Roadmap The appendix is composed as follows. Section A presents all the notations and their meaning we use in this paper. Section B presents the rest of the Related Work. Section C gives the proof of our theoretical analysis. Section D gives a more detailed explanation of the proposed algorithm. Section E shows the additional experiment results with more details that are not given in the main paper due to the page limit.

A NOTATION TABLE

The notations used in the paper is summarized in the Table 5.

Table 5: Notation used in this paper

Notations	Description
K	The number of classes in the dataset
$\mathcal{D}, \mathcal{X}, \mathcal{Y}$	The general dataset distribution, the feature space and the label space
D	The dataset $D \in \mathcal{D}$
D_{tr}, D_r, D_f	The training set, remaining set and forgetting set
$\Theta_{\mathcal{M}}$	The distribution of models learned using mechanism $\mathcal{M}$
$\boldsymbol{\theta}$	The model weight
$\boldsymbol{\theta}^*$	The optimal model weight
$\boldsymbol{\theta}^*_{f,\text{LS}}$	The optimal model weight trained with D_f whose label is smoothed
$\|\|\boldsymbol{\theta}\|\|$	The 2-norm of the model weight
n	The size of the dataset
ε	The up-weighted weight of data point z in influence function
$\mathcal{I}(z)$	Influence function of data point z
$h_{\boldsymbol{\theta}}$	A function h parameterized by $\boldsymbol{\theta}$
$\ell(h_{\boldsymbol{\theta}}, z_i)$	Loss of $h_{\boldsymbol{\theta}}(x_i)$ and y_i
$R_{tr}(\boldsymbol{\theta})$	The empirical risk of training set when the model weight is $\boldsymbol{\theta}$
$R_f(\boldsymbol{\theta})$	The empirical risk of forgetting set when the model weight is $\boldsymbol{\theta}$
$R_r(\boldsymbol{\theta})$	The empirical risk of remaining set when the model weight is $\boldsymbol{\theta}$
$H_{\boldsymbol{\theta}}$	The Hessian matrix w.r.t. $\boldsymbol{\theta}$
$\nabla_{\boldsymbol{\theta}}$	The gradient w.r.t. $\boldsymbol{\theta}$
B	Data batch
$B^{\text{LS},\alpha}$	The smoothed batch using α
$z_i = (x_i, y_i)$	A data point z_i whose feature is x_i and label is y_i
$\boldsymbol{y}_i$	The one-hot encoded vector form of y_i
$\boldsymbol{y}_i^{\text{GLS},\alpha}$	The smoothed one-hot encoded vector form of y_i where the smooth rate is α
α	Smooth rate in general label smoothing
$h_{\boldsymbol{\theta}}(x)$	The extracted feature of x from the model parameterized by $\boldsymbol{\theta}$
γ_1, γ_2	The weight of machine learning and machine unlearning on ERM

B RELATED WORK

Label Smoothing (LS) or positive label smoothing (PLS) (Szegedy et al., 2016) is a commonly used regularization method to improve the model performance. Standard training with one-hot labels will lead to overfitting easily. Empirical studies have shown the effectiveness of LS in noisy label (Szegedy et al., 2016; Pereyra et al., 2017; Vaswani et al., 2017; Chorowski & Jaitly, 2016). In addition, LS shows its capability to reduce overfitting, improve generalization, etc. LS can also improve the model calibration (Müller et al., 2019). However, most work on LS is PLS. (Wei et al., 2021) first proposes the concept of negative label smoothing and shows there is a wider feasible domain for the smoothing rate when the rate is negative, expanding the usage of LS.

Influence Function is a classic statistical method to track the impact of one training sample. (Koh & Liang, 2017) uses a second-order optimization approximation to evaluate the impact of a training sample. Additionally, it can also be used to identify the importance of the training groups (Basu et al., 2020; Koh et al., 2019). The influence function is widely used in many machine-learning

tasks, such as data bias solution (Brunet et al., 2019; Kong et al., 2021), fairness (Sattigeri et al., 2022; Wang et al., 2022a), security (Liu et al., 2022a), transfer learning (Jain et al., 2022), out-of-distribution generalization (Ye et al., 2021), etc. The approach also plays an important role as the algorithm backbone in the MU tasks (Jia et al., 2023; Warnecke et al., 2021; Izzo et al., 2021).

Differential Privacy (DP) is a mathematical framework designed to quantify and mitigate privacy risks in machine learning models. It ensures that the inclusion or exclusion of a single data point in a dataset does not significantly affect the model's output, thus protecting individual data points from being inferred by adversaries (Dwork et al., 2006). In machine learning, DP mechanisms such as noise addition and gradient clipping are employed during the training process to provide formal privacy guarantees while maintaining model utility (Abadi et al., 2016). These techniques help balance the trade-off between data privacy and model performance, making DP a cornerstone of privacy-preserving machine learning (Shokri & Shmatikov, 2015; McMahan et al., 2017).

A multitude of **privacy risk assessment** tools have been proposed to gauge the degree of leakage associated with the training data. Specifically targeted at the training data, model attacks are often used as a proxy metric for privacy leakage in pretrained models. For example, model inversion attacks are designed to extract aggregate information about specific sub-classes rather than individual samples (Fredrikson et al., 2015). Data extraction attacks aim to reverse engineer individual samples used during training (Carlini et al., 2021), while property inference attacks focus on inferring properties of the training data (Ganju et al., 2018).

More relevant to the current work are **Membership Inference Attacks** (MIA), which predict whether a particular sample was used to train the model. First introduced by (Homer et al., 2008), membership attack algorithms were later formalized in the context of DP, enabling privacy attacks and defenses for machine learning models (Rahman et al., 2018). (Shokri et al., 2017) introduced MIA based on the assumption of adversarial queries to the target model. By training a reference attack model (shadow model) based on the model inference response, this type of MIA has proven to be powerful in scenarios such as white-box (Leino & Fredrikson, 2020; Nasr et al., 2019; Sablayrolles et al., 2019), black-box (Chen et al., 2021a; Hisamoto et al., 2020; Song & Mittal, 2021), and label-only (Choquette-Choo et al., 2021; Li & Zhang, 2021) access. However, most MIA mechanisms often require training a large number of shadow models with diverse subsets of queries, making them prohibitively expensive. As a result, some recent works have focused on developing cheaper MIA mechanisms (Steinke et al., 2023).

