

Organization We provide in-depth descriptions for our algorithms, experimental setups, i.e. dataset configurations, implementation & training details, and additional experimental results & analysis that are not covered in the main document, as organized as follows:

- **Section A:** We provide the pseudo-code algorithms for Factorized-FL.
- **Section B:** We describe dataset configurations for label- and domain-heterogeneous scenarios.
- **Section C:** We elaborate on implementation and training details for our methods and the baselines.
- **Section D:** We provide additional experimental results and analysis.

A Factorized-FL Algorithms

In this section, we describe our pseudo-code algorithms for Factorized-FL α and Factorized-FL β in Algorithm 1 and 2. Our Factorized-FL α has strength for not only reducing the dimensionality of model parameters by factorizing them into rank 1 vector spaces and the additional highly-sparse matrices, but also effectively learning client-general and task-specific knowledge, performing well on domain heterogeneous scenario. Particularly, Factorized-FL α transmits a small portion of the models which are a set of \mathbf{u} (\mathcal{U}) and a single vector \mathbf{v}^{L-1} form the second last layer of neural networks, which significantly reduces communication costs while showing strong performance in label- and domain-heterogeneous scenarios, as shown in Section 5 in the main document.

Algorithm 1 Factorized-FL α	Algorithm 2 Factorized-FL β
1: R : number of rounds, E : number of epochs, K : number of clients, \mathcal{F} : a set of clients, $\Omega(\cdot)$: similarity matching function, σ : client-wise similarity score, L : number of layers in neural networks. \mathcal{U}_k , \mathcal{V}_k , and \mathcal{M}_k : factorized parameters. 2: Function RunServer() 3: initialize \mathcal{F} 4: for each round $r = 1, 2, \dots, R$ do 5: $\mathcal{F}^{(r)} \leftarrow$ select $K^{(r)}$ clients from \mathcal{F} 6: for each client $f_k^{(r)} \in \mathcal{F}^{(r)}$ in parallel do 7: if $r > 1$ then 8: $\{\sigma_i\}_{i \neq k}^{K^{(r)}} \leftarrow \Omega(\mathbf{v}_{f_k}^{L-1}, \mathbf{v}_{f_{i \neq k: K^{(r)}}}^{L-1})$ 9: $\mathcal{U}_k^{(r)} \leftarrow \frac{\exp(\epsilon \cdot \sigma_i)}{\sum_{i=1}^{K^{(r)}} \exp(\epsilon \cdot \sigma_i)} \sum_{i=1}^{K^{(r)}} \mathcal{U}_i^{(r)}$ 10: end if 11: $\mathcal{U}_k^{(r+1)}, \mathbf{v}_{f_k}^{L-1} \leftarrow$ RunClient($\mathcal{U}_k^{(r)}$) 12: end for 13: end for 14: Function RunClient(\mathcal{U}_k) 15: $\theta_k \leftarrow \mathcal{U}_k \times \mathcal{V}_k \oplus \mathcal{M}_k$ 16: for each local epoch e from 1 to E do 17: for minibatch $\mathcal{B} \in \mathcal{D}_k$ do 18: $\theta_k \leftarrow \theta_k - \eta \nabla \mathcal{L}(\mathcal{B}; \theta_k)$ 19: end for 20: end for 21: return $\mathcal{U}_k, \mathbf{v}_{f_k}^{L-1}$	1: Function RunServer() 2: initialize \mathcal{F} 3: for each round $r = 1, 2, \dots, R$ do 4: $\mathcal{F}^{(r)} \leftarrow$ select $K^{(r)}$ clients from \mathcal{F} 5: for each client $f_k^{(r)} \in \mathcal{F}^{(r)}$ in parallel do 6: if $r > 1$ then 7: $\{\sigma_i\}_{i \neq k}^{K^{(r)}} \leftarrow \Omega(\mathbf{v}_{f_k}^{L-1}, \mathbf{v}_{f_{i \neq k: K^{(r)}}}^{L-1})$ 8: $\mathcal{U}_k^{(r)} \leftarrow \frac{\exp(\epsilon \cdot \sigma_i)}{\sum_{i=1}^{K^{(r)}} \exp(\epsilon \cdot \sigma_i)} \sum_{i=1}^{K^{(r)}} \mathcal{U}_i^{(r)}$ 9: $\mathcal{V}_k^{(r)} \leftarrow \frac{\exp(\epsilon \cdot \sigma_i)}{\sum_{i=1}^{K^{(r)}} \exp(\epsilon \cdot \sigma_i)} \sum_{i=1}^{K^{(r)}} \mathcal{V}_i^{(r)}$ 10: $\mathcal{M}_k^{(r)} \leftarrow \frac{\exp(\epsilon \cdot \sigma_i)}{\sum_{i=1}^{K^{(r)}} \exp(\epsilon \cdot \sigma_i)} \sum_{i=1}^{K^{(r)}} \mathcal{M}_i^{(r)}$ 11: end if 12: $\mathcal{U}_k^{(r+1)}, \mathcal{V}_k^{(r+1)}, \mathcal{M}_k^{(r+1)} \leftarrow$ RunClient($\mathcal{U}_k^{(r)}, \mathcal{V}_k^{(r)}, \mathcal{M}_k^{(r)}$) 13: end for 14: end for 15: Function RunClient($\mathcal{U}_k, \mathcal{V}_k, \mathcal{M}_k$) 16: $\theta_k \leftarrow \mathcal{U}_k \times \mathcal{V}_k \oplus \mathcal{M}_k$ 17: for each local epoch e from 1 to E do 18: for minibatch $\mathcal{B} \in \mathcal{D}_k$ do 19: $\theta_k \leftarrow \theta_k - \eta \nabla \mathcal{L}(\mathcal{B}; \theta_k)$ 20: end for 21: end for 22: return $\mathcal{U}_k, \mathcal{V}_k, \mathcal{M}_k$

