On the geometry of recurrent spiking networks

Symmetry and Geometry in Neural Representations

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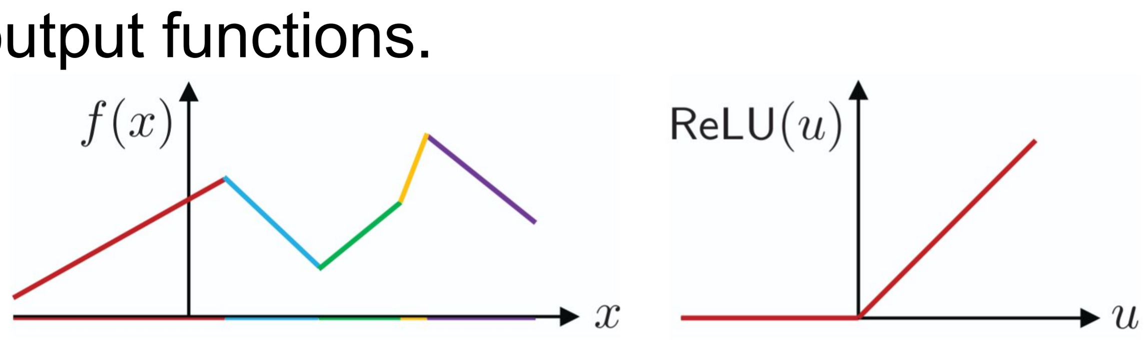
Background

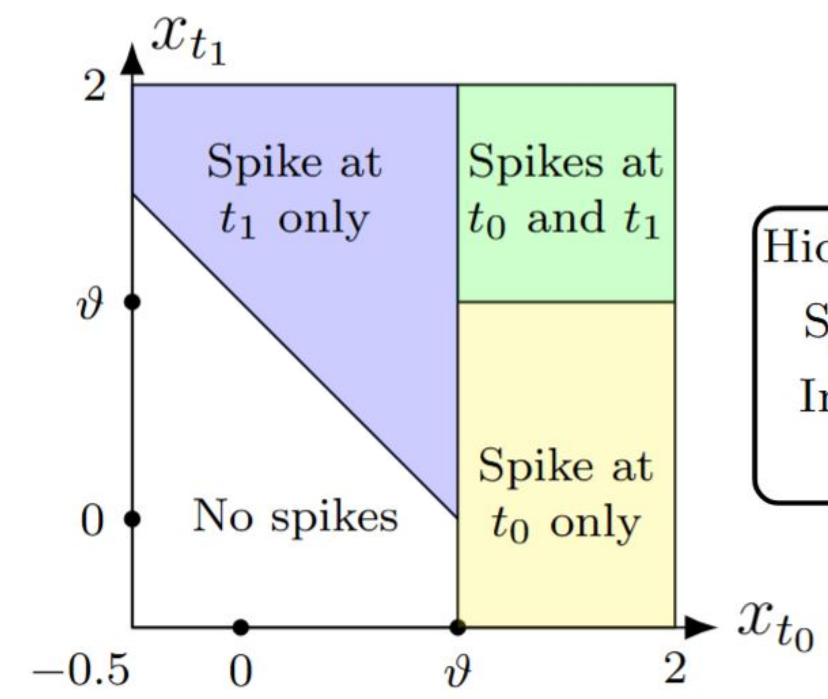
- ReLU networks are continuous piecewise-linear (CPWL) input-output functions.



- CPWL theory can explain robustness and normalization.
- Other works have analyzed spiking networks (SNNs) as piecewise functions either:
  - in *hidden* space (Machens lab), or
  - in *spike-timing* space, assuming each neuron spikes exactly once (Kutyniok lab), or
  - in *input* space (like us), but focusing on theoretical bounds of representational power instead of guiding empirical practice (Nguyen, Araya, Fono, and Kutyniok, 2025).
- SNNs are hard to train because it is unclear how best to apply traditional deep learning theory.

