IMITATION LEARNING FROM CORRUPTED DEMONSTRATIONS

Anonymous authors
Paper under double-blind review

ABSTRACT

We consider offline Imitation Learning from *corrupted demonstrations* where a constant fraction of data can be noise or even arbitrary outliers. Classical approaches such as Behavior Cloning assumes that demonstrations are collected by an presumably optimal expert, hence may fail drastically when learning from corrupted demonstrations. We propose a novel robust algorithm by minimizing a Median-of-Means (MOM) objective which guarantees the accurate estimation of policy, even in the presence of constant fraction of outliers. Our theoretical analysis shows that our robust method in the corrupted setting enjoys nearly the same error scaling and sample complexity guarantees as the classical Behavior Cloning in the expert demonstration setting. Our experiments on continuous-control benchmarks validate that existing algorithms are fragile under corrupted demonstration while our method exhibits the predicted robustness and effectiveness.

1 INTRODUCTION

Recent years have witnessed the success of using autonomous agent to learn and adapt to complex tasks and environments in a range of applications such as playing games (e.g. Mnih et al., 2015; Silver et al., 2018; Vinyals et al., 2019), autonomous driving (e.g. Kendall et al., 2019; Bellemare et al., 2020), robotics (Haarnoja et al., 2017), medical treatment (e.g. Yu et al., 2019) and recommendation system and advertisement (e.g. Li et al., 2011; Thomas et al., 2017).

Previous success for sequential decision making often requires two key components: (1) a careful design reward function that can provide the supervision signal during learning and (2) an unlimited number of online interactions with the real-world environment to query new unseen region. However, in many scenarios, both components are not allowed. For example, it is hard to define the reward signal in uncountable many extreme situations in autonomous driving; and it is dangerous and risky to directly deploy a learning policy on human to gather information in autonomous medical treatment. Therefore an *offline* sequential decision making algorithm without reward signal is in demand.

Offline Imitation Learning (IL) offers an elegant way to train intelligent agents for complex task without the knowledge of reward functions. Since the offline imitation learning does not interact with the environment, in order to guide intelligent agents to correct behaviors, it is crucial to have high quality expert demonstrations. The well-known Behavior Cloning (BC) algorithm (Pomerleau, 1988) requires that the demonstrations given for training are all *presumably optimal* and it aims to learn that mapping from state to action via expert demonstration data set.

However in real world scenario, since the demonstration is often collected from human, we cannot guarantee that *all* the demonstrations we have collected provide successful behaviors. An human expert tends to make mistakes by accident or due to the hardness of a complicated scenario (e.g., medical diagnosis). Furthermore, even an expert demonstrates a successful behavior, the recorder or the recording system can have a chance to contaminate the data by accident or on purpose (e.g. Eykholt et al., 2018; Neff & Nagy, 2016). This leads to the central question of the paper:

Can the presumably optimal assumption on demonstrations be weakened or even tolerate arbitrary outliers under offline imitation learning settings?

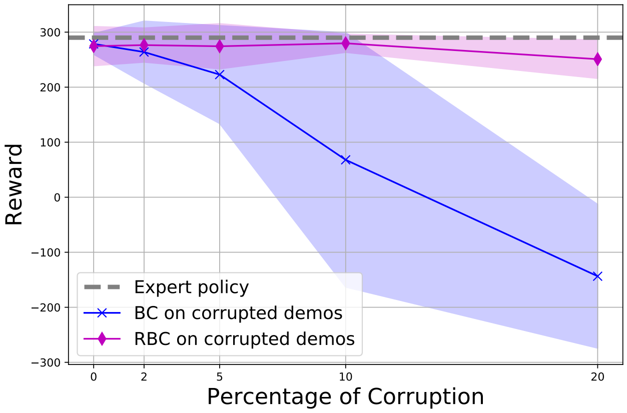

Figure 1: Reward vs. fraction of corruptions in Lunar Lander environment. Shaded region represents standard deviation for 20 trials. We fix the sample size $N = 4000$ for the demonstration data set, and vary the fraction of corruptions ϵ up to 20%. Our algorithm Robust Behavior Cloning (RBC) on corrupted demonstrations has nearly the same performance as BC on expert demonstrations (this is the case when $\epsilon = 0$), and barely change when ϵ grows larger. By contrast, the performance of vanilla BC on corrupted demos fails drastically.

More concretely, we consider *corrupted demonstrations* setting where the majority of the demonstration data is collected by an expert policy (presumably optimal), and the remaining data can be even arbitrary outliers (the formal definition is presented in Definition 2.1). This has great significance in many applications, such as automated medical diagnosis for healthcare (Yu et al. (2019)) and autonomous driving (Ma et al., 2018), where the historical data (demonstration) is often complicated and noisy which requires robustness consideration.

However, the classical offline imitation learning approaches such as Behavior Cloning (BC) fails drastically under this corrupted demonstration settings. We illustrated this phenomenon in Figure 1. We use BC on a continuous control environment, and the performance of the policy learned by BC drops drastically as the fraction of corruptions increases in the offline demonstration data set. However, our proposed algorithm – Robust Behavior Cloning (Algorithm 1) – is resilient to corruptions in the offline demonstrations. The detailed experimental setup is included in Section 5. We now summarize our contributions as follows.

1.1 MAIN CONTRIBUTIONS

- (Algorithm) We consider robustness in offline imitation learning where we have corrupted demonstrations. Our definition for corrupted demonstrations significantly weakens the presumably optimal assumption on demonstration data, and can tolerate a constant fraction of state-action pairs to be arbitrarily corrupted. We refer to Definition 2.1 for a more precise statement.

  To deal with this issue, we propose a novel algorithm Robust Behavior Cloning (Algorithm 1) for robust imitation learning. Our algorithm works in the offline setting, without any further interaction with the environment. The core ingredient of our robust algorithm is using a novel median of means objective in policy estimation compared to classical Behavior Cloning. Hence, it's simple to implement, and computationally efficient.
- (Theoretical guarantees) We analyze our Robust Behavior Cloning algorithm when there exists a constant fraction of outliers in the demonstrations under the offline setting. We show that our RBC achieves the same error scaling and sample complexity compared to vanilla BC with expert demonstrations. To this end, our algorithm guarantees robustness to corrupted demonstrations at no cost of statistical error. This is the content of Section 4.
- (Empirical support) We validate the predicted robustness and show the effectiveness of our algorithm on several continuous control benchmarks – the vanilla BC is fragile indeed with corrupted demonstrations, and our Robust Behavior Cloning achieves nearly the same performance compared to vanilla BC with expert demonstrations. This is the content of Section 5.

