

## Note to editor

The authors wish to thank the referees for their helpful comments and suggestions; we hope the changes that ensued have significantly improved the paper. We also thank the editor and referees both for their patience as we worked up to the deadline to answer all of the queries raised. In particular we altered some the theoretical results to make our claims more precise, and we added a brief new corollary to the main theorems in Section 3.1. A new discussion of the results immediately follows in Section 3.2 that explains how the present work builds on and extends the previous theory for PINNs applied to multiscale problems developed in a series of papers by Wang et al. Lastly, we conducted additional, more careful numerical experiments to bolster the claim that PINN training becomes harder as the problem scale separation increases, i.e. as  $\epsilon$  decreases.