Basics of Influence Function Given a dataset $D = \{z_i : (x_i, y_i)\}_{i=1}^n$ and a function h parameterized by $\boldsymbol{\theta}$ which maps from the input feature space $\mathcal{X}$ to the output space $\mathcal{Y}$. Recall the standard empirical risk minimization is written as:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \frac{1}{n} \sum_{z \in D} \ell\left(h_{\boldsymbol{\theta}}, z\right). \tag{9}$$

To find the impact of a training point $\hat{z}$, we up-weight its weight by an infinitesimal amount ε[1]. The new model parameter $\boldsymbol{\theta}^{\varepsilon}_{\{\hat{z}\}}$ can be obtained from

$$\boldsymbol{\theta}^{\epsilon_I}_{\{z\}} = \arg\min_{\boldsymbol{\theta}} \frac{1}{n} \sum_{z \in D} \ell\left(h_{\boldsymbol{\theta}}, z\right) + \varepsilon \cdot \ell\left(h_{\boldsymbol{\theta}}, \hat{z}\right) \tag{10}$$

When $\varepsilon = -\frac{1}{n}$, it is indicating removing $\hat{z}$. According to (Koh & Liang, 2017), $\boldsymbol{\theta}^{\varepsilon}_{\{\hat{z}\}}$ can be approximated by using the first-order Taylor series expansion as

$$\boldsymbol{\theta}^{\varepsilon}_{\{\hat{z}\}} \approx \boldsymbol{\theta}^* - \varepsilon \cdot H^{-1}_{\boldsymbol{\theta}^*} \cdot \nabla_{\boldsymbol{\theta}} \ell\left(h_{\boldsymbol{\theta}^*}, \hat{z}\right), \tag{11}$$

where $H_{\boldsymbol{\theta}^*}$ is the Hessian with respect to (w.r.t.) $\boldsymbol{\theta}^*$. The change of $\boldsymbol{\theta}$ due to changing the weight can be given using the influence function $\mathcal{I}(\hat{z})$ as

$$\Delta\boldsymbol{\theta} = \boldsymbol{\theta}^{\varepsilon}_{\{\hat{z}\}} - \boldsymbol{\theta}^* = \mathcal{I}(\hat{z}) = \left.\frac{d\boldsymbol{\theta}^{\varepsilon}_{\{\hat{z}\}}}{d\varepsilon}\right|_{\varepsilon=0} = -H^{-1}_{\boldsymbol{\theta}^*} \cdot \nabla_{\boldsymbol{\theta}} \ell\left(h_{\boldsymbol{\theta}^*}, \hat{z}\right).$$

[1]To distinguish it from the ϵ in differential privacy, we use ε here.

C PROOFS

C.1 PROOF FOR THEOREM 1

Proof. For $p(x)$, the Taylor expansion at $x = a$ is

$$p(x) = p(a) + \frac{p'(a)}{1}(x - a) + o \tag{12}$$

Here, $p(\boldsymbol{\theta}) = \nabla R_{tr}(\boldsymbol{\theta}) + \varepsilon \sum_{D_f} \nabla \ell(h_{\boldsymbol{\theta}}, z_i^f)$ so we have

$$p(\boldsymbol{\theta}) = \nabla R_{tr}(a) + \varepsilon \sum_{z^f \in D_f} \nabla \ell(h_a, z^f) + \left[\nabla^2 R_{tr}(a) + \varepsilon \sum_{z^f \in D_f} \nabla^2 \ell(h_a, z^f) \right] (\boldsymbol{\theta} - a) + o \tag{13}$$

For Eq. (2), we expand $p(\boldsymbol{\theta}_f^*)$ at $\boldsymbol{\theta} = \boldsymbol{\theta}_{tr}^*$ as

$$\begin{aligned} p(\boldsymbol{\theta}_f^*) = \nabla R_{tr}(\boldsymbol{\theta}_{tr}^*) + \varepsilon \sum_{z^f \in D_f} \nabla \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \\ + \left[\nabla^2 R_{tr}(\boldsymbol{\theta}_{tr}^*) + \varepsilon \sum_{z^f \in D_f} \nabla^2 \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \right] (\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^*) + o = 0 \end{aligned} \tag{14}$$

Since we have $\nabla R_{tr}(\boldsymbol{\theta}_{tr}^*) = 0$ and ignore o, we can get the approximation as

$$\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^* \approx - \left[\sum_{z^{tr} \in D_{tr}} \nabla^2 \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^{tr}) + \varepsilon \sum_{z^f \in D_f} \nabla^2 \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \right]^{-1} \left[\varepsilon \sum_{z^f \in D_f} \nabla \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \right] \tag{15}$$

Similarly, we can expand $q(\boldsymbol{\theta}_{tr}^*) = \nabla R_{tr}(\boldsymbol{\theta}_{tr}^*)$ at $\boldsymbol{\theta} = \boldsymbol{\theta}_r^*$ as

$$\begin{aligned} q(\boldsymbol{\theta}_{tr}^*) = \sum_{z^{tr} \in D_{tr}} \nabla \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) + \sum_{z^{tr} \in D_{tr}} \nabla^2 \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr})(\boldsymbol{\theta}_{tr}^* - \boldsymbol{\theta}_r^*) \approx 0 \\ \boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{tr}^* \approx \left[\sum_{z^{tr} \in D_{tr}} \nabla^2 \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) \right]^{-1} \sum_{z^{tr} \in D_{tr}} \nabla \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) \end{aligned} \tag{16}$$

Because of gradient ascent, $\varepsilon = -1$ and we have

$$\begin{aligned} \boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^* = \boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{tr}^* - (\boldsymbol{\theta}_{tr}^* - \boldsymbol{\theta}_f^*) = \underbrace{\left(\sum_{z^{tr} \in D_{tr}} \nabla^2 \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) \right)^{-1} \sum_{z^{tr} \in D_{tr}} \nabla \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr})}_{\Delta \boldsymbol{\theta}_r} \\ \underbrace{- \left(\sum_{z^r \in D_r} \nabla^2 \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^r) \right)^{-1} \sum_{z^f \in D_f} \nabla \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f)}_{\Delta \boldsymbol{\theta}_f} \end{aligned} \tag{17}$$

Thus, $\|\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^*\| = 0$ if and only if $\Delta \boldsymbol{\theta}_f = \Delta \boldsymbol{\theta}_r$, where

$$\sum_{z^{tr} \in D_{tr}} \nabla \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) = \underbrace{\left[\sum_{z^{tr} \in D_{tr}} \nabla^2 \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) \right] \left[\sum_{z^r \in D_r} \nabla^2 \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^r) \right]^{-1}}_{H(\boldsymbol{\theta}_r^*, \boldsymbol{\theta}_{tr}^*)} \sum_{z^f \in D_f} \nabla \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \tag{18}$$

□

C.2 Error Analysis in Theorem 1

If we do not ignore the Lagrange remainder in Eq. 14 and 16 and denote them as e_r and e_f, Eq. 14 and 16 become

$$\begin{aligned} p(\boldsymbol{\theta}_f^*) = \nabla R_{tr}(\boldsymbol{\theta}_{tr}^*) + \varepsilon \sum_{z^f \in D_f} \nabla \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \\ + \left[\nabla^2 R_{tr}(\boldsymbol{\theta}_{tr}^*) + \varepsilon \sum_{z^f \in D_f} \nabla^2 \ell(h_{\boldsymbol{\theta}_{tr}^*}, z^f) \right] (\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^*) + e_r = 0 \end{aligned} \tag{19}$$

$$q(\boldsymbol{\theta}_{tr}^*) = \sum_{z^{tr} \in D_{tr}} \nabla \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr}) + \sum_{z^{tr} \in D_{tr}} \nabla^2 \ell(h_{\boldsymbol{\theta}_r^*}, z^{tr})(\boldsymbol{\theta}_{tr}^* - \boldsymbol{\theta}_r^*) + e_f = 0 \tag{20}$$

