On Understanding Knowledge Graph Representation

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Paper under double-blind review

Abstract

Many methods have been developed to represent knowledge graph data, which implicitly exploit low-rank latent structure in the data to encode known information and enable unknown facts to be inferred. To predict whether a relationship holds between entities, their embeddings are typically compared in the latent space following a relation-specific mapping. Whilst link prediction has steadily improved, the latent structure, and hence why such models capture semantic information, remains unexplained. We build on recent theoretical interpretation of word embeddings as a basis to consider an explicit structure for representations of relations between entities. For identifiable relation types, we are able to predict properties and justify the relative performance of leading knowledge graph representation methods, including their often overlooked ability to make independent predictions.

1 Introduction

Knowledge graphs are large repositories of binary relations between words (or entities) in the form of fact triples *(subject, relation, object)*. Many models have been developed for learning representations of entities and relations in knowledge graphs, such that known facts can be recalled and previously unknown facts can be inferred, a task known as *link prediction*. Recent link prediction models (e.g. Bordes et al., 2013; Trouillon et al., 2016; Balažević et al., 2019b) learn entity representations, or *embeddings*, of far lower dimensionality than the number of entities, by capturing latent structure in the data. Relations are typically represented as a mapping from the embedding of a subject entity to its related object entity embedding(s). Although knowledge graph representation has advanced in terms of link prediction performance for nearly a decade, relatively little is understood of the latent structure that underpins low-rank models, which we seek to address in this work.

We start by drawing a parallel between entity embeddings in knowledge graphs and unsupervised word embeddings, as learned by algorithms such as Word2Vec (W2V) (Mikolov et al., 2013) and GloVe (Pennington et al., 2014). We assume that words have latent features, e.g. meaning(s), tense, grammatical type, that are innate and fixed, irrespective of what an embedding may capture (which may be only a part, subject to the embedding method and/or the data source); and that this same latent structure gives rise to patterns observed in the data, e.g. in word co-occurrence statistics and in which words are related to which. As such, an understanding of the latent structure from one embedding task (e.g. word embedding) might be useful to another (e.g. knowledge graph entity embedding).

Recent work (Allen & Hospedales, 2019; Allen et al., 2019) theoretically explains how semantic properties are encoded in word embeddings that (approximately) factorise a matrix of word co-occurrence *pointwise mutual information* (PMI), e.g. as is known for W2V (Levy & Goldberg, 2014). Semantic relationships between words (specifically similarity, relatedness, paraphrase and analogy) are proven to manifest as linear relationships between rows of the PMI matrix (subject to known error terms), of which word embeddings can be considered low-rank projections. This explains why similar words (e.g. synonyms) have similar embeddings; and embeddings of analogous word pairs share a common “vector offset”.

Importantly to our work, this insight provides a basis to identify geometric embedding relationships necessary for other semantic relations to hold, such as those of knowledge graphs. Such geometric relationships, or *relation requirements*, implicitly describe relation-specific mappings between entity embeddings, i.e. the essence of relation representation. We find that relations of popular knowledge graphs (e.g. WordNet (Miller, 1995), NELL (Carlson et al., 2010)) fall into a hierarchy of 3 relation

Table 1: Score functions of representative linear link prediction models. $\boldsymbol{R} \in \mathbb{R}^{d_e \times d_e}$ and $\boldsymbol{r} \in \mathbb{R}^d_e$ are the relation matrix and translation vector, $\mathbf{W} \in \mathbb{R}^{d_e \times d_r \times d_e}$ is the core tensor and $b_s, b_o \in \mathbb{R}$ are the entity biases.

Model		Linear Subcategory	Score Function
TransE	(Bordes et al., 2013)	additive	$-\|\boldsymbol{e}_s + \boldsymbol{r} - \boldsymbol{e}_o\|_2^2$
DistMult	(Yang et al., 2015)	multiplicative (diagonal)	$\boldsymbol{e}_s^\top \boldsymbol{R} \boldsymbol{e}_o$
TuckER	(Balažević et al., 2019b)	multiplicative	$\mathbf{W} \times_1 \boldsymbol{e}_s \times_2 \boldsymbol{r} \times_3 \boldsymbol{e}_o$
MuRE	(Balažević et al., 2019a)	multiplicative (diagonal) + additive	$-\|\boldsymbol{R}\boldsymbol{e}_s + \boldsymbol{r} - \boldsymbol{e}_o\|_2^2 + b_s + b_o$

types: highly related, (generalised) specialisation and (generalised) context-shift. Understanding the requirements of a relation representation provides a "blue-print" to consider knowledge graph representation models, which we can use to make several theoretical predictions and justify the relative performance of recent link prediction models for different relation types.

In summary, the key contributions of this work are:

- to use recent understanding of PMI-based word embeddings as a basis for developing new insight into knowledge graph relations and their representation;
- to derive a novel categorisation of relations based on identified *relation requirements*, i.e. what a relation representation must achieve to map a subject embedding to all related object embeddings;
- to show that the performance of several leading link prediction models on different relations depends on the ability of the model's architecture to represent that type of relation;
- to theoretically predict identifiable properties of knowledge graph models, e.g. that the strength of a relation's *relatedness* is reflected in the eigenvalues of its relation matrix; and
- noting how ranking metrics can be flawed, to provide novel insight into recent knowledge graph models from their per-relation prediction accuracy, an evaluation metric we recommend in future.

2 Background

Our work draws on knowledge graph representation and word embedding. Whilst related, these tasks differ materially in their training data. The former is restricted to datasets crafted by hand or automatically generated, the latter has the vast abundance of natural language text (e.g. Wikipedia).

2.1 Knowledge graph representation

Almost all recent knowledge graph models represent entities e_s, e_o as vectors $\boldsymbol{e}_s, \boldsymbol{e}_o \in \mathbb{R}^{d_e}$ of low dimension (e.g. $d_e = 200$) relative to the number of entities n_e (typically of order 10^4), and relations as transformations in the latent space from subject entity embedding to object. These models are distinguished by their *score function*, which defines (i) the form of the relation transformation, e.g. matrix multiplication, vector addition; and (ii) how "closeness" between the transformed subject embedding and an object embedding is evaluated, e.g. dot product, Euclidean distance. Score functions can be non-linear (e.g. Dettmers et al. (2018)), or linear and sub-categorised as additive, multiplicative or both. We focus on linear models due to their simplicity and strong performance at link prediction (including state-of-the-art). Table 1 shows the score functions of competitive linear models that span all linear sub-categories: TransE (Bordes et al., 2013), DistMult (Yang et al., 2015), TuckER (Balažević et al., 2019b) and MuRE (Balažević et al., 2019a).

Additive models typically use Euclidean distance and contain a relation-specific *translation* from a (possibly transformed) subject to a (possibly transformed) object entity embedding. A generic additive score function is given by $\phi(e_s, r, e_o) = -\|\boldsymbol{R}_s\boldsymbol{e}_s + \boldsymbol{r} - \boldsymbol{R}_o\boldsymbol{e}_o\|_2^2 + b_s + b_o = -\|\boldsymbol{e}_s^{(r)} + \boldsymbol{r} - \boldsymbol{e}_o^{(r)}\|_2^2 + b_s + b_o$. The simplest example is TransE for which $\boldsymbol{R}_s = \boldsymbol{R}_o = \boldsymbol{I}$ and $b_s = b_o = 0$. The score function of MuRE has $\boldsymbol{R}_o = \boldsymbol{I}$ and so combines multiplicative ($\boldsymbol{R}_s = \boldsymbol{R}$) and additive ($\boldsymbol{r}$) components.

Multiplicative models have the generic score function $\phi(e_s, r, e_o) = \boldsymbol{e}_s^\top \boldsymbol{R} \boldsymbol{e}_o = \langle \boldsymbol{e}_s^{(r)}, \boldsymbol{e}_o \rangle$, i.e. a *bilinear product* of the entity embeddings and a relation-specific matrix $\boldsymbol{R}$. DistMult is a simple example with diagonal $\boldsymbol{R}$ and so cannot model asymmetric relations (Trouillon et al., 2016). In TuckER, each relation-specific $\boldsymbol{R} = \mathbf{W} \times_3 \boldsymbol{r}$ is a linear combination of d_r "prototype" relation matrices in a core tensor $\mathbf{W} \in \mathbb{R}^{d_e \times d_r \times d_e}$ (where $\times_n$ denotes tensor product along mode n), facilitating *multi-task learning* across relations.

2.2 WORD EMBEDDING

Algorithms such as Word2Vec (Mikolov et al., 2013) and GloVe (Pennington et al., 2014) generate succinct low-rank word embeddings that perform well on downstream tasks (Baroni et al., 2014). Such models predict the context words (c_j) observed around each target word (w_i) in a text corpus using shallow neural networks. Whilst recent language models (e.g. Devlin et al. (2018); Peters et al. (2018)) create impressive *context-specific* word embeddings, we focus on the former embeddings since knowledge graph entities have no obvious context and, more importantly, they are *interpretable*.

