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759 A APPENDIX

761 A.1 TRAINING LOSS OF LLMS AND VLMs

763 To adapt a pretrained LLM or VLM to a specific domain or task, models are typically trained on a
 764 supervised dataset $\mathcal{D} = (\mathbf{x}_i, \mathbf{y}_i)_{i=1}^n$ of input-output pairs. Training is commonly performed using
 765 the standard teacher-forcing objective, which minimizes the negative log-likelihood of the target
 766 sequence:

$$768 \quad \mathcal{L}_{\text{SFT}}(\theta) = -\frac{1}{n} \sum_{i=1}^n \ln \pi_\theta(\mathbf{y}_i | \mathbf{x}_i) - \frac{1}{n} \sum_{i=1}^n \sum_{k=1}^{|\mathbf{y}_i|} \ln \pi_\theta(y_{i,k} | \mathbf{x}_i, \mathbf{y}_{i,<k}).$$

771 This objective maximizes the likelihood that the model generates the correct output sequence con-
 772 ditioned on the input and the ground-truth prefix at each step. The parameters are updated using
 773 gradient descent or its variants:

$$775 \quad \theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}_{\text{SFT}}(\theta), \quad \text{with } \theta_{t=0} = \theta_0,$$

777 where $\eta > 0$ is the learning rate. Teacher forcing stabilizes fine-tuning by supplying the true prefix
 778 $\mathbf{y}_{<k}$ during training, enabling the model to align its predictions closely with the target data distribu-
 779 tion in the new domain.

781 A.2 PROOF OF THEOREM I

783 In this section, we give the detailed proof of our Theorem I, we start by proving the following
 784 theorem:

785 **Theorem 2.** *For a data \mathbf{x}_v and its generation \mathbf{y}_v that await valuation, at any time $t \geq 0$ of training
 786 using a training data $(\mathbf{x}_i, \mathbf{y}_i), i \in [n]$, the training data exhibits larger value to the valuation data
 787 as the following increases:*

$$789 \quad \sum_{k=1}^{|\mathbf{y}_v|} \sum_{k'=1}^{|\mathbf{y}_i|} \alpha_{k,k'}(t) \cdot \left\langle \mathbf{h}_{\mathbf{x}_v, \mathbf{y}_{v,<k}}(t), \mathbf{h}_{\mathbf{x}_i, \mathbf{y}_{i,<k'}}(t) \right\rangle +$$

$$793 \quad \sum_{k=1}^{|\mathbf{y}_v|} \left\langle \mathbf{w}_{\mathbf{y}_{v,k}}(t) - \sum_{z \in \mathcal{V}} \pi_{\theta(t)}(z | \mathbf{x}_v) \cdot \mathbf{w}_z(t), (\mathbf{w}_{\mathbf{y}_{i,k}} - \sum_{z \in \mathcal{V}} \pi_{\theta(t)}(z | \mathbf{x}_v) \cdot \mathbf{w}_z(t)) \right\rangle \quad (3)$$

797 *Proof.*

$$799 \quad \frac{d}{dt} \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v) = \left\langle \nabla \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v), \frac{d}{dt} \theta(t) \right\rangle$$

$$801 \quad = \left\langle \nabla \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v), -\eta \nabla \mathcal{L}_D(\theta) \right\rangle$$

$$803 \quad = \left\langle \nabla \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v), \eta \sum_{i=1}^n \nabla \ln \pi_{\theta(t)}(y_i | \mathbf{x}_i) \right\rangle$$

807 As per the unconstrained features Assumption, the model's trainable parameters are

$$809 \quad \theta = \left(\mathbf{W}, \mathbf{h}_{\mathbf{x}_v}, \{ \mathbf{h}_{\mathbf{x}_v, \mathbf{y}_{v,<k}} \}_{k \in \{2, \dots, |\mathbf{y}_v|\}}, \{ \mathbf{h}_{\mathbf{x}_i, \mathbf{y}_{i,<k'}} \}_{i \in [n], k' \in \{1, \dots, |\mathbf{y}_i|\}} \right).$$

