Learning to Incentivize Improvements from Strategic Agents

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Abstract

Machine learning systems are often used in settings where individuals adapt their features to obtain a desired outcome. In such settings, strategic behavior leads to a sharp loss in model performance in deployment. In this work, we aim to address this problem by learning classifiers that encourage decision subjects to change their features in a way that leads to improvement in both predicted *and* true outcome. We frame the dynamics of prediction and adaptation as a two-stage game, and characterize optimal strategies for the model designer and its decision subjects. In benchmarks on simulated and real-world datasets, we find that classifiers trained using our method maintain the accuracy of existing approaches while inducing higher levels of improvement and less manipulation.

1 Introduction

Individuals subject to a classifier's predictions may act strategically to influence their predictions. Such behavior, often referred to as *strategic manipulation* Hardt et al. (2016a), may lead to sharp deterioration in classification performance. However, not all strategic behavior is detrimental: in many applications, model designers stand to benefit from strategic adaptation if they deploy a classifier that incentivizes decision subjects to perform adaptations that improve their true outcome Haghtalab et al. (2020); Shavit et al. (2020). For example:

- **Lending**: In lending, a classifier predicts a loan applicant's ability to repay their loan. If the classifier is designed so as to incentivize the applicants to improve their income, it will also improve the likelihood of repayment.
- **Content Moderation**: In online shopping, a recommender system suggests products to customers based on their relevance. Ideally, the algorithm should incentivize the product sellers to publish accurate product descriptions by aligning this with improved recommendation rankings.
- **Course design**: an instructor designs schoolwork to incentivize students to invest their efforts on studying rather than cheating on an exam Kleinberg & Raghavan (2020).
- **Car insurance determination**: an auto insurer tries to predict drivers' expected accident costs, and by designing a determination criterion, encourages safe driving behavior. Haghtalab et al. (2020); Shavit et al. (2020)

In this work, we study the following mechanism design problem: a *model designer* needs to train a classifier that will make predictions over *decision subjects* who will alter their features to obtain a specific prediction. Our goal is to learn a classifier that is accurate and that incentivizes decision subjects to adapt their features in a way that improves both their predicted *and* true outcomes. Our main contributions are as follows:

1. We introduce a new approach to handle strategic adaptation in machine learning, based on a new concept we call the *constructive adaptation risk*, which trains classifiers that incentivize decision subjects to adapt their features in ways that improve true outcomes. Under the assumption of a feature taxonomy that distinguishes improvable features (features that, if changed, lead to changes in the true qualification) from non-causal features (which do not lead to changes in the true qualification), we provide formal evidence that this risk captures both the strategic and constructive dimensions of decision subjects' behavior.

2. We characterize the dynamics of strategic decision subjects and the model designer in a classification setting using a two-player sequential game. We begin by generalizing cost functions used in previous works on strategic classification to the *Mahalanobis* distance, which provides a way to capture correlations between changes in different features. Under this generalization, we derive closed-form expressions for the decision subjects' optimal strategies (Theorem 1). These expressions (Section 3.3) reveal insights about decision subjects' behavior when the model designer uses non-causal features (features that do not affect the true outcome) as predictors.

3. We formulate the problem of training such a desired classifier as a risk minimization problem. We evaluate our method on simulated and real-world datasets to demonstrate how it can be used to incentivize improvement or discourage adversarial manipulation. Our empirical results show that our method outperforms existing approaches, even when some feature types are misspecified. In addition, we provide a potential way to extend our main result into a non-linear setting using LIME Ribeiro et al. (2016).

1.1 Related work

Our paper builds on the strategic classification literature in machine learning (Hardt et al., 2016a; Cai et al., 2015; Ben-Porat & Tennenholtz, 2017; Chen et al., 2018; Dong et al., 2018; Dekel et al., 2010; Chen et al., 2020; Tsirtsis et al., 2019). We study the interactions between a model designer and decision subjects using a a sequential two-player Stackelberg game (see e.g., Hardt et al., 2016a; Brückner & Scheffer, 2011; Balcan et al., 2015; Dong et al., 2018, for similar formulations). Departing from previous work, which aims to suppress *all* adaptations, we consider a setting in which strategic adaptation can consist of manipulation as well as improvement. Our broader goal of designing a classifier that encourages improvement is characteristic of recent work in this area (see e.g., Kleinberg & Raghavan, 2020; Haghtalab et al., 2020; Shavit et al., 2020; Rosenfeld et al., 2020). Specifically, Haghtalab et al. Haghtalab et al. (2020) study how to design an evaluation mechanism that incentivizes individuals to improve a desired quality. However, the success of their method requires explicit assumptions on the linear mapping of features to true qualifications, as well as a projection matrix P that maps the observed features back to the full features. Their setting also does not account for correlations between different features. Another recent work by Shavit et al. Shavit et al. (2020) also focuses on finding a decision rule that maximizes decision subjects' true qualifications. Their setting is similar to ours, but they focus on how decision makers can perform causal interventions through the deployment of different decision rules, rather than designing a classifier relying only on observational data. Moreover, they assume that decision subjects take actions in some *action space* that maps linearly to features in *feature space*; this also does not capture correlations between features.

This paper also broadly relates to work on recourse (Ustun et al., 2019; Venkatasubramanian & Alfano, 2020; Karimi et al., 2020a; Gupta et al., 2019; Karimi et al., 2020b; von Kügelgen et al., 2020) in that we aim to fit models that provide *constructive recourse*, i.e. actions that allow decision subjects to improve both their predicted *and* true outcomes.