Levy & Goldberg (2014) show that, for a dictionary of unique words $\mathbb{D}$ and embedding dimension $d \ll |\mathbb{D}|$, W2V's loss function is minimised when its weight matrices $\boldsymbol{W}, \boldsymbol{C} \in \mathbb{R}^{d\times|\mathbb{D}|}$ (whose columns are word embeddings $\boldsymbol{w}_i, \boldsymbol{c}_j$) factorise a word co-occurrence *pointwise mutual information* (PMI) matrix, subject to a shift term ($\text{PMI}(w_i, c_j) = \log \frac{P(w_i,c_j)}{P(w_i)P(c_j)}$). This connects W2V to earlier count-based embeddings and specifically to PMI, which has a long history in linguistic analysis (Turney & Pantel, 2010). From its loss function, GloVe can be seen to perform a comparable factorisation.

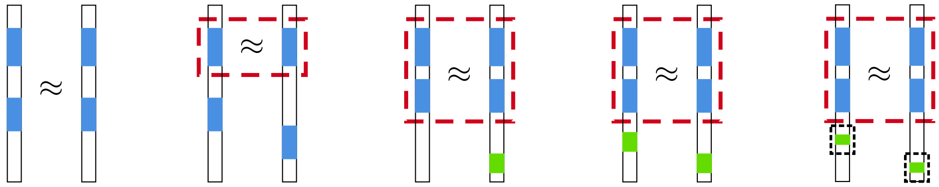
Recent work shows why word embeddings that factorise such PMI matrix encode semantic word relationships (Allen & Hospedales, 2019; Allen et al., 2019). The authors show that word embeddings can be seen as low-rank projections of high dimensional *PMI vectors* (rows of the PMI matrix), between which the semantic relationships of similarity, relatedness, paraphrase and analogy provably manifest as linear geometric relationships (subject to defined error terms), which are then preserved, under a sufficiently linear projection, between word embeddings. Thus *similar* words are known to have similar embeddings, and the embeddings of *analogous* word pairs share a common vector offset.

Specifically, the PMI vectors ($\boldsymbol{p}_x$ for word x) of an analogy "*man* is to *king* as *woman* is to *queen*" satisfy $\boldsymbol{p}_\text{Q} - \boldsymbol{p}_\text{W} \approx \boldsymbol{p}_\text{K} - \boldsymbol{p}_\text{M}$ because the difference between words associated with *king* and *man* (e.g. *reign*, *crown*) mirrors that between *queen* and *woman*. This leads to a common difference between their co-occurrence distributions (over all words), giving a common difference between their PMI vectors, which projects to a common difference between embeddings. Any discrepancy in the mirroring of word associations is shown to introduce error, weakening the analogy, as does a lack of statistical independence within certain word pairs (see (Allen & Hospedales, 2019)). The common difference in word co-occurrence distributions can be interpreted semantically as a common change in context (*context-shift*), e.g. the increased association with words {*reign*, *crown*, etc.}, that transforms *man* to *king* and *woman* to *queen*, can be seen to add a *royal* context. Under this interpretation, context can also be subtracted, e.g. "*king* is to *man* as *queen* is to *woman*" (minus *royal*); or both, e.g. "*boy* is to *king* as *girl* is to *queen*" (minus *youth* plus *royal*). Adding context can also be interpreted as *specialisation*, and subtracting context as *generalisation*. This establishes a correspondence between common word embedding vector offsets and semantic context-shifts.

Although the projection from PMI vectors to word embeddings preserves the *relative* relationships, and thus the above semantic interpretability of common embedding differences, a direct interpretation of dimensions themselves is obscured, not least because any embedding matrix can be arbitrarily scaled/rotated if the other is inversely transformed.

3 RELATIONSHIPS BETWEEN EMBEDDINGS OF RELATED WORDS

Our aim is to build on the understanding of PMI-based word embeddings (henceforth *word embeddings*), to identify what a knowledge graph relation representation needs to achieve to map all subject word embeddings to all related object word embeddings. We note that if a semantic relation between two words implies a particular geometric relationship between their embeddings, then the latter serves as a necessary quantitative condition for the former to hold (a *relation condition*). Relation conditions implicitly define a relation-specific mapping by which all subject embeddings are mapped to all related object embedding(s), allowing related entities to be identified by a proximity measure (e.g. Euclidean distance or dot product). Since this is the approach of many knowledge graph representation models, their performance can be contrasted with their ability to express mappings that satisfy required relation conditions. For example, similarity and context change relations respectively imply closeness and a relation-specific vector offset between embeddings (S2.2). Such relation conditions can be tested for by respectively making no change to the subject entity or adding the relation-specific offset, before measuring proximity with the object. We note that since relation conditions are not necessarily sufficient, they do not guarantee a relation holds, i.e. false positives may arise.



(a) Similarity **(b)** Relatedness **(c)** Specialisation **(d)** Context-shift **(e)** Gen. context-shift

Figure 1: Relationships between PMI vectors of entities under different relation types. Shaded regions indicate strong word associations (positive PMI values); red lines indicate relatedness; black lines denote context sets.

In general, the data from which knowledge graph embeddings are derived differs greatly to the co-occurrence data used for word embeddings, and the latter would not be anticipated to be learned by knowledge graph models. However, word embeddings provide a *known solution* (i.e. minimise the loss function) of any knowledge graph model able to express the required mapping(s) derived from relation conditions, where the loss function measures proximity between mapped entities.

The relation conditions for certain relation types (underlined) follow readily from S2.2:

• **Similarity:** Semantically similar words induce similar distributions over the words they co-occur with. Thus their PMI vectors (Fig 1a) and word embeddings are similar.

• **Relatedness:** The relatedness of two words can be defined in terms of the words with which both co-occur similarly ($\mathbb{S}\in\mathbb{D}$), which define the *nature* of relatedness, e.g. *milk* and *cheese* are related by $\mathbb{S}=\{$*dairy*, *breakfast*, ...$\}$; and $|\mathbb{S}|$ reflects the *strength* of relatedness. Since PMI vector components corresponding to $\mathbb{S}$ are similar (Fig 1b), embeddings of "$\mathbb{S}$-*related*" words have similar components in the subspace $\mathbb{V}_\mathbb{S}$ that spans the projected PMI vector dimensions corresponding to $\mathbb{S}$. The rank of $\mathbb{V}_\mathbb{S}$ might be expected to reflect relatedness strength. In general, relatedness is a weaker, more variable relation than similarity, its limiting case with $\mathbb{S}=\mathbb{D}$ and $\text{rank}(\mathbb{V}_\mathbb{S})=d$.

• **Context-shift:** In the context of word embeddings, *analogy* typically refers to *relational similarity* (Turney, 2006; Gladkova et al., 2016). More specifically, the relations within analogies that give a common vector offset between word embeddings require related words to have a common difference between their distributions of co-occurring words, defined as a *context-shifts* (see S2.2). These relations are strictly 1-to-1 and include an aspect of *relatedness* due to the word associations in common (Fig 1d). A *specialisation* relation is a context-shift in which context is only added (Fig 1c).

• **Generalised context-shift:** Context-shift relations are generalised to 1-to-many, many-to-1 and many-to-many relations by letting the fully-specified added or subtracted context be one of a (relation-specific) *context set* (Fig 1e), e.g. allowing an entity to be *any* colour or *anything* blue. The potential scope and size of each context set means these relations can vary greatly. The limiting case for small context sets has a single context in each, whereby the relation is an explicit context-shift (as above), and the *difference* between embeddings is a known vector offset. In the limiting case where context sets are large, the added/subtracted context is so loosely defined that, in effect, only the relatedness aspect of the relation and thus only the common subspace component of embeddings is known.

Link to set theory: Viewing PMI vectors as *sets of word associations* and taking intuition from Fig 1, the above relations can be seen to reflect set operations: similarity as set equality; relatedness as equality of a subset; and context-shift as the set difference equalling a relation-specific set. This highlights how the relatedness aspect of a relation reflects features that must be common, and context-shift reflects features that must differ. Whilst this mirrors an intuitive notion of "feature vectors", we emphasise that this is grounded in the co-occurrence statistics of PMI-based word embeddings.

3.1 Categorising knowledge graph relations

Analysing the relations of popular knowledge graph datasets with the above perspective, we find that they comprise (i) a relatedness aspect that reflects a common theme (e.g. both entities are animals or geographic terms); and (ii) specific word associations of the subject and/or object entities. Specifically, relations appear to fall under a hierarchy of three *relation types*: highly related (**R**); (generalised) specialisation (**S**); and (generalised) context-shift (**C**). As above, "generalised" indicates that any added/subtracted contexts can be from a set. From Fig 1, type R relations can be seen as a special case of S, which, in turn, is a special case of C. Type C is therefore a generalised case of all considered relations. Whilst there are several other ways to classify relations (e.g. by their

Table 2: Categorisation of WN18RR relations.

