We present additional experiments to provide a more comprehensive baseline comparison. Specifically, we apply various state-of-the-art optimization methods to Graphical Lasso (GLasso) and three distinct optimization methods to multi-objective optimization (MOO). Detailed descriptions of the additional algorithms for GLasso are provided in Table 1, and numerical results are presented in Table 2. The experiments conducted are as follows:

- **Experiment 1**: Synthetic data comprising 100 variables, 2 subgroups, and 1000 observations for each subgroup is generated following the procedure outlined in Appendix D.2 (Simulation Study of Fair GLasso). This experiment aims to validate the improved time complexity of faster optimization methods on both GLasso and MOO. All GLasso methods achieve the optimal loss, while all Fair GLasso methods maintain competitive performance in terms of the loss of graphical models and reduce pairwise graph disparity error, indicating enhanced fairness.
- **Experiment 2**: Experiment 1 is repeated on a synthetic dataset with an increased number of variables (200).
- **Experiment 3**: Synthetic data with 100 variables, 10 subgroups, and 1000 observations for each subgroup is generated to assess the improved efficiency when the number of sensitive groups is large. The adapted method, termed "Subset," randomly selects a subset of objectives in each iteration during MOO training. This method reduces the time required by the original method in the paper by one-third.

Table 1: Explanation of Algorithms for GLasso and Multi-Objective Optimization (MOO).

Algorithm	**Paper**
Optimization Algorithm for GLasso	
ISTA	Iterative Shrinkage-Thresholding Algorithm (Ours)
PISTA	PISTA: preconditioned Iterative Soft Thresholding Algorithm for Graphical Lasso
GISTA	Iterative Thresholding Algorithm for Sparse Inverse Covariance Estimation
OBN	Newton-Like Methods for Sparse Inverse Covariance Estimation
Optimization Algorithm for Multi-Objective Optimization (MOO)	
ISTA	An accelerated proximal gradient method for multiobjective optimization
FISTA	Fast Iterative Shrinkage-Thresholding Algorithm
Subset	On Learning Fairness and Accuracy on Multiple Subgroups

Table 2: Outcomes of additional baseline with different optimization algorithms applied to GLasso and Multi-Objective Optimization (MOO), measured in terms of the value of the objective function (F_1), the summation of the pairwise graph disparity error (Δ), and the average computation time in seconds ($\pm$ standard deviation) from 10 repeated experiments. "$\downarrow$" indicates that smaller values are better. Our method applies ISTA to both GLasso and MOO (first row in each experiment). All experiments are conducted using the same runtime environment on Google Colab.

Algorithm		$F_1 \downarrow$		$\%F_1 \uparrow$	$\Delta \downarrow$		$\%\Delta \uparrow$	**Runtime** $\downarrow$	
GLasso	**MOO**	**GLasso**	**Fair GLasso**		**GLasso**	**Fair GLasso**		**GLasso**	**Fair GLasso**
Synthetic Dataset 1 (2 Subgroups, 100 Variables, 1000 Observations in Each Group)									
ISTA	ISTA	97.17169	97.44911	-0.29%	7.81488	0.57942046	+92.59%	0.501 ($\pm$ 0.21)	85.48 ($\pm$ 1.92)
ISTA	FISTA	97.17169	97.43758	-0.27%	7.81488	0.88351416	+88.69%	0.297 ($\pm$ 0.12)	26.56 ($\pm$ 1.11)
PISTA	FISTA	97.17170	97.43800	-0.27%	7.81895	0.90841687	+88.38%	13.524 ($\pm$ 1.1)	59.66 ($\pm$ 2.65)
GISTA	FISTA	97.17169	97.43814	-0.27%	7.81489	0.90888418	+88.37%	0.426 ($\pm$ 0.16)	21.27 ($\pm$ 0.94)
OBN	FISTA	97.17169	97.43849	-0.27%	7.81336	0.91124797	+88.34%	0.483 ($\pm$ 0.16)	22.48 ($\pm$ 0.92)
Synthetic Dataset 2 (2 Subgroups, 200 Variables, 2000 Observations in Each Group)									
ISTA	ISTA	199.71222	200.70050	-0.49%	40.51080	1.48545236	+96.33%	2.622 ($\pm$ 1.28)	206.68 ($\pm$ 3.27)
ISTA	FISTA	199.71222	200.68234	-0.49%	40.51080	1.84854653	+95.44%	2.640 ($\pm$ 0.76)	108.08 ($\pm$ 2.42)
PISTA	FISTA	199.71222	200.67490	-0.48%	40.52060	1.94742524	+95.19%	39.162 ($\pm$ 2.3)	178.72 ($\pm$ 3.50)
GISTA	FISTA	199.71222	200.67559	-0.48%	40.51081	2.02596987	+95.00%	2.365 ($\pm$ 0.26)	78.99 ($\pm$ 3.07)
OBN	FISTA	199.71222	200.71791	-0.50%	40.51101	2.48346238	+93.87%	2.403 ($\pm$ 0.68)	53.11 ($\pm$ 2.17)
Synthetic Dataset 3 (10 Subgroups, 100 Variables, 1000 Observations in Each Group)									
ISTA	ISTA	95.33288	95.60252	-0.28%	11.39373	0.31083701	+97.27%	0.641 ($\pm$ 0.28)	224.07 ($\pm$ 2.29)
ISTA	Subset	95.33288	95.50581	-0.18%	11.39373	1.51333479	+86.72%	0.626 ($\pm$ 0.19)	143.19 ($\pm$ 2.28)