2 PROBLEM SETUP

2.1 REINFORCEMENT LEARNING AND IMITATION LEARNING

Markov Decision Process and Reinforcement Learning. We start the problem setup by introducing the Markov decision process (MDP). An MDP $M = \langle \mathcal{S}, \mathcal{A}, r, \mathsf{P}, \mu_0, \gamma \rangle$ consists of a state space $\mathcal{S}$, an action space $\mathcal{A}$, an unknown reward function $r : \mathcal{S} \times \mathcal{A} \to [0, \mathsf{R}_{\max}]$, an unknown transition kernel $\mathsf{P} : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$, an initial state distribution $\mu_0 \in \Delta(\mathcal{S})$, and a discounted factor $\gamma \in (0, 1)$. We use Δ to denote the probability distributions on the simplex.

An agent acts in a MDP following a policy $\pi(\cdot|\mathbf{s})$, which prescribes a distribution over the action space $\mathcal{A}$ given each state $\mathbf{s} \in \mathcal{S}$. Running the policy starting from the initial distribution $\mathbf{s}_1 \sim \mu_0$ yields a stochastic trajectory $\mathcal{T} := \{\mathbf{s}_t, \mathbf{a}_t, r_t\}_{1 \le t \le \infty}$, where $\mathbf{s}_t, \mathbf{a}_t, r_t$ represent the state, action, reward at time t respectively, with $\mathbf{a}_t \sim \pi(\cdot|\mathbf{s}_t)$ and the next state $\mathbf{s}_{t+1}$ follows the unknown transition kernel $\mathbf{s}_{t+1} \sim \mathsf{P}(\cdot|\mathbf{s}_t, \mathbf{a}_t)$. We denote $\rho_{\pi,t} \in \Delta(\mathcal{S} \times \mathcal{A})$ as the marginal joint stationary distribution for state, action at time step t, and we define $\rho_\pi = (1 - \gamma) \sum_{i=1}^{\infty} \gamma^t \rho_{\pi,t}$ as visitation distribution for policy π. For simplicity, we reuse the notation $\rho_\pi(s) = \int_{a \in \mathcal{A}} \rho_\pi(s, a) da$ to denote the marginal distribution over state. The goal of reinforcement learning is to find the best policy π to maximize the expected cumulative return $J_\pi = \mathbb{E}_{\mathcal{T} \sim \pi} \left[\sum_{i=1}^{\infty} \gamma^t r_t\right]$.

Imitation Learning. Imitation learning (IL) aims to obtain a policy to mimic expert's behavior with demonstration data set $\mathcal{D} = \{(\mathbf{s}_i, \mathbf{a}_i)\}_{i=1}^N$ where N is the sample size of $\mathcal{D}$. Note that we do not need any reward signal. Tradition imitation learning assumes perfect (or near-optimal) expert demonstration – for simplification we assume that each state, action pair $(\mathbf{s}_i, \mathbf{a}_i)$ is drawn from the joint stationary distribution of an expert policy π_E:

$$(\mathbf{s}_i, \mathbf{a}_i) \sim \rho_{\pi_{\mathrm{E}}} \tag{1}$$

The goal of *offline* IL is to learn a policy $\widehat{\pi}^{\mathrm{IL}} = \mathbb{A}(\mathcal{D})$ through an IL algorithm $\mathbb{A}$, given the demonstration data set $\mathcal{D}$, without further interaction with the unknown true transition dynamic P.

Behavior Cloning. The Behavior Cloning (BC) is the well known algorithm (Pomerleau, 1988) for IL which only uses offline demonstration data without any interaction with the environment. More specifically, BC solves the following Maximum Likelihood Estimation (MLE) problem, which minimizes the average Negative Log-Likelihood (NLL) for all samples in offline demonstrations $\mathcal{D}$:

$$\widehat{\pi}^{\mathrm{BC}} = \arg\min_{\pi \in \Pi} \frac{1}{N} \sum_{(s,a) \in \mathcal{D}} -\log(\pi(a|s)) \tag{2}$$

Recent work (Rajaraman et al., 2020) has shown that BC is optimal under the offline setting, and can only be improved with the knowledge of transition dynamic P. Also, a line of research considers improving BC with further online interaction of the environment (Brantley et al., 2019) or actively querying an expert (Ross et al., 2011).

2.2 LEARNING FROM CORRUPTED DEMONSTRATIONS

However, it is sometimes unrealistic to assume that the demonstration data set is collected through a presumably optimal expert policy. In this paper, we propose Definition 2.1 for the corrupted demonstrations, which tolerates gross corruption or model mismatch in offline data set.

Definition 2.1 (Corrupted Demonstrations)**.** *Let the state-action pair* $(\mathbf{s}_i, \mathbf{a}_i)_{i=1}^N$ *drawn from the joint stationary distribution of an expert policy* π_E*. The corrupted demonstration data* $\mathcal{D}$ *are generated by the following process: an adversary can choose an arbitrary* ϵ*-fraction of the samples in* $[N]$ *and modifies them with arbitrary values. After the corruption, we use* $\mathcal{D}$ *to denote the corrupted demonstration data set.*

This corruption process can represent gross corruptions or model mismatch in the demonstration data set. To the best of our knowledge, Definition 2.1 is the first definition for corrupted demonstrations in imitation learning which tolerates *arbitrary* corruptions. In the supervised learning, the well-known Huber's contamination model (Huber (1964)) considers $(\boldsymbol{x}, y) \overset{iid}{\sim} (1 - \epsilon)P + \epsilon Z$, where

$\boldsymbol{x} \in \mathbb{R}^d$ is the explanatory variable (feature) and $y \in \mathbb{R}$ is the response variable. Here, P denotes the *authentic* statistical distribution such as Normal mean estimation or linear regression model, and Z denotes the outliers. Dealing with corrupted $\boldsymbol{x}$ and y has a long history in the robust statistics community (e.g. Rousseeuw, 1984; Chen et al., 2013; 2017; Klivans et al., 2018; Yin et al., 2018; Prasad et al., 2020; Diakonikolas et al., 2019; Liu et al., 2019; 2020; Shen & Sanghavi, 2019; Lugosi & Mendelson, 2019; Lecué & Lerasle, 2020; Jalal et al., 2020). We note that in Imitation Learning, the data collecting process for the demonstrations does not obey i.i.d. assumption in traditional supervised learning due to the temporal dependency.