Type	Relation	Examples *(subject entity, object entity)*
R	verb_group	*(trim_down_VB_1, cut_VB_35), (hatch_VB_1, incubate_VB_2)*
	derivationally_related_form	*(lodge_VB_4, accommodation_NN_4), (question_NN_1, inquire_VB_1)*
	also_see	*(clean_JJ_1, tidy_JJ_1), (ram_VB_2, screw_VB_3)*
S	hypernym	*(land_reform_NN_1, reform_NN_1), (prickle-weed_NN_1, herbaceous_plant_NN_1)*
	instance_hypernym	*(yellowstone_river_NN_1, river_NN_1), (leipzig_NN_1, urban_center_NN_1)*
C	member_of_domain_usage	*(colloquialism_NN_1, figure_VB_5), (plural_form_NN_1, authority_NN_2)*
	member_of_domain_region	*(rome_NN_1, gladiator_NN_1), (usa_NN_1, multiple_voting_NN_1)*
	member_meronym	*(south_NN_2, sunshine_state_NN_1), (genus_carya_NN_1, pecan_tree_NN_1)*
	has_part	*(aircraft_NN_1, cabin_NN_3), (morocco_NN_1, atlas_mountains_NN_1)*
	synset_domain_topic_of	*(quark_NN_1, physics_NN_1), (harmonize_VB_3, music_NN_4)*

hierarchy, transitivity), by considering relation conditions, we delineate by the required mathematical form of their representation. Table 2 shows a categorisation of the relations of the WN18RR dataset (Dettmers et al., 2018), containing 11 relations between 40,943 entities (a subset of WordNet (Miller, 1995)).[1] An explanation for this category assignment is in Appx. A and that for NELL-995 (Xiong et al., 2017) and FB15k-237 (Toutanova et al., 2015) relations is in Appx. B.

3.2 Relations as mappings of embeddings

Given the relation conditions of a particular relation type, we can recognise mappings that meet them and thus which loss functions (that evaluate the proximity of mapped entity embeddings, e.g. by dot product or Euclidean distance) are able to identify relations of that type. Reviewing the form of relation representation in recent knowledge graph representation models (Table 1), we can contrast what they are able to express with that which we determine necessary for each relation type.

R: To evidence $\mathbb{S}$-relatedness, both entity embeddings $\boldsymbol{e}_s, \boldsymbol{e}_o$ must be projected onto a subspace $\mathbb{V}_{\mathbb{S}}$, where their images are compared. Projection requires multiplication by a matrix $\boldsymbol{P}_r \in \mathbb{R}^{d\times d}$ and cannot be achieved additively, except in the limiting case of similarity, when $\boldsymbol{P}_r = \boldsymbol{I}$ or vector $\boldsymbol{r} \approx \boldsymbol{0}$ is added. Comparison by dot product gives $(\boldsymbol{P}_r\boldsymbol{e}_s)^\top(\boldsymbol{P}_r\boldsymbol{e}_o) = \boldsymbol{e}_s^\top\boldsymbol{P}_r^\top\boldsymbol{P}_r\boldsymbol{e}_o = \boldsymbol{e}_s^\top\boldsymbol{M}_r\boldsymbol{e}_o$ (for a relation-specific symmetric $\boldsymbol{M}_r = \boldsymbol{P}_r^\top\boldsymbol{P}_r$). Euclidean distance gives $\|\boldsymbol{P}_r\boldsymbol{e}_s - \boldsymbol{P}_r\boldsymbol{e}_o\|^2 = (\boldsymbol{e}_s - \boldsymbol{e}_o)^\top\boldsymbol{M}_r(\boldsymbol{e}_s - \boldsymbol{e}_o) = \|\boldsymbol{P}_r\boldsymbol{e}_s\|^2 - 2\boldsymbol{e}_s^\top\boldsymbol{M}_r\boldsymbol{e}_o + \|\boldsymbol{P}_r\boldsymbol{e}_o\|^2$.

S/C: Evidencing these relations requires a test both for $\mathbb{S}$-relatedness and for relation-entity-specific embeddings component(s) ($\boldsymbol{v}_r^s$, $\boldsymbol{v}_r^o$). This can be achieved by (i) multiplying both entity embeddings by a relation-specific projection matrix $\boldsymbol{P}_r$ that projects onto the subspace that spans the low-rank projection of dimensions corresponding to $\mathbb{S}$, $\boldsymbol{v}_r^s$ and $\boldsymbol{v}_r^o$, (which tests for $\mathbb{S}$-relatedness whilst preserving any entity-specific embedding components); and (ii) adding a relation-specific vector $\boldsymbol{r} = \boldsymbol{v}_r^o - \boldsymbol{v}_r^s$ to the transformed subject entity embeddings. Comparison of the final transformed entity embeddings by dot product equates to $(\boldsymbol{P}_r\boldsymbol{e}_s + \boldsymbol{r})^\top\boldsymbol{P}_r\boldsymbol{e}_o$; and by Euclidean distance to $\|\boldsymbol{P}_r\boldsymbol{e}_s + \boldsymbol{r} - \boldsymbol{P}_r\boldsymbol{e}_o\|^2 = \|\boldsymbol{P}_r\boldsymbol{e}_s + \boldsymbol{r}\|^2 - 2(\boldsymbol{P}_r\boldsymbol{e}_s + \boldsymbol{r})^\top\boldsymbol{P}_r\boldsymbol{e}_o + \|\boldsymbol{P}_r\boldsymbol{e}_o\|^2$ (*cf* MuRE: $\|\boldsymbol{R}\boldsymbol{e}_s + \boldsymbol{r} - \boldsymbol{e}_o\|^2$).

For each relation type, minimising the Euclidean distance comparison term equates to maximising the dot product comparison term with l_2 regularisation of entity-specific terms.

Contrasting this insight with the loss functions of knowledge graph models (Table 1), we make the following predictions: (**P1**) the ability to learn the representation of a particular relation is expected to reflect the complexity of its type (R>S>C), and whether all relation conditions (e.g. additive or multiplicative interactions) can be met under a given model; (**P2**) relation matrices for relatedness (type R) relations are highly symmetric; (**P3**) offset vectors for relatedness relations have low norm; and (**P4**) as a proxy to the rank of $\mathbb{V}_{\mathbb{S}}$, the eigenvalues of a relation matrix reflect a relation's strength of relatedness. Specifically, P1 expresses that additive-only models (e.g. TransE) are not suited to identifying the relatedness aspect of relations (except in limiting cases of similarity, requiring a zero vector); and multiplicative-only models (e.g. DistMult) should perform well on type R but are not suited to identifying entity-specific features of type S/C, for which an asymmetric relation matrix may help compensate. Further, the loss function of MuRE closely resembles that identified for type C relations (which generalise all considered relations) and is expected to perform best overall.

[1] We omit the relation "similar_to" since its instances have no discernible structure, and only 3 occur in the test set, all of which are the inverse of a training example and trivial to predict.

Table 3: Hits@10 per relation on WN18RR.

Relation Name	Type	%	#	Khs	Max/Avg Path		TransE	MuRE$_{\boldsymbol{I}}$	DistMult	TuckER	MuRE
verb_group	R	1%	39	0.00	-	-	0.87	0.95	**0.97**	**0.97**	**0.97**
derivationally_related_form	R	34%	1074	0.04	-	-	0.93	0.96	0.96	0.96	**0.97**
also_see	R	2%	56	0.24	44	15.2	0.59	**0.73**	0.67	0.72	**0.73**
instance_hypernym	S	4%	122	1.00	3	1.0	0.22	0.52	0.47	0.53	**0.54**
synset_domain_topic_of	C	4%	114	0.99	3	1.1	0.19	0.43	0.42	0.45	**0.53**
member_of_domain_usage	C	1%	24	1.00	2	1.0	0.42	0.42	0.48	0.38	**0.50**
member_of_domain_region	C	1%	26	1.00	2	1.0	0.35	0.40	0.40	0.35	**0.46**
member_meronym	C	8%	253	1.00	10	3.9	0.04	0.38	0.30	**0.39**	**0.39**
has_part	C	6%	172	1.00	13	2.2	0.04	0.31	0.28	0.29	**0.35**
hypernym	S	40%	1251	0.99	18	4.5	0.02	0.20	0.19	0.20	**0.28**
all		100%	3134				0.38	0.52	0.51	0.53	**0.57**

Table 4: Hits@10 per relation on NELL-995.