810 Unfolding the gradients with respect to these parameters yields:
 811

$$\begin{aligned}
 812 \frac{d}{dt} \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v) &= \left\langle \nabla_{\mathbf{W}} \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v), \sum_i^n \nabla_{\mathbf{W}} \ln \pi_{\theta(t)}(\mathbf{y}_i | \mathbf{x}_i) \right\rangle \\
 813 &+ \sum_{k=1}^{|\mathbf{y}_v|} \underbrace{\left\langle \nabla_{\mathbf{h}_{\mathbf{x}_v, \mathbf{y}_{v, < k}}} \ln \pi_{\theta(t)}(\mathbf{y}_{v, k} | \mathbf{x}_v, \mathbf{y}_{v, < k}), \sum_{i'=1}^{n_k} \nabla_{\mathbf{h}_{\mathbf{x}_v, \mathbf{y}_{v, < k}}} \ln \pi_{\theta(t)}(\mathbf{y}_{i', k} | \mathbf{y}_{v, < k}) \right\rangle}_{\text{(II) Training data have the same } (\mathbf{x}_v, \mathbf{y}_{v, < k})} \\
 814 & \\
 815 & \\
 816 & \\
 817 & \\
 818 & \\
 819 & \\
 820 \end{aligned} \tag{4}$$

820 where n_k is the number of training data whose input and prediction before token k are the same as
 821 valuation data $(\mathbf{x}_v, \mathbf{y}_{v, < k})$. Since we have

$$\begin{aligned}
 822 \nabla_{\mathbf{W}} \ln \pi_{\theta(t)}(z | \mathbf{x}) &= \left(\mathbf{e}_z - \sum_{z' \in \mathcal{V}} \pi_{\theta(t)}(z' | \mathbf{x}) \cdot \mathbf{e}_{z'} \right) \mathbf{h}_{\mathbf{x}}^{\top}(t), \\
 823 \nabla_{\mathbf{h}_{\mathbf{x}}} \ln \pi_{\theta(t)}(z | \mathbf{x}) &= \mathbf{W}_z(t) - \sum_{z' \in \mathcal{V}} \pi_{\theta(t)}(z' | \mathbf{x}) \cdot \mathbf{W}_{z'}(t).
 \end{aligned}$$

824 Putting this back in (4) together with a few algebra steps, yields

$$\frac{d}{dt} \ln \pi_{\theta(t)}(\mathbf{y}_v | \mathbf{x}_v) = \text{(I)} + \text{(II)} \tag{5}$$

825 where:

$$\text{(I)} = \sum_{k=1}^{|\mathbf{y}_v|} \sum_{i=1}^n \sum_{k'=1}^{|\mathbf{y}_i|} \alpha_{k, k'}(t) \cdot \left\langle \mathbf{h}_{\mathbf{x}_v, \mathbf{y}_{v, < k}}(t), \mathbf{h}_{\mathbf{x}_i, \mathbf{y}_{i, < k'}}(t) \right\rangle \tag{6}$$

$$\text{(II)} = \sum_{k=1}^{|\mathbf{y}_v|} \left\langle \mathbf{w}_{\mathbf{y}_{v, k}}(t) - \sum_{z \in \mathcal{V}} \pi_{\theta(t)}(z | \mathbf{x}_v) \cdot \mathbf{w}_z(t), \sum_{i'=1}^{n_k} (\mathbf{w}_{\mathbf{y}_{i', k}} - \sum_{z \in \mathcal{V}} \pi_{\theta(t)}(z | \mathbf{x}_v) \cdot \mathbf{w}_z(t)) \right\rangle \tag{7}$$

826 where $\alpha_{k, k'}(t) = \left\langle \mathbf{e}_{\mathbf{y}_{v, k}} - \pi_{\theta(t)}(\cdot | \mathbf{x}, \mathbf{y}_{v, < k}), \mathbf{e}_{\mathbf{y}_{i, k'}} - \pi_{\theta(t)}(\cdot | \mathbf{x}, \mathbf{y}_{i, < k'}) \right\rangle$. By taking the i -th sample,
 827 we can obtain Theorem 2. \square

828 We observe the following:

829 (1) When the training input \mathbf{x}_i differs from the valuation input \mathbf{x}_v , its influence on the valuation
 830 target arises solely through Term (I), which captures the contribution of the token embeddings and
 831 all network parameters except the token unembedding layer.