Notations. Throughout this paper, we use $\{c_i\}_{i=1,2,3}$ to denote the universal positive constant. We utilize the big-O notation $f(n) = O(g(n))$ to denote that there exists a positive constant c_1 and a natural number n_0 such that, for all $n \geq n_0$, we have $f(n) \leq c_1 g(n)$.

3 OUR ALGORITHMS

It is well known that the Median-of-Means (MOM) estimator achieves sub-Gaussian concentration bound for one-dimensional mean estimation even though the underlying distribution only has second moment bound (heavy tailed distribution) (interested readers are referred to textbooks such as Nemirovsky & Yudin (1983); Jerrum et al. (1986); Alon et al. (1999)).

The vanilla MOM estimator for one-dimensional mean estimation works like following: (1) randomly partition N samples into M batches; (2) calculates the mean for each batch; (3) outputs the median of these batch mean. Very recently, MOM estimators are used for high dimensional robust regression (Brownlees et al., 2015; Hsu & Sabato, 2016) by applying MOM estimator on the *loss function* of empirical risk minimization process.

3.1 ROBUST BEHAVIOR CLONING

Motivated by using MOM estimators on the loss function, we propose Definition 3.1 which uses a MOM objective to handle arbitrary outliers in demonstration data set $(\mathbf{s}, \mathbf{a}) \in \mathcal{D}$.

Definition 3.1 (Robust Behavior Cloning)**.** *We split the corrupted demonstrations $\mathcal{D}$ into M batches randomly*[1]*: $\{B_j\}_{j=1}^M$. The Robust Behavior Cloning solves the following optimization*

$$\widehat{\pi}^{\mathrm{RBC}} = \arg\min_{\pi\in\Pi}\max_{\pi'\in\Pi}\operatorname*{median}_{1\leq j\leq M}\left(\ell_j(\pi) - \ell_j(\pi')\right), \tag{3}$$

where the loss function $\ell_j(\pi)$ is the average Negative Log-Likelihood in the batch B_j:

$$\ell_j(\pi) = \frac{1}{b}\sum_{(s,a)\in B_j} -\log(\pi(a|s)). \tag{4}$$

The workhorse of Definition 3.1 is eq. (3), which uses a novel variant of Median-of-Means (MOM) tournament procedure (Le Cam (2012); Lugosi & Mendelson (2019); Lecué & Lerasle (2020); Jalal et al. (2020)). In eq. (4), we calculate the average Negative Log-Likelihood (NLL) for a single batch, and $\widehat{\pi}^{\mathrm{RBC}}$ is the solution of a min-max formulation based on the batch loss $\ell_j(\pi)$. Though our algorithm minimizes the robust version of NLL, we do not utilize the traditional iid assumption in the supervised learning.

The intuition behind solving this min-max formulation is that the inner variable π' needs to get close to π_{E} to maximize the difference of loss function, and the outer variable π also need to get close to π_{E}. Hence we can guarantee that $\widehat{\pi}^{\mathrm{RBC}}$ will be close to π_{E}. In Section 4, we show that under corrupted demonstrations, $\widehat{\pi}^{\mathrm{RBC}}$ in eq. (3) has the same error scaling and sample complexity compared to π_{E}.

In Definition 3.1 the objective function eq. (3) is not convex (in general), hence we use Algorithm 1 as a computational heuristic to solve it. In each iteration of Algorithm 1, we randomly partition the demonstration data set $\mathcal{D}$ into M batches, and calculate the loss $\ell_j(\pi) - \ell_j(\pi')$ by eq. (4). We then pick the batch B_{Med} with the median loss, and evaluate the gradient on that batch. We use gradient

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[1] Without loss of generality, we assume that M exactly divides the sample size N

Algorithm 1 Robust Behavior Cloning.

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Input: Corrupted demonstrations $\mathcal{D}$
Output: Robust policy $\widehat{\pi}^{\mathrm{RBC}}$
Initialize $\pi$ and $\pi'$.
for $t = 0$ to $T - 1$, do
    Randomly partition $\mathcal{D}$ to $M$ batches. For each batch $j \in [M]$, calculate the loss $\ell_j(\pi) - \ell_j(\pi')$ by eq. (4).
    Pick the batch with median loss within $M$ batches
        $\underset{1 \le j \le M}{\mathrm{median}} \left( \ell_j(\pi) - \ell_j(\pi') \right),$
    and evaluate the gradient for $\pi$ and $\pi'$ using back-propagation on that batch
    (i) perform gradient descent on $\pi$.
    (ii) perform gradient ascent on $\pi'$.
end for
Return: Robust policy $\widehat{\pi}^{\mathrm{RBC}} = \pi$.
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descent on π for the $\arg\min$ part and gradient ascent on π' for the $\arg\max$ part. In Section 5, we empirically show that this gradient-based heuristic Algorithm 1 is able to minimize this objective and has good convergence properties. As for the time complexity, when using back-propagation on one batch of samples, our RBC incurs overhead costs compared to vanilla BC, in order to evaluate the loss function for all samples via forward propagation.

4 THEORETICAL ANALYSIS

In this section, we provide theoretical guarantees for our RBC algorithm. Since our method (Definition 3.1) directly estimates the conditional probability $\pi(a|s)$ over the offline demonstrations, our theoretical analysis provides guarantees on $\mathbb{E}_{s \sim \rho_{\pi_{\mathrm{E}}}} \left\| \widehat{\pi}^{\mathrm{RBC}}(\cdot|\mathbf{s}) - \pi_{\mathrm{E}}(\cdot|\mathbf{s}) \right\|_{\mathrm{TV}}^2$, which upper bounds the total variation norm compared to π_{E} under the expectation of $s \sim \rho_{\pi_{\mathrm{E}}}$. The ultimate goal of the learned policy is to maximize the expected cumulative return, thus we then provide an upper bound for the sub-optimality $J_{\pi_{\mathrm{E}}} - J_{\widehat{\pi}^{\mathrm{RBC}}}$.

