1 Derivation of Volume-form Sub-weight w_V for Anisotropic BRDFs

This section describes the derivation of the volume-form subweight w_V for anisotropic BRDFs. The volume-form subweight considers the transformation of Rusinkiewicz parameterization $(\theta_h, \phi_h, \theta_d, \phi_d)$ to incident and outgoing directions **i** and **o**. **i** and **o** are represented by $(\theta_h, \phi_h, \theta_d, \phi_d)$ as:

$$\mathbf{i} = \begin{pmatrix} i_x & i_y & i_z \end{pmatrix}^\top = \begin{pmatrix} (\sin\theta_h\cos\theta_d + \cos\theta_h\sin\theta_d\cos\phi_d)\cos\phi_h - \sin\theta_d\sin\phi_d\sin\phi_h \\ (\sin\theta_h\cos\theta_d + \cos\theta_h\sin\theta_d\cos\phi_d)\sin\phi_h + \sin\theta_d\sin\phi_d\cos\phi_h \\ \cos\theta_h\cos\theta_d - \sin\theta_h\sin\theta_d\cos\phi_d \end{pmatrix}, \tag{1}$$

$$\mathbf{o} = \begin{pmatrix} o_x & o_y & o_z \end{pmatrix}^\top = \begin{pmatrix} (\sin\theta_h\cos\theta_d - \cos\theta_h\sin\theta_d\cos\phi_d)\cos\phi_h + \sin\theta_d\sin\phi_d\sin\phi_h \\ (\sin\theta_h\cos\theta_d - \cos\theta_h\sin\theta_d\cos\phi_d)\sin\phi_h - \sin\theta_d\sin\phi_d\cos\phi_h \\ \cos\theta_h\cos\theta_d + \sin\theta_h\sin\theta_d\cos\phi_d \end{pmatrix}. \tag{2}$$

The Jacobian matrix of the transformation $T \in \mathbb{R}^{6\times 4}$ is represented as the following equation:

$$T = \begin{pmatrix} \frac{\partial i_x}{\partial\theta_h} & \frac{\partial i_x}{\partial\phi_h} & \frac{\partial i_x}{\partial\theta_d} & \frac{\partial i_x}{\partial\phi_d} \\ \frac{\partial i_y}{\partial\theta_h} & \frac{\partial i_y}{\partial\phi_h} & \frac{\partial i_y}{\partial\theta_d} & \frac{\partial i_y}{\partial\phi_d} \\ \frac{\partial i_z}{\partial\theta_h} & \frac{\partial i_z}{\partial\phi_h} & \frac{\partial i_z}{\partial\theta_d} & \frac{\partial i_z}{\partial\phi_d} \\ \frac{\partial o_x}{\partial\theta_h} & \frac{\partial o_x}{\partial\phi_h} & \frac{\partial o_x}{\partial\theta_d} & \frac{\partial o_x}{\partial\phi_d} \\ \frac{\partial o_y}{\partial\theta_h} & \frac{\partial o_y}{\partial\phi_h} & \frac{\partial o_y}{\partial\theta_d} & \frac{\partial o_y}{\partial\phi_d} \\ \frac{\partial o_z}{\partial\theta_h} & \frac{\partial o_z}{\partial\phi_h} & \frac{\partial o_z}{\partial\theta_d} & \frac{\partial o_z}{\partial\phi_d} \end{pmatrix}. \tag{3}$$

For example, the first row $\mathbf{t}_1$ of the Jacobian matrix T is expressed as:

$$\mathbf{t}_1 = \begin{pmatrix} \frac{\partial i_x}{\partial\theta_h} & \frac{\partial i_x}{\partial\phi_h} & \frac{\partial i_x}{\partial\theta_d} & \frac{\partial i_x}{\partial\phi_d} \end{pmatrix} = \begin{pmatrix} (\cos\theta_h\cos\theta_d - \sin\theta_h\sin\theta_d\cos\phi_d)\cos\phi_h \\ -(\sin\theta_h\cos\theta_d + \cos\theta_h\sin\theta_d\cos\phi_d)\sin\phi_h - \sin\theta_d\sin\phi_d\cos\phi_h \\ (-\sin\theta_h\sin\theta_d + \cos\theta_h\cos\theta_d\cos\phi_d)\cos\phi_h - \cos\theta_d\sin\phi_d\sin\phi_h \\ (-\cos\theta_h\sin\theta_d\sin\phi_d)\cos\phi_h - \sin\theta_d\cos\phi_d\sin\phi_h \end{pmatrix}^\top. \tag{4}$$