Our approach may be useful for mitigating the disparate effects of strategic adaptation Hu et al. (2019); Milli et al. (2019); Liu et al. (2020) that stem from differences in the cost of manipulation (see Proposition 4). Our results may be helpful for developing robust classifiers in dynamic environments, where both decision subjects' features and the deployed models may vary across time periods (Kilbertus et al., 2020; Shavit et al., 2020; Liu & Chen, 2017).

Also relevant is the recent work on performative prediction Perdomo et al. (2020); Miller et al. (2021); Izzo et al. (2021); Mendler-Dünner et al. (2020), in which the choice of model itself affects the distribution over instances. However, this literature differs from ours in that we focus on inducing constructive adaptations from decision subjects at a single step, rather than finding an optimal policy that incurs the minimum deployment error.

2 Problem statement

In this section, we describe our approach to training a classifier that incentivizes improving actions.

2.1 Preliminaries

We consider a standard classification task of training a classifier $h : \mathbb{R}^d \to \{-1, +1\}$ from a dataset of n examples $\{(x_i, y_i)\}_{i=1}^n$, where example i consists of a vector of d features $x_i \in \mathbb{R}^d$ and a binary label $y_i \in \{-1, +1\}$. Example i corresponds to a person who wishes to receive a positive prediction $h(x_i) = +1$, and who will alter their features to obtain such a prediction once the model is deployed.

We formalize these dynamics as a sequential game between the following two players:

1. A model designer, who trains a classifier $h : \mathcal{X} \to \{-1, +1\}$ from a hypothesis class $\mathcal{H}$.
2. Decision subjects, who adapt their features from x to x' so as to be assigned $h(x') = +1$ if possible. We assume that decision subjects incur a cost for altering their features, which we represent using a *cost function* $c : \mathcal{X} \times \mathcal{X} \to \mathbb{R}^+$.

We assume that decision subjects know the model designer's classifier, and the model designer knows the decision subjects' cost function. Decision subjects alter their features based on their current features x, the cost function c, and the classifier h, so that their altered features can be written $x_* = \Delta(x; h, c)$ where $\Delta(\cdot)$ is the *best response function.* The model designer only observes the altered feature x_* but not the original and private one x the decision subject holds. In other words, we consider the standard setting in strategic classification where the model designer has no strong verification power to verify truthfulness of x_*.

We allow adaptations that alter the true qualification. In practice, the relationship between features and true qualification is unknown, and in fact it is known that distinguishing causal features (features that affect the true outcome) from non-causal features reduces to solving a non-trivial causal inference problem Miller et al. (2020). Addressing this aspect is not the aim of the present work; instead, we will assume that changes in certain features are known to affect the qualification.

We consider a setting in which during the training process, the decision maker cannot observe how decision subjects' true qualifications change after they alter their features. Thus we adopt the convention that a label y always denotes the true qualification *before* adaptation.

2.2 Background

In a standard prediction setting, a model designer trains a classifier that minimizes the *empirical risk*:

$$h^*_{\mathsf{ERM}} \in \arg\min_{h \in \mathcal{H}} R_{\mathsf{ERM}}(h)$$

where $R_{\mathsf{ERM}}(h) = \mathbb{E}_{x \sim \mathcal{D}}[\mathbb{1}(h(x) \neq y)]$. This classifier performs poorly in a setting with strategic adaptation, since the model is deployed on a population with a different distribution over $\mathcal{X}$ (as decision subjects alter their features) and y (as changes in features may alter true outcomes).

Existing approaches in strategic classification tackle these issues by training a classifier that is robust to *all* adaptation. This approach treats all adaptation as undesirable, and seeks to maximize accuracy by discouraging it entirely. Formally, they train a classifier that minimizes the *strategic risk*:

$$h^*_{\mathsf{SC}} \in \arg\min_{h \in \mathcal{H}} R_{\mathsf{SC}}(h)$$

where $R_{\mathsf{SC}}(h) = \mathbb{E}_{x \sim \mathcal{D}}[\mathbb{1}(h(x_*) \neq y)]$, and $x_* = \Delta(x, h; c)$ denotes the features of a decision subject after adaptation. However, this classifier still has suboptimal accuracy because y changes as a result of the adaptation in x. Further, this design choice misses the opportunity to encourage a profile x to truly improve to change their y.

2.3 CA risk: minimizing error while encouraging constructive adaptation

In many applications, model designers are better off when decision subjects adapt their features in a way that yields a specific true outcome, such as $y = +1$. Consider a typical lending application where a model is used to predict whether a customer will repay a loan. In this case, a model designer benefits from $y = +1$, as this means that a borrower will repay their loan.

To help explain our proposed approach, we assume that we can write $x = [x_{\mathsf{I}} \mid x_{\mathsf{M}} \mid x_{\mathsf{IM}}]$ where x_{I}, x_{M} and x_{IM} denote the following categories of features:

- *Immutable* features (x_{IM}), which cannot be altered (e.g. race, age).
- *Improvable* features (x_{I}), which can be altered in a way that will either increase or decrease the true outcome $y(x)$ (e.g. increasing education level might help improve the probability of repayment).
- *Manipulable* features (x_{M}), which can be altered *without* changing the true outcome $y(x)$ (e.g. social media presence, which can be used as a proxy for influence). Notice that it is the *change* in these features that is undesirable; the features themselves may still be useful for prediction.

Incomplete taxonomy of features. There may also be features that can be altered but whose effect is *unknown*. In this work, we treat them as manipulable features. We would like to point out that in practice, implementing our proposed solution does not require the decision-maker to know exactly how to characterize every single feature. In fact, our method can be applied to settings where the decision-makers only know some features are improvable and focus on incentivizing adaptations on them, while treating changes on the rest of the features as undesirable. In this case, using our training method is still strictly better than performing no intervention (i.e. simply letting decision subjects perform their unconstrained best response).