We begin the theoretical analysis by Assumption 4.1, which simplifies our analysis and is common in literature (Agarwal et al., 2019; 2020). By assuming that the policy class Π is discrete, our upper bounds depend on the quantity $\log(|\Pi|)/N$, which matches the error rates and sample complexity for using BC with expert demonstrations (Agarwal et al., 2019; 2020).

Assumption 4.1. *We assume that the policy class Π is discrete, and realizable, i.e., $\pi_{\mathrm{E}} \in \Pi$.*

We first present Theorem 4.1, which shows that minimizing the MOM objective via eq. (3) guarantees the closeness of robust policy to optimal policy in total variation distance.

Theorem 4.1. *Suppose we have corrupted demonstration data set $\mathcal{D}$ with sample size N from Definition 2.1, and there exists a (sufficiently small) constant corruption ratio ϵ. Under Assumption 4.1, let τ to be the output objective value with $\widehat{\pi}^{\mathrm{RBC}}$ in the optimization eq. (3), then with probability at least $1 - c_1\delta$, we have*

$$\mathbb{E}_{s \sim \rho_{\pi_{\mathrm{E}}}} \left\| \widehat{\pi}^{\mathrm{RBC}}(\cdot|\mathbf{s}) - \pi_{\mathrm{E}}(\cdot|\mathbf{s}) \right\|_{\mathrm{TV}}^2 = O\left(\frac{\log(|\Pi|/\delta)}{N} + \tau \right). \tag{5}$$

The proof is collected in Appendix A. We note that the data collection process does not follow the iid assumption, hence we use martingale analysis similar to (Agarwal et al., 2019; 2020). The final objective value in the optimization eq. (3) τ includes two parts – the first part is inherent stochastic distribution of π_{E} and the fraction of corruption ϵ, which cannot be improved. The second part is the sub-optimality gap due to the solving the non-convex optimization. Our main theorem Theorem 4.1 guarantees that a small value of the final objective implies an accurate estimation of policy and hence we can certify estimation quality using the obtained final value of the objective.

Next, we present Theorem 4.2, which guarantees the reward performance of the learned robust policy $\widehat{\pi}^{\mathrm{RBC}}$.

Theorem 4.2. *Under the same setting as Theorem 4.1, we have*

$$J_{\pi_{\mathrm{E}}} - J_{\widehat{\pi}^{\mathrm{RBC}}} \leq O\left(\frac{1}{(1-\gamma)^2}\sqrt{\frac{\log(|\Pi|/\delta)}{N} + \tau}\right), \tag{6}$$

with probability at least $1 - c_1\delta$.

The proof is also collected in Appendix A. The error scaling and sample complexity of Theorem 4.1 and Theorem 4.2 matches the vanilla BC with expert demonstrations (Agarwal et al., 2019; 2020).

Remark 4.1. *The quadratic dependency on the effective horizon –* $\frac{1}{(1-\gamma)^2}$ *– is known as the compounding error or distribution shift in literature, which is due to the essential limitation of offline imitation learning setting. Recent work (Rajaraman et al., 2020) shows that this quadratic dependency cannot be improved without any further interaction with the environment or the knowledge of transition dynamic* P. *Hence BC is actually optimal under no-interaction setting. Also, a line of research considers improving BC by further online interaction with the environment or even active query of the experts (Ross et al., 2011; Brantley et al., 2019). Since our work, as a robust counterpart of BC, focuses on the robustness to the corruptions in the offline demonstrations, it can be naturally used in online algorithms such as DAGGER (Ross et al., 2011) and Brantley et al. (2019).*

5 EXPERIMENTS

In this section, we study the empirical performance of our Robust Behavior Cloning. We evaluate the robustness of Robust Behavior Cloning on several continuous control benchmarks simulated by PyBullet Coumans & Bai (2016) simulator: LunarLanderContinuous-v2 and HalfCheetahBulletEnv-v0. Actually, these tasks have true reward function already in the simulator. We will use *only* state observation and action for the imitation algorithm, and we then use the reward to evaluate the obtained policy when running in the simulator.

For each task, we collect the presumably optimal expert trajectories using pre-trained agents from Standard Baselines3[2]. In the experiment, we use TQC (Kuznetsov et al. (2020)) in the Standard Baselines3 pre-trained agents, which has the highest reward, and we consider it to be an expert.

For the continuous control environments, the action space are bounded. Hence we generate corrupted demonstration data set $\mathcal{D}$ as follows: we first randomly choose ϵ fraction of samples, and corrupt the action to the boundary (normally -1 or $+1$). We compare our RBC algorithm (Algorithm 1) to a number of natural baselines: the first baseline is directly using BC on the corrupted demonstration $\mathcal{D}$ without any robustness consideration. The second one is using BC on the *expert demonstrations* with the same sample size. In different settings, we fix the policy network as 2 hidden layer feed-forward Neural Network of size $\{400, 300\}$ with ReLU activation, which is standard in the baselines.

5.1 CONVERGENCE OF OUR ALGORITHM

In this subsection, we illustrate the convergence of our algorithm to support our theoretical analysis. We track the performance metric of different algorithms vs. epoch number in the whole training process. More specifically, we use the metrics as follows: (1) we track the Mean Squared Error $\|\mathbf{a} - \widehat{\mathbf{a}}\|_2^2$ in the validation data set, which is additional expert demonstrations not revealed to the training process; (2) We then evaluate current policy in the simulator for 20 trials, and obtain the mean and standard deviation of cumulative reward for each epoch. These metrics correspond to theoretical bounds in Theorem 4.1 and Theorem 4.2 respectively.

We first focus on the continuous control environment LunarLanderContinuous-v2, where the state space has dimension 8, and the action space has dimension 2 with boundary $[-1, 1]$. We fix the sample size as 4000, and the corruption fraction $\epsilon = 10\%$.