832 (2) The effect of the token unembeddings is concentrated in cases where the training and valuation
 833 data share the same input \mathbf{x} and exhibit overlapping output predictions \mathbf{y} .
 834 To eliminate this dependence on token unembeddings, we impose the following assumption:

835 **Assumption 2 (Distinct Input).** *The training dataset satisfies that no training input \mathbf{x}_i is identical
 836 to the valuation input \mathbf{x}_v .*

837 Under the Assumption 2, the contribution from token unembeddings (Term (II)) vanishes, so that
 838 the influence of the training data on the valuation data arises entirely through the shared representation
 839 features captured in Term (I). This assumption is mild, as training inputs typically differ from
 840 valuation inputs in practice — especially in vision-language datasets, where the input images are
 841 almost always distinct. Extending this result to cases where training examples share the same input
 842 but differ in their outputs \mathbf{y} is straightforward: the output prefix $\mathbf{y}_{< k}$ can be incorporated into the
 843 input \mathbf{x} , treating each unique pair $(\mathbf{x}, \mathbf{y}_{< k})$ as a distinct input, where $k - 1$ indicates the point at
 844 which the outputs begin to differ. Combining Theorem 2 and Assumption 2 then yields Theorem 1.

845 A.3 ADDITIONAL DETAILS OF INFLUENTIAL AND MISLABLED DATA DETECTION

846 **Training setting for baselines.** While For-Value requires only a single forward pass, the influence
 847 function-based baselines Hessian-free and DataInf require fine-tuning the models to

864 convergence. For text generation tasks, we follow the training setup in [Kwon et al. \(2024\)](#), except
 865 to llama-2-13B, we use float16 weights instead of 8-bit quantization. For image-to-text generation
 866 tasks, we apply LoRA to every query and value matrix within the model’s attention layers. To fine-
 867 tune VLMs, we use a learning rate of 2×10^{-4} , LoRA hyperparameters $r = 8$ and $\alpha = 32$, float16
 868 model weights, a batch size of 32, and train for 20 epochs.

869 **Efficiency details.** For larger 32B and 72B models in Fig. 4, we employ 4 A100 GPUs for inference
 870 and a single A100 for value computation. Baseline methods requiring training are fine-tuned on
 871 up to 8 GPUs, with the 32B model quantized to 8-bit to enable valuation on a single A100. Due
 872 to their long runtime, we restrict baselines to the sentence transformation task and, for 14B/32B
 873 models, sample 10% of valuation data—scaling time by a factor of 10 to estimate totals. Despite
 874 these adjustments, **For-Value** achieves substantially lower runtime without quantization and with
 875 fewer GPUs.

877 A.4 ADDITIONAL RESULTS

879 **Complexity Analysis.** Tab. 6 compares the training, computational, and memory costs of different
 880 methods. Traditional approaches such as IF, Hessian-free, HyperINF, and DataInf rely on
 881 gradient traces or Hessian computations, resulting in high costs that scale poorly with model size.
 882 In contrast, Emb and **For-Value** are training-free and algorithm-agnostic, which significantly
 883 reduces overhead. Although HyperINF is the strongest baseline in terms of accuracy, its cubic
 884 complexity makes it impractical for large LLMs—requiring about 6 hours for a Qwen-32B model
 885 (Fig. 4b). Although Emb achieves the best runtime efficiency, its performance lags behind other
 886 methods, as demonstrated in Tab. 1 and Tab. 2. Our method, **For-Value**, maintains strong perfor-
 887 mance while remaining highly efficient. Since $|\hat{\mathcal{V}}|$ is typically small (often under 2k), **For-Value**
 888 achieves much lower computational and memory costs than baselines.