B Dataset Configurations

In this section, we describe detailed configurations for datasets that we used in label- and domain-heterogeneous scenarios.

Table 2: **Label permutations for label-heterogeneous scenario** We provide permutations of labels for each dataset. These permutations are randomly generated based on different seeds.

Dataset	Class	Original Labels	Client No.																			
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
CIFAR-10	Airplane	0	2	5	3	0	8	2	4	4	2	2	0	6	1	8	4	0	0	6	9	7
	Automobile	1	8	4	1	5	1	8	1	0	9	7	7	3	6	2	0	4	1	8	1	4
	Bird	2	3	0	5	3	4	9	9	5	4	1	5	5	3	0	1	6	3	7	6	5
	Cat	3	5	9	0	7	9	3	8	8	7	6	2	7	8	7	6	7	4	4	3	8
	Deer	4	6	2	6	9	6	5	2	1	0	9	6	1	7	3	7	5	5	3	8	9
	Dog	5	4	1	4	8	5	6	6	6	3	4	9	4	4	4	5	9	8	1	2	0
	Frog	6	9	3	2	1	2	0	3	2	6	3	3	0	5	6	2	1	7	2	5	1
	Horse	7	0	7	9	4	3	7	0	3	5	0	1	2	9	5	3	2	9	5	0	6
	Ship	8	1	8	7	6	7	4	5	7	8	5	8	9	0	9	8	3	6	0	7	2
	Truck	9	7	6	8	2	0	1	7	9	1	8	4	8	2	1	9	8	2	9	4	3
SVHN	Digit 0	10	2	5	3	0	8	2	4	4	2	2	0	6	1	8	4	0	0	6	9	7
	Digit 1	1	8	4	1	5	1	8	1	0	9	7	7	3	6	2	0	4	1	8	1	4
	Digit 2	2	3	0	5	3	4	9	9	5	4	1	5	5	3	0	1	6	3	7	6	5
	Digit 3	3	5	9	0	7	9	3	8	8	7	6	2	7	8	7	6	7	4	4	3	8
	Digit 4	4	6	2	6	9	6	5	2	1	0	9	6	1	7	3	7	5	5	3	8	9
	Digit 5	5	4	1	4	8	5	6	6	6	3	4	9	4	4	4	5	9	8	1	2	0
	Digit 6	6	9	3	2	1	2	0	3	2	6	3	3	0	5	6	2	1	7	2	5	1
	Digit 7	7	0	7	9	4	3	7	0	3	5	0	1	2	9	5	3	2	9	5	0	6
	Digit 8	8	1	8	7	6	7	4	5	7	8	5	8	9	0	9	8	3	6	0	7	2
	Digit 9	9	7	6	8	2	0	1	7	9	1	8	4	8	2	1	9	8	2	9	4	3