In Figure 2a, the validation error of vanilla BC on corrupted demonstrations $\mathcal{D}$ barely converge. Instead, the validation error of our RBC has good convergence properties compared to the "oracle ver-

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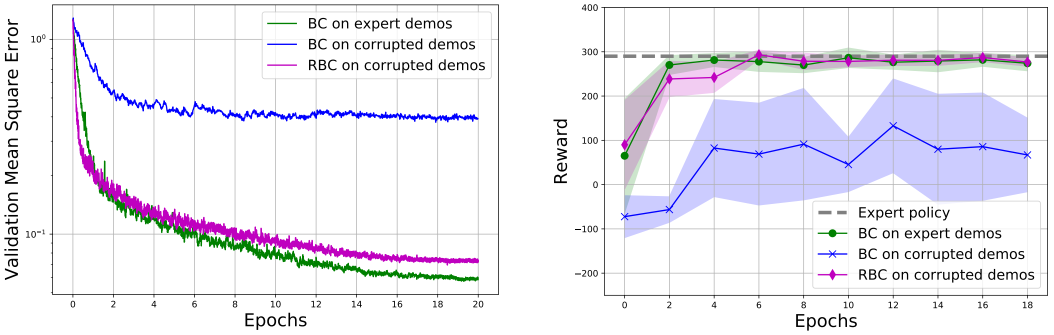
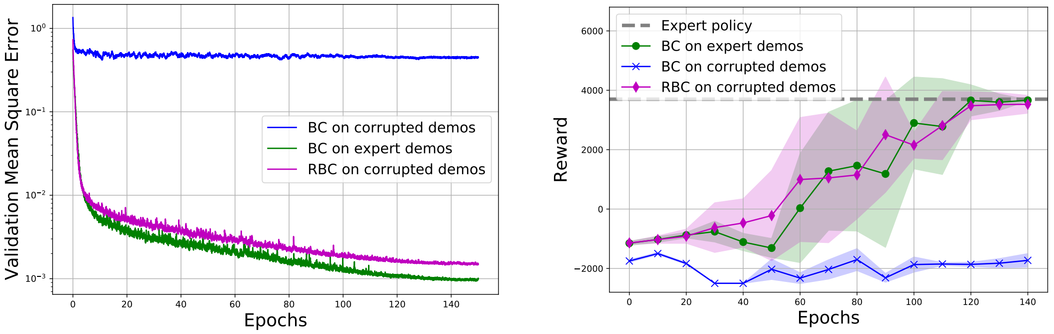
[2] The pre-trained agents were cloned from the following repositories: `https://github.com/DLR-RM/stable-baselines3`, `https://github.com/DLR-RM/rl-baselines3-zoo`.



(a) MSE on validation set vs. epoch number for different baselines.

(b) Reward vs. epoch for different baselines. Shaded region represents standard deviation for 20 trials.

Figure 2: Offline Imitation Learning on Lunar Lander task with demonstration data of size 4000. We fix the corruption ration $\epsilon = 10\%$, and run algorithms with 20 epochs. Vanilla BC on corrupted demonstrations fails to converge to expert policy. Using the robust counterpart Algorithm 1 on corrupted demonstrations has good convergence properties. Surprisingly, our RBC on corrupted data set has nearly the same validation error in Figure 2a and reward performance in Figure 2b compared to using BC on expert data set.



(a) MSE on validation set vs. epoch number for different baselines.

(b) Reward vs. epoch for different baselines. Shaded region represents standard deviation for 20 trials.

Figure 3: Offline Imitation Learning on Half Cheetah task with demonstration data of size 40000. We fix the corruption ration $\epsilon = 1\%$, and run algorithms with 150 epochs. Directly running BC on corrupted demonstration data set fails drastically with no improvement over a random policy. Using the robust counterpart Algorithm 1 on corrupted demonstrations has good convergence properties. Surprisingly, in Figure 3b, our RBC on corrupted data set has nearly the same performance of using BC on expert data set.

sion" which uses BC on expert demonstrations. This validates our theoretical results Theorem 4.1, which guarantees the closeness of robust policy to the expert policy π_{E}.

In Figure 2b, we track the cumulative reward for the current policy after each epoch. The BC on corrupted demonstrations cannot converge to the expert policy, yet surprisingly, our robust method has nearly the same performance compared to the BC on *expert demonstrations*.

Then, we move to another continuous control task HalfCheetahBulletEnv-v0, which has significantly higher dimension, where the observation space has dimension 27 and the action space has dimension 8 with boundary $[-1, 1]$ on each dimension. The results for Half Cheetah is presented in Figure 3, and we observe similar results as in Figure 2. The vanilla BC on corrupted demonstrations is fragile indeed, and fails drastically. Instead, our robust algorithm achieves nearly the same results as the BC on expert demonstrations.

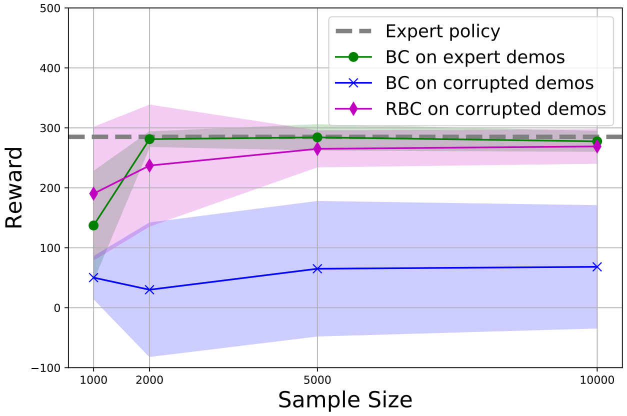


Figure 4: Reward vs. different sample size in Lunar Lander environment. Shaded region represents standard deviation for 20 trials. We fix $\epsilon = 10\%$, and vary the sample size in $\mathcal{D}$. Our RBC on corrupted demonstrations has nearly the same performance as BC on expert demonstrations. BC on corrupted demos fails drastically, and cannot converge to expert policy with large sample size.

5.2 PERFORMANCE UNDER DIFFERENT SETUPS

In this subsection, we present the effectiveness and predicted robustness of our Robust Behavior Cloning by evaluating on multiple runs under different setups. We keep using the continuous control environment LunarLanderContinuous-v2, and focus on different settings for the demonstrations $\mathcal{D}$: the sample size N and the fraction of corruptions ϵ.