The second and third rows $\mathbf{t}_2$ and $\mathbf{t}_3$ are expressed as:

$$\mathbf{t}_2 = \begin{pmatrix} \frac{\partial i_y}{\partial\theta_h} & \frac{\partial i_y}{\partial\phi_h} & \frac{\partial i_y}{\partial\theta_d} & \frac{\partial i_y}{\partial\phi_d} \end{pmatrix} = \begin{pmatrix} (\cos\theta_h\cos\theta_d - \sin\theta_h\sin\theta_d\cos\phi_d)\sin\phi_h \\ (\sin\theta_h\cos\theta_d + \cos\theta_h\sin\theta_d\cos\phi_d)\cos\phi_h - \sin\theta_d\sin\phi_d\sin\phi_h \\ (-\sin\theta_h\sin\theta_d + \cos\theta_h\cos\theta_d\cos\phi_d)\sin\phi_h + \cos\theta_d\sin\phi_d\cos\phi_h \\ (-\cos\theta_h\sin\theta_d\sin\phi_d)\sin\phi_h + \sin\theta_d\cos\phi_d\cos\phi_h \end{pmatrix}^\top, \tag{5}$$

$$\mathbf{t}_3 = \begin{pmatrix} \frac{\partial i_z}{\partial\theta_h} & \frac{\partial i_z}{\partial\phi_h} & \frac{\partial i_z}{\partial\theta_d} & \frac{\partial i_z}{\partial\phi_d} \end{pmatrix} = \begin{pmatrix} -\sin\theta_h\cos\theta_d - \cos\theta_h\sin\theta_d\cos\phi_d \\ 0 \\ -\cos\theta_h\sin\theta_d - \sin\theta_h\cos\theta_d\cos\phi_d \\ \sin\theta_h\sin\theta_d\sin\phi_d \end{pmatrix}^\top. \tag{6}$$

The rest rows $\mathbf{t}_4$, $\mathbf{t}_5$, and $\mathbf{t}_6$ can be expressed similarly.

The volume-form subweight w_V is then calculated from the Jacobian matrix T as:

$$w_V = \sqrt{\det(T^\top T)}\, d\theta_h d\phi_h d\theta_d d\phi_d. \tag{7}$$

Let us define the 4×4 matrix $M = T^\top T$, and M is expressed as:

$$M = \begin{pmatrix} 2\cos^2\theta_d + 2\sin^2\theta_d\cos^2\phi_d & 2\sin\theta_h\sin^2\theta_d\sin\phi_d\cos\phi_d & 0 & 0 \\ 2\sin\theta_h\sin^2\theta_d\sin\phi_d\cos\phi_d & 2\sin^2\theta_h\cos^2\theta_d + 2\cos_h^\theta\sin^2\theta_d\cos^2\phi_d + 2\sin^2\theta_d\sin^2\phi_d & 0 & 2\cos\theta_h\sin^2\theta_d \\ 0 & 0 & 2 & 0 \\ 0 & 2\cos\theta_h\sin^2\theta_d & 0 & 2\sin^2\theta_d \end{pmatrix}. \tag{8}$$

Then the determinant of M, $\det(M)$, is calculated as:

$$\det(M) = 16\cos^2\theta_d\sin^2\theta_d\sin^2\theta_h. \tag{9}$$

Please note that $\det(M)$ hindges only on θ_h and θ_d. By substituting this into Eq (7), the volume-form subweight w_V is expressed as:

$$w_V = 4\cos\theta_d\sin\theta_d\sin\theta_h d\theta_h d\phi_h d\theta_d d\phi_d. \tag{10}$$

2 Implementation Details

Our method calculates the diffuse component ρ_d, the specular component ρ_s, NDF D, Fresnel term F, and GAF G by solving alternating weighted least square method. Specifically, we first calculate NDF D by considering other factors as constant. Then the Fresnel term F, GAF G, ρ_d and ρ_s are fitted in the same way. For GAF G, we first fit $G(\theta_i, \phi_i)$, then $G(\theta_o, \phi_o)$ is calculated. $G(\theta, \phi)$ is calculated by averaging $G(\theta_i, \phi_i)$ and $G(\theta_o, \phi_o)$. After averaging $G(\theta_i, \phi_i)$ and $G(\theta_o, \phi_o)$, smoothing operation is applied for stable optimization. NDF D, Fresnel term F, and GAF G are calculated by using Dynamic Range (DR) clamping that normalizes the maximum value of each factor to 1 and clamps the minimum value to $1e^{-6}$.