B.1 Label Heterogeneous Scenario

We use CIFAR-10 and SVHN for the label-heterogeneous scenario. We first split each dataset into train, validation, and test sets for CIFAR-10 (48,000/6,000/6,000) and SVHN (79,431/9,929/9,929). We then split the train set into K local partitions $\mathcal{P}_{1:20}$ ($K=20$) for IID partitions (all instances in each class are evenly distributed to all clients) or for the non-IID partitions (instances in each class are sampled from Dirichlet distribution with $\alpha=0.5$). We further permute the labels for each class per local partition \mathcal{P}_k for permuted IID and permuted non-IID scenarios. We use different random seed per client, i.e. `fixed global seed + client id`, for example, `1234 + 0` for Client 1 and `1234 + 19` for Client 20. We provide permutations of labels that we used for each dataset in Table 2.

B.2 Domain Heterogeneous Scenario

We use CIFAR-100 datasets (60,000) and create five sub-datasets grouped by 10 similar classes, such as Fruits&Foods (6,000), Transport (6,000), Household Objects (6,000), Animals (6,000), Trees&Flowers (6,000). We then split train (4,800), test (600), validation (600) sets for each sub-dataset. To have 20 clients in total, we assign four clients per subdataset, and split each train set into 4 partitions, making a single partition contains 1,200 instances. Additionally, we further permute the labels for those 20 partitions to simulate more realistic scenarios where labeling schemes are not synchronized across all clients even in the same domain (sub-dataset). We provide class division and label permutation information in Table 3.

C Training Details & Implementations

In this section, we provide detailed implementation and training details that are not described in the main document.

C.1 Baseline Models

Stand-Alone does not share their locally learned knowledge with other clients. It shows the pure model performance on the data partitions. In our extremely heterogeneous scenarios, where knowledge collapse may happen severely, which even deteriorates the local knowledge, this model shows the comparable performance amongst the existing models.

FedAvg [18] performs weighted aggregation of the model parameters, considering the size of the local training set. This model is considered as the standard baseline of many federated learning algorithms.

Table 3: **Class division and label permutation information for domain-heterogeneous scenario** We provide class division information and label permutation details for each domain. These permutations are randomly generated based on the same method used in label-heterogeneous scenario using different seeds.