Different sample size in $\mathcal{D}$. In this experiment, we fix the fraction of the corruption $\epsilon = 10\%$, and vary the sample size of the demonstration data set. It is expected in Theorem 4.2 that larger sample size of $\mathcal{D}$ leads to smaller suboptimality gap in value function. Figure 4 validates our theory: Our RBC on corrupted demonstrations has nearly the same reward as BC on expert demonstrations, and the sub-optimality gap gets smaller as N grows larger. However, directly using BC on corrupted demonstrations cannot improve as sample size grows larger.

Different fraction of corruption. This is the experiment we have shown in Section 1. In this experiment, we fix the sample size $N = 4000$ as usual, and then vary the fraction of corruptions ϵ. As expected, Figure 1 shows that our RBC is resilient to a constant-fraction of outliers in the demonstrations ranging from 0 to 20%, and it achieves nearly the same performance as the BC on expert demonstrations. In contrast, directly using BC on corrupted demonstrations obtains worse reward performance as the fraction of outliers grows.

In all our experiments, BC on expert demonstrations is a *strong baseline* which achieves reward performance at expert level with very few transition samples in demonstrations. This suggests that compounding error or distribution shift may be less of a problem in these environments. This is consistent with the findings in Brantley et al. (2019). Since our RBC is a robust counterpart of BC, which is resilient to outliers in *offline data*, it is straightforward to combine RBC with online Imitation Learning algorithms (such as DAGGER (Ross et al., 2011), and ensemble based method (Brantley et al., 2019), etc.). It would be interesting to examine the performance of our algorithm in the online interaction setting, and we leave it as future works.

6 DISCUSSIONS

6.1 RELATED WORK

Imitation Learning. Behavior Cloning (BC) is the most widely-used imitation learning algorithm (Pomerleau, 1988; Osa et al., 2018) due to its simplicity, effectiveness and scalability, and has been widely used in practice. From a theoretical viewpoint, it has been showed that BC achieves informational optimality in the offline setting (Rajaraman et al., 2020) with *no further online interactions*.

With online interaction, there's a line of research focusing on improving BC – for example, Ross et al. (2011) proposed DAGGER (Data Aggregation) by querying the expert policy in the online setting. Brantley et al. (2019) proposed using an ensemble of BC as uncertainty measure and interacts with the environment to improve BC by taking the uncertainty into account, without the need to query the expert.

Besides BC, there are other imitation learning algorithms: Ho & Ermon (2016) uses generative adversarial networks for distribution matching to learn a reward function; Reddy et al. (2019) provides a reinforcement learning framework to deal with imitation learning by artificially setting the reward; Ghasemipour et al. (2020) unified several existing imitation learning algorithm as minimizing distribution divergence between learned policy and expert demonstration, just to name a few.

Offline RL. Reinforcement learning leverages the signal from reward function to train the policy. Different from IL, offline RL often does not require the demonstration to be expert demonstration (e.g. Fujimoto et al., 2019; Fujimoto & Gu, 2021; Kumar et al., 2020) (interested readers are referred to (Levine et al., 2020)), and even expects the offline data with higher coverage for different suboptimal policies (Buckman et al., 2020; Jin et al., 2021; Rashidinejad et al., 2021). Behavior-agnostic setting (Nachum et al., 2019; Mousavi et al., 2020) even does not require the collected data from a single policy.

The closest relation between offline RL and IL is the learning of stationary visitation distribution, where learning such visitation distribution does not involve with reward signal, similar to IL. A line of recent research especially for off-policy evaluation tries to learn the stationary visitation distribution of a given target policy (e.g. Liu et al., 2018; Nachum et al., 2019; Tang et al., 2020; Mousavi et al., 2020; Dai et al., 2020). Especially Kostrikov et al. (2020) leverages the off-policy evaluation idea to IL area.

Robustness in IL and RL. There are several recent papers consider corruption-robust in either RL or IL. In RL, Zhang et al. (2021b) considers that the adversarial corruption may corrupt the whole episode in the online RL setting while a more recent one (Zhang et al., 2021a) considers offline RL where ϵ-fraction of the whole data set can be replaced by the outliers. Many other papers consider perturbations, heavy tails, or corruptions in either reward function (Bubeck et al., 2013) or in transition dynamic (Xu & Mannor, 2012; Tamar et al., 2014; Roy et al., 2017).

The most related papers follow a similar setting of robust IL are (Wu et al., 2019; Tangkaratt et al., 2020; 2021), where they consider imperfect or noisy observations in imitation learning. However, their algorithms require online interactions with the environment, and cannot handle outliers in the demonstrations. Our algorithm achieves robustness guarantee from purely *offline* demonstration, without the potentially costly or risky interaction with the real world environment.

6.2 SUMMARY AND FUTURE WORKS

In this paper, we considered the corrupted demonstrations issues in imitation learning, and proposed a novel robust algorithm, Robust Behavior Cloning, to deal with the corruptions in offline demonstration data set. The core technique is replacing the vanilla Maximum Likelihood Estimation with a Median-of-Means (MOM) objective which guarantees the policy estimation and reward performance in the presence of constant fraction of outliers. Our algorithm has strong robustness guarantees and works well in practice.

There are several avenues for future work: since our work focuses on the corruption in offline data set, any improvement in *online imitation learning* which utilizes Behavior Cloning would benefit from the corruption-robustness guarantees by our *offline* Robust Behavior Cloning. Also, it would also be of interest to apply our algorithm for real-world environment, such as automated medical diagnosis and autonomous driving.

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A PROOFS

The analysis of maximum likelihood estimation is standard in i.i.d. setting for the supervised learning setting (van de Geer, 2000). In our proofs of the robust offline imitation learning algorithm, the analysis for the sequential decision making leverages the martingale analysis technique from (Zhang, 2006; Agarwal et al., 2020).