Table 1: The average relMSE, MAPE, and PSNR of all wavelengths in Figs. 1, 2, and 3.

	relMSE↓	MAPE↓	PSNR↑
brushed_aluminium_1	0.257639	23.5623	44.3065
copper_sheet	0.128014	26.9692	40.3821
green_pvc	0.061847	22.5857	43.4143
metallic_paper_copper	0.037910	21.0128	44.1411
metallic_paper_gold	0.044599	24.1465	43.5306
miro_7	0.033316	25.6066	43.5056
morpho_melenaus	0.207516	19.7027	30.2176
sari_silk_2color	0.021250	1308.11	39.4889
darth_vader_pants	0.52347	10.9871	42.2086
tarkin_tunic	1.29527	7.38509	28.5561
brushed_steel_satin_pink	0.106422	15.963	35.4093

Table 2: The numbers of truncated singular values for NDF D and GAF G

	NDF			GAF		
	99%	95%	90%	99%	95%	90%
brushed_aluminium_1	3	1	1	56	8	3
copper_sheet	5	2	1	54	8	4
green_pvc	8	2	2	18	6	4
metallic_paper_copper	5	2	1	30	8	4
metallic_paper_gold	4	2	1	27	7	3
miro_7	3	1	1	30	9	5
morpho_melenaus	3	2	2	22	7	5
sari_silk_2color	25	1	1	22	9	6
darth_vader_pants	26	4	2	22	6	4
tarkin_tunic	13	3	2	20	5	3
brushed_steel_satin_pink	3	1	1	19	3	2

3 Additional Results

Figs. 1, 2, and 3 show the multi-spectral rendering results from 423.48*nm* to 958.04*nm*. For each material, the top row shows the rendering results using the original BRDF (reference), the center row shows those using our non parametric factor representation (ours), and the bottom row visualizes the relative mean square error (relMSE). Fig. 4 shows the first three PCA components of NDF D and GAF G. Table. 1 lists the average relMSE, MAPE, and PSNR of all wavelengths in Figs. 1, 2, and 3. Table. 2 shows the number of truncated singular values of NDF D and GAF G with 99%, 95%, and 90% cummulative ratios.