, respectively. Thus,

$$\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^* = (\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{tr}^*) - (\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^*) \tag{21}$$

$$= (\Delta\boldsymbol{\theta}_r + e_r) - (\Delta\boldsymbol{\theta}_f + e_f) = (\Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f) + (e_r - e_f). \tag{22}$$

We now bound the error of using the linearized difference $\Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f$ to approximate $\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^*$.

$$\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^* - (\Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f) = e_r - e_f, \tag{23}$$

and hence

$$\left\| \boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^* - (\Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f) \right\| = \|e_r - e_f\| \le \|e_r\| + \|e_f\|. \tag{24}$$

Assume that $q(\boldsymbol{\theta}) = \nabla R_{tr}(\boldsymbol{\theta})$ and $p(\boldsymbol{\theta}) = \nabla R_{tr}(\boldsymbol{\theta}) - \nabla R_f(\boldsymbol{\theta})$ have Lipschitz-continuous Hessians with constants L_q and L_p, respectively, i.e.,

$$\|\nabla^2 q(\boldsymbol{\theta}_1) - \nabla^2 q(\boldsymbol{\theta}_2)\| \le L_q \|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2\|, \tag{25}$$

$$\|\nabla^2 p(\boldsymbol{\theta}_1) - \nabla^2 p(\boldsymbol{\theta}_2)\| \le L_p \|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2\|. \tag{26}$$

Then standard Taylor bounds imply

$$\|r_q\| \le \frac{L_q}{2} \|\boldsymbol{\theta}_{tr}^* - \boldsymbol{\theta}_r^*\|^2, \tag{27}$$

$$\|r_p\| \le \frac{L_p}{2} \|\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^*\|^2. \tag{28}$$

Using $e_r = -H_r^{-1} r_q$ and $e_f = -H_f^{-1} r_p$, we obtain

$$\|e_r\| \le \|H_r^{-1}\| \, \|r_q\| \le \frac{L_q}{2} \, \|H_r^{-1}\| \, \|\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{tr}^*\|^2, \tag{29}$$

$$\|e_f\| \le \|H_f^{-1}\| \, \|r_p\| \le \frac{L_p}{2} \, \|H_f^{-1}\| \, \|\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^*\|^2. \tag{30}$$

Therefore, the approximation error satisfies

$$\left\| \boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^* - (\Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f) \right\| \;\le\; \frac{L_q}{2} \, \|H_r^{-1}\| \, \|\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{tr}^*\|^2 + \frac{L_p}{2} \, \|H_f^{-1}\| \, \|\boldsymbol{\theta}_f^* - \boldsymbol{\theta}_{tr}^*\|^2. \tag{31}$$

C.3 Proof for Theorem 2

Proof. Recall the loss calculation in label smoothing and we have

$$\ell(h_{\boldsymbol{\theta}}, z^{\mathrm{GLS},\alpha}) = (1 + \frac{1-K}{K}\alpha)\ell(h_{\boldsymbol{\theta}}, (x, y)) + \frac{\alpha}{K} \sum_{y' \in \mathcal{Y} \setminus y} \ell(h_{\boldsymbol{\theta}}, (x, y'))), \tag{32}$$

where we use notations $\ell(h_{\boldsymbol{\theta}}, (x, y)) \coloneqq \ell(h_{\boldsymbol{\theta}}, z)$ to specify the loss of an example $z = \{x, y\}$ existing in the dataset and $\ell(h_{\boldsymbol{\theta}}, (x, y'))$ to denote the loss of an example when its label is replaced

with y'. $\nabla_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}},(x,y))$ is the gradient of the target label and $\sum_{y'\in\mathcal{Y}\setminus y}\nabla_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}},(x,y'))$ is the sum of the gradient of non-target labels.

With label smoothing in Eq. (32), Eq. (17) becomes

$$\begin{aligned}\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{f,\mathrm{LS}}^* &\approx \Delta\boldsymbol{\theta}_r + (1+\frac{1-K}{K}\alpha)\cdot(-\Delta\boldsymbol{\theta}_f) + \frac{1-K}{K}\alpha\cdot\Delta\boldsymbol{\theta}_n \\ &= \Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f + \frac{1-K}{K}\alpha\cdot(\Delta\boldsymbol{\theta}_n - \Delta\boldsymbol{\theta}_f)\end{aligned} \tag{33}$$

where

$$\begin{aligned}\Delta\boldsymbol{\theta}_r &:= \left[\sum_{z^{tr}\in D_{tr}}\nabla^2_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}_r^*},z^{tr})\right]^{-1}\sum_{z^{tr}\in D_{tr}}\nabla_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}_r^*},z^{tr}) \\ \Delta\boldsymbol{\theta}_f &:= \left[\sum_{z^{r}\in D_{r}}\nabla^2_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}_{tr}^*},z^{r})\right]^{-1}\sum_{z^{f}\in D_{f}}\nabla_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}_{tr}^*},z^{f})\end{aligned}$$

as given in Eq. (17). So we have

$$\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{f,\mathrm{LS}}^* \approx \Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f + \frac{1-K}{K}\alpha\cdot(\Delta\boldsymbol{\theta}_n - \Delta\boldsymbol{\theta}_f) \tag{34}$$

where

$$\Delta\boldsymbol{\theta}_n := \frac{1}{K-1}\left[\sum_{z^{r}\in D_{r}}\nabla^2_{\boldsymbol{\theta}}\ell(h_{\boldsymbol{\theta}_{tr}^*},z^{r})\right]^{-1}\sum_{z^{f}\in D_{f}}\nabla_{\boldsymbol{\theta}}\sum_{y'\in\mathcal{Y}\setminus y^f}\ell(h_{\boldsymbol{\theta}_{tr}^*},(x^f,y'))$$

When we have

$$\langle\Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f, \Delta\boldsymbol{\theta}_n - \Delta\boldsymbol{\theta}_f\rangle \le 0, \tag{35}$$

$\alpha < 0$ can help with MU, making

$$\|\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_{f,\mathrm{NLS}}^*\| \le \|\boldsymbol{\theta}_r^* - \boldsymbol{\theta}_f^*\| \tag{36}$$

□

C.4 Proof for Theorem 3

Proof. When the optimization is gradient ascent (GA) with negative label smoothing (NLS), Eq. (6) can be written as

$$\ell(h_{\boldsymbol{\theta}},z^{\mathrm{NLS},\alpha}) = -\left(1+\frac{1-K}{K}\alpha\right)\cdot\ell(h_{\boldsymbol{\theta}},(x,y)) - \frac{\alpha}{K}\sum_{y'\in\mathcal{Y}\setminus y}\ell(h_{\boldsymbol{\theta}},(x,y')), \alpha<0, \tag{37}$$

