Theorem 3 (PNG Finds Local Optimum). *Under Assumption 1, we have*

If $\theta_t \notin \mathcal{P}_e$ *is a fixed point of the algorithm, that is,* $\frac{\mathrm{d}\theta_t}{\mathrm{d}t} = -v_t = 0$, *and* F, $\boldsymbol{\ell}$ *are convex in a neighborhood* θ_t, *then* θ_t *is a local minimum of* F *in the Pareto closure* $\overline{\{\theta_t\}}$, *that is, there exists a neighborhood of* θ_t *in which there exists no point* θ' *such that* $F(\theta') < F(\theta_t)$ *and* $\boldsymbol{\ell}(\theta') \preceq \boldsymbol{\ell}(\theta_t)$.

If $\theta_t \in \mathcal{P}_e$, *we have* $v_t = \nabla F(\theta_t)$, *and hence a fixed point with* $\frac{\mathrm{d}\theta_t}{\mathrm{d}t} = -v_t = 0$ *is an unconstrained local minimum of* F *when* F *is locally convex on* θ_t.