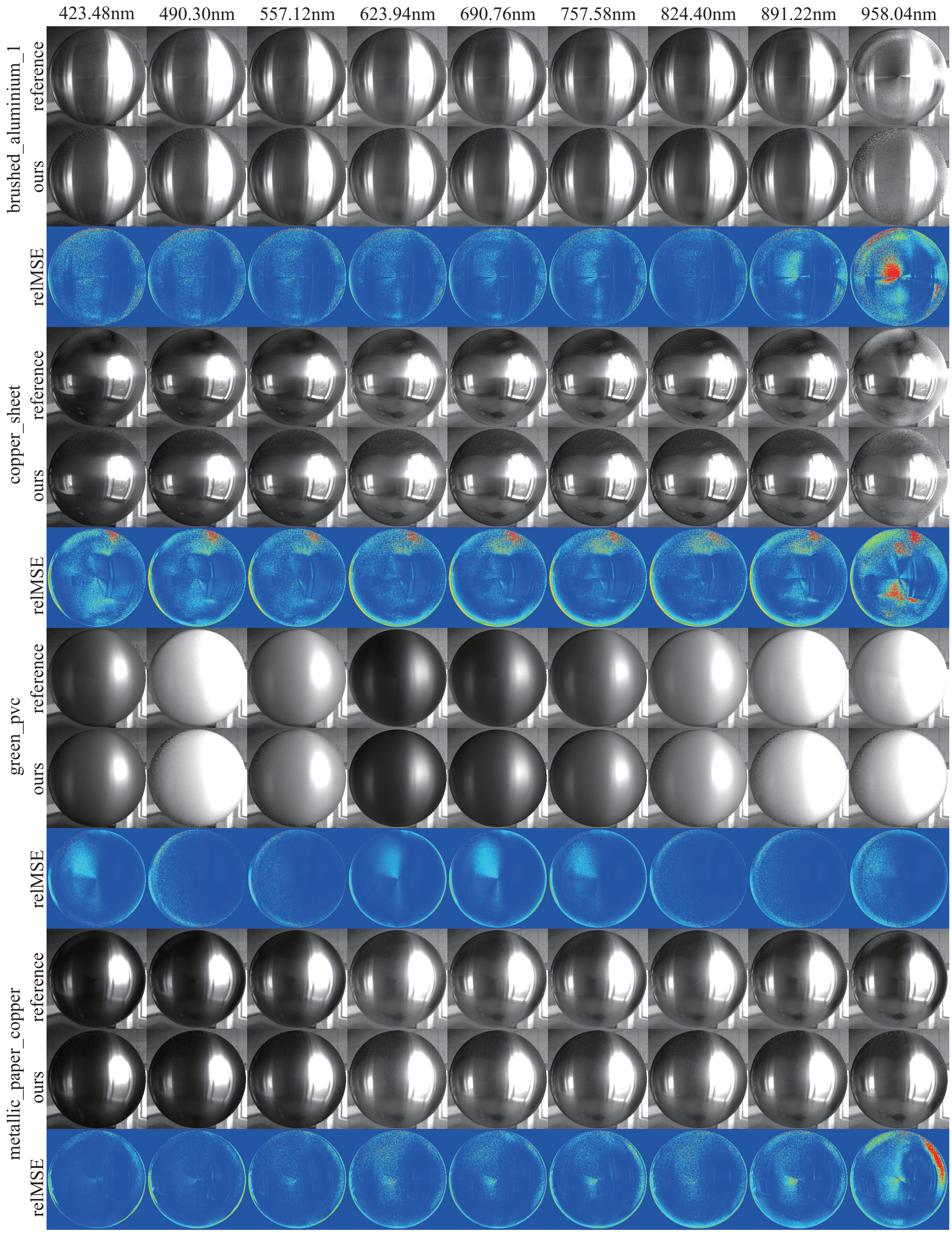

Figure 1: Multi-spectral rendering results.