Domain	Class	Original Labels	Client No.																			
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Household Objects	Bed	5	2	5	3	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Chair	20	8	4	1	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Couch	22	3	0	5	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Table	25	5	9	0	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Wardrobe	39	6	2	6	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Clock	40	4	1	4	8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Keyboard	84	9	3	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Lamp	86	0	7	9	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Telephone	87	1	8	7	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Television	94	7	6	8	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
Fruits & Foods	Apple	0	-	-	-	-	8	2	4	4	-	-	-	-	-	-	-	-	-	-	-	-
	Mushroom	9	-	-	-	-	1	8	1	0	-	-	-	-	-	-	-	-	-	-	-	-
	Orange	10	-	-	-	-	4	9	9	5	-	-	-	-	-	-	-	-	-	-	-	-
	Pear	16	-	-	-	-	9	3	8	8	-	-	-	-	-	-	-	-	-	-	-	-
	Sweet Pepper	28	-	-	-	-	6	5	2	1	-	-	-	-	-	-	-	-	-	-	-	-
	Bottle	51	-	-	-	-	5	6	6	6	-	-	-	-	-	-	-	-	-	-	-	-
	Bowl	53	-	-	-	-	2	0	3	2	-	-	-	-	-	-	-	-	-	-	-	-
	Can	57	-	-	-	-	3	7	0	3	-	-	-	-	-	-	-	-	-	-	-	-
	Cup	61	-	-	-	-	7	4	5	7	-	-	-	-	-	-	-	-	-	-	-	-
Plate	83	-	-	-	-	0	1	7	9	-	-	-	-	-	-	-	-	-	-	-	-	
Trees & Flowers	Orchid	47	-	-	-	-	-	-	-	2	2	0	6	-	-	-	-	-	-	-	-	-
	Poppy	52	-	-	-	-	-	-	-	9	7	7	3	-	-	-	-	-	-	-	-	-
	Rose	54	-	-	-	-	-	-	-	4	1	5	5	-	-	-	-	-	-	-	-	-
	Sunflower	56	-	-	-	-	-	-	-	7	6	2	7	-	-	-	-	-	-	-	-	-
	Tulip	59	-	-	-	-	-	-	-	0	9	6	1	-	-	-	-	-	-	-	-	-
	Maple Tree	62	-	-	-	-	-	-	-	3	4	9	4	-	-	-	-	-	-	-	-	-
	Oak Tree	70	-	-	-	-	-	-	-	6	3	3	0	-	-	-	-	-	-	-	-	-
	Palm Tree	82	-	-	-	-	-	-	-	5	0	1	2	-	-	-	-	-	-	-	-	-
	Pine Tree	92	-	-	-	-	-	-	-	8	5	8	9	-	-	-	-	-	-	-	-	-
Willow Tree	96	-	-	-	-	-	-	-	1	8	4	8	-	-	-	-	-	-	-	-	-	
Transport	Lawn Mower	8	-	-	-	-	-	-	-	-	-	-	-	1	8	4	0	-	-	-	-	-
	Rocket	13	-	-	-	-	-	-	-	-	-	-	-	6	2	0	4	-	-	-	-	-
	Streetcar	41	-	-	-	-	-	-	-	-	-	-	-	3	0	1	6	-	-	-	-	-
	Tank	48	-	-	-	-	-	-	-	-	-	-	-	8	7	6	7	-	-	-	-	-
	Tractor	58	-	-	-	-	-	-	-	-	-	-	-	7	3	7	5	-	-	-	-	-
	Bicycle	69	-	-	-	-	-	-	-	-	-	-	-	4	4	5	9	-	-	-	-	-
	Bus	81	-	-	-	-	-	-	-	-	-	-	-	5	6	2	1	-	-	-	-	-
	Motorcycle	85	-	-	-	-	-	-	-	-	-	-	-	9	5	3	2	-	-	-	-	-
	Pickup Truck	89	-	-	-	-	-	-	-	-	-	-	-	0	9	8	3	-	-	-	-	-
Train	90	-	-	-	-	-	-	-	-	-	-	-	2	1	9	8	-	-	-	-	-	
Animals	Fox	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	6	9	7	-	-
	Porcupine	34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	8	1	4	-	-
	Possum	42	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	7	6	5	-	-
	Raccoon	43	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	4	3	8	-	-
	Skunk	63	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	3	8	9	-	-
	Bear	64	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	1	2	0	-	-
	Leopard	66	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	2	5	1	-	-
	Lion	75	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	5	0	6	-	-
	Tiger	88	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	0	7	2	-	-
Wolf	97	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	9	4	3	-	-	

FedProx [15] uses proximal regularization term for alleviating divergence between global parameters and the local parameters. This model is devised to tackle data heterogeneity (also known as the non-iid problems) across the clients.

Clustered-FL [23] performs bi-partitioning process for the participants when certain conditions are satisfied (calculating the norm of the gradient updates and comparing it with given threshold values). It continues to split the given pool of the clients into two novel clusters and performs knowledge aggregations for each cluster.

pFedPara [20] uses the Hadamard product of two sets of low rank matrices and reconstructs the kernel parameters. It shares one of the sets of low rank matrices with other clients and remains another set of low rank matrices for the personalization.

Per-FedAvg [5] adopts Model-Agnostic Meta Learning (MAML) [6] approach to federated learning algorithm to search the initial global model which participants can easily adapt to their local data by being trained with only one or a few steps of gradient descent.