Relation Name	Type	%	#	Khs	Max/Avg Path		TransE	MuRE$_{\boldsymbol{I}}$	DistMult	TuckER	MuRE
teamplaysagainstteam	R	2%	243	0.11	10	3.5	0.76	0.84	**0.90**	0.89	0.89
clothingtogowithclothing	R	1%	132	0.17	5	2.6	0.72	0.80	**0.88**	0.85	0.84
professionistypeofprofession	S	1%	143	0.38	7	2.5	0.37	0.55	0.62	0.65	**0.66**
animalistypeofanimal	S	1%	103	0.68	9	3.1	0.50	0.56	0.64	**0.68**	0.65
athleteplayssport	C	1%	113	1.00	1	1.0	0.54	0.58	0.58	0.60	**0.64**
chemicalistypeofchemical	S	1%	115	0.53	6	3.0	0.23	0.43	0.49	0.51	**0.60**
itemfoundinroom	C	2%	162	1.00	1	1.0	0.39	0.57	0.53	0.56	**0.59**
agentcollaborateswithagent	R	1%	119	0.51	14	4.7	0.44	0.58	**0.64**	0.61	0.58
bodypartcontainsbodypart	C	1%	103	0.60	7	3.2	0.30	0.38	0.54	**0.58**	0.55
atdate	C	10%	967	0.99	4	1.1	0.41	0.50	0.48	0.48	**0.52**
locationlocatedwithinlocation	C	2%	157	1.00	6	1.9	0.35	0.37	0.46	**0.48**	**0.48**
atlocation	C	1%	294	0.99	6	1.4	0.22	0.33	0.39	0.43	**0.44**
all		100%	20000				0.36	0.48	0.51	**0.52**	**0.52**

4 Comparing knowledge graph models

We test the predictions made on the basis of word embeddings by comparing the performance of competitive knowledge graph models, TransE, DistMult, TuckER and MuRE (see S2), which entail different forms of relation representation, on all WN18RR relations and a similar number of NELL-995 relations (selected to represent each relation type). Since applying the logistic sigmoid to the score function of TransE does not give a probabilistic interpretation comparable to other models, we include MuRE$_{\boldsymbol{I}}$, a constrained variant of MuRE with $\boldsymbol{R}_s = \boldsymbol{R}_o = \boldsymbol{I}$, as a proxy to TransE for a fairer comparison. Implementation details are included in Appx. D. For evaluation, we generate $2n_e$ *evaluation triples* for each test triple (for the number of entities n_e) by fixing the subject entity e_s and relation r and replacing the object entity e_o with all possible entities and then keeping e_o and r fixed and varying e_s. The obtained scores are ranked to give the standard metric hits@10 (Bordes et al., 2013), i.e. the fraction of times a true triple appears in the top 10 ranked evaluation triples.

4.1 Performance per relation type

Tables 3 and 4 report results (hits@10) for each relation and include our relation categorisation and known confounding influences: percentage of relation instances in the training and test sets (approximately equal), number of instances in the test set, Krackhardt hierarchy score (see Appx. E) (Krackhardt, 2014; Balažević et al., 2019a) and maximum and average shortest path between any two related nodes. A further confounding effect is dependence between relations. Balažević et al. (2019b) and Lacroix et al. (2018) show that constraining the rank of relation representations benefits datasets with many relations (particularly when the number of instances per relation is low) due to multi-task learning, which is expected to benefit TuckER on the NELL-995 dataset (200 relations). Note that all models have a comparable number of free parameters.

P1: As predicted, all models tend to perform best at type R relations, with a clear performance gap to other relation types. Also, performance on type S relations appears higher in general than type C. Additive-only models (TransE, MuRE$_{\boldsymbol{I}}$) perform most poorly on average, in line with prediction since all relation types involve a relatedness component. They achieve their best results on type R relations, where the relation vector can be zero/small. Multiplicative-only DistMult performs well, sometimes best, on type R relations, fitting expectation as it can fully represent those relations and has no additional parameters that may overfit to noise (which may explain where MuRE performs slightly worse). As expected, MuRE, performs best on average (particularly on WN18RR), and most strongly on S and C type relations that require both multiplicative and additive components. The comparable performance of TuckER on NELL-995 is explained by its ability for multi-task learning.

Other unexpected results also tend to align with confounding factors, e.g. that all models perform poorly on the *hypernym* relation, despite it having type S and a relative abundance of training data (40% of all instances), might be explained by its hierarchical nature (Khs $\approx$ 1 and long paths). The same may explain the reduced performance on type R relations *also_see* and *agentcollaborateswithagent*. As found previously, none of the models are well suited to modelling hierarchical structures (Balažević et al., 2019a). We also note that the percentage of training instances of a relation does not seem to correlate with its performance, as might typically be expected.

P2/P3: Table 5 shows the symmetry score ($\in$ [-1, 1] indicating perfect anti-symmetry to symmetry; see Appx. F) for the relation matrix of TuckER and the norm of relation vectors of TransE, MuRE$_{\boldsymbol{I}}$ and MuRE on the WN18RR dataset. As expected, type R relations have high symmetry, whereas both other relation types have lower scores, fitting the expectation that TuckER compensates for having no additive component. All additive models learn relation vectors of a noticeably lower norm for type R relations (where, in the extreme, no additive component is required) than for types S and C.

Table 5: Relation matrix symmetry score [-1.1] for TuckER; and relation vector norm for TransE, MuRE$_{\boldsymbol{I}}$ and MuRE (WN18RR).

		Symmetry Score	Vector Norm		
Relation	Type	TuckER	TransE	MuRE$_{\boldsymbol{I}}$	MuRE
verb_group	R	0.52	5.65	0.76	0.89
derivationally_related_form	R	0.54	2.98	0.45	0.69
also_see	R	0.50	7.20	0.97	0.97
instance_hypernym	S	0.13	18.26	2.98	1.88
member_of_domain_usage	C	0.10	11.24	3.18	1.88
member_of_domain_region	C	0.06	12.52	3.07	2.11
synset_domain_topic_of	C	0.12	23.29	2.65	1.52
member_meronym	C	0.12	4.97	1.91	1.97
has_part	C	0.13	6.44	1.69	1.25
hypernym	S	0.04	9.64	1.53	1.03

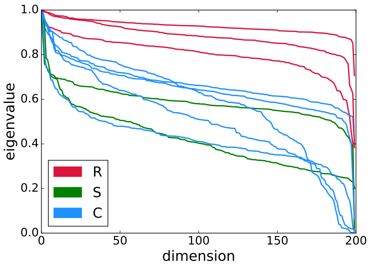


Figure 2: Eigenvalue magnitudes of relation-specific matrices $\boldsymbol{R}$ for MuRE by relation type (WN18RR).

P4: Fig 2 shows eigenvalue magnitudes (scaled relative to their largest and ordered) of the relation-specific matrices $\boldsymbol{R}$ of MuRE, labelled by relation type. Predicted to reflect strength of a relation's relatedness, they should be highest for type R relations, as observed. For relation types S and C the profiles are more varied, fitting the expectation that the relatedness of those types has greater variability in both choice and size of $\mathbb{S}$, i.e. in the nature and strength of relatedness.

The results above confirm all of our predictions about the ability of each model to represent different relational mappings. Our analysis identifies the best model for each relation type (multiplicative-only DistMult for type R, additive-multiplicative MuRE for types S/C), providing a basis for *dataset-dependent model selection*. Separately, the per-relation insight into where models perform poorly (e.g. on hierarchical relations) can be used to aid future model design.

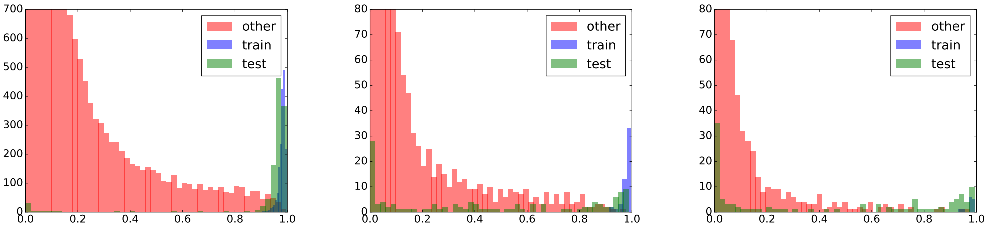
4.2 Knowledge graph model predictions

Even though in practice we want to know whether a particular triple is true or false, such independent predictions are not commonly reported or evaluated. Despite many recent link prediction models being able to independently predict the truth of each triple, it is common practice to report ranking-based metrics, e.g. mean reciprocal rank, hits@k, which compare the prediction of a test triple to those of all evaluation triples (see S4). Not only is this computationally costly, the evaluation is flawed if entities are related to more than k others and does not evaluate a model's ability to independently predict whether "*a* is related to *b*". We address this by considering actual model predictions.

Since for each relation there are n_e^2 possible entity-entity relationships, we sub-sample by computing predictions for all (e_s, r, e_o) triples only for each e_s, r pair seen in the test set. We split positive predictions ($\sigma(\phi(e_s, r, e_o)) > 0.5$) between (i) *known truths*, either training or test/validation; and (ii) *other*, the truth of which is not known. Per relation, we then compute accuracy over the true training instances (train) and the true test/validation instances (test); and the average number of other truths predicted per e_s, r pair. Table 6 shows results for MuRE$_{\boldsymbol{I}}$, DistMult, TuckER and MuRE. All

Table 6: Per relation prediction accuracy for MuRE$_I$ (M$_I$), (D)istMult, (T)uckER and (M)uRE (WN18RR).