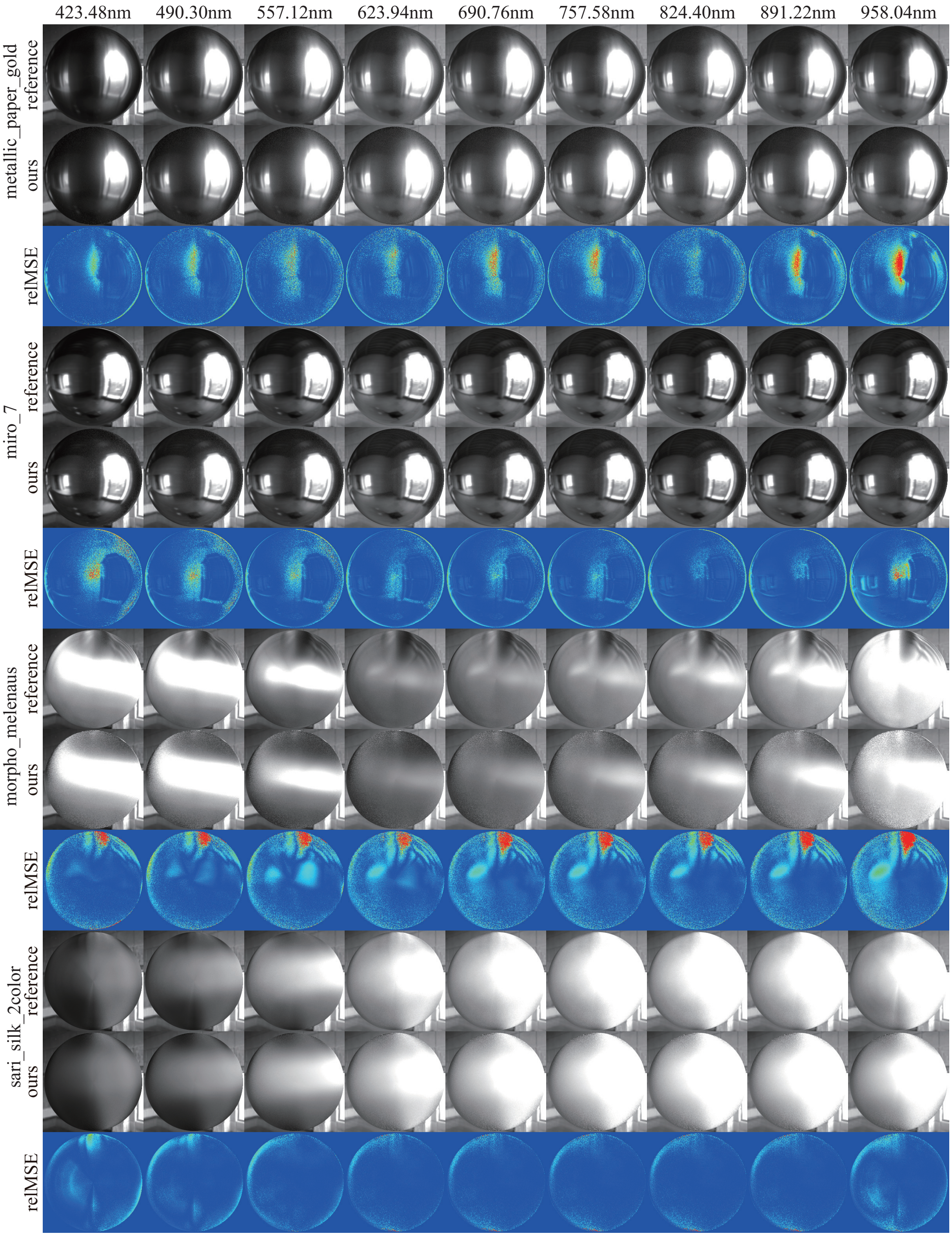

Figure 2: Multi-spectral rendering results.

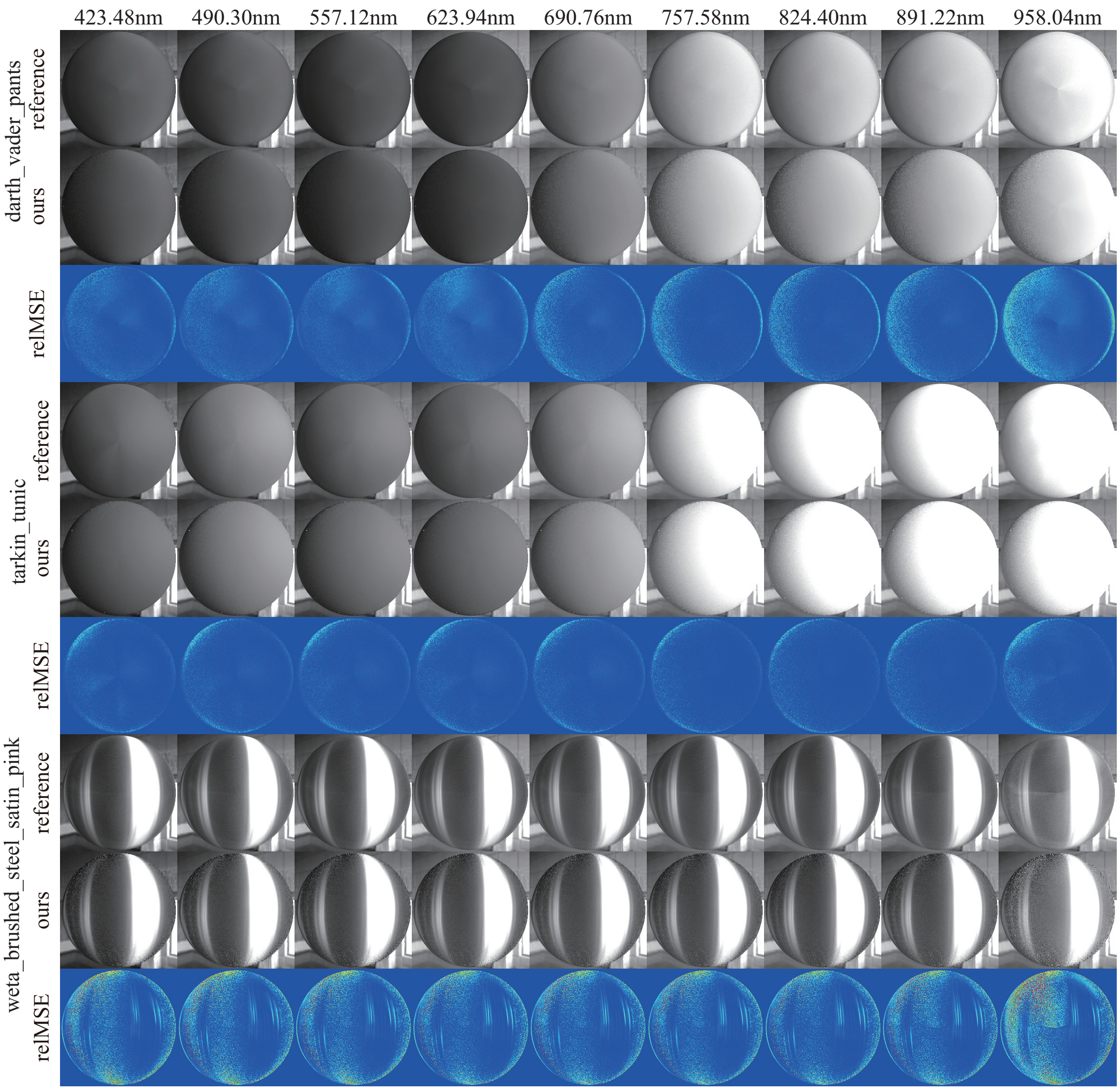

Figure 3: Multi-spectral rendering results.

