	1.07 ± 0.22	1.36 ± 0.37	2.69 ± 0.28	3.90 ± 0.41	4.75 ± 0.32	6.08 ± 0.48
	gini	.864 ± .015	.851 ± .039	.821 ± .038	.779 ± .049	.646 ± .062	.569 ± .064	.633 ± .045
	time	0.7 ± 0.0	0.8 ± 0.1	0.9 ± 0.2	2.0 ± 0.3	5.0 ± 0.4	9.3 ± 0.8	16.7 ± 1.0
sonar	acc	.534 ± .144	.759 ± .091	.798 ± .072	.789 ± .052	.774 ± .046	.779 ± .081	.812 ± .058
	nodes	0.0 ± 0.0	2.0 ± 0.4	7.6 ± 1.4	18.7 ± 2.6	29.5 ± 4.3	39.3 ± 3.3	50.4 ± 6.3
	path len	0.00 ± 0.00	1.64 ± 0.28	3.38 ± 0.29	4.70 ± 0.36	5.30 ± 0.50	5.77 ± 0.44	6.24 ± 0.67
	gini	.000 ± .000	.931 ± .014	.905 ± .020	.844 ± .024	.770 ± .029	.743 ± .031	.731 ± .037
	time	0.3 ± 0.0	1.0 ± 0.1	2.4 ± 0.5	4.9 ± 0.6	7.3 ± 1.0	9.6 ± 0.8	12.0 ± 1.5
pendigits	acc	.094 ± .003	.783 ± .057	.854 ± .024	.881 ± .019	.912 ± .012	.939 ± .009	.949 ± .011
	nodes	0.0 ± 0.0	8.2 ± 0.7	12.8 ± 1.2	30.2 ± 3.7	69.7 ± 10.1	166.7 ± 10.7	372.4 ± 23.3
	path len	0.00 ± 0.00	4.10 ± 0.47	4.56 ± 0.56	5.28 ± 0.34	6.58 ± 0.27	7.93 ± 0.34	9.49 ± 0.18
	gini	.000 ± .000	.770 ± .029	.684 ± .043	.475 ± .090	.317 ± .030	.353 ± .045	.354 ± .031
	time	0.4 ± 0.0	4.7 ± 0.3	6.6 ± 0.5	12.2 ± 1.2	24.2 ± 2.8	62.9 ± 4.2	179.6 ± 18.3
ionosphere	acc	.903 ± .045	.892 ± .057	.920 ± .045	.909 ± .035	.897 ± .092	.917 ± .052	.906 ± .054
	nodes	1.1 ± 0.3	1.8 ± 0.6	3.8 ± 1.1	10.5 ± 2.3	23.3 ± 4.2	40.5 ± 7.4	58.8 ± 6.4
	path len	1.03 ± 0.10	1.42 ± 0.34	2.29 ± 0.46	4.35 ± 0.66	5.96 ± 0.88	7.48 ± 0.88	9.28 ± 1.87
	gini	.894 ± .012	.887 ± .029	.839 ± .033	.706 ± .022	.616 ± .050	.596 ± .051	.594 ± .058
	time	0.7 ± 0.1	0.9 ± 0.2	1.4 ± 0.3	2.9 ± 0.6	6.1 ± 0.9	13.2 ± 2.3	18.9 ± 1.9

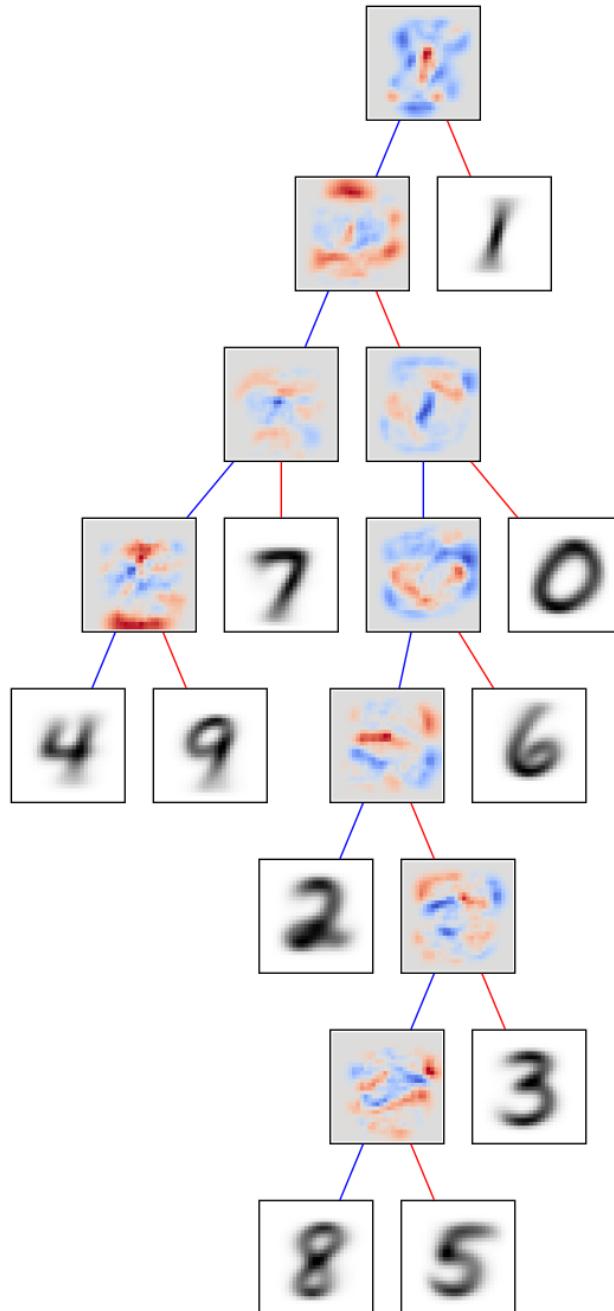
Ours: prototype features, L1 regularization, random initialization, crisp