Method	Training Free	Algorithm Agnostic	Training Complexity	Computational Complexity	Memory Complexity
Original IF	✗	-	$O(nEd_{in}dL)$	$O(nd_{in}^2d^2L + d_{in}^3d^3L)$ $\tilde{O}(nd_{in}dL)$	$O(D^2L + nDL)$
Hessian-free	✗	✗	$O(nEd_{in}dL)$	$O(nd_{in}dL)$	$O(nd_{in}dL)$
DataInf	✗	✗	$O(nEd_{in}dL)$	$O(nd_{in}dL)$	$O(nd_{in}dL)$
HyperINF	✗	✗	$O(nEd_{in}dL)$	$O(nd^3L)$	$O(nd^2L)$
Emb	✓	✓	0	$O(nd)$	$O(nd)$
For-Value (ours)	✓	✓	0	$O(nd \hat{\mathcal{V}})$	$O(nd \hat{\mathcal{V}})$

894 Table 6: Comparison on complexity of the Influence Function (IF), Hessian-free, DataInf,
 895 Emb, and **For-Value**. Complexities are given assuming a multilayer perceptron (MLP) with L
 896 layers, each containing $d_{in} \times d$ neurons where d_{in} is input dimension and d is the output embedding
 897 dimension, trained for E epochs on n training samples. The parameter count is identical across
 898 layers ($D \in \mathbb{N}$), and the in-batch vocabulary size is $|\hat{\mathcal{V}}|$. Overall, **For-Value** achieves higher
 899 computational and memory efficiency than baseline methods.

900 **Discussion on Parallel Computing:** While previous studies focus on using a single GPU for fair
 901 comparison, we would like to highlight that **For-Value** can further improve efficiency through
 902 parallel computing with a large batch size, as it only requires forward calculations. In contrast, base-
 903 line methods require computing the gradient for each individual data sample, which restricts them
 904 to a batch size of one and makes scaling up challenging.

905 **Qualitative Demonstration.** Beyond quantitative results, we present qualitative examples identified
 906 by **For-Value**. Fig. 5 shows a target valuation sample alongside its most and least influential training
 907 samples as ranked by **For-Value**. Specifically, **For-Value** successfully identifies highly
 908 relevant training points — for example, selecting samples from the same reverse order of words task
 909 for sentence transformation, or matching the same subject or artistic style in image-to-text tasks.
 910 In contrast, the least influential samples are clearly less relevant and often differ entirely in task or
 911 content from the target valuation data.

914 A.5 ADDITIONAL DETAILS OF SELECT DATA FOR FINETUNING

915 **Mathematics: GSM8K** As the baseline methods require LoRA, we begin with a one-epoch warmup
 916 training on Llama3-8B [Meta \(2024\)](#) using the whole training set to avoid utilizing gradients from
 917 randomly initialized LoRA modules (with a rank of $r = 32$). Next, we calculate influence scores

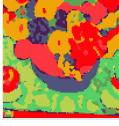
918	Valuation Sample	Most Influential Sample	Least Influential Sample
919	Task: Math Problem with reasoning (Qwen-2.5-1.5B) Solve the following math problem. Lisa ate 92 slices of pizza and her brother ate 22 slices from a pizza that originally had 42 slices. How many slices of the pizza are left? > Reason: Combined slices eaten = 92 + 22. Left = 42 - (92 + 22). Answer: -72</s>	Solve the following math problem. Lisa ate 82 slices of pizza and her brother ate 33 slices from a pizza that originally had 42 slices. How many slices of the pizza are left? > Reason: Combined slices eaten = 82 + 33. Left = 42 - (82 + 33). Answer: -73</s>	Solve the following math problem. Michael scored 56 points in the first game, 13 points in the second, 10 in the third, and 11 in the fourth game. What is his total points? > Reason: Total points = 56 + 13 + 10 + 11. Answer: 90</s>
920			
921			
922			
923	Task: Sentence Transformation (Qwen-2.5-1.5B) Cempoi is a chatbot that performs a specific transformation on : Reverse Order of Words For example: Feathers float on dreams. > dreams. on float Feathers</s>	Cempoi is a chatbot that performs a specific transformation on sentences: Reverse Order of Words For example:n Children chase fleeting dreams. > dreams. fleeting chase Children</s>	Ojzlw is a chatbot that performs a specific transformation on sentences: Remove All Vowels For example:n Moonlight serenades the night. > Mnlight srnds th nght.</s>
924			
925			
926	Task: Subject Generation (Llama-3.2-11B-Vision)  Q: Describe this image. A: It is a backpack.	 Q: Describe this image. A: It is a backpack.	 Q: Describe this image. A: It is a vase.
927			
928			
929			
930			
931	Task: Style Generation (Llama-3.2-11B-Vision)  Q: Describe this image. A: This is an image in a specific pixelart style. a gauguinesque, impressionist painting of flowers and fruit on a table cloth on a cloth, by alexej von jawlensky, trending on flickr, fauvism, fauvism, picasso, painterly.	 Q: Describe this image. A: This is an image in a specific pixelart style. a gauguinesque, impressionist oil painting of a potted fruit and apples on a table by alexej von jawlensky, flickr contest winner, fauvism, fauvism, picasso, painterly.	 Q: Describe this image. A: This is an image in a specific black and white line sketch style. Man on horse in desert.
932			
933			
934			
935			