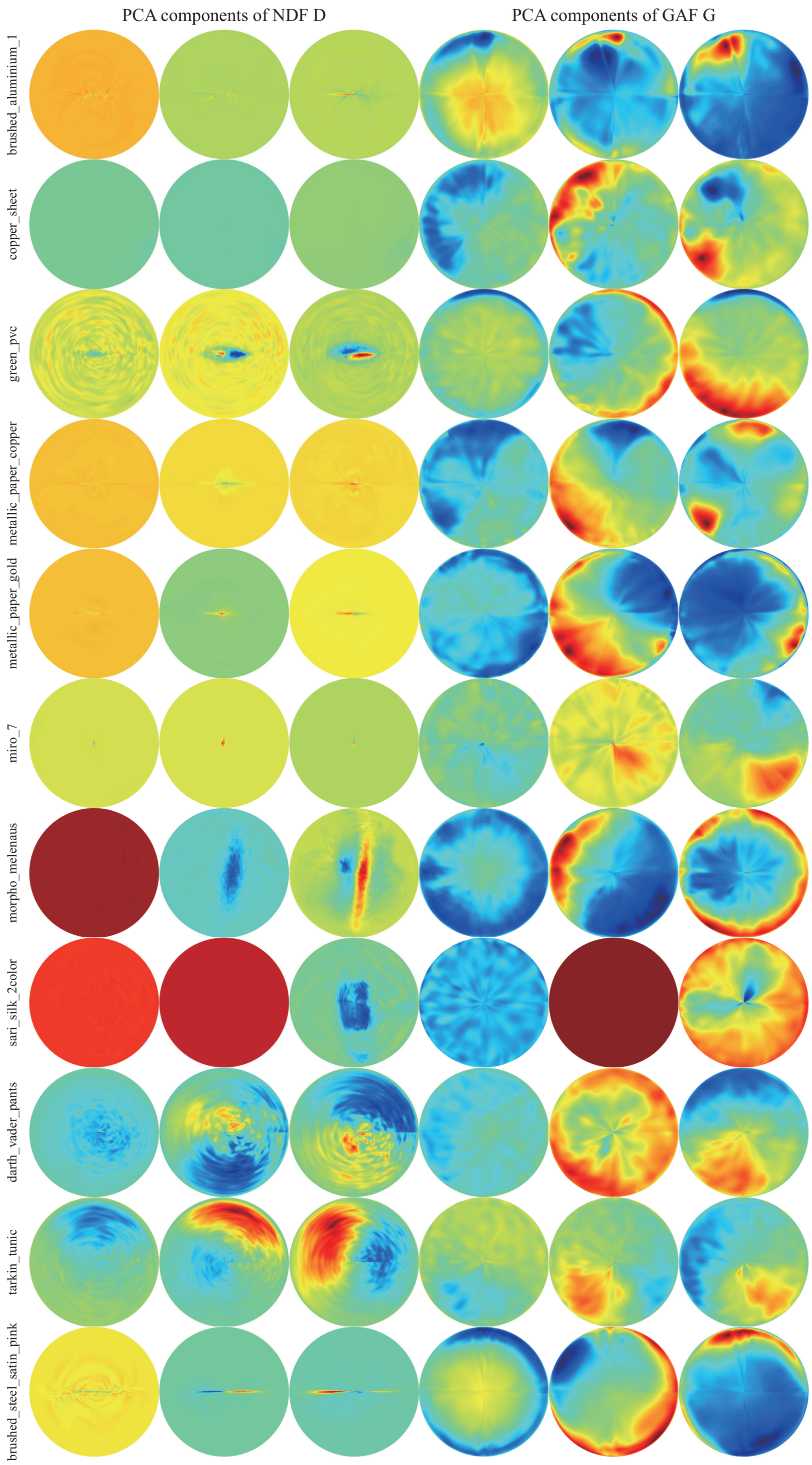

Figure 4: The first three PCA components of NDF D and GAF G.