FedFOMO [28] leverages other clients’ knowledge for improving their local models. It downloads a few random models from other clients and validates them on their own validation set at each client. When some parameters show better validation performance compared to their own local performance, it aggregates such helpful parameters with their own local parameters.

C.2 Training Details

As default, all training configurations are equally set across all models, unless otherwise stated to ensure stricter fairness. We use ResNet-9 architecture as local backbone networks and train them on 32×32 sized images with 256 for batch size. We apply data augmentations, i.e. cropping, flipping, jittering, etc, during training. Optimizer that we used is Stochastic Gradient Descent (SGD). We set 1e-3 for learning rate, 1e-6 for weight decay, and 0.9 for momentum. For baseline models, we use the reported hyper-parameters as default, or we adjust hyper-parameters so that they show the best performance for fairness. For ours and pFedPara, the model capacity is adjusted to around 90% - 99% of the original size, as we fairly compare with other methods that use full capacity (2.57M number of parameters). For ours, we use [5e-4, 1e-3] for $\lambda_{\text{sparsity}}$, [0-0.75] for τ , [1, 20] for ϵ . We use 8 GPUs (NVIDIA Titan Xp) for experiments.

C.3 ResNet-9 Architecture

Table 4: Detailed ResNet-9 Architecture

We use ResNet-9 architecture consisting of eight convolutional layers and one fully connected layer as a classifier, as described in Table 4. We use max pooling with size 2 after Conv 5 and an adaptive max pooling after Conv 8 to make output width 1 for the following FC layer. The total number of parameters of the model is 2.57M. As we use

Layer	Input	Output	Filter Size	Stride	Dimension of \mathbf{W}^l
Conv 1	3	64	3	1	$64 \times 3 \times 3 \times 3$
Conv 2	64	128	5	2	$128 \times 64 \times 5 \times 5$
Conv 3	128	128	3	1	$128 \times 128 \times 3 \times 3$
Conv 4	128	128	3	1	$128 \times 128 \times 3 \times 3$
Conv 5	128	256	3	1	$256 \times 128 \times 3 \times 3$
Conv 6	256	256	3	1	$256 \times 256 \times 3 \times 3$
Conv 7	256	256	3	1	$256 \times 256 \times 3 \times 3$
Conv 8	256	256	3	1	$256 \times 256 \times 3 \times 3$
FC 1	256	C	-	-	$256 \times C$

PyTorch framework for implementation and the default data type of tensor of the framework is 32-bits floating point, the model size can be calculated as $2.57 \times 4 = 10.28$ Mbytes.

C.4 Calculation of Communication Cost

We measure the communication cost by $\{(P_{S2C} + P_{C2S}) \times 4\}_{\text{byte}} \times K \times R$, where P_{S2C} is number of server-to-client transmitted parameters and P_{C2S} is number of client-to-server transmitted parameters. Depending on the FL algorithms, P_{S2C} and P_{C2S} are differently calculated. For example, FedFOMO downloads a few random models from the server (10 by default as reported in the paper) but sends only a single local model to the server. Our Factorized-FL α only sends the small portion of model parameters, \mathcal{U} and \mathbf{v}^{L-1} , to server, while receiving a single set of \mathcal{U} from server.

D Additional Experimental Results

Table 5: The Approximated Space-Time Analysis We provide the approximated space-time analysis for training, inference, client-to-server communication costs, and server-to-client communication costs.

Method	Training	Inference	C2S Cost	S2C Cost
FedAvg	$W \cdot N$	$W \cdot N$	W	W
pFedPara	$(U + V)^2 \cdot N$	$(U + V)^2 \cdot N$	$U + V$	$U + V$
FedFOMO	$W \cdot N + H \cdot W \cdot N_{\text{val}}$	$W \cdot N$	W	$(H + 1) \cdot W$
Factorized-FL α	$(U \cdot V + M) \cdot N$	$(U \cdot V + M) \cdot N$	$U + \mathbf{v}$	U
Factorized-FL β	$(U \cdot V + M) \cdot N$	$(U \cdot V + M) \cdot N$	$U + V + M$	$U + V + M$