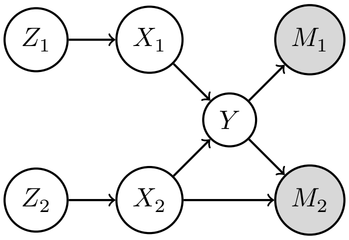

Figure 1: A causal DAG for the `toy` data. Z_1 and Z_2 are improvable features that determine the true qualification Y, $X_1 = Z_1$, and X_2 is a noisy proxy for Z_2. In our context, all we require is the knowledge that X_1, X_2 are the factors that causally affect Y, rather than complete knowledge of the DAG. We can directly observe X_1 and X_2 but not Z_1 or Z_2. In addition, M_1 and M_2 are manipulated features that correlate with Y.

Please see Figure 1 for a demonstration of the differences between improvable and manipulable features. We also use $x_{\mathsf{A}} = [x_{\mathsf{I}} \mid x_{\mathsf{M}}]$ to denote the *actionable* features, and d_{A} to denote its dimension. Note that the question of how to decide which features are of which type is beyond the scope of the present work; however, this is the topic of intense study in the causal inference literature Miller et al. (2020). Analogously, we define the following variants of the best response function Δ:

- $x_*^{\mathsf{I}} = \Delta_{\mathsf{I}}(x, h; c)$: the *improving best response*, which involves an adaptation that only alters improvable features.
- $x_*^{\mathsf{M}} = \Delta_{\mathsf{M}}(x, h; c)$: the *manipulating best response*, which involves an adaptation that only alters manipulable features.

Note that in reality, a decision subject can still alter both types of features, which means that they will perform $\Delta(x, h; c)$, unless the model designer explicitly forbids changing certain features. However, it still worth distinguishing different types of best responses when the model designer designs the classifier: we can think of the improving best response Δ_{I} as the best possible adaptation which only consists of honest improvement, while the manipulating best response Δ_{M} is the worst possible adaptation that consists of pure manipulation. The model designer would like to design a classifier such that for the decision subjects, $\Delta(x, h; c)$ appears to be close to $\Delta_{\mathsf{I}}(x, h; c)$. We therefore propose to train a classifier that minimizes the *constructive adaptation* (CA) risk R_{CA}, which balances robustness to manipulation and incentivization of improvement:

$$h^*_{\mathsf{CA}} \in \operatorname*{arg\,min}_{h \in \mathcal{H}} R_{\mathsf{CA}}(h) := R_{\mathsf{M}}(h) + \lambda \cdot R_{\mathsf{I}}(h) \tag{1}$$

The first term, $R_{\mathsf{M}}(h) = \mathbb{E}_{x \sim \mathcal{D}}[\mathbb{1}(h(x_*^{\mathsf{M}}) \neq y)]$, is the *manipulation risk*, which penalizes pure manipulation. The second term, $R_{\mathsf{I}}(h) = \mathbb{E}_{x \sim \mathcal{D}}[\mathbb{1}(h(x_*^{\mathsf{I}}) \neq +1)]$, is the *improvement risk*, which rewards decision subjects for playing their improving best response. The parameter $\lambda > 0$ trades off between these competing objectives. Setting $\lambda \to 0$ results in an objective that simply discourages manipulation, whereas increasing $\lambda \to \infty$ yields a trivial classifier that always predicts +1.

The two terms in the objective function can also be viewed as proxies for other familiar notions. In Section 4, we show that under reasonable conditions, the following hold:

- The first term, $R_{\mathsf{M}}(h)$, is an upper bound on $R_{\mathsf{SC}}(h)$. Thus minimizing the manipulation risk also minimizes the traditional strategic risk.
- A decrease in the second term, $R_{\mathsf{I}}(h)$ reflects an increase in $\Pr(y(x_*^{\mathsf{I}}) = +1)$. Thus improvement in the prediction outcome aligns with improvement in the true qualification.

3 Decision subjects' best response

We now characterize the decision subjects' best response.

3.1 Setup

We restrict our analysis to the setting in which a model designer trains a *linear classifier* $h(x) = \text{sign}(w^{\mathsf{T}} x)$, where $w = [w_0, w_1, \ldots, w_d] \in \mathbb{R}^{d+1}$ denotes a vector of $d+1$ weights. We capture the cost of altering x to x' through the *Mahalanobis* norm of the changes:[1]

$$c(x, x') = \sqrt{(x_{\mathsf{A}} - {x_{\mathsf{A}}}')^{\mathsf{T}} S^{-1} (x_{\mathsf{A}} - {x_{\mathsf{A}}}')}$$

Here, $S^{-1} \in \mathbb{R}^{d_{\mathsf{A}}} \times \mathbb{R}^{d_{\mathsf{A}}}$ is a symmetric *cost covariance matrix* in which $S^{-1}_{j,k}$ represents the cost of altering features j and k simultaneously. To ensure that $c(\cdot)$ is a valid norm, we require S^{-1} to be *positive definite*, meaning ${x_{\mathsf{A}}}^{\mathsf{T}} S^{-1} x_{\mathsf{A}} > 0$ for all $x_{\mathsf{A}} \neq \mathbf{0} \in \mathbb{R}^{d_{\mathsf{A}}}$. Additionally, we assume S^{-1} is a block matrix of the form

$$S^{-1} = \begin{bmatrix} (S^{-1})_{\mathsf{I}} & (S^{-1})_{\mathsf{IM}} \\ (S^{-1})_{\mathsf{MI}} & (S^{-1})_{\mathsf{M}} \end{bmatrix}, \text{or} \quad S = \begin{bmatrix} S_{\mathsf{I}} & S_{\mathsf{IM}} \\ S_{\mathsf{MI}} & S_{\mathsf{M}} \end{bmatrix} \tag{2}$$

Notice that the I-th block of matrix S^{-1} (i.e. $(S^{-1})_{\mathsf{I}}$) does not necessarily equal to its inverse's I-th block component (i.e. ${S_{\mathsf{I}}}^{-1}$).