Our Robust Behavior Cloning (Definition 3.1) solves the following optimization

$$\widehat{\pi}^{\mathrm{RBC}} = \arg\min_{\pi\in\Pi}\max_{\pi'\in\Pi}\operatorname*{median}_{1\le j\le M}\left(\ell_j(\pi)-\ell_j(\pi')\right), \tag{7}$$

where the loss function $\ell_j(\pi)$ is the average Negative Log-Likelihood in the batch B_j:

$$\ell_j(\pi) = \frac{1}{b}\sum_{(\mathbf{s},\mathbf{a})\in B_j} -\log(\pi(\mathbf{a}|\mathbf{s})). \tag{8}$$

This can be understood as a robust counterpart for the maximum likelihood estimation in sequential decision process.

With a slight abuse of notation, we use x_i and y_i to denote the observation and action, and the underlying unknown expert distribution is $y_i \sim p(\cdot|x_i)$ and $p(y|x) = f^*(x,y)$. Following Assumption 4.1, we have the realizable $f^* \in \mathcal{F}$, and the discrete function class satisfies $|\mathcal{F}| < \infty$.

Let $\mathcal{D}$ denote the data set and let $\mathcal{D}'$ denote a tangent sequence $\{x_i', y_i'\}_{i=1}^{|\mathcal{D}|}$. The tangent sequence is defined as $x_i' \sim \mathscr{D}_i(x_{1:i-1}, y_{1:i-1})$ and $y_i' \sim p(\cdot|x_i')$. Note here that x_i' follows from the distribution $\mathscr{D}_i$, and depends on the original sequence, hence the tangent sequence is independent conditional on $\mathcal{D}$.

For this martingale process, we first introduce a decoupling Lemma from Agarwal et al. (2020).

Lemma A.1. *[Lemma 24 in Agarwal et al. (2020)] Let $\mathcal{D}$ be a dataset, and let $\mathcal{D}'$ be a tangent sequence. Let $\Gamma(f,\mathcal{D}) = \sum_{(x,y)\in\mathcal{D}} \phi(f,(x,y))$ be any function which can be decomposed additively across samples in $\mathcal{D}$. Here, ϕ is any function of f and sample (x,y). Let $\widehat{f} = \widehat{f}(\mathcal{D})$ be any estimator taking the dataset $\mathcal{D}$ as input and with range $\mathcal{F}$. Then we have*

$$\mathbb{E}_{\mathcal{D}}\left[\exp\left(\Gamma(\widehat{f},\mathcal{D}) - \log\mathbb{E}_{\mathcal{D}'}\exp(\Gamma(\widehat{f},\mathcal{D}')) - \log|\mathcal{F}|\right)\right] \le 1.$$

Then we present a Lemma which upper bounds the TV distance via a loss function closely related to KL divergence. Such bounds for probabilistic distributions are discussed extensively in literature such as Tsybakov (2009).

Lemma A.2. *[Lemma 25 in Agarwal et al. (2020)] For any two conditional probability densities f_1, f_2 and any state distribution $\mathscr{D} \in \Delta(\mathcal{X})$ we have*

$$\mathbb{E}_{x\sim\mathscr{D}}\|f_1(x,\cdot) - f_2(x,\cdot)\|_{\mathrm{TV}}^2 \le -2\log\mathbb{E}_{x\sim\mathscr{D},y\sim f_2(\cdot|x)}\exp\left(-\frac{1}{2}\log\frac{f_2(x,y)}{f_1(x,y)}\right).$$

A.1 PROOF OF THEOREM 4.1

With these Lemmas in hand, we are now equipped to prove our main theorem (Theorem 4.1), which guarantees the solution $\widehat{\pi}^{\mathrm{RBC}}$ of eq. (3) is close to the optimal policy π_{E} in TV distance.

Theorem A.1 (Theorem 4.1)**.** *Suppose we have corrupted demonstration data set $\mathcal{D}$ with sample size N from Definition 2.1, and there exists a (sufficiently small) constant corruption ratio ϵ. Under*

Assumption 4.1, let τ to be the output objective value with $\widehat{\pi}^{\text{RBC}}$ in the optimization eq. (3), then with probability at least $1 - c_1\delta$, we have

$$\mathbb{E}_{s\sim\rho_{\pi_{\text{E}}}} \left\| \widehat{\pi}^{\text{RBC}}(\cdot|\mathbf{s}) - \pi_{\text{E}}(\cdot|\mathbf{s}) \right\|_{\text{TV}}^2 = O\left(\frac{\log(|\Pi|/\delta)}{N} + \tau \right).$$

Proof of Theorem 4.1. En route to the proof of Theorem 4.1, we keep using the notations in Lemma A.1 and Lemma A.2, where the state observation is x, the action is y, and the discrete function class is $\mathcal{F}$.

Similar to Agarwal et al. (2020), we first note that Lemma A.1 can be combined with a simple Chernoff bound to obtain an exponential tail bound. With probability at least $1 - c_1\delta$, we have

$$-\log \mathbb{E}_{\mathcal{D}'} \exp(\Gamma(\widehat{f}, \mathcal{D}')) \le -\Gamma(\widehat{f}, \mathcal{D}) + \log|\mathcal{F}| + \log(1/\delta). \tag{9}$$

Our proof technique relies on lower bounding the LHS of eq. (9), and upper bounding the RHS eq. (9).

Let the batch size b as a constant in Definition 3.1, then there exists the number of batches $M \ge 2\epsilon N$ such that at least 50% batches have no corruptions.

In the definition of RBC (Definition 3.1), we solve

$$\widehat{\pi}^{\text{RBC}} = \arg\min_{\pi\in\Pi} \max_{\pi'\in\Pi} \operatorname*{median}_{1\le j\le M} \left(\ell_j(\pi) - \ell_j(\pi')\right). \tag{10}$$

Notice that since π_{E} is one feasible solution of the inner maximization step eq. (10), we can choose $\pi' = \pi_{\text{E}}$. Now we consider the objective function which is the difference of Negative Log-Likelihood between f and f^*, i.e., $\ell_j(f) - \ell_j(f^*)$, defined in eq. (4) where

$$\ell_j(\pi) = \frac{1}{b} \sum_{(\mathbf{s},\mathbf{a})\in B_j} -\log(\pi(\mathbf{a}|\mathbf{s})).$$

Hence, we choose $\Gamma(f, \mathcal{D})$ in Lemma A.1 as

$$\begin{aligned}\Gamma_j(f, \mathcal{D}) &= \frac{N}{b} \sum_{i\in B_j} -\frac{1}{2} \log \frac{f^*(x_i, y_i)}{f(x_i, y_i)} \\ &= \frac{N}{2b} \sum_{i\in B_j} \left(\log f(x_i, y_i) - \log f^*(x_i, y_i)\right),\end{aligned}$$

which is the difference of Negative Log-Likelihood $N(\ell_j(f^*) - \ell_j(f))/2$ evaluated on a single batch $B_j, j \in [M]$. This is actually the objective function on a single batch appeared in eq. (3).

