Simplicial 2-Complex Convolutional Neural Networks

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Abstract

Recently, neural network architectures have been developed to accommodate when the data has the structure of a graph or, more generally, a hypergraph. While useful, graph structures can be potentially limiting. Hypergraph structures in general do not account for higher order relations between their hyperedges. Simplicial complexes offer a middle ground, with a rich theory to draw on. We develop a convolutional neural network layer on simplicial 2-complexes.

Simplicial 2-complex S and boundary maps $\mathbf{B}_k : C_k \to C_{k-1}$

Feature maps X_0, X_1, X_2

Simplicial Complex Convolution layer

$$X_0^{(h+1)} = \sigma\left(\mathbf{D}_1^{-1}\mathbf{B}_1 X_1^{(h)} W_{0,1}^{(h)} + \widetilde{\mathbf{A}}_0^u X_0^{(h)} W_{0,0}^{(h)}\right)$$

$$X_1^{(h+1)} = \sigma\left(\mathbf{B}_2\mathbf{D}_3 X_2^{(h)} W_{1,2}^{(h)} + (\widetilde{\mathbf{A}}_1^d + \widetilde{\mathbf{A}}_1^u) X_1^{(h)} W_{1,1}^{(h)} + \mathbf{D}_2\mathbf{B}_1^*\mathbf{D}_1^{-1} X_0^{(h)} W_{1,0}^{(h)}\right)$$

$$X_2^{(h+1)} = \sigma\left(\widetilde{\mathbf{A}}_2^d X_2^{(h)} W_{2,2}^{(h)} + \mathbf{D}_4\mathbf{B}_2^*\mathbf{D}_5^{-1} X_1^{(h)} W_{2,1}^{(h)}\right)$$

Ablation study on MNIST classification

Model	acc. $\pm$ std.
CONV1D – FC	87.40 ± 0.70
GCONV – CONV1D – FC	87.42 ± 0.59
SCCONV – CONV1D – FC	**91.10** $\pm$ 0.40

Topological Data Analysis and Beyond Workshop at NeurIPS 2020