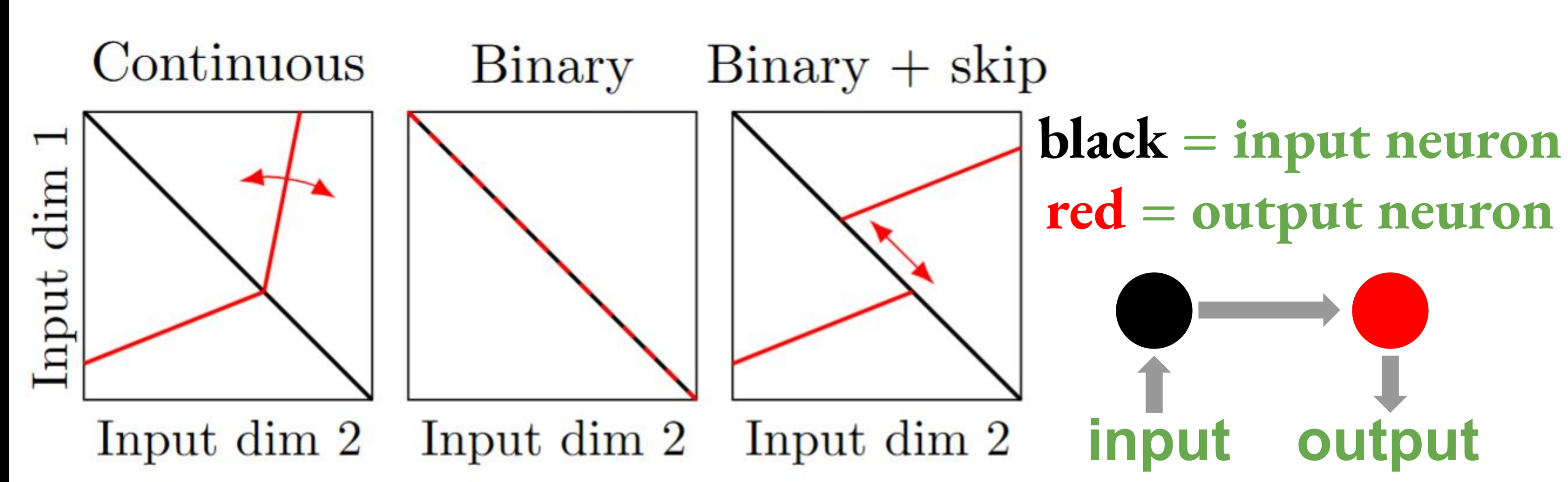
How do SNN design parameters affect unrolled input-output CPWL geometry?

Piecewise spiking neuron

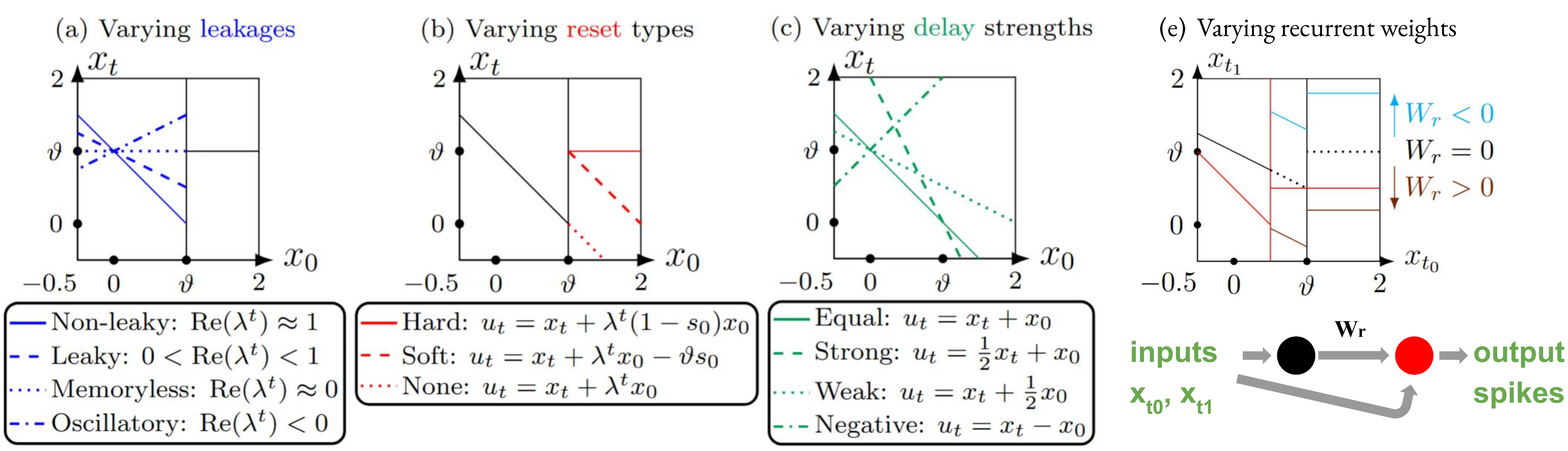


Hidden: $u_t = \lambda(1 - s_{t-1})u_{t-1} + Wx_t$
Spike: $s_t = \text{Heaviside}(\text{Re}(u_t) - \vartheta)$
Input: x — t_0, t_1, t