			Accuracy (train)				Accuracy (test)				# Other "True"			
Relation Name	$\#_{train}$	$\#_{test}$	M$_I$	D	T	M	M$_I$	D	T	M	M$_I$	D	T	M
verb_group	15	39	1.00	1.00	1.00	1.00	0.97	0.97	0.97	0.97	8.3	1.7	0.9	2.7
derivationally_related_form	1714	1127	1.00	1.00	1.00	1.00	0.96	0.94	0.95	0.95	8.8	0.5	0.6	1.7
also_see	95	61	1.00	1.00	1.00	1.00	0.64	0.64	0.61	0.59	7.9	1.6	0.9	1.9
instance_hypernym	52	122	1.00	1.00	1.00	1.00	0.32	0.32	0.23	0.43	1.3	0.4	0.3	0.9
member_of_domain_usage	545	43	0.98	1.00	1.00	1.00	0.02	0.00	0.02	0.00	1.5	0.6	0.0	0.3
member_of_domain_region	543	42	0.88	0.89	1.00	0.93	0.02	0.02	0.00	0.02	1.0	0.4	0.8	0.7
synset_domain_topic_of	13	115	1.00	1.00	1.00	1.00	0.42	0.10	0.14	0.47	0.7	0.6	0.1	0.2
member_meronym	1402	307	1.00	1.00	1.00	1.00	0.22	0.02	0.01	0.22	7.9	3.4	1.5	5.6
has_part	848	196	1.00	1.00	1.00	1.00	0.24	0.05	0.09	0.22	7.1	2.4	1.3	3.9
hypernym	57	1254	1.00	1.00	1.00	1.00	0.15	0.02	0.02	0.22	3.7	1.2	0.0	1.7
all	5284	3306	0.99	0.99	1.00	0.99	0.47	0.37	0.37	0.50	5.9	1.2	0.5	2.1



(a) derivationally_related_form (R) **(b)** instance_hypernym (S) **(c)** synset_domain_topic_of (C)

Figure 3: Histograms of MuRE predictions for an example WN18RR relation of each type, split into true training, true test/validation and other instances.

models achieve almost perfect training accuracy. The additive-multiplicative MuRE gives best test set performance, followed (surprisingly) closely by MuRE$_I$, with multiplicative models (DistMult and TuckER) performing poorly on all but type R relations. Analysing a sample of "other" positive predictions for a relation of each type (see Appx. G), we estimate that TuckER is relatively accurate but pessimistic ($\sim$0.3 correct of the 0.5 predictions$\approx 60\%$), MuRE$_I$ is optimistic but inaccurate ($\sim$2.3 of 7.5$\approx$31%), whereas MuRE is both optimistic and accurate ($\sim$1.1 of 1.5$\approx$73%).

Fig 3 shows histograms of MuRE prediction probabilities for the same sample relations, split by known truths (training and test/validation) and other instances. There is a clear distinction between relation types: for type R, most train and test triples are classified correctly with high confidence; for types S and C, an increasing majority of incorrect test predictions are far below the decision boundary, i.e. the model is confidently incorrect. For relations where the model is less accurate, fewer positive predictions are made overall and the prediction distribution is more peaked towards zero.

This predictive performance analysis shows more specifically how models have difficulty representing S/C relations and, given it provides new insight compared to ranking-based metrics, we recommend its inclusion in future link prediction work.

5 Conclusion

Many methods for learning low-rank representations of knowledge graphs have been developed, yet little is known about the latent structure they learn. This work focuses on understanding why existing knowledge graph representation models work and identifying their limitations, with the ultimate goal of improving representation. With this in mind, we build on recent understanding of PMI-based word embeddings to establish *relation conditions*, i.e. what a relation representation needs to achieve to map one word embedding to the other. Those conditions provide a framework to consider knowledge graph models, since if they are satisfied by the relation representations defined by a model's architecture, then word embedding parameters are a known solution for its entity embeddings. This way, the encoding of semantic relations is not only understood – it is built into the geometry of entity and relation representation. We use this understanding to categorise knowledge graph relations according to their relation conditions. Our analysis confirms that a model's ability to represent a specific relation type depends on the limitations imposed by the model architecture with respect to satisfying the necessary relation conditions. Finally, we evaluate several existing knowledge graph models based on their predictive performance.

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A Categorising WordNet relations

Table 7 describes how each WN18RR relation was assigned to its respective category.

Table 7: Explanation for the WN18RR relation category assignment.

Type	Relation	Relatedness	Subject Specifics	Object Specifics
R	verb_group	both verbs; similar in meaning	-	-
	derivationally_related_form	different syntactic categories; semantically related	-	-
	also_see	semantically similar	-	-
S	hypernym	semantically similar	instance of the object	-
	instance_hypernym	semantically similar	instance of the object	-
C	member_of_domain_usage	loosely semantically related (word usage features)	usage descriptor	broad feature set
	member_of_domain_region	loosely semantically related (regional features)	region descriptor	broad feature set
	member_meronym	semantically related	collection of objects	part of the subject
	has_part	semantically related	collection of objects	part of the subject
	synset_domain_topic_of	semantically related	broad feature set	domain descriptor

B Analysing NELL-995 and FB15k-237 relations

NELL-995: We categorise a random subsample of 12 relations from the NELL-995 dataset (Xiong et al., 2017) containing 75,492 entities and 200 relations (a subset of NELL (Carlson et al., 2010)), which span our identified relation types (see Table 8). Explanation for the relation category assignment is shown in Table 9.

Table 8: Categorisation of NELL-995 relations.

Type	Relation	Examples *(subject entity, object entity)*
R	teamplaysagainstteam	*(sportsteam_rangers, sportsteam_mariners)*, *(sportsteam_phillies, sportsteam_tampa_bay_rays)*
	clothingtogowithclothing	*(clothing_shirts, clothing_trousers)*, *(clothing_shoes, clothing_black_shirt)*
	agentcollaborateswithagent	*(musicartist_white_stripes, musicartist_jack_white)*, *(politician_obama, politician_hillary_clinton)*
S	professionistypeofprofession	*(profession_trial_lawyers, profession_attorneys)*, *(profession_engineers, profession_experts)*
	animalistypeofanimal	*(mammal_cats, mammal_small_animals)*, *(bird_chickens, agriculturalproduct_livestock)*
	chemicalistypeofchemical	*(chemical_moisture, chemical_gas)*, *(chemical_oxide, chemical_materials)*
C	athleteplayssport	*(athlete_joe_smith, sport_baseball)*, *(athlete_chris_cooley, sport_football)*
	itemfoundinroom	*(bedroomitem_bed, room_den)*, *(hallwayitem_refrigerator, visualizablething_kitchen_area)*
	bodypartcontainsbodypart	*(bodypart_system002, braintissue_eyes)*, *(bodypart_blood, bodypart_left_ventricle)*
	atdate	*(currency_scotland, date_n2009)*, *(governmentorganization_wto, date_n2003)*
	locationlocatedwithinlocation	*(city_medellin, country_colombia), (city_jackson, stateorprovince_wyoming)*
	atlocation	*(city_ogunquin, stateorprovince_maine)*, *(city_palmer_lake, stateorprovince_colorado)*

Table 9: Explanation for the NELL-995 relation category assignment.

Type	Relation	Relatedness	Subject Specifics	Object Specifics
R	teamplaysagainstteam	both sport teams	-	-
	clothingtogowithclothing	both items of clothing that go together	-	-
	agentcollaborateswithagent	both people or companies; related industries	-	-
S	professionistypeofprofession	semantically related (both profession types)	instance of the object	-
	animalistypeofanimal	semantically related (both animals)	instance of the object	-
	chemicalistypeofchemical	semantically related (both chemicals)	instance of the object	-
C	athleteplayssport	semantically related (sports features)	athlete descriptor	sport descriptor
	itemfoundinroom	semantically related (room/furniture features)	item descriptor	room descriptor
	bodypartcontainsbodypart	emantically related (specific body part features)	collection of objects	part of the subject
	atdate	loosely semantically related (start date features)	broad feature set	date descriptor
	locationlocatedwithinlocation	semantically related (geographical features)	part of the subject	collection of objects
	atlocation	semantically related (geographical features)	part of the subject	collection of objects

FB15k-237: Reviewing the FB15k-237 dataset (Toutanova et al., 2015) shows that the vast majority of relations are of type C (e.g. */people/person/profession*, */location/location/contains*), preventing contrast between relation types being drawn, so we consider that dataset no further.

C Splitting the NELL-995 dataset

The test set of NELL-995 created by Xiong et al. (2017) contains only 10 out of 200 relations present in the training set. To ensure a fair representation of all training set relations in the validation and test sets, we create new validation and test set splits by combining the initial validation and test sets with the training set and randomly selecting 20,000 triples each from the combined dataset.

D Implementation details

All algorithms are re-implemented in PyTorch with the Adam optimizer (Kingma & Ba, 2015) that minimises binary cross-entropy loss, using hyper-parameters that work well for all models (learning rate: 0.001, batch size: 128, number of negative samples: 50). Entity and relation embedding dimensionality is set to $d_e = d_r = 200$ for all models except TuckER, for which $d_r = 30$ (Balažević et al., 2019b).