D.1 The Space-Time Analysis

We provide the approximated space-time complexity in Table 5, compared against those of the essential baselines. Let W be the model size, N be the number of instances, H be the number of other models (for FedFOMO algorithms), U be the size of the rank-1 vectors (or matrices for pFedPara) in \mathcal{U} , V be that of \mathcal{V} , M be the that of \mathcal{M} , \mathbf{v} be the vector for the second last layer from \mathcal{V} . For training procedure of FedFOMO, it requires validation step for H number of other model weights on their validation set at each local client. Note that, for communication costs of our methods (Fac. -FL), $(U \cdot V + M) \approx W$ and thus $(U + V + M) < W$ for both Client-to-Server (C2S) and Server-to-Client (S2C) communication costs. Please also refer to the actual size of the data transmission for the communication cost in Table 1.

D.2 Sparsity Analysis on FL Scenarios

In the main document, we show the effect of model size and sparsity controlled by $\lambda_{\text{sparsity}}$ for a single model. In this section, we analyze it under federated learning scenario. In Figure 8 (a), we show the performance over model size in domain heterogeneous scenarios. As shown, our method shows superior performance even with around 65% of the model size over the baseline model that achieves the best performance (Per-FedAvg) amongst other baseline models. With 50% sparsity, ours still shows competitive performance compared to Clustered-FL and FedAvg, while it starts being significantly degenerated when sparsity becomes over 50%.

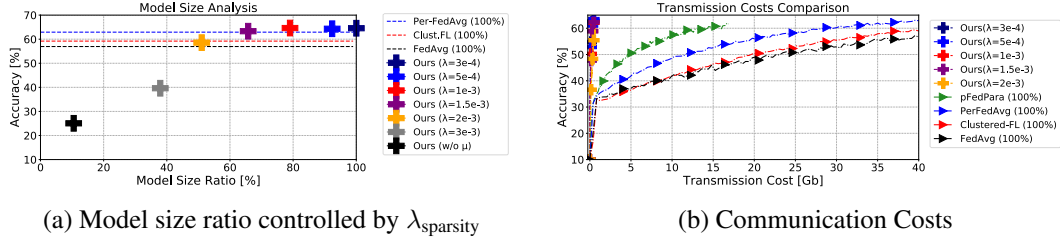


Figure 8: **Model size and communication costs comparison** (a) we plot accuracy over model size on domain heterogeneous scenario. (b) we plot accuracy over transmission costs on domain heterogeneous scenario.

In Figure 8 (b), we show accuracy over communication costs. Note that, in our method, the model size is not really related to the communication costs since we send very small portion of model parameters. For example, even though we use almost full model size ($\lambda_{\text{sparsity}}=3e-4$), our communication cost is significantly lesser than the other baseline models, as shown in the figure.

D.3 Test Accuracy Curves and Plots for Communication Costs

For label-heterogeneous FL scenario (Table 1 (Top)), we provide all test accuracy curves over communication rounds and transmission costs for results of CIFAR-10 and SVHN with standard IID/non-IID and permuted IID/non-IID partitions in Figure 10. For domain-heterogeneous FL scenario (Table 1 (Bottom)), we provide performance of 20 clients in Figure 11.