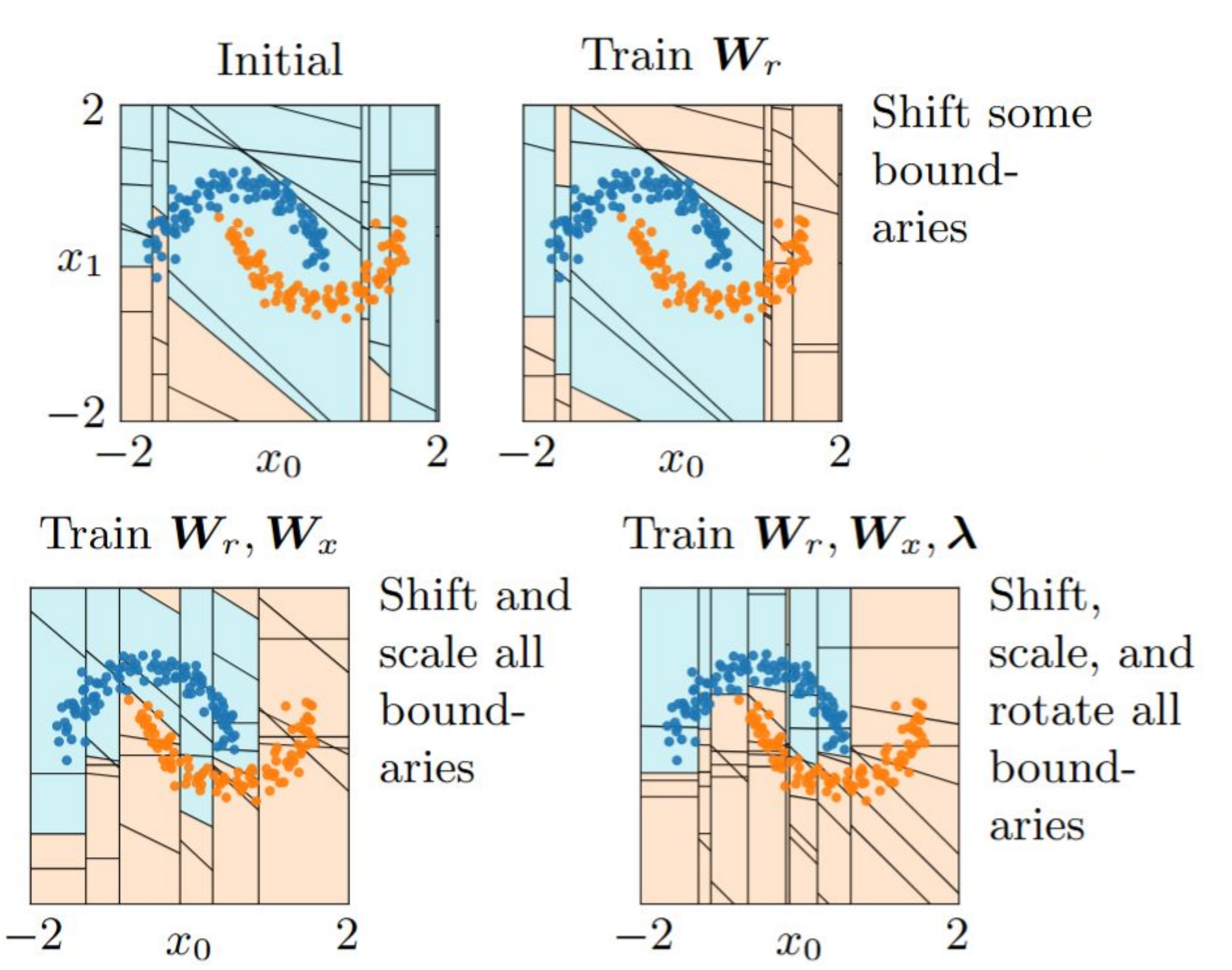
Skip connections are crucial for spiking networks



Many params define boundaries, recurrent weights shift them



Exact geometry visualization to probe plasticity



Summary + Future Work

We study SNNs as piecewise input-output functions, and devise a new algorithm to visualize them as such. We show that skip connections, delays, and leakages majorly influence their geometry, suggesting future plasticity methods should focus on them.

Future Work:

- Expand visualization algorithm to incorporate complex leakages (resonators) and skip connections
- **Quantify** input-output geometry complexity vs plasticity:
  - **Normalization** *should align geometry around inputs*
  - **Membrane regularization** *may also align geometry around inputs, like* normalization
  - **STDP** *may reduce geometric complexity around inputs, inducing* robustness / generalization
- **??? (your name here!)**