E Krackhardt hierarchy score

The Krackhardt hierarchy score measures the proportion of node pairs (x, y) where there exists a directed path $x \to y$, but not $y \to x$; and it takes a value of one for all directed acyclic graphs, and zero for cycles and cliques (Krackhardt, 2014; Balažević et al., 2019a).

Let $\boldsymbol{M} \in \mathbb{R}^{n \times n}$ be the binary *reachability matrix* of a directed graph $\mathcal{G}$ with n nodes, with $\boldsymbol{M}_{i,j} = 1$ if there exists a directed path from node i to node j and 0 otherwise. The Krackhardt hierarchy score of $\mathcal{G}$ is defined as:

$$\text{Khs}_{\mathcal{G}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{1}(\boldsymbol{M}_{i,j} == 1 \wedge \boldsymbol{M}_{j,i} == 0)}{\sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{1}(\boldsymbol{M}_{i,j} == 1)}. \tag{1}$$

F Symmetry score

The symmetry score $\in [-1, 1]$ (Hubert & Baker, 1979) for a relation matrix $\boldsymbol{R} \in \mathbb{R}^{d_e \times d_e}$ is defined as:

$$s = \frac{\sum\sum_{i \neq j} \boldsymbol{R}_{ij} \boldsymbol{R}_{ji} - \frac{(\sum\sum_{i \neq j} \boldsymbol{R}_{ij})^2}{d_e(d_e - 1)}}{\sum\sum_{i \neq j} \boldsymbol{R}_{ij}^2 - \frac{(\sum\sum_{i \neq j} \boldsymbol{R}_{ij})^2}{d_e(d_e - 1)}}, \tag{2}$$

where 1 indicates a symmetric and -1 an anti-symmetric matrix.

G "Other" predicted facts

Tables 10 to 13 shows a sample of the unknown triples (i.e. those formed using the WN18RR entities and relations, but not present in the dataset) for the *derivationally_related form* (R), *instance_hypernym* (S) and *synset_domain_topic_of* (C) relations at a range of probabilities ($\sigma(\phi(e_s, r, e_o)) \approx \{0.4, 0.6, 0.8, 1\}$), as predicted by each model. True triples are indicated in bold; instances where a model predicts an entity is related to itself are indicated in blue.

Table 10: “Other” facts as predicted by MuRE$_I$.

Relation (Type)	$\sigma(\phi(e_s, r, e_o)) \approx 0.4$	$\sigma(\phi(e_s, r, e_o)) \approx 0.6$	$\sigma(\phi(e_s, r, e_o)) \approx 0.8$	$\sigma(\phi(e_s, r, e_o)) \approx 1$
derivationally_related_form (R)	*(equalizer_NN_2, set_off_VB_5)*	*(extrapolation_NN_1, maths_NN_1)*	*(sewer_NN_2, stitcher_NN_1)*	*(trail_VB_2, trail_VB_2)*
	(constellation_NN_2, satellite_NN_3)	*(spread_VB_5, circularize_VB_3)*	*(lard_VB_1, vegetable_oil_NN_1)*	*(worship_VB_1, worship_VB_1)*
	(shrink_VB_3, subtraction_NN_2)	*(flaunt_NN_1, showing_NN_2)*	***(snuggle_NN_1, draw_close_VB_3)***	*(steer_VB_1, steer_VB_1)*
	(continue_VB_10, proceed_VB_1)	***(extrapolate_VB_3, synthesis_NN_3)***	***(train_VB_3, training_NN_1)***	*(sort_out_VB_1, sort_out_VB_1)*
	(support_VB_6, defend_VB_5)	*(strategist_NN_1, machination_NN_1)*	***(scratch_VB_3, skin_sensation_NN_1)***	*(make_full_VB_1, make_full_VB_1)*
	(shutter_NN_1, fill_up_VB_3)	*(crush_VB_4, grind_VB_2)*	*(scheme_NN_5, schematization_NN_1)*	*(utilize_VB_1, utilize_VB_1)*
	(yawning_NN_1, patellar_reflex_NN_1)	*(spike_VB_5, steady_VB_2)*	*(ordain_VB_3, vest_VB_1)*	*(geology_NN_1, geology_NN_1)*
	(yaw_NN_1, spiral_VB_1)	*(licking_NN_1, vanquish_VB_1)*	*(lie_VB_1, front_end_NN_1)*	*(zoology_NN_2, zoology_NN_2)*
	(stratum_NN_2, social_group_NN_1)	***(synthetical_JJ_1, synthesizer_NN_2)***	*(tread_NN_1, step_NN_9)*	*(uranology_NN_1, uranology_NN_1)*
	(duel_VB_1, scuffle_NN_3)	*(realization_NN_2, embodiment_NN_3)*	***(register_NN_3, file_away_VB_1)***	*(travel_VB_1, travel_VB_1)*
instance_hypernym (S)	*(thomas_aquinas_NN_1, martyr_NN_2)*	***(taiwan_NN_1, asian_nation_NN_1)***	***(prophets_NN_1, gospels_NN_1)***	***(helsinki_NN_1, urban_center_NN_1)***
	(volcano_islands_NN_1, volcano_NN_2)	***(st._gregory_of_n._NN_1, canonization_NN_1)***	*(malcolm_x_NN_1, passive_resister_NN_1)*	*(mannheim_NN_1, stockade_NN_2)*
	(cape_horn_NN_1, urban_center_NN_1)	*(st._gregory_of_n._NN_1, saint_VB_2)*	*(taiwan_NN_1, national_capital_NN_1)*	*(nippon_NN_1, nippon_NN_1)*
	(bergen_NN_1, national_capital_NN_1)	*(mccormick_NN_1, find_VB_8)*	*(truth_NN_5, abolitionism_NN_1)*	*(victor_hugo_NN_1, novel_NN_1)*
	(marshall_NN_2, generalship_NN_1)	***(st._gregory_i_NN_1, bishop_NN_1)***	***(thomas_aquinas_NN_1, saint_VB_2)***	*(regiomontanus_NN_1, uranology_NN_1)*
	(nansen_NN_1, venturer_NN_2)	*(richard_buckminster_f._NN_1, technological_JJ_2)*	*(central_america_NN_1, s._am._nation_NN_1)*	***(prophets_NN_1, book_NN_10)***
	(wisconsin_NN_2, state_capital_NN_1)	*(thomas_aquinas_NN_1, archbishop_NN_1)*	*(de_mille_NN_1, dance_VB_1)*	*(thomas_aquinas_NN_1, church_father_NN_1)*
	(prussia_NN_1, stockade_NN_2)	***(marshall_NN_2, general_officer_NN_1)***	*(st._gregory_i_NN_1, apostle_NN_3)*	*(woody_guthrie_NN_1, minstrel_VB_1)*
	(de_mille_NN_1, dancing-master_NN_1)	*(newman_NN_2, primateship_NN_1)*	*(fertile_crescent_NN_1, asian_nation_NN_1)*	*(central_america_NN_1, c._am._nation_NN_1)*
	(aegean_sea_NN_1, aegean_island_NN_1)	*(thomas_the_apostle_NN_1, sanctify_VB_1)*	*(robert_owen_NN_1, industry_NN_1)*	*(aegean_sea_NN_1, island_NN_1)*
synset_domain_topic_of (C)	*(write_VB_8, tape_VB_3)*	*(draw_VB_6, creative_person_NN_1)*	*(libel_NN_1, sully_VB_3)*	***(libel_NN_1, disparagement_NN_1)***
	(introvert_NN_1, scientific_discipline_NN_1)	***(suborder_NN_1, taxonomic_group_NN_1)***	*(relationship_NN_4, relationship_NN_4)*	***(roll-on_roll-off_NN_1, transport_NN_1)***
	(libel_NN_1, slur_NN_2)	*(draw_VB_6, draw_VB_6)*	***(etymologizing_NN_1, linguistics_NN_1)***	***(prance_VB_4, equestrian_sport_NN_1)***
	(etymologizing_NN_1, law_NN_1)	***(first_sacker_NN_1, ballgame_NN_2)***	***(turn_VB_12, cultivation_NN_2)***	***(libel_NN_1, traducement_NN_1)***
	(temple_NN_4, place_of_worship_NN_1)	*(alchemize_VB_1, modify_VB_3)*	*(brynhild_NN_1, mythologize_VB_2)*	***(sell_VB_1, selling_NN_1)***
	(trial_impression_NN_1, proof_VB_1)	*(sermon_NN_1, sermon_NN_1)*	***(brynhild_NN_1, myth_NN_1)***	*(trot_VB_2, ride_horseback_VB_1)*
	(friend_of_the_court_NN_1, war_machine_NN_1)	***(saint_VB_2, catholic_church_NN_1)***	***(assist_NN_2, am._football_game_NN_1)***	*(prance_VB_4, ride_horseback_VB_1)*
	(multiv._analysis_NN_1, applied_math_NN_1)	*(male_JJ_1, masculine_JJ_2)*	*(mitzvah_NN_2, human_activity_NN_1)*	*(gallop_VB_1, ride_horseback_VB_1)*
	(sell_VB_1, transaction_NN_1)	*(fire_VB_3, zoology_NN_2)*	*(drive_NN_12, drive_VB_8)*	***(brynhild_NN_1, mythology_NN_2)***
	(draw_VB_6, represent_VB_9)	*(sell_VB_1, sell_VB_1)*	***(relationship_NN_4, biology_NN_1)***	***(drive_NN_12, badminton_NN_1)***

Table 11: “Other” facts as predicted by DistMult.