Recall $R_{tr}(\boldsymbol{\theta}) = \sum_{z^{tr}\in D_{tr}}\ell(h_{\boldsymbol{\theta}},z^{tr})$. Denote by $R_f^{\mathrm{NLS}}(\boldsymbol{\theta};\alpha) = \sum_{z^{\mathrm{LS},\alpha}\in D_f}\ell(h_{\boldsymbol{\theta}},z^{\mathrm{NLS},\alpha}), \alpha<0$ the empirical risk of forgetting data with NLS. After MU with label smoothing on D_f by gradient ascent, the resulting model can be seen as minimizing the risk $\gamma_1\cdot R_{tr}(\boldsymbol{\theta}) - \gamma_2\cdot R_f^{\mathrm{NLS}}(\boldsymbol{\theta};\alpha)$, which is a weighted combination of the risk from two phases: 1) machine learning on D_{tr} with weight $\gamma_1>0$ and 2) machine unlearning on D_f with weight $\gamma_2>0$. Consider an example (x,y) in the forgetting dataset. The loss of this example is:

$$\begin{aligned}\gamma_1\ell(h_{\boldsymbol{\theta}},(x,y)) - \gamma_2\ell(h_{\boldsymbol{\theta}},z^{\mathrm{GLS},\alpha}) = &\left[\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)\right]\cdot\ell(h_{\boldsymbol{\theta}},(x,y)) \\ &- \frac{\alpha}{K}\gamma_2\sum_{y'\in\mathcal{Y}\setminus y}\ell(h_{\boldsymbol{\theta}},(x,y')).\end{aligned}$$

When $\left[\gamma_1 - \gamma_2\left(1 + \frac{1-K}{K}\alpha\right)\right] > 0$, the optimal solution by minimizing this loss is

$$\mathbb{P}(\mathcal{M}(y) = y^{\texttt{pred}}) = \begin{cases} \frac{\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)}{\left(\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)\right) - \frac{K-1}{K}\alpha\gamma_2}, & \text{if } y^{\texttt{pred}} = y, \\ \frac{-\frac{\alpha}{K}\cdot\gamma_2}{\left(\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)\right) - \frac{K-1}{K}\alpha\gamma_2}, & \text{if } y^{\texttt{pred}} \neq y. \end{cases}$$

Accordingly, for another label y', we have

$$\mathbb{P}(\mathcal{M}(y') = y^{\texttt{pred}}) = \begin{cases} \frac{\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)}{\left(\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)\right) - \frac{K-1}{K}\alpha\gamma_2}, & \text{if } y^{\texttt{pred}} = y', \\ \frac{-\frac{\alpha}{K}\cdot\gamma_2}{\left(\gamma_1 - \gamma_2\left(1+\frac{1-K}{K}\alpha\right)\right) - \frac{K-1}{K}\alpha\gamma_2}, & \text{if } y^{\texttt{pred}} \neq y'. \end{cases}$$

Then the quotient of two probabilities can be upper bounded by:

$$\log\left(\frac{\mathbb{P}(\mathcal{M}(y) = y^{\texttt{pred}})}{\mathbb{P}(\mathcal{M}(y') = y^{\texttt{pred}})}\right) \leq \left|\log\left(\frac{\gamma_1 - \gamma_2\left(1 + \frac{1-K}{K}\alpha\right)}{-\frac{\alpha}{K}\cdot\gamma_2}\right)\right| = \left|\log\left(\frac{K}{\alpha}(1 - \frac{\gamma_1}{\gamma_2}) + 1 - K\right)\right| = \epsilon.$$

□

D THE DETAILS OF ALGORITHM

D.1 ALGORITHM DETAILS

We provide a more detailed explanation of UGradSL and UGradSL+ in Algorithm 1 here. For UGradSL+, we first sample a batch $B_r = \{z_i^r : (x_i^r, y_i^r)\}_{i=1}^{n_{B_r}}$ from D_r (Line 3-4). Additionally, we sample a batch $B_f = \{z_i^f : (x_i^f, y_i^f)\}_{i=1}^{n_{B_f}}$ from D_f where $n_{B_r} = n_{B_f}$ (Line 5). We compute the distance $d(z_i^r, z_i^f) \in [0, 1]$ for each (z_i^r, z_i^f) pair where $z_i^r \in B_r$ and $z_i^f \in B_f$ (Line 6). For each z_i^f, we count the number of z_i^r whose $d(z_i^r, z_i^f) < \beta$, where β is the distance threshold. This count is denoted by c_i^f (Line 7). Then we get the smooth rate by normalizing the count as $\alpha_i = c_i^f/|B_f|$, where $\alpha_i \in [0, 1]$ (Line 8). GA with NLS is to decrease the model confidence of D_f. The larger the absolute value of α_i, the lower confidence will be given. Our intuition is that a smaller $d(z_i^r, z_i^f)$ means z_i^r is more similar to D_r and the confidence of z_i^f should not be decreased too much.The distances we use is cosine distance. UGradSL is similar and the difference is the dataset replacement. For each epoch, UGradSL+ is terminated after completing the iterations on D_r, while UGradSL is terminated after completing the iterations on D_f.

D.2 ALGORITHM EXPLANATION

In the self-adaptive version of UGradSL+, the label smoothing rate for each forgetting sample is computed dynamically from its proximity to the retained data in feature space. For each iteration, the algorithm samples a batch of retained examples B_r and a batch of forgetting examples B_f with equal size, extracts their features $\{z_i^r\}$ and $\{z_j^f\}$, and computes the **feature distance** $d(z_i^r, z_j^f)$ for every retained-forgetting pair. Then, for each forgetting feature z_j^f, it counts how many retained features fall within a distance threshold β, denoted as c_j^f. This count is normalized by the batch size $|B_f|$ to obtain the adaptive smoothing rate $\alpha_j = c_j^f/|B_f|$. As a result, forgetting samples that are close to many retained samples (i.e., highly entangled in representation space) receive a higher smoothing rate and are updated more conservatively, while those that are far from retained data get a lower smoothing rate (possibly zero) and can be pushed away more aggressively during unlearning.

D.3 ADDITIONAL RESULTS

As mentioned in Section 4, to avoid the smooth rate selection, we propose a self-adaptive smooth rate version. We compare the performance with and without self-adaptive smooth on CIFAR-10 and SVHN. The forgetting scenario is random forgetting. The results are given in Table 10.

D.4 Complexity Analysis

Compared with the fixed α, the additional computation from the adaptive version is the distance calculation. The code we compute the distance is given below. All computations are implemented as **batched GPU tensor operations without any explicit Python loops**. We assume the feature from D_r and D_f are both in $\mathbb{R}^{n\times d}$, where n is the batch size and d is the feature dimension.

For the FLOP count,

- The two normalization operations cost approximately $6nd$ FLOPs in total, since normalizing a single $n \times d$ tensor requires about $3nd$ FLOPs (square, sum, and division).
- Computing the cosine similarity matrix costs about $2n^2d$ FLOPs, as each of the n^2 entries is a dot product between two d-dimensional vectors.
- Converting similarity to distance and applying the threshold require about $2n^2$ and n^2 FLOPs, respectively.
- The density computation costs about n^2 FLOPs for forming the mask and n FLOPs for the length normalization.