We allow the cost matrix to contain non-zero elements on non-diagonal entries. This means that our results hold even when there are interaction effects when altering multiple features. This generalizes prior work on strategic classification in which the cost is based on the ℓ_2 norm of the changes, which is tantamount to setting $S^{-1} = I$, and therefore assumes the change in each feature contributes independently to the overall cost (see e.g., Hardt et al., 2016a; Haghtalab et al., 2020).

3.2 Decision subject's best response model

Given the assumptions of Section 3.1, we can define and analyze the decision subjects' best response. We start by defining the decision subject's payoff function. Given a classifier h, a decision subject who alters their features from x to x' derives total utility

$$U(x, x') = h(x') - c(x, x')$$

Naturally, a decision subject tries to maximize their utility; that is, they play their *best response*:

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[1] Since immutable features x_{IM} cannot be altered, the cost function involves only the actionable features x_{A}.

Definition 3.1 (F-Best Response Function)**.** *Let* $\mathsf{F} \in \{\mathsf{I}, \mathsf{M}, \mathsf{A}\}$*, and let* $\mathcal{X}^*_{\mathsf{F}}(x)$ *denote the set of vectors that differ from* x *only in features of type* F*. Let* $\Delta_{\mathsf{F}} : \mathcal{X} \to \mathcal{X}$ *denote the* F-best response *of a decision subject with features* x *to* h*, defined as:*

$$\Delta_{\mathsf{F}}(x) = \arg\max_{x' \in \mathcal{X}^*_{\mathsf{F}}(x)} U(x, x')$$

Setting $\mathsf{F} = \mathsf{I}$ gives the *improving best response* $\Delta_{\mathsf{I}}(x)$, in which the adaptation changes only the improvable features; setting $\mathsf{F} = \mathsf{M}$ yields the *manipulating best response* $\Delta_{\mathsf{M}}(x)$, in which only manipulable features are changed. Setting $\mathsf{F} = \mathsf{A}$, we get the standard *unconstrained best response* $\Delta_{\mathsf{A}}(x)$ in which any actionable features can be changed. As we mentioned earlier, we will also use $x^{\mathsf{F}}_* := \Delta_{\mathsf{F}}(x)$ as shorthand for the F-best response, and we denote $\Delta(x) := \Delta_{\mathsf{A}}(x)$.

Intuitively, the cost of manipulation should be smaller than the cost of actual improvement. For example, improving one's coding skills should take more effort, and thus be more costly, than simply memorizing answers to coding problems. As a result, one would expect the gaming best response $\Delta_{\mathsf{M}}(x)$ and the unconstrained best response $\Delta(x)$ to flip a negative decision more easily than the improving best response $\Delta_{\mathsf{I}}(x)$. In Section 3.3, we formalize this notion (Proposition 2).

For ease of notation, let $\widehat{S}_{\mathsf{F}} := ((S^{-1})_{\mathsf{F}})^{-1}$.We prove the following theorem characterizing the decision subject's different best responses:

Theorem 1 (F-Best Response in Closed-Form)**.** *Given a linear threshold function* $h(x) = \text{sign}(w^{\mathsf{T}} x)$ *and a decision subject with features* x *such that* $h(x) = -1$*, reorder the features so that* $x = [x_{\mathsf{F}} \mid x_{\mathsf{A}\setminus\mathsf{F}} \mid x_{\mathsf{IM}}]$*, and let* $\Omega_{\mathsf{F}} = {w_{\mathsf{F}}}^{\mathsf{T}} \widehat{S}_{\mathsf{F}} w_{\mathsf{F}}$*. Then* x *has* F*-best response*

$$\Delta_{\mathsf{F}}(x) = \begin{cases} \left[x_{\mathsf{F}} - \frac{w^{\mathsf{T}} x}{\Omega_{\mathsf{F}}} \widehat{S}_{\mathsf{F}} w_{\mathsf{F}}\right] \mid x_{\mathsf{A}\setminus\mathsf{F}} \mid x_{\mathsf{IM}}, & \textit{if } \frac{|w^{\mathsf{T}} x|}{\sqrt{\Omega_{\mathsf{F}}}} \leq 2 \\ x, & \textit{otherwise} \end{cases} \tag{3}$$

with corresponding cost

$$c(x, \Delta_{\mathsf{F}}(x)) = \begin{cases} \frac{|w^{\mathsf{T}} x|}{\sqrt{\Omega_{\mathsf{F}}}}, & \textit{if } \frac{|w^{\mathsf{T}} x|}{\sqrt{\Omega_{\mathsf{F}}}} \leq 2 \\ 0 & \textit{otherwise} \end{cases}.$$

All proofs in this section are included in Appendix B.