Lower bound for the LHS of eq. (9). We apply the concentration bound eq. (9) for such uncorrupted batches, and the majority of all batches satisfies eq. (9). For those batches, the LHS of eq. (9)

can be lower bounded by the TV distance according to Lemma A.2.

$$\begin{aligned} & -\log \mathbb{E}_{\mathcal{D}'}\left[\exp\left(\frac{N}{b}\sum_{i\in B_j} -\frac{1}{2}\log\left(\frac{f^\star(x_i',y_i')}{\widehat{f}(x_i',y_i')}\right)\right)\Big|\mathcal{D}\right] \\ & \overset{(i)}{=} -\frac{N}{b}\sum_{i\in B_j}\log \mathbb{E}_{x,y\sim\mathscr{D}_i}\exp\left(-\frac{1}{2}\log\frac{f^\star(x,y)}{\widehat{f}(x,y)}\right) \\ & \overset{(ii)}{\geq} \frac{N}{2b}\sum_{i\in B_j}\mathbb{E}_{x\sim\mathscr{D}_i}\left\|\widehat{f}(x,\cdot)-f^\star(x,\cdot)\right\|_{\mathrm{TV}}^2, \end{aligned} \tag{11}$$

where (i) follows from the independence between $\widehat{f}$ and $\mathcal{D}'$ due to the decoupling technique, and (ii) follows from Lemma A.2, which is an upper bound of the Total Variation distance.

Upper bound for the RHS of eq. (9). Note that the objective is the median of means of each batches and f^* is one feasible solution of the inner maximization step eq. (10). Since τ is the output objective value with $\widehat{\pi}^{\mathrm{RBC}}$ in the optimization eq. (3), this implies that $\ell_{\mathrm{Med}}(\pi)-\ell_{\mathrm{Med}}(\pi')\leq\tau$ for the median batch B_{Med}, which is equivalent to $-\Gamma_{\mathrm{Med}}(f,\mathcal{D})\leq N\tau/2$.

Hence for the median batch B_{Med}, the RHS of eq. (9) can be upper bounded by

$$-\Gamma_{\mathrm{Med}}(\widehat{f},\mathcal{D})+\log|\mathcal{F}|+\log(1/\delta)\leq\log|\mathcal{F}|+\log(1/\delta)+N\tau/2. \tag{12}$$

Putting together the pieces eq. (11) and eq. (12) for B_{Med}, we have

$$\mathbb{E}_{s\sim\rho_{\pi_{\mathrm{E}}}}\left\|\widehat{\pi}^{\mathrm{RBC}}(\cdot|s)-\pi_{\mathrm{E}}(\cdot|s)\right\|_{\mathrm{TV}}^2 = O\left(\frac{\log(|\mathcal{F}|/\delta)}{N}+\tau\right),$$

with probability at least $1-c_1\delta$.

□

A.2 PROOF OF THEOREM 4.2

With the supervised learning guarantees Theorem 4.1 in hand, which provides an upper bound for $\mathbb{E}_{s\sim\rho_{\pi_{\mathrm{E}}}}\left\|\widehat{\pi}^{\mathrm{RBC}}(\cdot|s)-\pi_{\mathrm{E}}(\cdot|s)\right\|_{\mathrm{TV}}^2$, we are now able to present the suboptimality guarantee of the reward for $\widehat{\pi}^{\mathrm{RBC}}$. This bound directly corresponds to the reward performance of a policy.

Theorem A.2 (Theorem 4.2)**.** *Under the same setting as Theorem 4.1, we have*

$$J_{\pi_E}-J_{\widehat{\pi}^{\mathrm{RBC}}}\leq O\left(\frac{1}{(1-\gamma)^2}\sqrt{\frac{\log(|\mathcal{F}|/\delta)}{N}+\tau}\right),$$

with probability at least $1-c_1\delta$.

Proof of Theorem 4.2. This part is similar to Agarwal et al. (2019), and we have

$$\begin{aligned} (1-\gamma)(J_{\pi_E}-J_{\widehat{\pi}^{\mathrm{RBC}}}) &= \mathbb{E}_{s\sim\rho_{\pi_{\mathrm{E}}}}\mathbb{E}_{a\sim\pi_{\mathrm{E}}(\cdot|s)}A^{\widehat{\pi}^{\mathrm{RBC}}}(\mathbf{s},\mathbf{a}) \\ &\leq \frac{1}{1-\gamma}\sqrt{\mathbb{E}_{s\sim\rho_{\pi_{\mathrm{E}}}}\|\widehat{\pi}^{\mathrm{RBC}}(\cdot|s)-\pi_{\mathrm{E}}(\cdot|s)\|_1^2} \\ &= \frac{2}{1-\gamma}\sqrt{\mathbb{E}_{s\sim\rho_{\pi_{\mathrm{E}}}}\|\widehat{\pi}^{\mathrm{RBC}}(\cdot|s)-\pi_{\mathrm{E}}(\cdot|s)\|_{\mathrm{TV}}^2}, \end{aligned}$$

where we use the fact that $\sup_{s,a,\pi}|A^{\pi}(\mathbf{s},\mathbf{a})| \leq \frac{1}{1-\gamma}$ for the advantage function and the reward is always bounded between 0 and 1.

Combining Theorem 4.1, we have

$$J_{\pi_E} - J_{\widehat{\pi}^{\text{RBC}}} \leq O\left(\frac{1}{(1-\gamma)^2}\sqrt{\frac{\log(|\mathcal{F}|/\delta)}{N} + \tau}\right),$$

with probability at least $1 - c_1\delta$. □