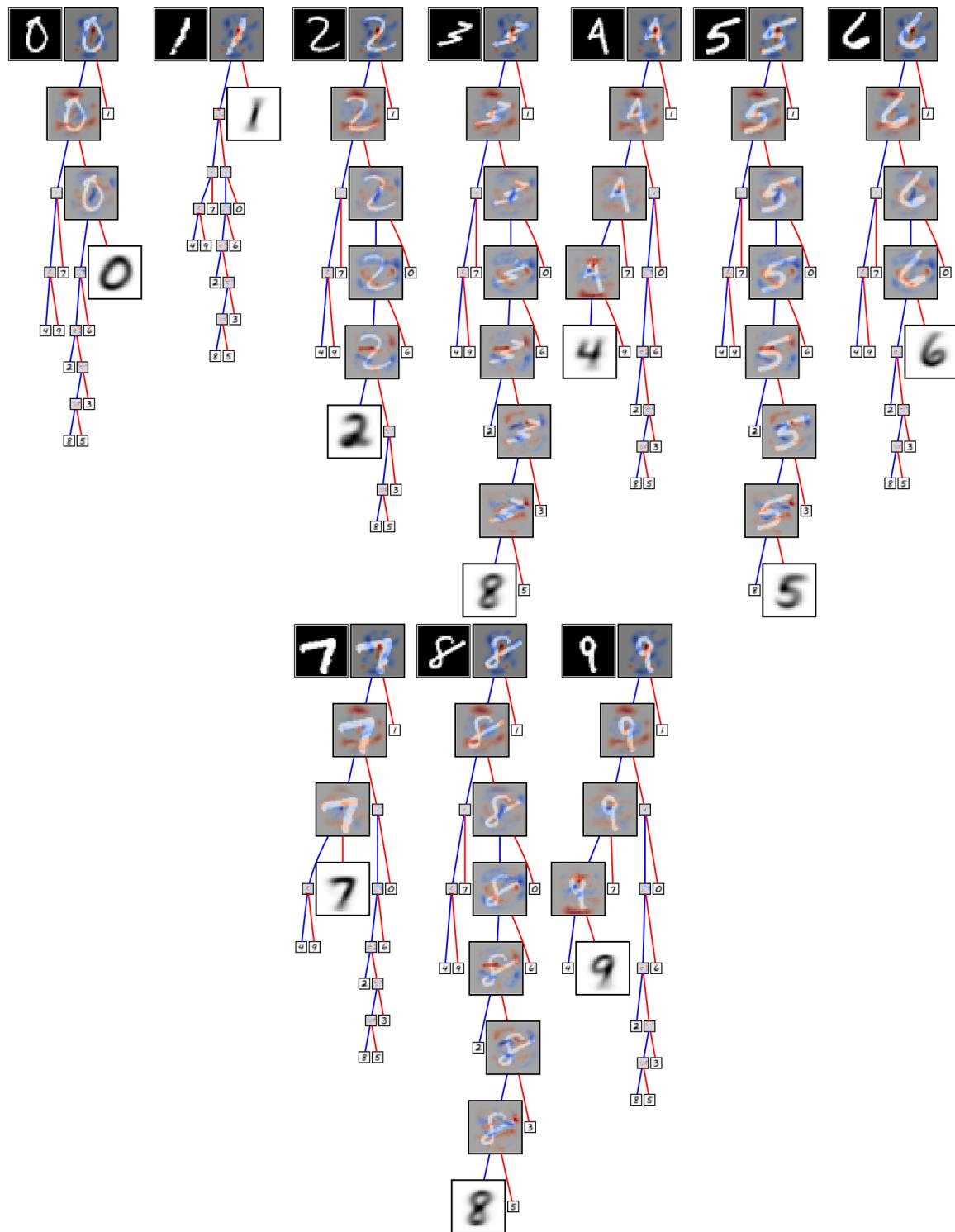
data	metric	1e-1	3e-2	1e-2	3e-3	1e-3	3e-4	1e-4
iris	acc	.947 ± .065	.940 ± .047	.947 ± .050	.947 ± .050	.947 ± .058	.927 ± .063	.947 ± .050
	nodes	2.0 ± 0.0	2.0 ± 0.0	3.1 ± 0.7	6.2 ± 1.1	9.5 ± 2.2	18.3 ± 4.4	26.3 ± 7.3
	path len	1.67 ± 0.09	1.67 ± 0.09	2.03 ± 0.21	2.56 ± 0.33	2.89 ± 0.53	4.00 ± 0.61	4.43 ± 0.71
	gini	.616 ± .043	.646 ± .040	.604 ± .058	.574 ± .061	.575 ± .095	.538 ± .056	.578 ± .048
	time	1.0 ± 0.0	1.0 ± 0.0	1.4 ± 0.2	1.7 ± 0.2	2.4 ± 0.5	5.3 ± 1.1	7.4 ± 1.9
heart	acc	.809 ± .055	.802 ± .036	.772 ± .061	.743 ± .066	.737 ± .082	.750 ± .061	.792 ± .041
	nodes	1.0 ± 0.0	1.2 ± 0.4	3.9 ± 1.1	13.5 ± 1.7	45.9 ± 5.0	71.1 ± 6.8	97.1 ± 9.0
	path len	1.00 ± 0.00	1.09 ± 0.17	2.19 ± 0.37	3.84 ± 0.48	5.47 ± 0.53	7.09 ± 0.80	7.81 ± 0.98
	gini	.936 ± .003	.935 ± .002	.929 ± .005	.900 ± .012	.852 ± .022	.840 ± .020	.828 ± .031
	time	0.7 ± 0.0	0.7 ± 0.1	1.3 ± 0.2	3.3 ± 0.4	9.4 ± 0.9	14.1 ± 1.3	19.1 ± 1.8
dry-bean	acc	.612 ± .065	.861 ± .035	.887 ± .011	.890 ± .007	.892 ± .007	.894 ± .006	.893 ± .017
	nodes	2.6 ± 0.5	5.6 ± 0.5	7.5 ± 1.0	16.1 ± 1.9	37.1 ± 2.8	79.7 ± 7.0	190.1 ± 15.1
	path len	1.93 ± 0.31	2.84 ± 0.18	3.14 ± 0.29	4.39 ± 0.20	5.65 ± 0.33	7.07 ± 0.40	9.45 ± 1.11
	gini	.724 ± .052	.718 ± .026	.596 ± .078	.383 ± .059	.357 ± .046	.313 ± .045	.308 ± .045
	time	2.5 ± 0.3	4.1 ± 0.2	5.0 ± 0.5	8.6 ± 0.7	15.8 ± 1.0	27.2 ± 2.6	56.9 ± 4.7
wine	acc	.949 ± .040	.960 ± .036	.939 ± .058	.944 ± .066	.933 ± .054	.922 ± .067	.916 ± .037
	nodes	2.0 ± 0.0	2.2 ± 0.4	3.1 ± 0.5	8.3 ± 2.2	14.1 ± 3.2	25.2 ± 6.4	35.1 ± 9.5
	path len	1.67 ± 0.11	1.74 ± 0.20	2.02 ± 0.26	3.26 ± 0.52	4.22 ± 0.55	5.25 ± 0.76	5.76 ± 0.75
	gini	.845 ± .012	.847 ± .020	.815 ± .026	.612 ± .110	.529 ± .085	.420 ± .094	.403 ± .071
	time	0.8 ± 0.1	0.9 ± 0.1	1.1 ± 0.1	2.2 ± 0.4	3.3 ± 0.6	5.4 ± 1.2	7.2 ± 1.8
car	acc	.700 ± .044	.771 ± .053	.832 ± .044	.953 ± .017	.969 ± .013	.974 ± .015	.982 ± .009
	nodes	0.0 ± 0.0	1.1 ± 0.7	2.8 ± 1.2	12.9 ± 2.9	26.5 ± 6.8	49.8 ± 12.4	70.4 ± 13.1