Figure 5: Qualitative examples of data influence identified by For-Value. For each target valuation sample (left column), the most influential (middle column) and least influential (right column) training samples are shown. For-Value correctly retrieves training samples that share relevant task characteristics (e.g., same reasoning type, sentence transformation rule, subject, or style) and filters out unrelated or mismatched examples.

for both the baselines and For-Value. To ensure consistency and performance, we also perform a one-epoch warm-up but with full-parameter finetuning on the entire dataset. Finally, we select the top 5% of data based on these influence scores to further finetune the model with learning rate $1e-5$ and batch size 64 on 4 H100 GPU for 4 epochs.

Medicine: Noise-Huatuo-Complex-CoT As the baseline methods utilize LoRA, we begin with a one-epoch training on Llama3-8B-Instruction [Meta (2024)] using the whole training set to avoid using gradients from randomly initialized LoRA modules (with a rank of $r = 16$). Next, we calculate influence scores for both the baselines and our approach. Considering the training data is noisy, we select the top 5% high value training data based on these scores and finetune the original pretrained model using full-parameter finetuning for 5 epochs, with a learning rate of 1×10^{-6} , a batch size of 16 and gradient accumulation 8 on 8 H100 GPUs. We follow [Wu et al. (2025)] using greedy decoding to evaluate the model on 5 held out datasets MedQA [Jin et al. (2021)], MedMCQA [Pal et al. (2022)], PubMedQA [Jin et al. (2019)], MMLU-Pro-Med [Wang et al. (2024b)], GPQA-Med [Rein et al. (2024)].

Medicine: Noise-Huatuo-Complex-CoT Similarly, we start with a one-epoch warm-up on the entire training set to prevent using gradients from randomly initialized LoRA modules (with a rank of $r = 16$). Then, we compute influence scores for the baseline methods. For our method, since the pretrained model already demonstrates sufficient medical knowledge (as shown by adequate test accuracy in Table 2), we directly use the original pretrained model to assess data value. Finally, we finetune the pretrained Qwen2.5-3B-VL model [Bai et al. (2025a)] with full-parameter finetuning for 3 epochs, using a learning rate of 1×10^{-5} , a batch size of 16, and gradient accumulation of 8 on 8 H100 GPUs. We evaluate the model with greedy decoding on 6 held out datasets: PMC [Zhang et al. (2023)], MMMU [Yue et al. (2024)], MedX-M [Zuo et al. (2025)], PathVQA [He et al. (2020)], SLAKE [Liu et al. (2021)], VQA-Rad [Lau et al. (2018)].

A.6 ADDITIONAL ANALYSIS ON SELECT DATA FOR FINETUNING

Medicine: Noise-Huatuo-Complex-CoT. As indicated in Tab. 4, baseline methods struggle to effectively select high-quality data from noisy training datasets. This is primarily because these methods rely on assumptions of uniqueness or convergence to an optimal solution [Bae et al. (2024)], which are difficult to satisfy in the presence of noisy data. To illustrate this, we evaluated the proportion of high-quality data within the top 10% of high-value data, as shown in Tab. 7. The results reveal

Llama-3.1-8B	Detection Accuracy
Hessian-free	48.2
HyperINF	15.1
DataInf	33.2
For-Value	84.4

Table 7: High quality data detection accuracy

that baseline methods generally lack the capability to accurately identify noisy data, whereas our proposed method (For-Value) achieves significantly higher accuracy in detecting clean data.

Table 8: Description of the sentence transformation task templates. We consider 10 different types of sentence transformations. For each sentence transformation, unique identifying “chatbot” names were additionally prepended to the task prompt to assist the model in training.