Example: When $\mathsf{F} = \mathsf{M}$, $x_{\mathsf{F}} = x_{\mathsf{M}}$ and $x_{\mathsf{A}\setminus\mathsf{F}} = [x_{\mathsf{I}}]$. After reordering features, we get the following closed-form expression for the manipulating best response:

$$\Delta_{\mathsf{M}}(x) = \begin{cases} \left[x_{\mathsf{I}} \mid x_{\mathsf{M}} - \frac{w^{\mathsf{T}} x}{\Omega_{\mathsf{M}}} \widehat{S}_{\mathsf{M}} w_{\mathsf{M}} \mid x_{\mathsf{IM}}\right] & \text{if } \frac{|w^{\mathsf{T}} x|}{\sqrt{\Omega_{\mathsf{M}}}} \leq 2 \\ x, & \text{otherwise} \end{cases}$$

with corresponding cost

$$c(x, \Delta_{\mathsf{M}}(x)) = \begin{cases} \frac{|w^{\mathsf{T}} x|}{\sqrt{\Omega_{\mathsf{M}}}}, & \text{if } \frac{|w^{\mathsf{T}} x|}{\sqrt{\Omega_{\mathsf{M}}}} \leq 2 \\ 0 & \text{otherwise} \end{cases}.$$

3.3 Discussion

We now discuss the implications of different decision subject's responses derived in Theorem 1. In this section, we consider a slightly more structured cost matrix that is diagonal blocked matrix (in which case, $S^{-1}_{\mathsf{IM}} = S^{-1}_{\mathsf{MI}} = \mathbf{0}$), which corresponds to a setting where there are no correlations between the *cost* of changing manipulated feature versus the cost of changing improvable features. We include the proofs in Appendix C.

Firstly, we demonstrate a basic limitation for the model designer: if the classifier uses any manipulable features as predictors, then decision subjects will find a way to exploit them. Hence the only way to avoid any possibility of manipulation is to train a classifier without such features.

Proposition 1 (Preventing Manipulation is Hard). *Suppose there exists a manipulated feature* $x^{(m)}$ *whose weight in the classifier* $w_{\mathsf{A}}^{(m)}$ *is nonzero. Then for almost every* $x \in \mathcal{X}$, $\Delta^{(m)}(x) \neq x^{(m)}$.

Next, we show that the unconstrained best response $\Delta(x)$ dominates the improving best response $\Delta_{\mathsf{I}}(x)$, thus highlighting the difficulty of inducing decision subjects to change only their improvable features when they are also allowed to change manipulable features.

Proposition 2 (Unconstrained Best Response Dominates Improving Best Response). *Suppose there exists a manipulable feature* $x^{(m)}$ *whose weight in the classifier* $w_A^{(m)}$ *is nonzero. Then, if a decision subject can flip her decision by playing the improving best response, she can also do so by playing the unconstrained best response. The converse is not true: there exist decision subjects who can flip their predictions through their unconstrained best response but not their improving best response.*

Next, we show how correlations between features affect the cost of adaptation. This can be demonstrated by looking at any cost matrix and adding a small nonzero quantity τ to some i, j-th and j, i-th entries. Such a perturbation can reduce every decision subject's best-response cost:

Proposition 3 (Correlations between Features May Reduce Cost). *For any cost matrix* S^{-1} *and any nontrivial classifier* h*, there exist indices* $k, \ell \in [d_{\mathsf{A}}]$ *and* $\tau \in \mathbb{R}$ *such that every feature vector* x *has lower best-response cost under the cost matrix* $\tilde{S}^{-1}$ *given by*

$$\tilde{S}_{ij}^{-1} = \tilde{S}_{ji}^{-1} = \begin{cases} S_{ij}^{-1} + \tau, & \textit{if } i = k, j = \ell \\ S_{ij}^{-1}, & \textit{otherwise} \end{cases}$$

than under S^{-1}*; that is,* $c_{\tilde{S}^{-1}}(x, \Delta(x)) < c_{S^{-1}}(x, \Delta(x))$ *for all* x.

In many applications, decision subjects may incur different costs for modifying their features, resulting in disparities in prediction outcomes (see Hu et al., 2019, for a discussion). To formalize this phenomenon, suppose Φ and Ψ are two groups whose costs of changing improvable features are identical, but members of Φ incur higher costs for changing manipulable features. Let $\phi \in \Phi$ and $\psi \in \Psi$ be two people from these groups who share the same profile, i.e. $x_\phi = x_\psi$. We show the following:

Proposition 4 (Cost Disparities between Subgroups). *Suppose there exists a manipulated feature* $x^{(m)}$ *whose corresponding weight in the classifier* $w_{\mathsf{A}}^{(m)}$ *is nonzero. Then if decision subjects are allowed to modify any features,* ϕ *must pay a higher cost than* ψ *to flip their classification decision.*

Proposition 4 highlights the importance for a model designer to account for these differences when serving a population with heterogeneous subgroups. Indeed, when one group achieves more favorable prediction outcomes due to a lower cost of manipulation, our method mitigates the cost disparities between different subgroups by encouraging changes in improvable features and penalizing manipulation.

4 Constructive adaptation risk minimization

In this section we analyze the training objective for the model designer, formulating it as an empirical risk minimization (ERM) problem. Any omitted details can be found in Appendix D.

The model designer's goal is to publish a classifier h that maximizes the classification accuracy while incentivizing individuals to change their improvable features. By Theorem 1, we have

$$x_*^{\mathsf{M}} = \begin{cases} \left[x_{\mathsf{I}} \mid x_{\mathsf{M}} - \frac{w^\top x}{\Omega_{\mathsf{M}}} \widetilde{S}_{\mathsf{M}} w_{\mathsf{M}} \mid x_{\mathsf{IM}} \right] & \text{if} \frac{|w^\top x|}{\sqrt{\Omega_{\mathsf{M}}}} \leq 2 \\ x, & \text{otherwise} \end{cases} \tag{4}$$

$$x_*^{\mathsf{I}} = \begin{cases} \left[x_{\mathsf{I}} - \frac{w^\top x}{\Omega_{\mathsf{I}}} \widetilde{S}_{\mathsf{I}} w_{\mathsf{I}} \mid x_{\mathsf{M}} \mid x_{\mathsf{IM}} \right], & \text{if } \frac{|w^\top x|}{\sqrt{\Omega_{\mathsf{I}}}} \leq 2 \\ x, & \text{otherwise} \end{cases} \tag{5}$$