	path len	0.00 ± 0.00	0.97 ± 0.39	1.61 ± 0.39	2.85 ± 0.39	3.38 ± 0.54	3.70 ± 0.59	4.01 ± 0.49
	gini	.000 ± .000	.700 ± .236	.816 ± .040	.769 ± .035	.707 ± .056	.658 ± .035	.596 ± .038
	time	0.3 ± 0.0	0.8 ± 0.2	1.2 ± 0.3	3.4 ± 0.6	6.3 ± 1.3	10.9 ± 2.2	14.8 ± 2.5
wdbc	acc	.961 ± .025	.956 ± .024	.961 ± .027	.958 ± .026	.947 ± .025	.951 ± .020	.937 ± .037
	nodes	1.0 ± 0.0	1.1 ± 0.3	1.8 ± 0.7	6.3 ± 1.3	19.1 ± 1.8	37.4 ± 3.1	68.8 ± 4.4
	path len	1.00 ± 0.00	1.07 ± 0.22	1.36 ± 0.37	2.69 ± 0.30	3.88 ± 0.40	4.71 ± 0.35	6.13 ± 0.61
	gini	.864 ± .015	.851 ± .039	.821 ± .038	.780 ± .050	.645 ± .062	.568 ± .063	.632 ± .044
	time	0.7 ± 0.0	0.7 ± 0.1	0.9 ± 0.2	1.8 ± 0.3	4.4 ± 0.4	7.9 ± 0.7	13.8 ± 0.8
sonar	acc	.534 ± .144	.769 ± .115	.813 ± .062	.798 ± .046	.778 ± .057	.788 ± .066	.803 ± .072
	nodes	0.0 ± 0.0	2.0 ± 0.4	7.6 ± 1.4	18.7 ± 2.6	29.5 ± 4.3	39.3 ± 3.3	50.4 ± 6.3
	path len	0.00 ± 0.00	1.65 ± 0.27	3.44 ± 0.30	4.73 ± 0.41	5.34 ± 0.54	5.85 ± 0.48	6.31 ± 0.71
	gini	.000 ± .000	.931 ± .014	.906 ± .020	.844 ± .023	.770 ± .028	.744 ± .032	.730 ± .035
	time	0.3 ± 0.0	0.9 ± 0.1	2.1 ± 0.3	4.3 ± 0.5	6.4 ± 0.8	8.5 ± 1.1	10.5 ± 1.6
pendigits	acc	.094 ± .003	.773 ± .047	.844 ± .024	.865 ± .015	.892 ± .013	.919 ± .010	.930 ± .014
	nodes	0.0 ± 0.0	8.3 ± 0.6	12.8 ± 1.2	30.2 ± 3.7	69.7 ± 10.1	166.7 ± 10.7	372.4 ± 23.3
	path len	0.00 ± 0.00	3.93 ± 0.45	4.57 ± 0.58	5.28 ± 0.34	6.58 ± 0.27	7.93 ± 0.37	9.51 ± 0.18
	gini	.000 ± .000	.743 ± .034	.684 ± .043	.475 ± .090	.317 ± .030	.352 ± .045	.354 ± .031
	time	0.4 ± 0.0	4.2 ± 0.3	5.7 ± 0.5	10.4 ± 1.0	20.2 ± 2.4	51.1 ± 3.4	152.9 ± 8.7
ionosphere	acc	.903 ± .045	.892 ± .057	.923 ± .048	.906 ± .043	.909 ± .079	.900 ± .057	.883 ± .079
	nodes	1.1 ± 0.3	1.8 ± 0.6	3.8 ± 1.1	10.5 ± 2.3	23.3 ± 4.2	40.5 ± 7.4	58.8 ± 6.4
	path len	1.03 ± 0.10	1.42 ± 0.35	2.29 ± 0.48	4.38 ± 0.68	6.01 ± 0.88	7.67 ± 0.88	10.00 ± 2.49
	gini	.894 ± .012	.887 ± .029	.839 ± .032	.705 ± .022	.615 ± .051	.596 ± .051	.594 ± .060
	time	0.7 ± 0.1	0.8 ± 0.1	1.2 ± 0.2	2.6 ± 0.5	5.4 ± 0.9	11.4 ± 1.9	16.2 ± 1.6