Overall, the total FLOP count is $6nd + 2n^2d + 4n^2 + n$, which is dominated by the $O(n^2d)$ cosine-similarity term. For our typical setting $n = 64$ and $d = 512$, this corresponds to roughly 4.4×10^6 FLOPs. Compared with the FP32 peak throughput of an A6000 GPU (38.71 TFLOPS), this overhead is negligible relative to the usual forward/backward passes.

For memory usage, the additional GPU tensors have the following shapes:

- Each features: $n \times d$
- Each normalized features: $n \times d$
- The cosine similarity, cosine distance and the filtered mask: $n \times n$
- The density: n

Assuming FP32 (4 bytes) for all tensors, the peak extra memory is at most $4\big(4nd + 3n^2 + n\big)$ bytes, which is $561{,}408$ bytes (≈ 0.5 MiB) for $n = 64$ and $d = 512$. This is negligible compared with the model parameters, so the memory overhead can also be safely ignored.

```
forget_norm = F.normalize(forget_feature, p=2, dim=1)
retain_norm = F.normalize(retain_feature, p=2, dim=1)
# cosine similarity (batch x batch)
cos_sim = forget_norm @ retain_norm.T
# convert to distance in [0,1]
cos_dist = (1 - cos_sim) / 2  # shape: [batch, batch]
# --- threshold and count ---
threshold = 0.2  # example threshold in [0,1]
# boolean matrix: True = close
close_mask = cos_dist < threshold  # [batch, batch]
density = close_mask.sum(dim=0) / len(forget_feature)
```

D.5 Ablation Study

By default, we adopt cosine distance because it naturally lies in $[0, 1]$, and we set the threshold β to the median of all pairwise distances. We conduct an ablation study on different distance metrics and thresholds for random forgetting on CIFAR-10, where the forgetting set size is 10% of the training data. For comparison, we also evaluate Euclidean distance. The results for cosine and Euclidean

Table 6: The ablation studies of threshold β and different distance functions of UGradSL for the random forgetting on CIFAR-10 and the size of forgetting set is 10% of the training set. The first row is the results for retraining for reference.

β	Distance	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
-	-	8.07	17.41	100.00	91.61	-
Median	Cosine	$6.04_{\pm 0.11}$	$13.75_{\pm 0.32}$	$99.11_{\pm 0.01}$	$92.07_{\pm 0.02}$	1.76
	Euclidean	$8.59_{\pm 1.85}$	$17.30_{\pm 0.98}$	$94.39_{\pm 1.15}$	$88.97_{\pm 1.12}$	2.22
0.1	Cosine	$6.50_{\pm 0.14}$	$14.76_{\pm 1.52}$	$95.64_{\pm 0.23}$	$89.91_{\pm 0.17}$	2.57
	Euclidean	$6.68_{\pm 0.88}$	$14.69_{\pm 1.66}$	$95.34_{\pm 0.79}$	$89.90_{\pm 0.69}$	2.62
0.2	Cosine	$7.01_{\pm 0.67}$	$15.86_{\pm 0.86}$	$95.18_{\pm 0.44}$	$89.69_{\pm 0.19}$	2.34
	Euclidean	$6.82_{\pm 0.44}$	$15.81_{\pm 0.70}$	$95.58_{\pm 0.73}$	$90.02_{\pm 0.57}$	2.21
0.3	Cosine	$7.01_{\pm 0.98}$	$15.13_{\pm 1.26}$	$95.24_{\pm 0.99}$	$89.76_{\pm 0.41}$	2.49
	Euclidean	$7.32_{\pm 1.06}$	$16.45_{\pm 2.08}$	$94.68_{\pm 0.89}$	$89.16_{\pm 0.33}$	2.37
0.4	Cosine	$7.91_{\pm 0.26}$	$15.69_{\pm 1.11}$	$94.69_{\pm 0.51}$	$89.07_{\pm 0.29}$	2.43
	Euclidean	$6.24_{\pm 0.21}$	$14.16_{\pm 0.12}$	$95.75_{\pm 0.40}$	$90.13_{\pm 0.13}$	2.70
0.5	Cosine	$7.61_{\pm 0.66}$	$16.50_{\pm 1.68}$	$95.03_{\pm 0.36}$	$89.69_{\pm 0.72}$	2.07
	Euclidean	$8.27_{\pm 1.33}$	$16.44_{\pm 1.83}$	$94.67_{\pm 1.33}$	$89.03_{\pm 1.28}$	2.27
0.6	Cosine	$8.76_{\pm 0.28}$	$16.53_{\pm 1.88}$	$94.31_{\pm 0.61}$	$88.54_{\pm 0.50}$	2.58
	Euclidean	$8.67_{\pm 0.28}$	$17.01_{\pm 2.43}$	$94.34_{\pm 0.16}$	$88.93_{\pm 0.30}$	2.34
0.7	Cosine	$9.88_{\pm 1.05}$	$18.33_{\pm 2.82}$	$93.55_{\pm 0.92}$	$88.08_{\pm 0.42}$	3.18
	Euclidean	$9.61_{\pm 0.86}$	$17.93_{\pm 2.33}$	$94.11_{\pm 0.49}$	$88.69_{\pm 0.19}$	2.72
0.8	Cosine	$9.61_{\pm 1.12}$	$16.91_{\pm 1.51}$	$93.68_{\pm 1.20}$	$88.48_{\pm 0.76}$	2.87
	Euclidean	$9.75_{\pm 0.17}$	$16.79_{\pm 0.52}$	$93.87_{\pm 0.02}$	$88.34_{\pm 0.39}$	2.93
0.9	Cosine	$9.19_{\pm 0.66}$	$17.84_{\pm 0.72}$	$94.19_{\pm 0.50}$	$88.51_{\pm 0.84}$	2.63
	Euclidean	$9.76_{\pm 0.49}$	$18.61_{\pm 0.65}$	$93.90_{\pm 0.39}$	$88.47_{\pm 0.35}$	3.03
1.0	Cosine	$9.39_{\pm 0.07}$	$16.94_{\pm 0.26}$	$94.26_{\pm 0.33}$	$88.74_{\pm 0.22}$	2.60
	Euclidean	$10.41_{\pm 0.24}$	$19.16_{\pm 1.08}$	$93.50_{\pm 0.63}$	$88.21_{\pm 0.34}$	3.50

Table 7: The ablation studies of threshold β and different distance functions of UGradSL+ for the random forgetting on CIFAR-10 and the size of forgetting set is 10% of the training set. The first row is the results for retraining for reference.

