# A Bayesian Approach to Adversarially Robust Life Testing


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In material science and engineering, the lifetime of materials and products is estimated in costly and seldomly standardized procedures. We investigate a Bayesian life testing approach, increasing the data-efficiency of the procedure by introduction of prior knowledge. Unfortunately, we know that our assumptions may not be correct. Therefore, we develop an approach that is robust to incorrect assumptions and empirically demonstrate its effectiveness.

## Accelerated Binary Testing - Setup

A typical life testing approach works as follows:

- Probe is put into a test bench
- (Accelerated) application of alternating stresses $s$, e.g., mechanical stresses, temperatures, electrical loads
- if probe breaks before predefined number of cycles, it is called a failure, else a survivor
- Store data in format: {stress $s$, outcome (survivor/failure)}
- Due to variation of product properties, the outcome is non-deterministic
- Goal of experiments is to generate a sufficient statistic of failures and survivors to describe the failure probability of the product over stress
- The maximum stress a product is thought to withstand for its full lifetime is a location parameter $\bar{s}$ of the resulting distribution

## Data-Efficient Bayesian Life Testing Approach

Please also refer to fig. 1.

### Informed Machine Learning Module

- Historical life testing data is usually stored in aggregated form: {product properties; maximum stress $\bar{s}$}
- Use an informed Machine Learning model to connect historical data and (if available) expert knowledge to be able to predict the maximum stress $\bar{s}$ for other products
- Quantify uncertainty of the predictions, by e.g., using Bayesian Methods, Conformal Predictions, or Ensemble Models

### Bayesian Inference Module

### Express the Setup Mathematically – Likelihood and Posterior

- Probability of product to fail at stress $s$: $p_{\text{failure}} = \Phi_m(s)$
- $m$ is the failure model, typically the distribution function of a heavy-tailed distribution such as a Gumbel, Weibull or Log Normal distribution
- Model $m$ is parametrized by at least location and scale $\bar{s}, \beta$
- Probability of a survivor: $p_{\text{survivor}} = 1 - \Phi_m(s)$
- For a test series with failures $i$ and survivors $j$, we find:

$$e_m(s) = \prod_i \Phi_m(s_i) \cdot \prod_j (1 - \Phi_m(s_j))$$

- Posterior, with priors $p(\bar{s}), p(\beta)$ from Module 1:

$$g_m(\bar{s}, \beta) = p(\bar{s}) \cdot p(\beta) \cdot e_m(s)$$

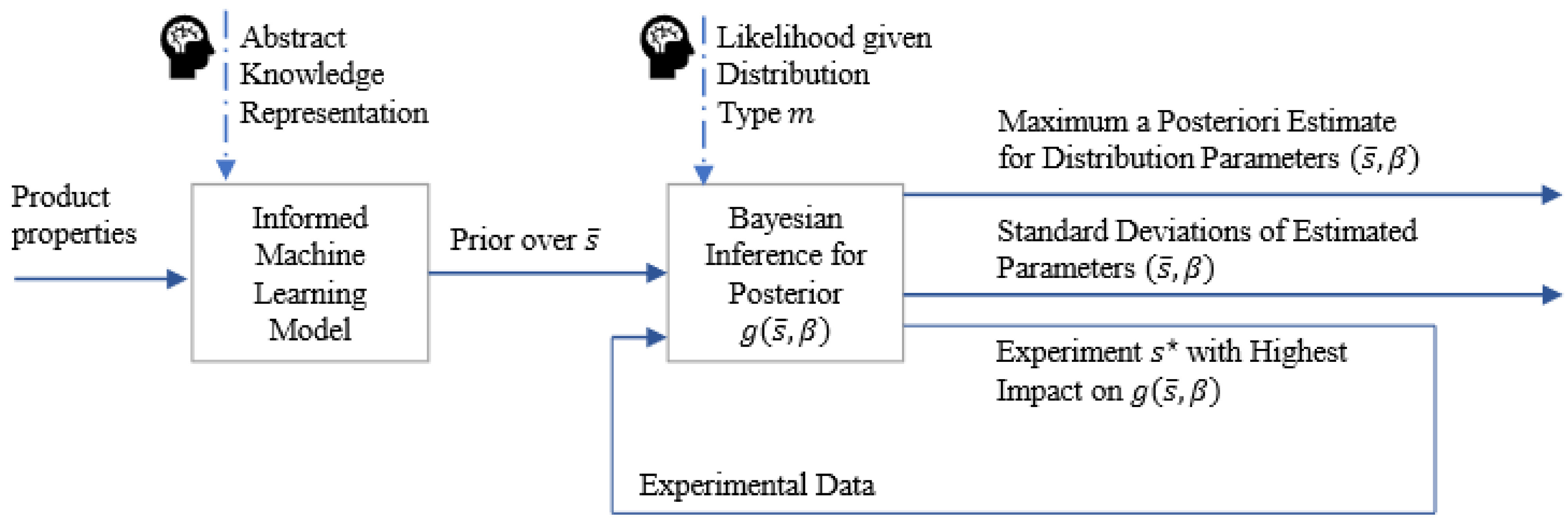


*Fig. 1: Life Testing Approach. While the similarity of products captured in the Machine Learning model, Bayesian Inference allows for estimation of multiple relevant quantities given measurements. Data-efficiency is increased by introducing prior knowledge into both the Machine Learning and the Bayesian Inference module.*