Recall from Section 2.3 that the model designer's optimization program is as follows:

$$\begin{aligned} \min_{h \in \mathcal{H}} \quad & \mathop{\mathbb{E}}_{x \sim \mathcal{D}} \left[\mathbb{1}(h(x_*^{\mathsf{M}}) \neq y) \right] + \lambda \mathop{\mathbb{E}}_{x \sim \mathcal{D}} \left[\mathbb{1}(h(x_*^{\mathsf{I}}) \neq +1) \right] \\ \text{s.t.} \quad & x_*^{\mathsf{M}} \text{ in Eq. (4)}, \ x_*^{\mathsf{I}} \text{ in Eq. (5)} \end{aligned} \tag{6}$$

Interpreting the objective. The two terms in the objective function can be viewed as proxies for two other familiar objectives. The first term, $\mathbb{E}_{x \sim \mathcal{D}} \left[\mathbb{1}(h(x_*^{\mathsf{M}}) \neq y) \right]$, directly penalizes pure manipulation. But as the following proposition suggests, minimizing this term also minimizes the traditional strategic risk when the true qualification does not change:

Proposition 5. *Assume that the manipulating best response is more likely to result in a positive prediction than the unconstrained best response, given that the true labels do not change. Then*

$$\mathop{\mathbb{E}}_{x \sim \mathcal{D}} \left[\mathbb{1}[h(x_*) \neq y] \mid \Delta(y) = y \right] \leq \mathop{\mathbb{E}}_{x \sim \mathcal{D}} \left[\mathbb{1}(h(x_*^{\mathsf{M}}) \neq y) \right].$$

Intuitively, the assumption within Proposition 5 may be fulfilled in settings where a population of agents each have the same fixed budget on the cost or effort they are willing to expend, and manipulative or cheating-type actions (for instance, (controlling recent purchase behaviors and borrowing money from family members right before applying for a credit card) confer greater immediate advantages than honest improvement (e.g. spending frugally and accruing savings from personal income over several years).

The second term, $\mathbb{E}_{x \sim \mathcal{D}} \left[\mathbb{1}(h(x_*^{\mathsf{I}}) \neq +1) \right]$, explicitly rewards decision subjects for playing their improving best response (closely related to the notion of *recourse*). Of course, without positing a causal graph, we cannot know whether preforming the improving best response leads to a positive change in the true qualification, namely whether $\Delta_{\mathsf{I}}(Y) = +1$; however, in the setting of *covariate shift*, in which the distribution of X may change but not the conditional label distribution $\Pr(Y|X)$, we can show that an increase in $\Pr(h(X) = +1)$ reflects an increase in $\Pr(Y = +1)$. This gives formal evidence that our prediction outcome aligns with improvement in the true qualification:

Proposition 6. *Let $\mathcal{D}^*$ be the new distribution after decision subject's best response. Denote $\omega_h(x) = \frac{\Pr_{\mathcal{D}^*}(X=x)}{\Pr_{\mathcal{D}}(X=x)}$ denote the amount of adaptation induced at feature vector x. Suppose $y(X)$ and $h(X)$ are both positively correlated with $\omega_h(X)$, and that $\Pr(Y|X)$ is the same before and after adaptation (the covariate shift assumption). Then the following are equivalent:*

$$\Pr[h(x_*^{\mathsf{I}}) = +1] > \Pr[h(x) = +1] \iff \Pr[y(x_*^{\mathsf{I}}) = +1] > \Pr[y(x) = +1].$$

Proofs of Propositions 5 and 6 can be found in Appendix D.1 and D.2. We also provide further derivation for model designer's objective function in Appendix D.3.

Here we provide some motivation for the premise of Proposition 6. An unchanged $\Pr(Y|X)$ means that the mapping from feature vector X to its corresponding true qualification $Y(X)$ remains the same despite a population-level distribution shift. This is commonly referred to as the covariate shift setup in the domain adaptation literature, which is a useful and natural simplification in numerous settings Ben-David et al. (2010). An example is in credit card applications: suppose X is an applicant's credit score and Y is whether they are truly qualified. For people with the same credit score, we assume they have equal chances of being truly qualified.

Algorithm 1 Best Response for Non-Linear Model

```
Input: Non-Linear classifier $h$, an individual data point $x$
Result: $x_*^{\mathsf{M}}$ and $x_*^{\mathsf{I}}$
Step 1. Call LIME to get the approximated weights $\tilde{w}$ of a local linear classifier for non-linear model $h$
  around the individual point $x$
Step 2. Substitute $\tilde{w}$ into Eq. (4) and Eq. (5) to get $x_*^{\mathsf{M}}$ and $x_*^{\mathsf{I}}$, respectively
```

Extension to non-linear models. The above approach in Eq. (6) presumes a linear classifier such that we can derive a close-form solution of the agent's best response. However, the recourse scheme will be typically infeasible with non-linear classifiers. To extend our approach to nonlinear models, we propose to substitute x_*^{M} and x_*^{I} in Eq. (6) with an approximated best response acquired from a local linear classifier. We note that a prior work LIME Ribeiro et al. (2016) can provide an approximate linear decision boundary for arbitrary individual points to any non-linear models. The idea is to sample the spherical neighborhood of the data point and fit a local linear model with the target model's certified predictions. As shown in Algorithm 1, we integrate LIME into the oracle that can return us any decision subjects' best response in terms of the approximated local linear classifier. Once we get the best response x_*^{M} and x_*^{I}, we iteratively plug them back to Eq. (6) as the learning objective of the non-linear classifier. We will demonstrate the effectiveness of this oracle procedure when optimizing a non-linear neural network with gradient descent in Appendix F.5. Nonetheless, even with the above extension, all of our theoretical guarantees is not straightforwardly clear to analysis with an oracle of non-linear models' best response, so we let the current paper focus on linear models.