Relation (Type)	$\sigma(\phi(e_s, r, e_o)) \approx 0.4$	$\sigma(\phi(e_s, r, e_o)) \approx 0.6$	$\sigma(\phi(e_s, r, e_o)) \approx 0.8$	$\sigma(\phi(e_s, r, e_o)) \approx 1$
derivationally_ related_form (R)	**(stag_VB_3, undercover_work_NN_1)**	(dish_NN_2, stew_NN_2)	(shrink_NN_1, pedology_NN_1)	**(alliterate_VB_1, versifier_NN_1)**
	(print_VB_4, publisher_NN_2)	**(expose_VB_3, show_NN_1)**	**(finish_VB_6, finishing_NN_2)**	**(geology_NN_1, structural_JJ_5)**
	(crier_NN_3, pitchman_NN_2)	(system_NN_9, orderliness_NN_1)	**(play_VB_26, playing_NN_3)**	**(resect_VB_1, amputation_NN_2)**
	(play_VB_26, turn_NN_10)	**(spread_NN_4, strew_VB_1)**	**(centralization_NN_1, unite_VB_6)**	(nutrition_NN_3, man_NN_4)
	(count_VB_4, recite_VB_2)	(take_down_VB_2, put_VB_2)	(existence_NN_1, living_NN_3)	**(saint_NN_3, sanctify_VB_1)**
	(vividness_NN_2, imbue_VB_3)	**(wrestle_VB_4, wrestler_NN_1)**	(mouth_VB_3, sassing_NN_1)	(right_fielder_NN_1, leftfield_NN_1)
	(sea_mew_NN_1, larus_NN_1)	**(autotr._organism_NN_1, epiphytic_JJ_1)**	(constellation_NN_2, star_NN_1)	**(list_VB_4, slope_NN_2)**
	(alkali_NN_2, acidify_VB_2)	**(duel_VB_1, slugfest_NN_1)**	**(print_VB_4, publishing_house_NN_1)**	(lieutenancy_NN_1, captain_NN_1)
	(see_VB_17, understand_VB_2)	(vocal_NN_2, rock_star_NN_1)	**(puzzle_VB_2, secret_NN_3)**	**(tread_NN_1, step_VB_7)**
	(shun_VB_1, hedging_NN_2)	**(smelling_NN_1, scent_VB_1)**	(uranology_NN_1, tt_NN_1)	**(exenteration_NN_1, enucleate_VB_2)**
instance_ hypernym (S)	(wisconsin_NN_2, urban_center_NN_1)	(mississippi_river_NN_1, american_state_NN_1)	(deep_south_NN_1, south_NN_1)	**(helsinki_NN_1, urban_center_NN_1)**
	(marshall_NN_2, lieutenant_general_NN_1)	(r._e._byrd_NN_1, commissioned_officer_NN_1)	**(capital_of_gambia_NN_1, urban_center_NN_1)**	(the_nazarene_NN_1, save_VB_7)
	(abidjan_NN_1, cote_d'ivoire_NN_1)	**(kobenhavn_NN_1, urban_center_NN_1)**	(south_west_africa_NN_1, africa_NN_1)	**(irish_capital_NN_1, urban_center_NN_1)**
	(world_war_i_NN_1, urban_center_NN_1)	(the_gambia_NN_1, africa_NN_1)	(brandenburg_NN_1, urban_center_NN_1)	**(r._e._byrd_NN_1, inspector_general_NN_1)**
	(st._paul_NN_2, evangelist_NN_2)	(tirich_mir_NN_1, urban_center_NN_1)	(sierra_nevada_NN_1, urban_center_NN_1)	**(r._e._byrd_NN_1, chief_of_staff_NN_1)**
	(deep_south_NN_1, urban_center_NN_1)	**(r._e._byrd_NN_1, military_advisor_NN_1)**	**(malcolm_x_NN_1, emancipationist_NN_1)**	(central_america_NN_1, c._am._nation_NN_1)
	(nuptse_NN_1, urban_center_NN_1)	(r._e._byrd_NN_1, aide-de-camp_NN_1)	(north_platte_river_NN_1, urban_center_NN_1)	(malcolm_x_NN_1, environmentalist_NN_1)
	(ticino_NN_1, urban_center_NN_1)	(tampa_bay_NN_1, urban_center_NN_1)	**(oslo_NN_1, urban_center_NN_1)**	**(the_nazarene_NN_1, christian_JJ_1)**
	(aegean_sea_NN_1, aegean_island_NN_1)	(tidewater_region_NN_1, south_NN_1)	(zaire_river_NN_1, urban_center_NN_1)	(thomas_aquinas_NN_1, church_father_NN_1)
	(cowpens_NN_1, war_of_am._ind._NN_1)	**(r._e._byrd_NN_1, executive_officer_NN_1)**	(transylvanian_alps_NN_1, urban_center_NN_1)	(the_nazarene_NN_1, el_nino_NN_2)
synset_domain_ topic_of (C)	(limitation_NN_4, trammel_VB_2)	**(roll-on_roll-off_NN_1, transport_NN_1)**	**(etymologizing_NN_1, explanation_NN_1)**	**(flat_JJ_5, matte_NN_2)**
	(light_colonel_NN_1, colonel_NN_1)	(hizb_ut-tahrir_NN_1, asia_NN_1)	**(ferry_VB_3, travel_VB_1)**	(etymologizing_NN_1, derive_VB_3)
	(nurse_VB_1, nursing_NN_1)	**(slugger_NN_1, softball_game_NN_1)**	(public_prosecutor_NN_1, prosecute_VB_2)	**(hole_out_VB_1, hole_NN_3)**
	(sermon_NN_1, prophesy_VB_2)	(sermon_NN_1, sermonize_VB_1)	(alchemize_VB_1, modify_VB_3)	**(relationship_NN_4, relation_NN_1)**
	(libel_NN_1, practice_of_law_NN_1)	**(draw_VB_6, drawer_NN_3)**	(libel_NN_1, libel_VB_1)	**(drive_NN_12, badminton_NN_1)**
	(slugger_NN_1, baseball_player_NN_1)	**(turn_VB_12, plow_NN_1)**	(turn_VB_12, till_VB_1)	(etymologizing_NN_1, etymologize_VB_2)
	(rna_NN_1, chemistry_NN_1)	**(assist_NN_2, softball_game_NN_1)**	(hit_NN_1, hit_VB_1)	(matrix_algebra_NN_1, diagonalization_NN_1)
	(metrify_VB_1, versify_VB_1)	**(council_NN_2, assembly_NN_4)**	**(fire_VB_3, flaming_NN_1)**	**(cabinetwork_NN_2, woodworking_NN_1)**
	(trial_impression_NN_1, publish_VB_1)	**(throughput_NN_1, turnout_NN_4)**	(ring_NN_4, chemical_chain_NN_1)	(cabinetwork_NN_2, bottom_VB_1)
	(turn_VB_12, plowman_NN_1)	**(cream_VB_1, cream_NN_2)**	**(libidinal_energy_NN_1, charge_NN_9)**	(cabinetwork_NN_2, upholster_VB_1)

Table 12: “Other” facts as predicted by TuckER.