Sentence transformations	Example transformation of “Sunrises herald hopeful tomorrows”:
Reverse Order of Words	tomorrows. hopeful herald Sunrises
Capitalize Every Other Letter	sUnRiSeS hErAlD hOpEfUi tOmOrRowS.
Insert Number 1 Between Every Word	Sunrises 1herald 1hopeful 1tomorrows.
Replace Vowels with *	S*nr*s*s h*r*ld h*p*f*l t*m*m*rr*ws.
Double Every Consonant	SSunrriisseess hheraldd hhopefull ttomorrows.
Capitalize Every Word	Sunrises Herald Hopeful Tomorrows.
Remove All Vowels	Snrss hrld hpfl tmrrws.
Add ‘ly’ To End of Each Word	Sunrisesly heraldly hopefully tomorrowslly
Remove All Consonants	uie ea oeu ooo.
Repeat Each Word Twice	Sunrises Sunrises herald herald hopeful hopeful tomorrows. tomorrows.

A.7 DETAILED TASK DESCRIPTION

A.7.1 LLM INFLUENCE EVALUATION TASKS

Following (Kwon et al., 2024), we evaluate the performance of For-Value on three text generation tasks for large language models (LLMs) to identify influential data points:

- **Sentence Transformations:** This task requires transforming input sentences into alternative forms while preserving meaning (e.g., active to passive voice). The dataset comprises 10 distinct classes (e.g., declarative to interrogative), each with 100 examples, split into 90 training and 10 test examples per class. Data examples see Tab. 8.
- **Math Word Problems (Without Reasoning):** These problems involve direct numerical computation from textual descriptions (e.g., basic arithmetic). The dataset has 10 classes based on operation types, with 100 examples per class (90 training, 10 test). Data examples see Tab. 9.
- **Math Word Problems (With Reasoning):** These require multi-step reasoning (e.g., solving word problems involving algebra or logic). Similar to the previous task, the dataset includes 10 classes with 100 examples each (90 training, 10). Data examples see Tab. 9.

A.7.2 VLM INFLUENCE EVALUATION TASKS

For VLMs, we adapt text-to-image generation tasks from (Kwon et al., 2024) into image-to-text (captioning) tasks to evaluate influence:

- **Style Generation:** This task involves generating captions for images in specific styles: cartoons (Norod78 (2023)), pixel art (Jainr3 (2023)), and line sketches (Zoheb (2023)). Each style dataset contains 200 training and 50 test image-text pairs, totaling 600 training and 150 test samples across three styles. Data examples see Fig. 5.
- **Subject Generation:** Using the DreamBooth dataset (Ruiz et al., 2023), this task generates captions for images of 30 distinct subjects (e.g., specific objects or animals). Each subject provides 3 training samples, with the remaining samples used for valuation. Data examples see Fig. 5.

1026 Table 9: Description of the math problem task templates. We consider 10 different types of math
 1027 word problems.

1028

1029 Math Word Problems	1030 Template prompt question
1031 Remaining pizza slices	1032 Lisa ate A slices of pizza and her brother ate B slices from a 1033 pizza that originally had C slices. How many slices of the pizza are left? Reason: Combined slices eaten = A + B. Left = C - (A + B).
1034 Chaperones needed for trip	1035 For every A students going on a field trip, there are B adults 1036 needed as chaperones. If C students are attending, how many adults are needed? Reason: Adults needed = (B * C) // A.
1037 Total number after purchase	1038 In an aquarium, there are A sharks and B dolphins. If they bought 1039 C more sharks, how many sharks would be there in total? Reason: 1040 Total sharks = A + C.
1041 Total game points	1042 Michael scored A points in the first game, B points in the second, 1043 C in the third, and D in the fourth game. What is his total points? Reason: Total points = A + B + C + D.
1044 Total reading hours	1045 Emily reads for A hours each day. How many hours does she 1046 read in total in B days? Reason: Total hours read = A * B.
1047 Shirt cost after discount	1048 A shirt costs A. There's a B-dollar off sale. How much does the 1049 shirt cost after the discount? Reason: Cost after discount = A - B.
1050 Area of a garden	1051 A rectangular garden has a length of A meters and a width of B 1052 meters. What is its area? Reason: Area = A * B.
1053 Total savings	1054 If Jake saves A each week, how much will he save after B weeks? Reason: Total savings = A * B.
1055 Number of cupcake boxes	1056 A bakery sells cupcakes in boxes of A. If they have B cupcakes, 1057 how many boxes can they fill? Reason: Boxes filled = B // A.
1058 Interest earned	1059 John invests A at an annual interest rate of B%. How much in- 1060 terest will he earn after C years? Reason: Interest = (A * B * C) // 100.