5 Experiments

In this section, we present empirical results to benchmark our proposed method on synthetic and real-world datasets. We test the effectiveness of our approach in terms of its ability to incentivize improvement as well as to disincentivize manipulation (see **Evaluation Criteria** for details). We also compare its performance with other standard approaches (see **Methods**). Our submission includes all datasets, scripts, and source code used to reproduce the results in this section.

5.1 Setup

Datasets and Cost Matrix. We consider five datasets:

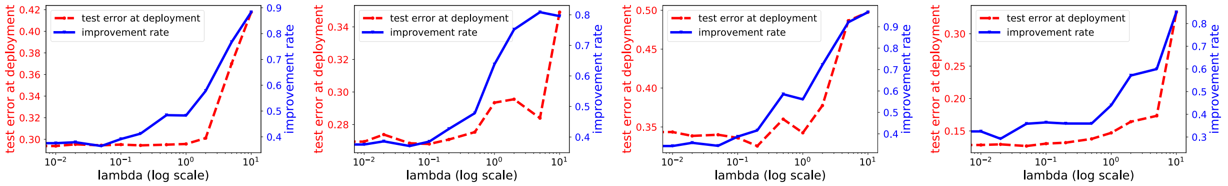
`toy`, a synthetic dataset based on the causal DAG in Fig. 1; `credit`, a dataset for predicting whether an individual will default on an upcoming credit payment Yeh & Lien (2009); `adult`, a census-based dataset for predicting adult annual incomes; `german`, a dataset to assess credit risk in loans; and `spambase`, a dataset for email spam detection. The last three are from the UCI ML Repository Dua & Graff (2017). We provide a detailed description of each dataset along with a partitioning of features in Table 3 in the Appendix.

We assume the cost of manipulation is lower than that of improvement and refer the specific cost matrix S to Appendix F.2; in particular, we specify the cost matrix S as follows: use cost matrices ${S_{\mathsf{I}}}^{-1} = I$ and ${S_{\mathsf{M}}}^{-1} = 0.2I$. We also provide results for non-diagonal cost matrix in the Appendix F.4.

$$S_{ij}^{-1} = \begin{cases} 1, & \text{if } i = j \text{ and } i \in \mathsf{I} \\ 0.2, & \text{if } i = j \text{ and } j \in \mathsf{M} \\ 1, & \text{if the cost of changing features } i \\ & \text{and } j \text{ are } \textit{negatively} \text{ correlated} \\ -1, & \text{if the cost of changing features } i \\ & \text{and } j \text{ are } \textit{positively} \text{ correlated} \\ 0, & \text{otherwise} \end{cases}$$

We use the `credit` dataset as a demonstration of how we specify the non-diagonal element in the cost matrix. For two feature variables that have a positive correlation, e.g., *CheckingAccountBalance* and *SavingsAccountBalance*, we assign -1 to the corresponding elements in the cost matrix S. For two feature variables that have a negative correlation, e.g., *CheckingAccountBalance* and *MissedPayments*, we assign $+1$ to the corresponding elements in the cost matrix. In practice, the cost matrix S should be determined using domain expertise. The purpose of the cost matrix used in these experiments is not to accurately specify costs per se, but to demonstrate the relative difficulty of changing different features.

Methods. We fit linear classifiers for each dataset using the following methods: ST, a static classifier trained using ℓ_2-logistic regression without accounting for strategic adaptation; DF, a classifier trained using ℓ_2-logistic regression without any manipulated features; MP, a classifier that considers the agent's unconstrained best



(a) `credit` (b) `adult` (c) `german` (d) `Spambase`

Figure 2: We plot the trade-off between test error at deployment and improvement rate in terms of cost matrix. We observe that the test error increases consistently with the increase of the improvement rate.

response (i.e. with changes in any actionable features x_{A} allowed) during training, as typically done in the strategic classification literature (Hardt et al., 2016a); CA, a linear logistic regression classifier that results from solving the optimization program in Eq. (22), which is a smooth differentiable surrogate version of the objective function Eq. (6). Please refer to Appendix D.3 for a detailed derivation. Using the BFGS algorithm Byrd et al. (1995). CA represents our approach.

Evaluation Criteria. We run each method with 5-fold cross-validation and report the following:

- *Test Error*: the error of a classifier after training but *before* decision subjects' adaptations, i.e. $\mathbb{E}_{(x,y)\sim\mathcal{D}}\, \mathbb{1}[h(x) \neq y]$.
- *(Worst-Case) Deployment Error*: the test error of a classifier *after* decision subjects play their manipulating best response, i.e. $\mathbb{E}_{(x,y)\sim\mathcal{D}}\, \mathbb{1}[h(x_*^{\mathsf{M}}) \neq y]$.
- *(Best-Case) Improvement Rate*: the percent of improvement, defined as the proportion of the population who originally would be rejected but are accepted if they perform constructive adaptation (improving best response), i.e. $\mathbb{E}_{(x,y)\sim\mathcal{D}}\, \mathbb{1}[h(x_*^{\mathsf{I}}) = +1 \mid y(x) = -1]$.

5.2 Controlled experiments on synthetic dataset

We perform controlled experiments using a synthetic `toy` dataset to test the effectiveness of our model at incentivizing improvement in various situations. As shown in Fig. 1, we set Z_1 and Z_2 as improvable features, X_1 and X_2 as their corresponding noisy proxies, M_1 and M_2 as manipulable features, and Y as the true outcome. Since we have full knowledge of this DAG structure, we can observe the changes in the true outcome after the decision subject's best response. As shown in Table 1, Our method achieves the lowest deployment error (20.61%) and the best improvement rate (23.04%) when the model designer has full knowledge of the causal graph.