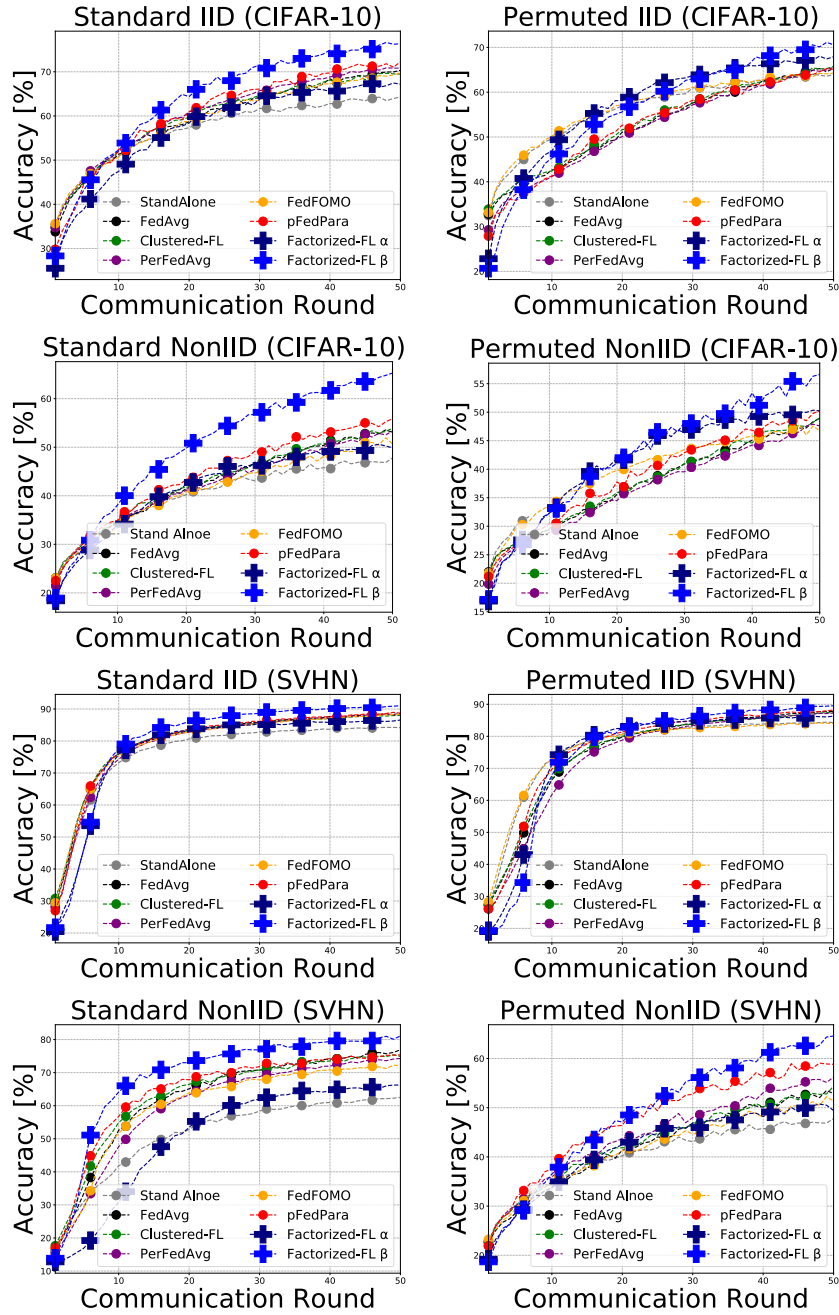


Figure 9: Test accuracy curves over communication round for standard federated learning and label-heterogeneous FL scenario: We provide test accuracy curves on CIFAR-10 and SVHN in standard iid/non-iid and permuted iid/non-iid partitions ($E=5, R=50$).