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1058 A.7.3 INFLUENTIAL DATA DETECTION METRICS

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We adopt two metrics from (Kwon et al., 2024) to assess influence:

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- **AUC Score:** For each test data point, we assign pseudo labels to training points (1 if the training point's label matches the test point's, 0 otherwise). We compute the Area Under the Curve (AUC) between data values (influence scores) and pseudo labels, averaging across all test points. A higher AUC indicates better identification of influential points.
- **Recall:** For each test point, we calculate the percentage of influential training points (top-ranked by influence score) that share the same class as the test point. This measures the relevance of identified influential points.

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1074 A.7.4 MISLABLED DATA DETECTION DATA & METRICS

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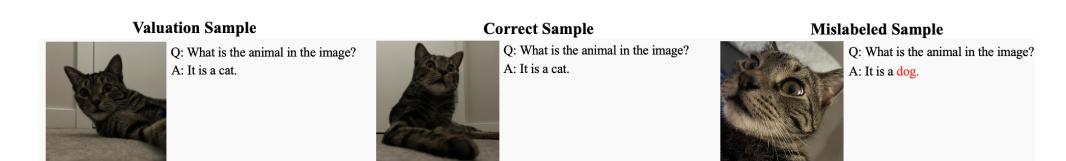


Figure 6: Description of the mislabeled data detection task. We utilize a cat versus dog classification dataset and intentionally introduce noise by randomly swapping the labels of 50% of the data.

1080 For mislabeled detection, we transform the dataset into a visual-language question answering task
 1081 with the template "What is the animal in the image? It is a [label]" with demonstration³ in Fig. 6.
 1082 We then select the first 400 images for both dogs and cats, flipping 50% of the labels to introduce
 1083 noise. For valuation, we use 200 images, with each class containing 100 images. For evaluation, we
 1084 also calculate the AUC and Recall but with the pseudo labels to training points being 1 if the training
 1085 point's label matches the test point's and it is clean data, 0 otherwise.
 1086

1087 A.8 NOISE-HUATUO-COT DATA EXAMPLE

1088 We construct the Noise-Huatuo-Complex-CoT dataset by randomly sampling 5,000 examples with-
 1089 out replacement and injecting noise into 40% of them through random insertion or deletion of irrel-
 1090 evant words, as illustrated in Fig. 7.

Random Drop	Random Insert
<p>Question: A nucleic acid was analyzed and found to contain 32% adenine, 18% guanine, 17% cytosine and 33% thymine. The nucleic acid must be:</p> <p>A. Single-stranded RNA B. Single-stranded DNA C. Double-stranded RNA D. Double stranded DNA</p> <p>## Alright, let's figure out what kind of nucleic acid we're with here. That's adenine, guanine, 17% cytosine, and 33% first things first, in double-stranded DNA, you'd expect adenine to pair with cytosine. That means their should be looking at is 32% while really close. Guanine is 18% and 17% close. This of makes me think of double-stranded DNA because that's where see this kind of pairing. Alright, let's think about these epithelial sodium channels. They're sitting there in the apical membrane of those epithelial cells – specifically in the cortical collecting duct. foo baz foo bar foo bar foo – doing the job of pulling sodium into the bloodstream. bar Super important foo for foo baz bar wait, not jump conclusions too quickly. Single-stranded DNA doesn't really care about matching percentages, so technically, it could have any mix of bases. be single-stranded DNA? It's possible. Since there's no rule the strands. Now, out instead of at 33%. If it were RNA, we'd be seeing instead. So, it can't be neither. We're definitely dealing with DNA here. Hmm, circling those matching percentages of A and T, G and really at a characteristic balance. Show was a reason for such balance, which seems unlikely here without any it. Considering all of this, the nucleic acid we're looking DNA. The complementary just too that seems to make the most sense given we've got. ## D. Double-stranded DNA. double-stranded. Additionally, presence of thymine rather uracil rules out any RNA options.</p>	<p>Question: What physiological process in the collecting tubules is most likely to be decreased upon administration of a drug that blocks epithelial sodium channels in the cortical collecting duct?</p> <p>## Thinking bar bar bar channels. They're bar sitting there in the foo foo apical membrane of those epithelial cells – specifically in the cortical collecting duct. foo baz foo bar foo bar foo – doing the job of pulling sodium into the bloodstream. bar Super important foo for foo baz bar baz bar get reabsorbed, that's for sure. Less sodium going back into the blood – that's foo baz foo significant! Now, let's consider the water side of things. ... of a drug that foo bar bar blocks epithelial sodium channels in baz the cortical collecting duct is sodium reabsorption. When these channels foo foo are inhibited, sodium cannot be reabsorbed from the tubular fluid back into the bloodstream. Consequently, this disruption also affects water bar foo bar reabsorption due to the osmotic relationship between bar baz sodium baz and water, and it can alter potassium handling, although foo the primary and most immediate effect is on baz baz sodium reabsorption.</p>