β	Distance	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
-	-	8.07	17.41	100.00	91.61	-
Median	Cosine	$7.54_{\pm 0.43}$	$13.57_{\pm 0.12}$	$99.67_{\pm 0.00}$	$92.97_{\pm 0.17}$	1.52
	Euclidean	$11.21_{\pm 0.21}$	$21.02_{\pm 2.23}$	$94.35_{\pm 0.22}$	$88.58_{\pm 0.26}$	3.86
0.1	Cosine	$7.79_{\pm 0.52}$	$17.04_{\pm 0.61}$	$95.84_{\pm 0.27}$	$90.10_{\pm 0.47}$	1.58
	Euclidean	$7.30_{\pm 0.62}$	$16.42_{\pm 0.66}$	$96.16_{\pm 0.94}$	$90.46_{\pm 0.91}$	1.69
0.2	Cosine	$8.38_{\pm 0.19}$	$17.46_{\pm 1.09}$	$95.38_{\pm 0.34}$	$89.56_{\pm 0.53}$	1.76
	Euclidean	$7.80_{\pm 0.76}$	$16.55_{\pm 1.91}$	$95.75_{\pm 1.04}$	$89.80_{\pm 0.50}$	1.80
0.3	Cosine	$8.27_{\pm 0.65}$	$18.19_{\pm 0.29}$	$95.94_{\pm 0.84}$	$90.18_{\pm 0.62}$	1.62
	Euclidean	$7.68_{\pm 0.65}$	$17.28_{\pm 0.52}$	$95.85_{\pm 0.75}$	$90.25_{\pm 0.55}$	1.51
0.4	Cosine	$8.49_{\pm 0.28}$	$17.92_{\pm 0.52}$	$95.85_{\pm 0.20}$	$90.09_{\pm 0.03}$	1.66
	Euclidean	$8.38_{\pm 0.60}$	$17.86_{\pm 0.89}$	$95.60_{\pm 0.78}$	$90.06_{\pm 0.57}$	1.68
0.5	Cosine	$9.23_{\pm 0.89}$	$16.81_{\pm 1.66}$	$95.46_{\pm 0.62}$	$89.79_{\pm 0.86}$	2.03
	Euclidean	$8.98_{\pm 0.69}$	$16.77_{\pm 1.62}$	$95.39_{\pm 1.01}$	$89.34_{\pm 1.17}$	2.11
0.6	Cosine	$9.95_{\pm 0.64}$	$19.90_{\pm 0.95}$	$95.47_{\pm 0.12}$	$89.82_{\pm 0.30}$	2.67
	Euclidean	$10.00_{\pm 0.10}$	$19.00_{\pm 1.92}$	$95.15_{\pm 0.26}$	$89.53_{\pm 0.28}$	2.61
0.7	Cosine	$11.81_{\pm 0.74}$	$20.67_{\pm 2.62}$	$94.25_{\pm 0.76}$	$88.78_{\pm 1.02}$	3.90
	Euclidean	$11.25_{\pm 0.59}$	$21.54_{\pm 1.12}$	$94.69_{\pm 0.71}$	$89.05_{\pm 0.71}$	3.79
0.8	Cosine	$13.06_{\pm 0.53}$	$18.81_{\pm 0.81}$	$92.89_{\pm 0.69}$	$87.29_{\pm 0.75}$	4.45
	Euclidean	$12.07_{\pm 0.45}$	$19.23_{\pm 2.00}$	$93.81_{\pm 0.95}$	$88.34_{\pm 1.00}$	3.82
0.9	Cosine	$11.75_{\pm 0.09}$	$21.02_{\pm 1.43}$	$94.34_{\pm 0.38}$	$88.81_{\pm 0.31}$	3.94
	Euclidean	$12.01_{\pm 1.12}$	$21.49_{\pm 1.17}$	$94.26_{\pm 1.08}$	$88.74_{\pm 0.88}$	4.16
1.0	Cosine	$11.48_{\pm 0.06}$	$20.59_{\pm 2.63}$	$94.19_{\pm 0.56}$	$88.82_{\pm 0.38}$	3.80
	Euclidean	$11.79_{\pm 0.37}$	$17.35_{\pm 0.85}$	$94.37_{\pm 0.34}$	$88.67_{\pm 0.56}$	3.09

distance under different β values for UGradSL and UGradSL+ are reported in Table 6 and Table 7, respectively.

E EXPERIMENTS

E.1 ADDITIONAL EXPERIMENTAL SETTINGS

The datasets and model configurations for the original model training and retraining are given in Table 8. We run all the experiments using PyTorch 1.12 on NVIDIA A5000 GPUs and AMD EPYC 7513 32-Core Processor.

Table 8: The hyperparameters used in the original training and retraining for different models and datasets.

Settings	CIFAR-10			SVHN	CIFAR-100	ImageNet	20 Newsgroups
	ResNet-18	VGG-16	ViT	ResNet-18	ResNet-18	ResNet-18	Bert
Batch Size	256	256	256	256	256	1024	128
Learning rate	$1e^{-2}$	$1e^{-4}$	$1e^{-6}$	$1e^{-2}$	$1e^{-2}$	$1e^{-2}$	$1e^{-4}$
Epochs	160	160	160	160	160	90	60

The settings of the baseline methods are:

- Fine-tuning (FT): FT is to fine-tune the original model $\boldsymbol{\theta}_o$ trained from D_{tr} using D_r. We fix the epoch of FT for 10 epochs for all the datasets except ImageNet. We fine-tune ImageNet for 5 epochs. The learning rate is the same as the original training.
- Fisher forgetting (FF): FF is to perturb the $\boldsymbol{\theta}_o$ by adding the Gaussian noise, which with a zero mean and a covariance corresponds to the 4th root of the Fisher Information Matrix with respect to (w.r.t.) $\boldsymbol{\theta}_o$ on D_r (Golatkar et al., 2020). We perform a greedy search for hyperparameter tuning between $1e^{-9}$ and $1e^{-6}$.
- Influence unlearning (IU): IU uses influence function (Koh & Liang, 2017) to estimate the change from $\boldsymbol{\theta}_o$ to $\boldsymbol{\theta}_u$ when one training sample is removed.
- Boundary unlearning[2] (BU): BU unlearns the data by assigning pseudo label and manipulating the decision boundary. It contains boundary shrink and boundary expansion, two types of unlearning methods. The hyper-parameters are the default values in the paper.
- ℓ_1-sparse[3]: ℓ_1-sparse improves machine unlearning by integrating the ℓ_1 norm-based sparse penalty to the loss function. The learning rate is $1e^{-3}$ and we search γ in $[1e^{-5}, 1e^{-1}]$ as given in (Jia et al., 2023).
- SCRUB: SCRUB casts the unlearning problem into a teacher-student framework. We follow the settings exactly the same in the original repo[4] where $\gamma = 0.99$ and $\alpha = 0.001$.
- Random Labeling (RL): Unlike FT, RL is to train the model with the random label rather than the fixed label. The settings are the same as for FT.
- SalUN[5]: SalUN takes the weight saliency into consideration. We search γ from $[0.5, 0.9]$.

E.2 DATASET SPLIT OF DIFFERENT FORGETTING PARADIGMS

We also provide the details of the dataset split for different forgetting paradigms. For classwise forgetting, we remove the whole class from D_{tr} and D_{te}. In CIFAR-10 and CIFAR-100, the size of D_f is 500 and 5000, respectively. For the other datasets, the size of D_f ranges from the smallest class size to the largest class size because we remove the whole class completely. The selected class to be forgotten is totally random. For random forgetting, we randomly select 10% data from D_{tr} as D_f. We make sure the distribution of D_f is the same as D_{tr}. For CIFAR-20 in group forgetting, each fine-grained class is in the same size which is 500. The coarse class is 2500.