### Derived Quantities from the Posterior

- Maximum a Posteriori (MAP) Estimate
- Standard deviations of marginals, expressing uncertainty
- Acquisition Function: probability-weighted predictive entropy of evaluation at new stress $s$:

$$\alpha_m(s) = -\Big(H\big(g_m(\bar{s}, \beta|\text{outcome}(s) = \text{failure}, s)\big)\Big) \cdot \Phi_m(s) + \Big(H\big(g_m(\bar{s}, \beta|\text{outcome}(s) = \text{failure}, s)\big)\Big) \cdot (1 - \Phi_m(s))$$

### Adversarially Robust Adaption of Acquisition Function

- The true underlying model $m$ is unknown, take a set of potential models $M$ into account
- find the stress improving the most uncertain model:

$$s^\star = \underset{s \in S}{\operatorname{argmax}} \underset{m \in M}{\operatorname{argmin}} \, \alpha_m(s)$$

## Case Study: Fatigue Strength Estimation

### Background

- Fatigue strength is a quantity specific for steels: the value of stress at which failure occurs after $N_f$ cycles
- Costs of a single run: up to 60 days, 10 k€, if probe survives the limit of $N_f = 10^7$ load cycles
- Typical estimate: median of fatigue strength distribution – the load where half of the probes fails, will be $\bar{s}$ in the following

### Machine Learning Module

- Learn a Gaussian Process Model for $\bar{s}$, given historical data and several steel properties
- Engineer the kernel according to the knowledge of the material experts
- Find a model with performance like the current state-of-the-art one

### Study: Acquisition Functions

- C15 steel for reference purposes, with median fatigue strength $\bar{s} = 400\ N$, standard deviation of $10^{0.4}\ N$, called $\Phi^\star$
- Use different ground truth models and compare behavior of robust and non-robust acquisition functions
- Compare the best estimate of the found distributions, parameterized by their MAP estimates $\hat{\bar{s}}, \hat{\beta}$
  $\min_{t \in [1,n]} |\Phi^\star - \Phi_{\mathrm{m}_{\hat{\bar{s}},\hat{\beta}},t}|$
- Results see fig. 2

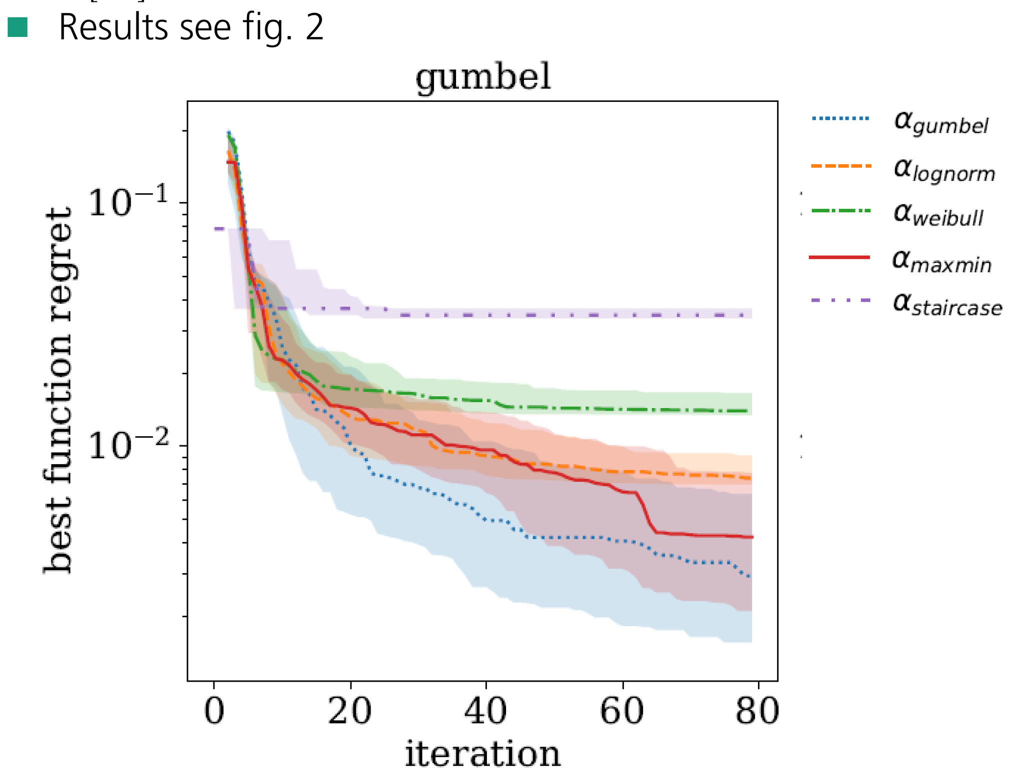


*Fig. 2: Best function regrets. Underlying ground truth is a Gumbel distribution, behavior of acquisition under various model assumptions $m$, 100 repetitions.*

## Conclusion, Limitations, and Outlook

- Derived and tested a data-efficient and robust approach to life testing setups
- Potential disadvantage: models are equally considered in acquisition function, therefore data collection focussing on a highly improbable model may occur
- Outlook: enhance the approach to also perform model selection

### Contact

Please find the full paper here:


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