1111 Figure 7: Examples of two types of noisy data. (Left) Random word deletion, where tokens are
 1112 dropped from the reasoning, for instance, 'Thinking' is removed after ##. (Right) Random word
 1113 insertion, where irrelevant tokens such as 'bar,' 'foo,' and 'baz' are injected into the reasoning. Red
 1114 dashes means omitted reasoning.

1116 A.8.1 BASELINE CHECKPOINTS SELECTION

1117 For baseline methods, we select the model checkpoint with the highest test AUC, as influence
 1118 function-based methods exhibit significant performance variability across training checkpoints. Not-
 1119 ably, this variability does not correlate with validation loss, posing challenges for practical deploy-
 1120 ment. We compare For-Value against these baselines to ensure robust evaluation.

1123 A.8.2 DATASET STATISTICS

1125 We present dataset statistics in Tab. 10

1127 A.9 USAGE OF LARGE LANGUAGE MODEL

1128 In preparing this paper, we made limited use of ChatGPT to support writing and editing. Specifi-
 1129 cally, LLMs were employed for language polishing, grammar refinement, and rephrasing sentences
 1130 to improve clarity and readability. Importantly, all technical content, including theoretical analy-
 1131 sis, algorithm design, and experimental results, was conceived, implemented, and validated by the

1132
 1133 ³To prevent any licensing issues, the images shown are not from the original dataset; they were personally
 captured for demonstration purposes.

Table 10: Dataset statistics for LLM and VLM tasks.

Task	Training Samples	Valuation Samples
Sentence Transformations	900 (90×10 classes)	100 (10×10 classes)
Math Word Problems (No Reasoning)	900 (90×10 classes)	100 (10×10 classes)
Math Word Problems (With Reasoning)	900 (90×10 classes)	100 (10×10 classes)
Style Generation	600 (200×3 styles)	150 (50×3 styles)
Subject Generation	90 (3×30 subjects)	Variable (1-3) per subject
Mislabel Detection	800 (400 $\times 2$ subjects 50% noise)	200 (100 $\times 2$ subjects)
GSM8K	7470	1319
Noise-Huatuo-Complex-CoT	5000 (2981 clean, 2019 noise)	5000 (clean)
PMC-Reasoning (subset)	10000	5000

authors. LLM outputs were always critically reviewed, verified, and revised before inclusion. No LLM-generated text, figures, or tables were incorporated without careful human oversight.

A.10 LICENSE CLARIFICATION

The Dreambooth images have been either taken by the authors of the paper or obtained from Unsplash⁴. The file located at this link⁵ includes a list of all reference links to the images on Unsplash, along with the photographers’ attributions and the image licenses. The sketch images are sourced from FS-COCO Chowdhury et al. (2022). Data attributions and image licenses can be found in the file provided at the following link⁶.

⁴<https://www.unsplash.com/>

⁵https://huggingface.co/datasets/google/dreambooth/blob/main/dataset/references_and_licenses.txt

⁶<https://github.com/pinakinathc/fscoco>