3 Visualization of MNIST Trees

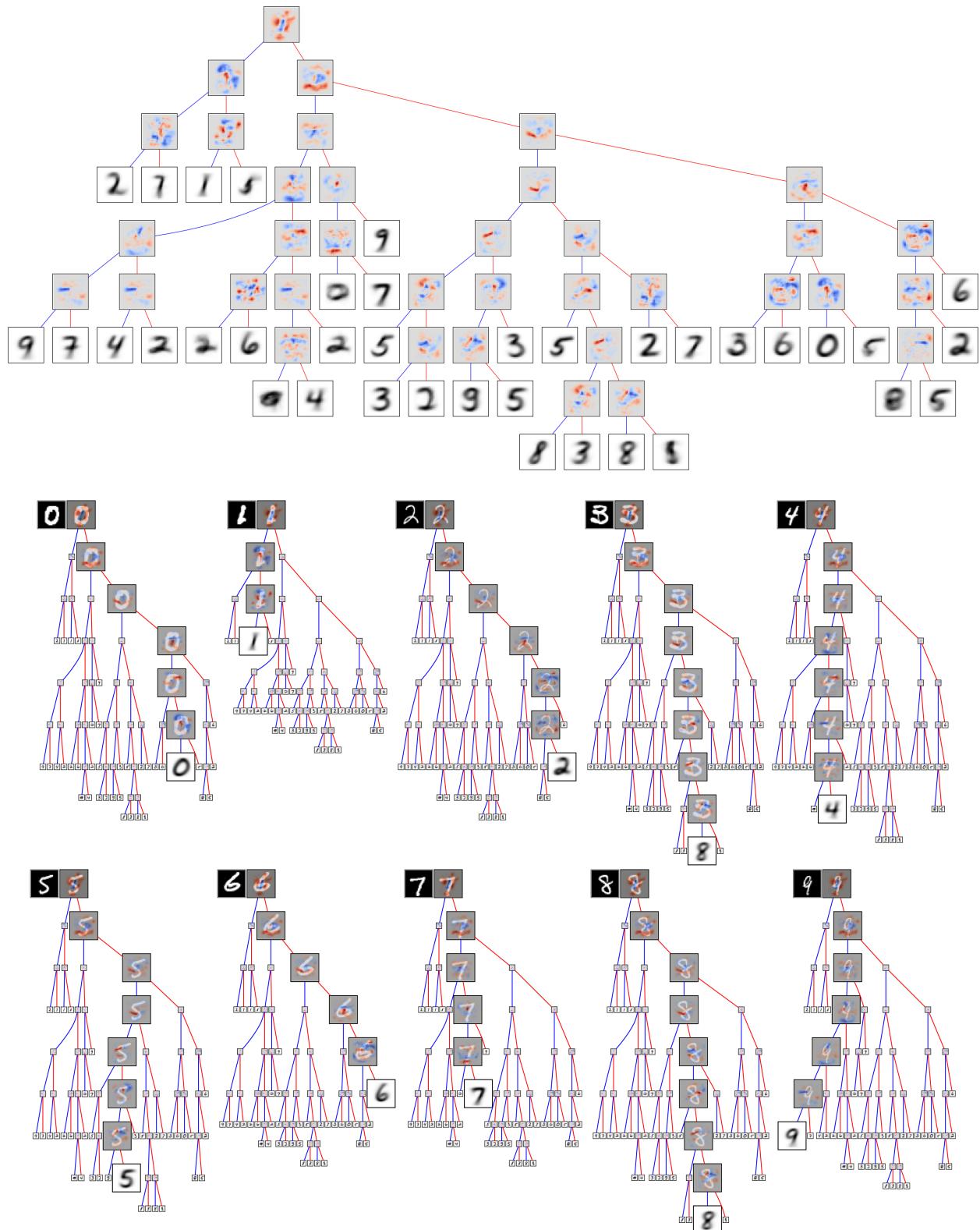
In this section, we show the full decision trees trained on MNIST and Fashion-MNIST that were reported in the main paper, along with one randomly selected prediction from the training set for each class. The trees with $\alpha = 10^{-5}$ are not shown because of their large size. While the larger trees may be too small to be legible in print, the image resolutions are high, so zooming in to a digital view of this document will make the details visible.