[2] https://github.com/TY-LEE-KR/Boundary-Unlearning-Code

[3] https://github.com/OPTML-Group/Unlearn-Sparse

[4] https://github.com/meghdadk/SCRUB/tree/main

[5] https://github.com/OPTML-Group/Unlearn-Saliency

Table 9: The experiment results of class-wise forgetting in 20 Newsgroups and SVHN datasets.

20 Newsgroups	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$98.31_{\pm 2.56}$	$81.95_{\pm 1.69}$	-	26.25
FT	$4.14_{\pm 2.11}$	$9.23_{\pm 3.40}$	$98.83_{\pm 0.86}$	$82.63_{\pm 0.73}$	46.96	1.77
GA	$17.12_{\pm 9.48}$	$62.03_{\pm 5.84}$	$99.99_{\pm 0.01}$	$85.41_{\pm 0.37}$	31.50	**0.37**
IU	$0.00_{\pm 0.00}$	$0.25_{\pm 0.12}$	$100.00_{\pm 0.00}$	$85.58_{\pm 0.20}$	51.27	1.52
BS	$78.33_{\pm 3.47}$	$92.63_{\pm 2.19}$	$97.28_{\pm 0.99}$	$90.93_{\pm 0.81}$	9.76	1.42
UGradSL	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$96.31_{\pm 4.02}$	$78.54_{\pm 5.10}$	1.35	0.39
UGradSL+	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$99.76_{\pm 0.23}$	$84.21_{\pm 0.41}$	**0.93**	2.13
SVHN	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$100.00_{\pm 0.01}$	$95.94_{\pm 0.11}$	-	37.05
FT	$6.49_{\pm 1.49}$	$99.98_{\pm 0.04}$	$100.00_{\pm 0.01}$	$96.08_{\pm 0.01}$	23.42	2.42
GA	$87.49_{\pm 1.94}$	$99.85_{\pm 0.09}$	$99.52_{\pm 0.03}$	$95.27_{\pm 0.21}$	3.45	**0.15**
IU	$93.55_{\pm 2.78}$	$100.00_{\pm 0.00}$	$99.54_{\pm 0.03}$	$95.64_{\pm 0.31}$	1.80	0.23
BE	$85.56_{\pm 3.07}$	$99.98_{\pm 0.02}$	$99.55_{\pm 0.01}$	$95.53_{\pm 0.07}$	3.83	3.17
BS	$96.62_{\pm 1.14}$	$99.95_{\pm 0.09}$	$99.99_{\pm 0.00}$	$95.39_{\pm 0.18}$	1.00	3.91
ℓ_1-sparse	$99.78_{\pm 0.31}$	$100.00_{\pm 0.00}$	$98.63_{\pm 0.01}$	$97.36_{\pm 0.18}$	0.75	2.91
RL	$99.99_{\pm 0.01}$	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$95.44_{\pm 0.13}$	0.13	3.53
EU-k	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$99.61_{\pm 0.08}$	$65.56_{\pm 2.38}$	7.69	4.93
CF-k	$0.09_{\pm 0.03}$	$2.18_{\pm 2.21}$	$99.34_{\pm 0.02}$	$69.87_{\pm 4.13}$	56.12	5.02
SCRUB	$99.99_{\pm 0.02}$	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$95.79_{\pm 0.26}$	**0.04**	4.97
SalUN	$99.74_{\pm 0.39}$	$100.00_{\pm 0.00}$	$99.53_{\pm 0.02}$	$95.00_{\pm 1.50}$	0.42	4.77
UGradSL	$90.71_{\pm 4.08}$	$99.90_{\pm 0.16}$	$99.54_{\pm 0.04}$	$95.64_{\pm 0.25}$	2.54	0.23
UGradSL+	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$99.82_{\pm 0.62}$	$94.35_{\pm 0.70}$	0.44	4.56

E.3 Additional Class-Wise Forgetting Results

We present the performance of class-wise forgetting in 20 Newsgroups and SVHN datasets in Table 9. The observation is similar in CIFAR-100 and ImageNet given in Table 1. UGradSL and UGradSL+ can improve the MU performance with acceptable time increment, showing the generalization of the proposed method in different modalities and different dataset sizes.

E.4 Additional Random Forgetting Results

We present the performance of random forgetting in CIFAR-10 and SVHN datasets in Table 10. The observation is similar in CIFAR-100 and Tiny ImageNet given in Table 2.

E.5 MU with the Other Classifier

To validate the generalization of the proposed method, we also try the other classification models. We test vision transformer (ViT) and VGG-16 on the task of class-wise forgetting and random forgetting using CIFAR-10, respectively. The results are given in Tables 11 and 12. The observation is similar in Tables 1 and 2, respectively.

Table 11: The experiment results of class-wise forgetting in CIFAR-10 using ViT.

CIFAR-10	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	$100.00_{\pm 0.00}$	$100.00_{\pm 0.00}$	$61.41_{\pm 0.81}$	$58.94_{\pm 1.09}$	-	189.08
FT	$3.97_{\pm 0.87}$	$7.60_{\pm 1.76}$	$98.29_{\pm 0.05}$	$80.44_{\pm 0.22}$	61.70	2.99
GA	$33.77_{\pm 6.36}$	$40.47_{\pm 6.63}$	$89.47_{\pm 4.21}$	$71.65_{\pm 2.79}$	41.63	0.32
IU	$1.74_{\pm 0.09}$	$2.16_{\pm 0.61}$	$73.96_{\pm 0.01}$	$68.88_{\pm 0.00}$	54.65	0.24
BE	$85.56_{\pm 3.07}$	$99.98_{\pm 0.02}$	$99.55_{\pm 0.01}$	$95.53_{\pm 0.07}$	22.30	3.17
UGradSL	$68.11_{\pm 11.03}$	$73.84_{\pm 9.58}$	$84.11_{\pm 2.70}$	$68.33_{\pm 1.69}$	22.54	**0.22**
UGradSL+	$99.99_{\pm 0.01}$	$99.99_{\pm 0.02}$	$94.46_{\pm 1.06}$	$77.26_{\pm 1.19}$	**12.85**	5.86

Table 10: The experiment results of random forgetting in CIFAR-10 and SVHN.