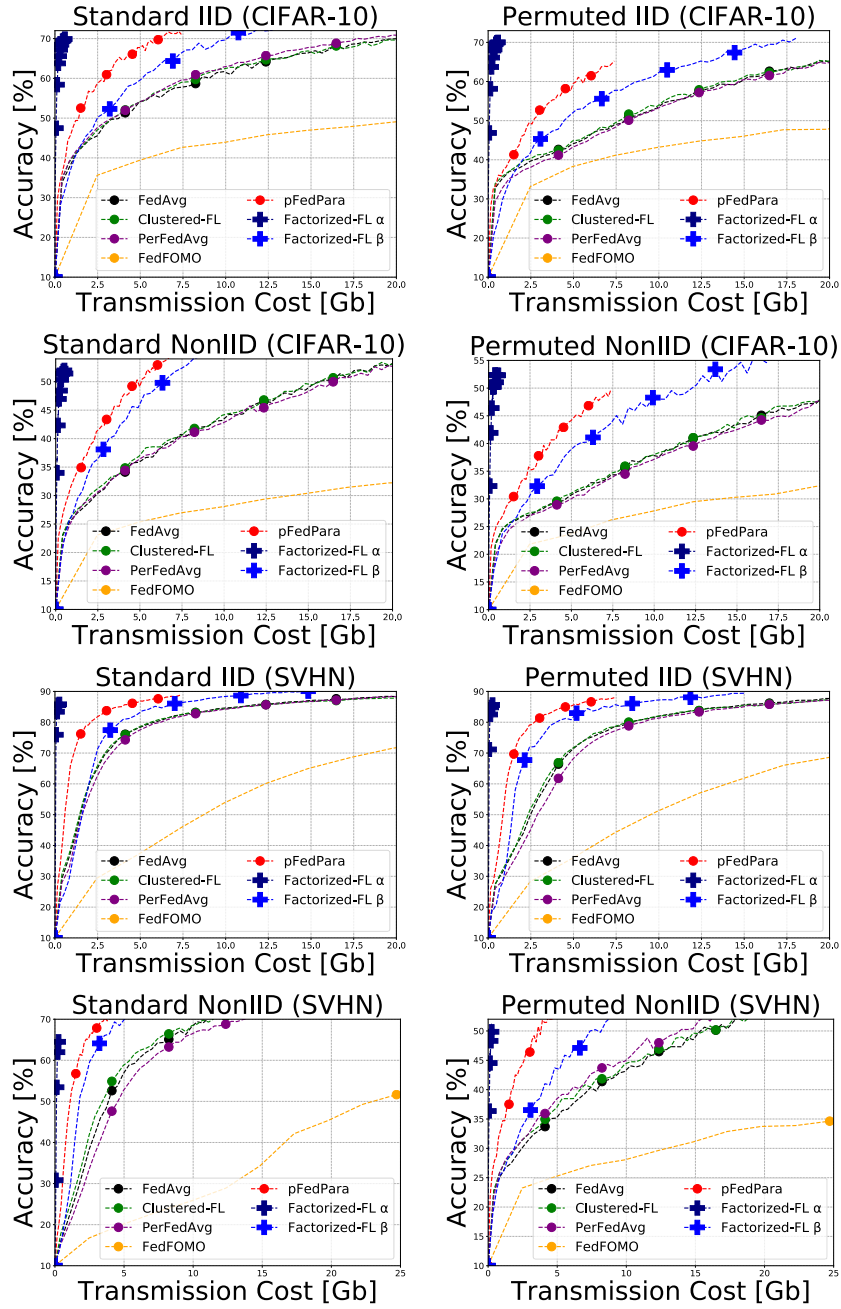


Figure 10: **Test accuracy over communication costs for standard federated learning and label-heterogeneous FL scenario:** We provide test accuracy curves on CIFAR-10 and SVHN in standard iid/non-iid and permuted iid/non-iid partitions ($E=5, R=50$).

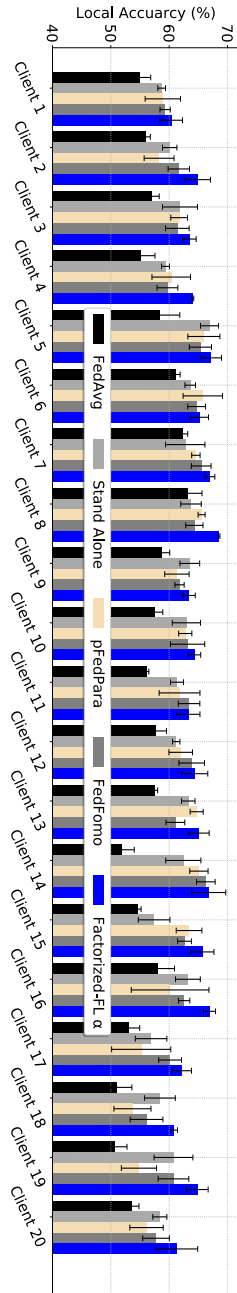


Figure 11: **Performance of all 20 clients in domain heterogeneous scenario:** We plot performance of 20 clients in domain-heterogeneous scenario, of which results are corresponding to Table 1 (Bottom).