3.1 MNIST $\alpha = 10^{-2}$

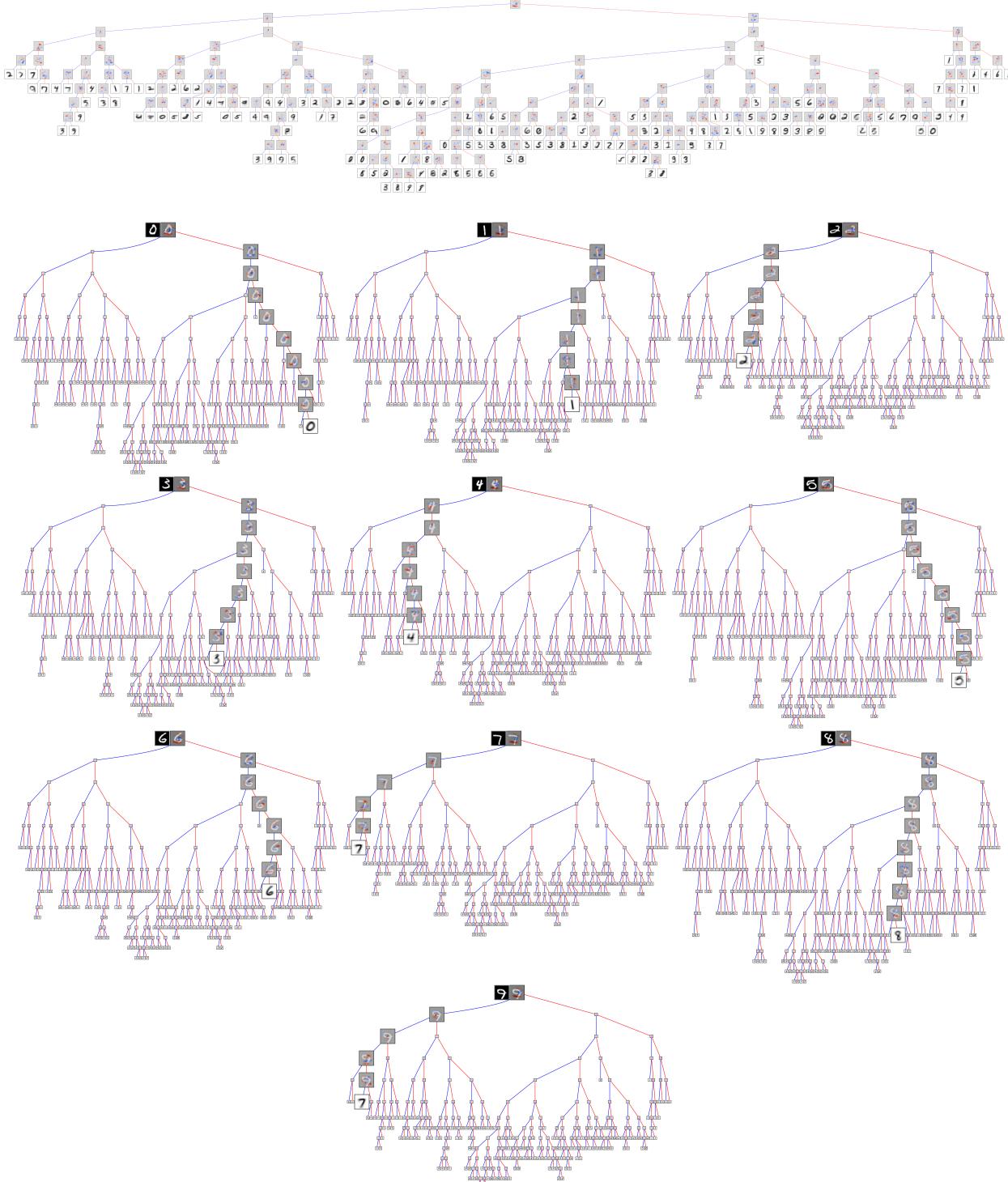




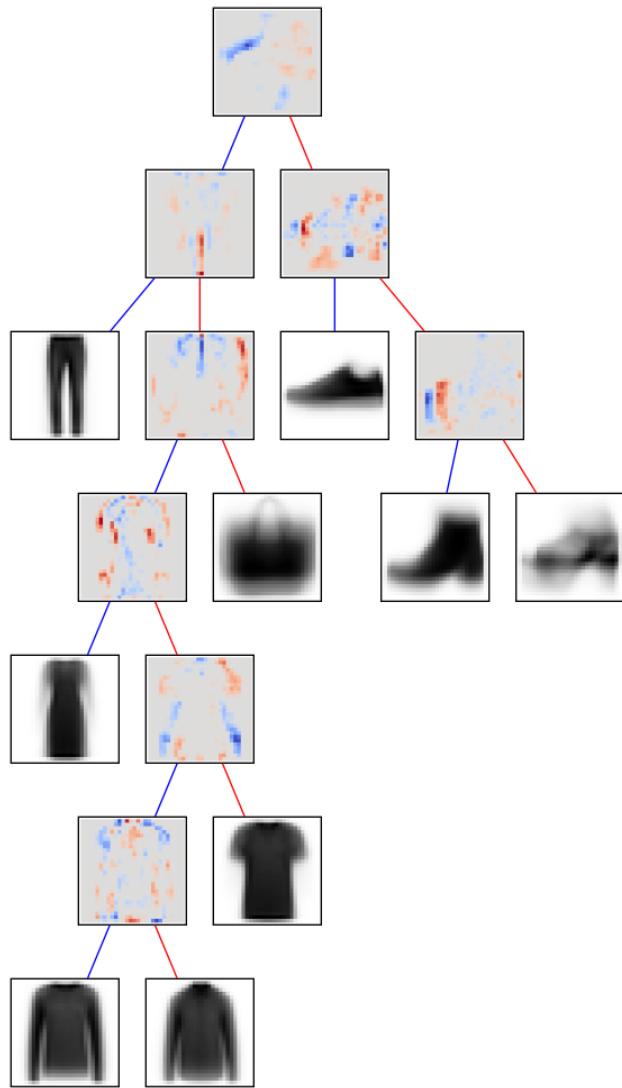
3.2 MNIST $\alpha = 10^{-3}$

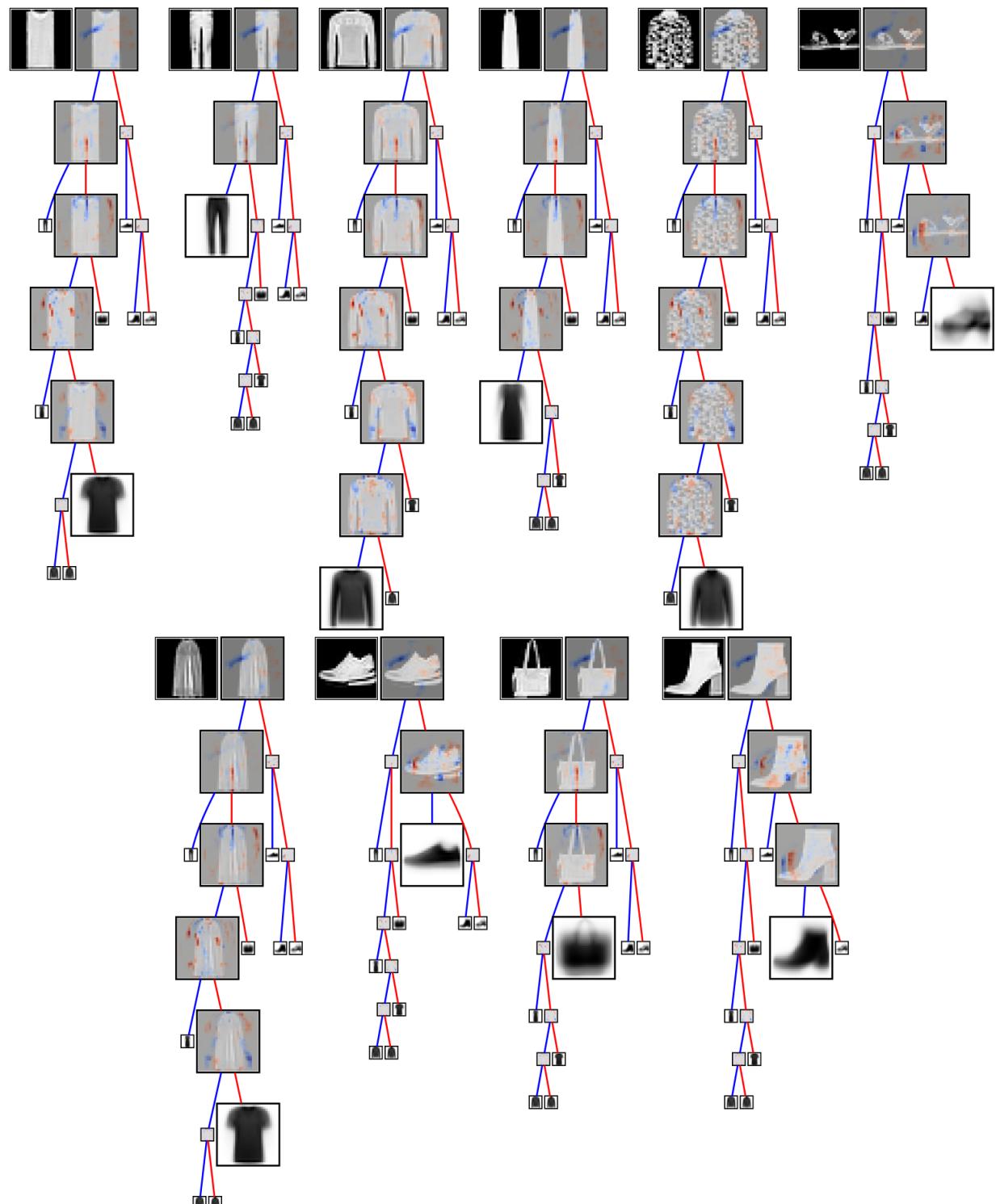


3.3 MNIST $\alpha = 10^{-4}$

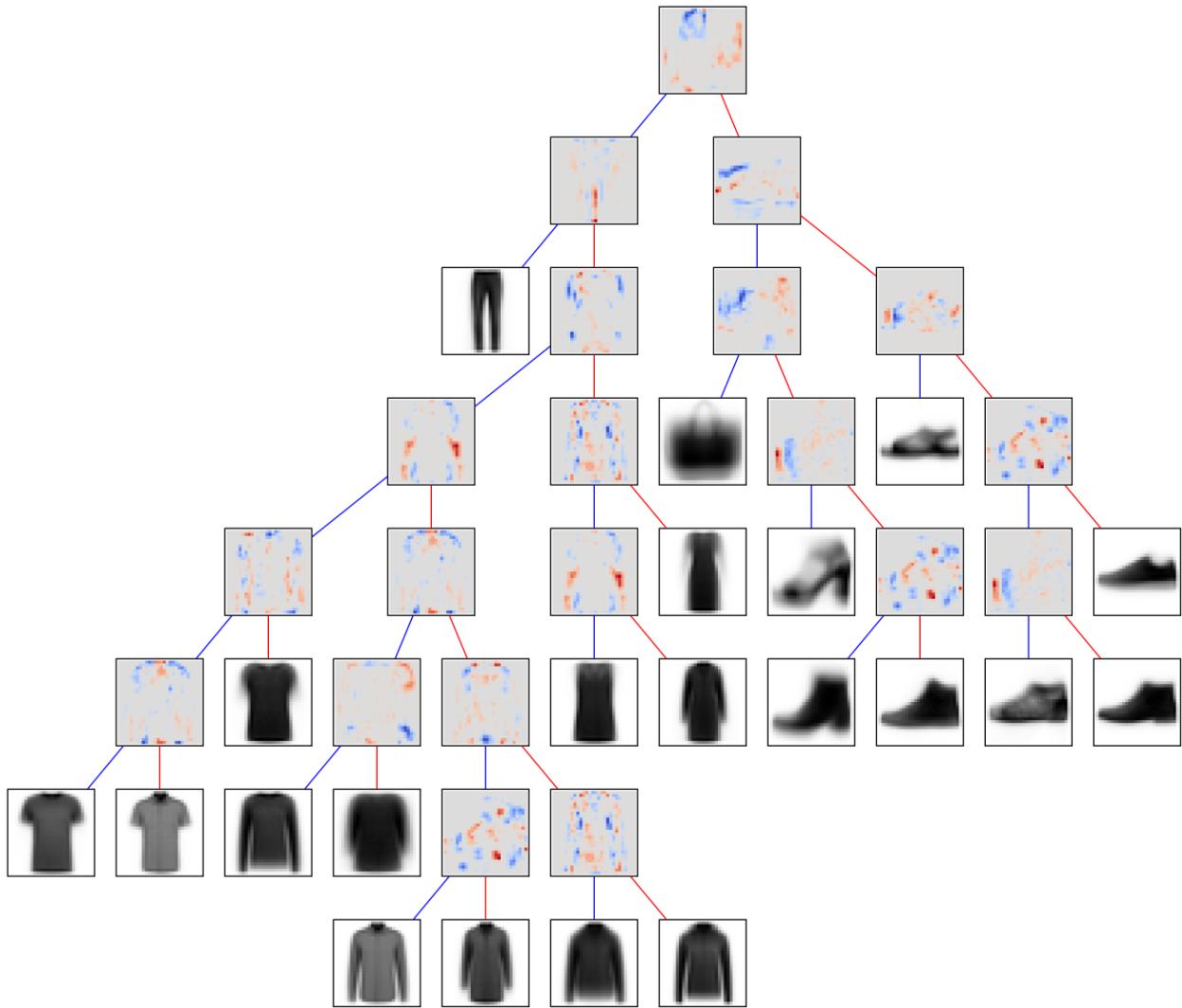


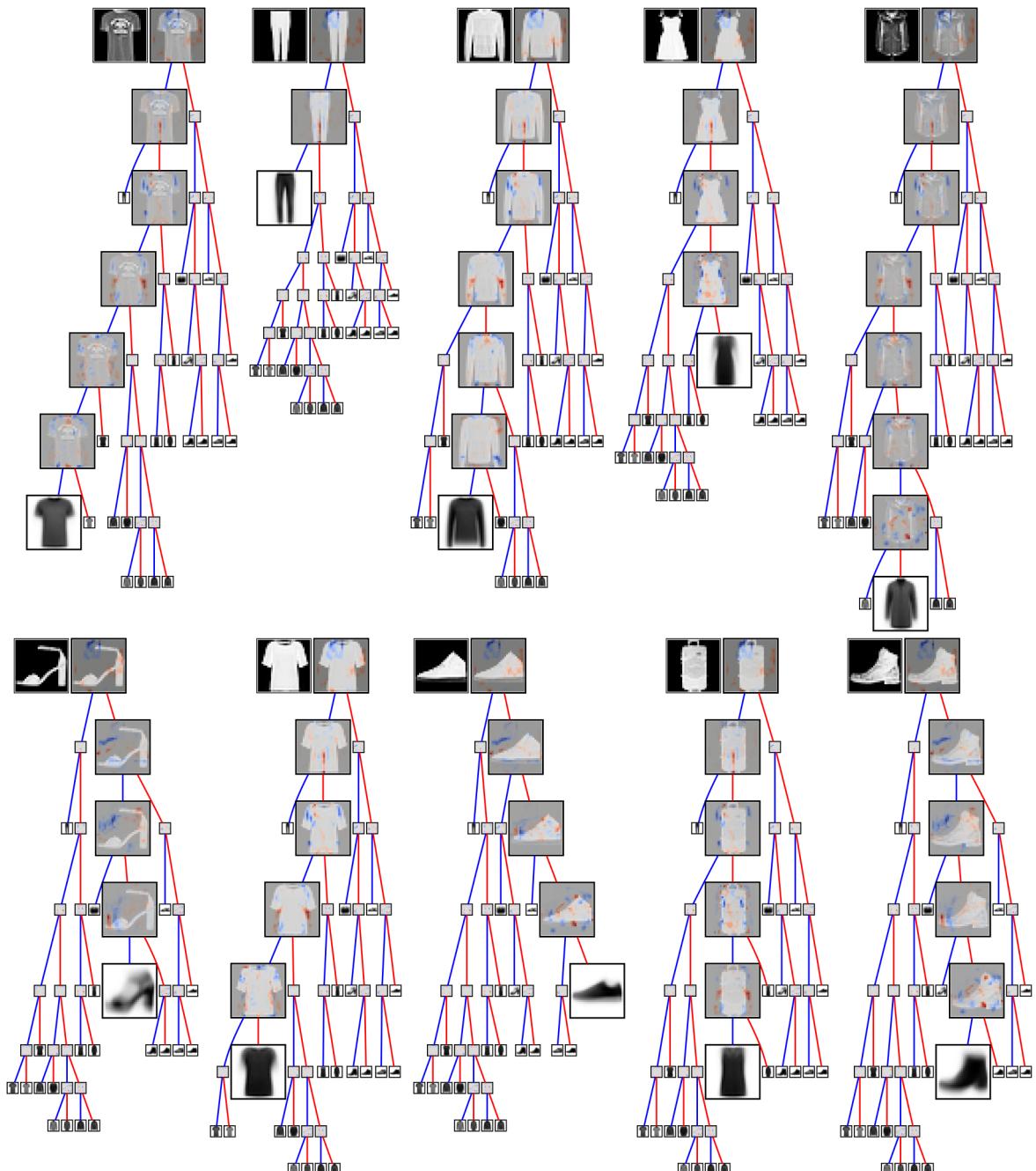
3.4 Fashion-MNIST $\alpha = 10^{-2}$