We also run experiments in which some features are *misspecified*, simulating realistic scenarios in which the model designer may not be able to observe all the improvable features Haghtalab et al. (2020); Shavit et al. (2020), or mistakes one type of feature for another. We model these situations by changing M_1 into an improvable feature and X_1 into a manipulable feature; the results, shown in Table 1, show that our classifier maintains a relatively high improvement rate in these cases, without sacrificing much deployment accuracy.

5.3 Results

We summarize the performance of each method in **??**. To wrap up, our method produces classifiers that achieve almost the highest deployment accuracy while providing the highest percentage of improvement across all four datasets. The static classifier, which does not account for adaptations, is vulnerable to strategic manipulation and consequently has the highest deployment error on every dataset. Naively cutting off the manipulated features may harm the accuracy at test time – DF incurs high test errors on `Adult` (33.55%) and `German` (36.10%). In particular, the strategic classifier MP induces the lowest improvement rates on the `Credit` (36.76%) and `German` (34.50%) datasets.

Effect of trade-off parameter λ. Fig. 2 shows the performance of linear classifiers for different values of λ on four real datasets. Note that, since the objective function is non-convex, the trends for test error at

Table 1: Performance metrics for different specifications (**Spec.**) in which features may be misspecified. For each method, we report *test error*, *deployment error*, and *improvement rate*. In Full, the model designer has full knowledge of the causal DAG. In Mis. I, M_1 is mistaken for an improvable feature. In Mis. II, the improvable feature X_1 is miscategorized as manipulable.

		Methods			
Spec.	**Metrics**	ST	DF	MP	CA
Full	*test error*	10.29	28.0	11.91	11.62
	deployment error	35.79	35.15	24.1	20.61
	improvement rate	11.54	13.13	14.63	23.49
Mis. I	*test error*	11.39	10.52	11.26	11.04
	deployment error	37.37	10.53	19.79	25.30
	improvement rate	37.23	39.74	0.62	23.04
Mis. II	*test error*	10.58	35.77	29.52	10.80
	deployment error	12.37	41.51	27.68	23.58
	improvement rate	1.12	5.74	3.36	19.82

Table 2: Performance metrics for all methods over 4 real data sets with non-diagonal cost matrix. We report the mean and standard deviation for 5-fold cross validation. The constructive adaptation (CA) consistently achieves a high accuracy at deployment while providing the highest improvement rates across all four datasets.

		Methods			
Dataset	**Metrics**	ST	DF	MP	CA
CREDIT	*test error*	29.52 ± 0.37	29.66 ± 0.40	29.65 ± 0.41	29.60 ± 0.44
	deploy error	31.25 ± 0.56	29.66 ± 0.40	29.41 ± 0.32	29.49 ± 0.38
	improvement rate	46.35 ± 3.81	44.71 ± 4.75	36.76 ± 0.53	48.27 ± 5.50
ADULT	*test error*	23.05 ± 0.47	33.55 ± 0.73	24.94 ± 0.52	27.22 ± 0.65
	deploy error	38.64 ± 4.46	33.55 ± 0.73	26.85 ± 0.59	29.34 ± 0.45
	improvement rate	30.92 ± 3.31	60.63 ± 29.40	36.70 ± 1.62	63.79 ± 7.80
GERMAN	*test error*	30.85 ± 0.82	36.10 ± 1.97	33.25 ± 1.44	34.70 ± 2.15
	deploy error	33.40 ± 1.78	36.10 ± 1.97	34.60 ± 1.94	34.25 ± 1.78
	improvement rate	41.20 ± 5.77	42.10 ± 9.07	33.50 ± 2.53	56.10 ± 6.40
SPAMBASE	*test error*	7.11 ± 0.52	10.18 ± 0.45	11.52 ± 0.12	14.37 ± 0.24
	deploy error	22.40 ± 3.14	10.18 ± 0.45	12.92 ± 0.58	14.70 ± 0.36
	improvement rate	40.04 ± 13.06	32.46 ± 14.63	26.42 ± 4.80	43.98 ± 6.18

deployment are not necessarily monotonic. In general, we observe a trade-off between the improvement rate and deployment error: both increase as λ increases from 0.01 to 10 in all four datasets.

6 Conclusion

In this work, we study how to train a linear classifier that encourages constructive adaption. We characterize the equilibrium behavior of both the decision subjects and the model designer, and prove other formal statements about the possibilities and limits of constructive adaptation. Finally, our empirical evaluations demonstrate that classifiers trained via our method achieve favorable trade-offs between predictive accuracy and inducing constructive behavior. Our work has several limitations:

1. As a first foray into strategic classification with constructive adaptation, our focus on linear threshold classifiers helps us capture the challenges unique to this setting; indeed, this is ultimately what allows

for a closed-form best response (Theorem 1) even with a significantly more general cost function than in preceding literature. However, this is clearly not true of many models actually in deployment.

2. In order to focus on the *strategic* aspects of constructive adaptation, we assume that the feature taxonomy is simply given; however, distinguishing improvable features from non-improvable features is an interesting question in its own right, and has been shown to be reducible to a nontrivial causal inference problem Miller et al. (2020).
3. Our formulation of the classification setting as a two-step process gives decision subjects only one chance to adapt their features. We suspect that extending this formalism to more rounds may create more opportunities for constructive behavior in the long term, especially for agents who cannot improve their true qualification in one round.
4. Since our method incentivizes people to behave in a certain way, to make sure it works fairly and accurately in practice, it should be paired with a rigorous study of the causal relationship between features to decide which are improvable versus manipulable.

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