Relation (Type)	$\sigma(\phi(e_s, r, e_o)) \approx 0.4$	$\sigma(\phi(e_s, r, e_o)) \approx 0.6$	$\sigma(\phi(e_s, r, e_o)) \approx 0.8$	$\sigma(\phi(e_s, r, e_o)) \approx 1$
derivationally_ related_form (R)	*(tympanist_NN_1, gong_NN_2)*	***(mash_NN_2, mill_VB_2)***	*(take_chances_VB_1, venture_NN_1)*	*(venturer_NN_2, venturer_NN_2)*
	(indication_NN_1, signalize_VB_2)	*(walk_VB_9, zimmer_frame_NN_1)*	*(shutter_NN_1, fill_up_VB_3)*	*(dynamitist_NN_1, dynamitist_NN_1)*
	(turn_over_VB_3, rotation_NN_3)	***(use_VB_5, utility_NN_2)***	***(exit_NN_3, leave_VB_1)***	***(love_VB_3, lover_NN_2)***
	(date_VB_5, geological_dating_NN_1)	***(musical_instrument_NN_1, write_VB_6)***	***(trembler_NN_1, vibrate_VB_1)***	***(snuggle_NN_1, squeeze_VB_8)***
	(set_VB_23, emblem_NN_2)	***(lining_NN_3, wrap_up_VB_1)***	***(motivator_NN_1, trip_VB_4)***	***(departed_NN_1, die_VB_2)***
	(tyro_NN_1, start_VB_5)	***(scrap_VB_2, struggle_NN_2)***	***(support_VB_6, indorsement_NN_1)***	***(position_VB_1, placement_NN_1)***
	(identification_NN_1, name_VB_5)	***(tape_VB_3, tape_recorder_NN_1)***	***(federate_VB_2, confederation_NN_1)***	*(repentant_JJ_1, repentant_JJ_1)*
	(stabber_NN_1, thrust_VB_5)	***(vindicate_VB_2, justification_NN_2)***	***(take_over_VB_6, return_NN_7)***	***(tread_NN_1, step_VB_7)***
	(justification_NN_1, apology_NN_2)	***(leaching_NN_1, percolate_VB_3)***	***(wait_on_VB_1, supporter_NN_3)***	*(stockist_NN_1, stockist_NN_1)*
	(manufacture_VB_1, prevarication_NN_1)	***(synchronize_VB_2, synchroscope_NN_1)***	***(denote_VB_3, promulgation_NN_1)***	*(philanthropist_NN_1, philanthropist_NN_1)*
instance_ hypernym (S)	*(deep_south_NN_1, south_NN_1)*	*(thomas_aquinas_NN_1, bishop_NN_1)*	***(cowpens_NN_1, siege_NN_1)***	*(r._e._byrd_NN_1, siege_NN_1)*
	(st._paul_NN_2, organist_NN_1)	***(irish_capital_NN_1, urban_center_NN_1)***	*(mccormick_NN_1, arms_manufacturer_NN_1)*	***(shaw_NN_3, women’s_rightist_NN_1)***
	(helsinki_NN_1, urban_center_NN_1)	***(thomas_the_apostle_NN_1, apostle_NN_2)***	*(thomas_the_apostle_NN_1, evangelist_NN_2)*	*(aegean_sea_NN_1, aegean_island_NN_1)*
	(malcolm_x_NN_1, emancipationist_NN_1)	*(st._paul_NN_2, apostle_NN_3)*	***(mccormick_NN_1, technologist_NN_1)***	*(thomas_aquinas_NN_1, church_father_NN_1)*
	(thomas_the_apostle_NN_1, church_father_NN_1)	*(mccormick_NN_1, painter_NN_1)*	***(st._gregory_i_NN_1, church_father_NN_1)***	
	(st._gregory_of_n._NN_1, sermonizer_NN_1)	*(thomas_the_apostle_NN_1, troglodyte_NN_1)*		
	(robert_owen_NN_1, movie_maker_NN_1)	*(mccormick_NN_1, electrical_engineer_NN_1)*		
	(theresa_NN_1, monk_NN_1)	*(mississippi_river_NN_1, american_state_NN_1)*		
	(st._paul_NN_2, philosopher_NN_1)			
	(ibn-roshd_NN_1, pedagogue_NN_1)			
synset_domain_ topic_of (C)	***(roll-on_roll-off_NN_1, motorcar_NN_1)***	***(drive_NN_12, badminton_NN_1)***		
	(libel_NN_1, legislature_NN_1)			
	(roll-on_roll-off_NN_1, passenger_vehicle_NN_1)			

Table 13: “Other” facts as predicted by MuRE.

Relation (Type)	$\sigma(\phi(e_s, r, e_o)) \approx 0.4$	$\sigma(\phi(e_s, r, e_o)) \approx 0.6$	$\sigma(\phi(e_s, r, e_o)) \approx 0.8$	$\sigma(\phi(e_s, r, e_o)) \approx 1$
derivationally_ related_form (R)	***(surround_VB_1, wall_NN_1)***	***(word_picture_NN_1, sketch_VB_2)***	***(smelling_NN_1, wind_VB_4)***	***(spoliation_NN_2, sack_VB_1)***
	(unpleasant_JJ_1, unpalatableness_NN_1)	***(develop_VB_10, adjustment_NN_4)***	*(try_out_VB_1, somatic_cell_nuclear_transplantation_NN_1)*	***(desire_NN_2, hope_VB_2)***
	(love_VB_3, enjoyment_NN_2)	*(gloss_VB_3, commentary_NN_1)*	***(lighting_NN_4, set_on_fire_VB_1)***	***(snuffle_VB_3, whine_NN_1)***
	(magnitude_NN_1, tall_JJ_1)	***(violate_VB_2, violation_NN_3)***	*(symphalangus_NN_1, one-half_NN_1)*	***(nasalization_NN_1, sound_out_VB_1)***
	(testify_VB_2, information_NN_1)	***(suffocate_VB_1, strangler_tree_NN_1)***	***(just_JJ_3, validity_NN_1)***	***(tread_NN_1, step_VB_7)***
	(connect_VB_6, converging_NN_1)	***(number_VB_3, point_NN_12)***	*(reprove_VB_1, talking_to_NN_1)*	***(yearn_VB_1, pining_NN_1)***
	(connect_VB_6, connexion_NN_4)	***(develop_VB_10, organic_process_NN_1)***	***(sustain_VB_5, beam_NN_2)***	***(unreliableness_NN_1, arbitrary_JJ_1)***
	(operate_VB_4, psyop_NN_1)	***(plication_NN_1, twist_VB_4)***	***(spring_NN_6, hurdle_VB_1)***	*(travesty_NN_2, travesty_NN_2)*
	(market_VB_1, trade_NN_4)	***(split_up_VB_3, separation_NN_5)***	***(spark_NN_1, scintillate_VB_1)***	***(spark_NN_1, sparkle_VB_1)***
	(operate_VB_4, mission_NN_2)	***(plication_NN_1, wrinkle_VB_2)***	***(utility_NN_2, functional_JJ_1)***	*(stockist_NN_1, stockist_NN_1)*
instance_ hypernym (S)	*(malcolm_x_NN_1, hipster_NN_1)*	*(central_america_NN_1, central_america_NN_1)*	*(theresa_NN_1, monk_NN_1)*	
	(the_nazarene_NN_1, judaism_NN_2)	***(st._gregory_i_NN_1, church_father_NN_1)***	*(nippon_NN_1, european_nation_NN_1)*	
	(old_line_state_NN_1, river_NN_1)	*(south_korea_NN_1, african_nation_NN_1)*	*(great_lakes_NN_1, river_NN_1)*	
	(r._e._byrd_NN_1, commissioned_officer_NN_1)	*(malcolm_x_NN_1, passive_resister_NN_1)*	***(r._e._byrd_NN_1, noncommissioned_officer_NN_1)***	
	(south_korea_NN_1, peninsula_NN_1)	*(malcolm_x_NN_1, birth-control_reformer_NN_1)*	***(world_war_i_NN_1, pitched_battle_NN_1)***	
	(st._gregory_of_n._NN_1, vicar_of_christ_NN_1)	***(los_angeles_NN_1, port_NN_1)***	***(irish_capital_NN_1, urban_center_NN_1)***	
	(nippon_NN_1, italian_region_NN_1)	*(great_lakes_NN_1, canadian_province_NN_1)*	*(volcano_islands_NN_1, urban_center_NN_1)*	
	(robert_owen_NN_1, tycoon_NN_1)	*(transylvanian_alps_NN_1, urban_center_NN_1)*	*(nippon_NN_1, american_state_NN_1)*	
	(mandalay_NN_1, national_capital_NN_1)	***(gettysburg_NN_2, siege_NN_1)***	***(helsinki_NN_1, urban_center_NN_1)***	
	(nan_ling_NN_1, urban_center_NN_1)	***(wisconsin_NN_2, geographical_region_NN_1)***	***(capital_of_gambia_NN_1, urban_center_NN_1)***	
synset_domain_ topic_of (C)	***(libel_NN_1, criminal_law_NN_1)***	***(write_VB_8, transcription_NN_5)***	***(assist_NN_2, am._football_game_NN_1)***	
	(brynhild_NN_1, mythology_NN_2)	*(temple_NN_4, muslimism_NN_2)*	***(drive_NN_12, court_game_NN_1)***	
	(slugger_NN_1, sport_NN_1)	***(assist_NN_2, hockey_NN_1)***	*(sell_VB_1, offense_NN_3)*	
	(sell_VB_1, law_NN_1)	***(relationship_NN_4, biology_NN_1)***	***(slugger_NN_1, softball_game_NN_1)***	
	(semitic_deity_NN_1, mythology_NN_1)	***(apostle_NN_3, western_church_NN_1)***	***(drive_NN_12, badminton_NN_1)***	
	(nuclear_deterrence_NN_1, law_NN_1)	***(assist_NN_2, sport_NN_1)***		
	(reception_NN_5, baseball_game_NN_1)	***(trot_VB_2, equestrian_sport_NN_1)***		
	(photosynthesis_NN_1, chemistry_NN_1)	***(rna_NN_1, chemistry_NN_1)***		
	(isolde_NN_1, parable_NN_1)	***(assist_NN_2, soccer_NN_1)***		
	(assist_NN_2, court_game_NN_1)	***(assist_NN_2, football_game_NN_1)***		