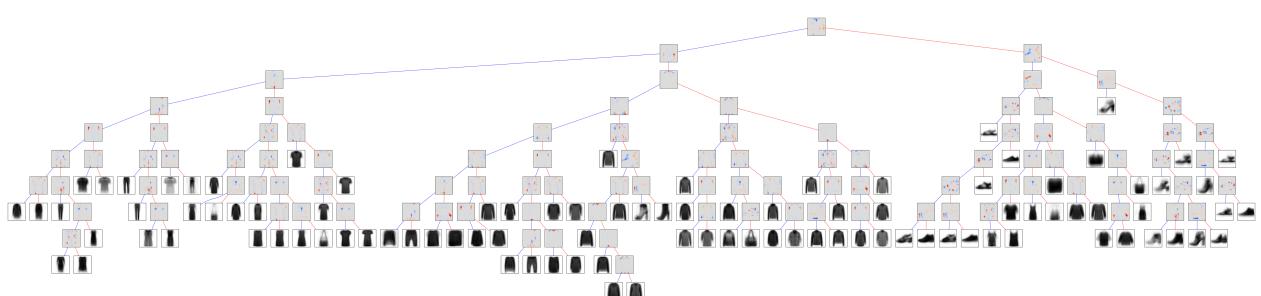


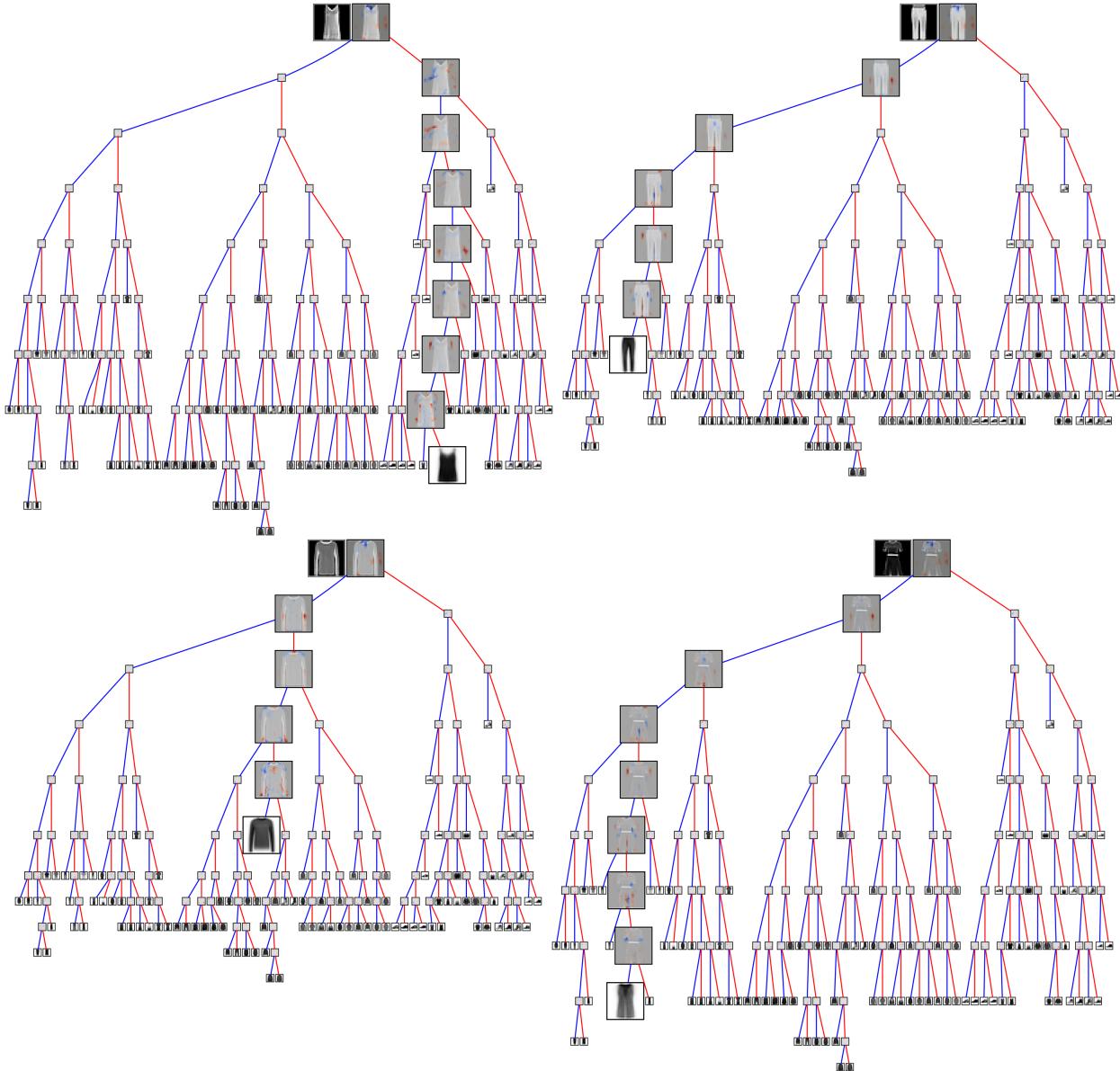
3.5 Fashion-MNIST $\alpha = 10^{-3}$

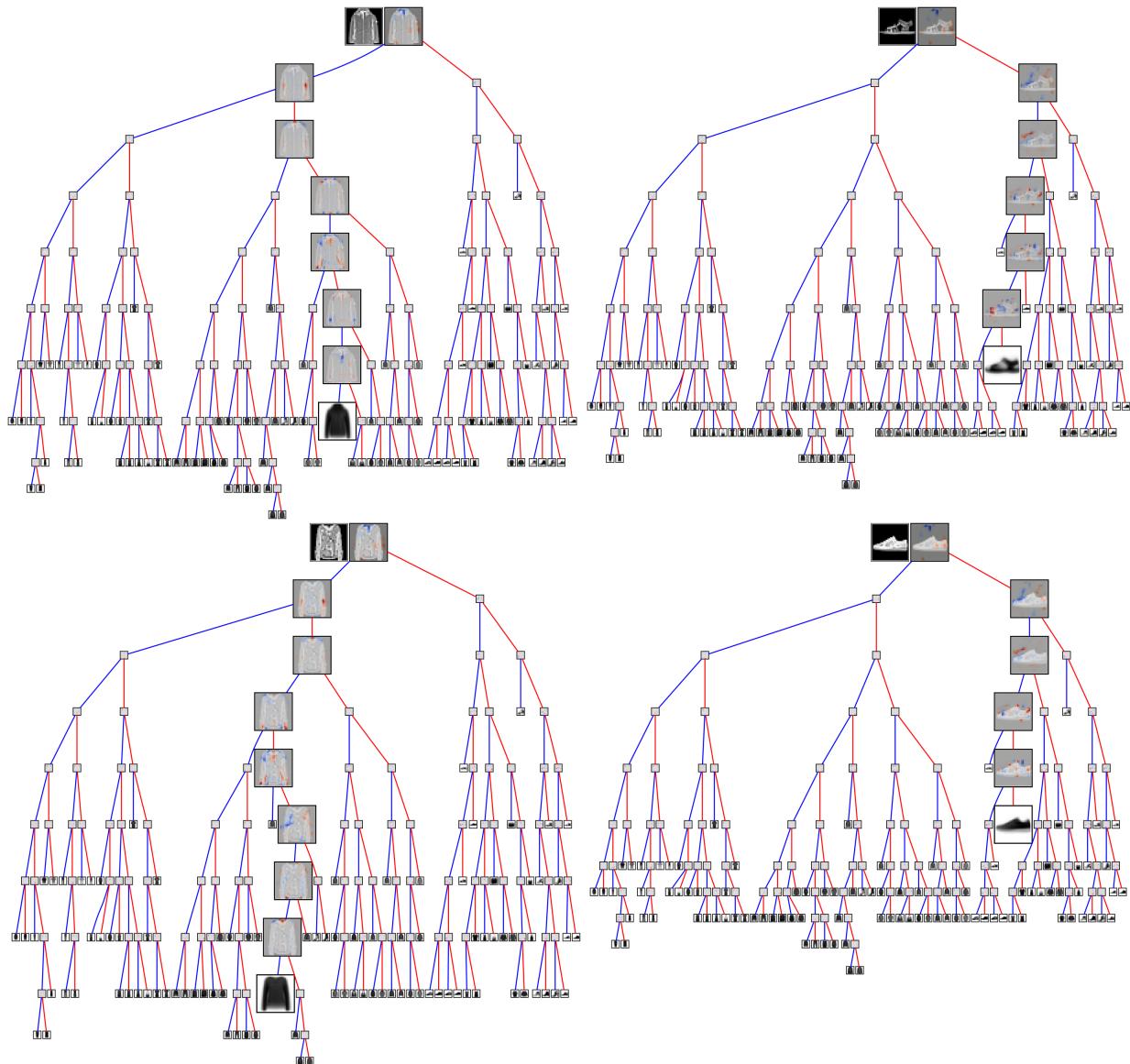


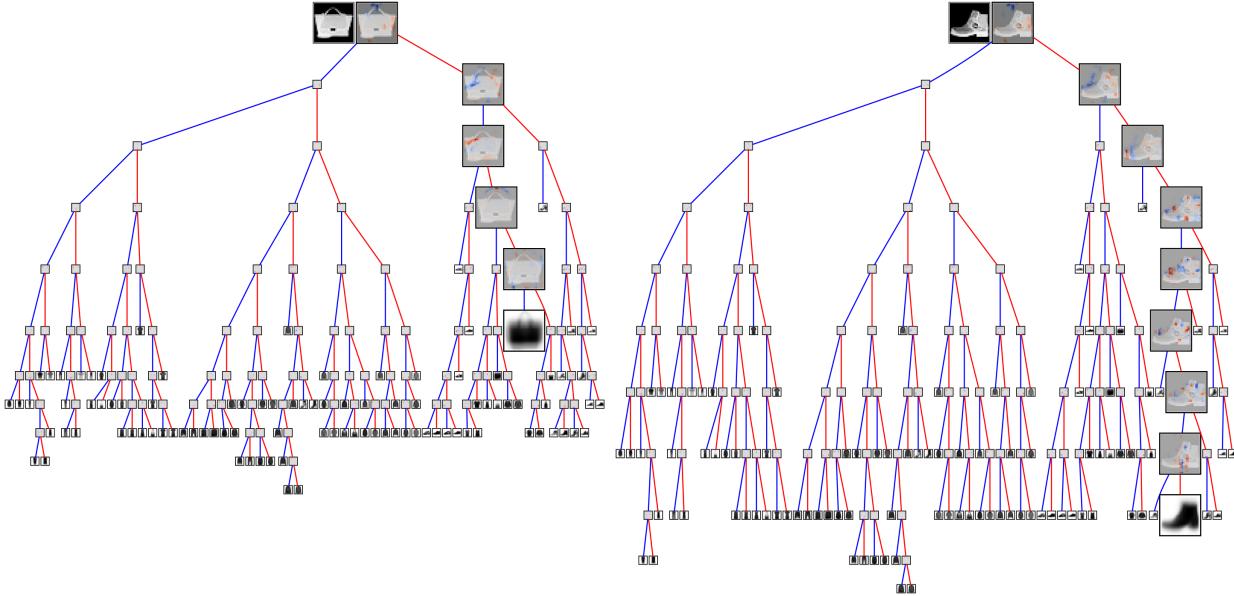


3.6 Fashion-MNIST $\alpha = 10^{-4}$









References

- [1] Dheeru Dua and Casey Graff. UCI machine learning repository, 2017.
- [2] Niall Hurley and Scott Rickard. Comparing measures of sparsity. *IEEE Transactions on Information Theory*, 55(10):4723–4741, 2009.
- [3] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.
- [4] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Kopf, Edward Yang, Zachary DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. Pytorch: An imperative style, high-performance deep learning library. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d'Alché-Buc, E. Fox, and R. Garnett, editors, *Advances in Neural Information Processing Systems*, volume 32. Curran Associates, Inc., 2019.
- [5] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay. Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, 12:2825–2830, 2011.