CIFAR-10	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	$8.07_{\pm 0.47}$	$17.41_{\pm 0.69}$	$100.00_{\pm 0.01}$	$91.61_{\pm 0.24}$	-	24.66
FT	$1.10_{\pm 0.19}$	$4.06_{\pm 0.41}$	$99.83_{\pm 0.03}$	$93.70_{\pm 0.10}$	5.65	1.58
GA	$0.56_{\pm 0.01}$	$1.19_{\pm 0.05}$	$99.48_{\pm 0.02}$	$94.55_{\pm 0.05}$	6.80	**0.31**
IU	$17.51_{\pm 2.19}$	$21.39_{\pm 1.70}$	$83.28_{\pm 2.44}$	$78.13_{\pm 2.85}$	10.91	1.18
BE	$0.00_{\pm 0.00}$	$0.26_{\pm 0.02}$	$100.00_{\pm 0.00}$	$95.35_{\pm 0.18}$	7.24	3.17
BS	$0.48_{\pm 0.07}$	$1.16_{\pm 0.04}$	$99.47_{\pm 0.01}$	$94.58_{\pm 0.03}$	6.84	1.41
ℓ_1-sparse	$1.21_{\pm 0.38}$	$4.33_{\pm 0.52}$	$97.39_{\pm 0.31}$	$95.49_{\pm 0.18}$	6.61	1.82
SCRUB	$0.70_{\pm 0.59}$	$3.88_{\pm 1.25}$	$99.59_{\pm 0.34}$	$94.22_{\pm 0.26}$	5.98	4.05
Random Label	$2.80_{\pm 0.37}$	$18.59_{\pm 3.48}$	$99.97_{\pm 0.01}$	$94.08_{\pm 0.12}$	2.24	1.98
UGradSL	$5.87_{\pm 0.51}$	$13.33_{\pm 0.70}$	$98.82_{\pm 0.28}$	$92.17_{\pm 0.23}$	2.01	0.45
UGradSL+	$6.03_{\pm 0.17}$	$10.65_{\pm 0.13}$	$99.79_{\pm 0.03}$	$93.64_{\pm 0.16}$	2.76	3.07
UGradSL (Adp)	$6.04_{\pm 0.11}$	$13.75_{\pm 0.32}$	$99.11_{\pm 0.01}$	$92.07_{\pm 0.02}$	1.76	1.35
UGradSL+ (Adp)	$7.54_{\pm 0.43}$	$13.57_{\pm 0.12}$	$99.67_{\pm 0.00}$	$92.97_{\pm 0.17}$	**1.52**	9.23
SVHN	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	$4.95_{\pm 0.03}$	$15.59_{\pm 0.93}$	$99.99_{\pm 0.01}$	$95.61_{\pm 0.22}$	-	35.65
FT	$0.45_{\pm 0.14}$	$2.30_{\pm 0.04}$	$99.99_{\pm 0.00}$	$95.78_{\pm 0.01}$	4.49	2.76
GA	$0.58_{\pm 0.04}$	$1.13_{\pm 0.02}$	$99.56_{\pm 0.01}$	$95.62_{\pm 0.01}$	4.82	0.31
FF	$0.45_{\pm 0.09}$	$1.30_{\pm 0.12}$	$99.55_{\pm 0.01}$	$95.49_{\pm 0.03}$	4.84	6.02
BE	$0.00_{\pm 0.02}$	$0.02_{\pm 0.17}$	$100.00_{\pm 0.01}$	$96.14_{\pm 0.02}$	5.27	1.03
BS	$0.45_{\pm 0.14}$	$1.13_{\pm 0.05}$	$99.57_{\pm 0.03}$	$95.66_{\pm 0.01}$	4.86	4.24
ℓ_1-sparse	$3.73_{\pm 0.78}$	$8.44_{\pm 0.34}$	$97.84_{\pm 0.28}$	$96.18_{\pm 0.33}$	2.77	**0.07**
SCRUB	$0.35_{\pm 0.20}$	$4.96_{\pm 0.93}$	$99.94_{\pm 0.02}$	$95.36_{\pm 0.23}$	3.88	3.24
RL	$8.00_{\pm 0.64}$	$29.40_{\pm 11.92}$	$98.72_{\pm 0.45}$	$94.04_{\pm 1.10}$	4.93	1.79
UGradSL	$3.29_{\pm 2.53}$	$14.32_{\pm 4.56}$	$99.89_{\pm 0.02}$	$94.38_{\pm 0.28}$	1.07	0.57
UGradSL+	$5.77_{\pm 2.93}$	$15.95_{\pm 2.26}$	$100.00_{\pm 0.00}$	$95.12_{\pm 0.50}$	0.42	4.44
UGradSL (Adp)	$3.97_{\pm 0.29}$	$14.63_{\pm 2.15}$	$99.89_{\pm 0.01}$	$94.40_{\pm 0.12}$	0.81	2.20
UGradSL+ (Adp)	$5.07_{\pm 0.34}$	$15.89_{\pm 1.03}$	$100.00_{\pm 0.00}$	$95.21_{\pm 0.44}$	**0.21**	14.33

Table 12: The experiment results of random forgetting across all classes in CIFAR-10 using VGG-16

CIFAR-10	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	$11.41_{\pm 0.41}$	$11.97_{\pm 0.50}$	$74.65_{\pm 0.23}$	$66.13_{\pm 0.16}$	-	9.48
FT	$1.32_{\pm 0.13}$	$3.48_{\pm 0.13}$	$74.24_{\pm 0.04}$	$67.04_{\pm 0.10}$	4.98	0.60
GA	$1.35_{\pm 0.08}$	$2.18_{\pm 0.66}$	$73.95_{\pm 0.01}$	$66.88_{\pm 0.01}$	5.33	**0.14**
IU	$1.74_{\pm 0.09}$	$2.16_{\pm 0.61}$	$73.96_{\pm 0.01}$	$68.88_{\pm 0.00}$	5.73	0.24
FF	$1.35_{\pm 0.09}$	$2.21_{\pm 0.58}$	$73.95_{\pm 0.02}$	$66.87_{\pm 0.04}$	5.32	1.02
BE	$0.01_{\pm 0.01}$	$0.23_{\pm 0.05}$	$99.98_{\pm 0.00}$	$94.04_{\pm 0.21}$	19.10	1.09
BS	$0.01_{\pm 0.01}$	$0.22_{\pm 0.03}$	$99.98_{\pm 0.01}$	$94.00_{\pm 0.14}$	19.09	3.17
ℓ_1-sparse	$1.27_{\pm 1.13}$	$3.60_{\pm 2.41}$	$98.97_{\pm 1.13}$	$92.18_{\pm 1.46}$	17.22	0.08
SCRUB	$61.16_{\pm 50.89}$	$44.65_{\pm 43.31}$	$39.26_{\pm 50.57}$	$36.95_{\pm 46.68}$	36.75	0.91
UGradSL	$13.45_{\pm 0.63}$	$11.77_{\pm 0.54}$	$65.05_{\pm 0.48}$	$58.52_{\pm 0.38}$	**4.86**	0.19
UGradSL+	$12.41_{\pm 0.32}$	$14.96_{\pm 0.52}$	$65.90_{\pm 0.52}$	$58.58_{\pm 0.35}$	5.13	1.08

E.6 STREISAND EFFECT

From the perspective of security, it is important to make the predicted distributions are almost the same from the forgetting set D_f and the testing set D_{te}, which is called Streisand effect. We investigate this effect in the *random forgetting* on CIFAR-10 by plotting confusion matrix as shown in Figure 5. It can be found that our method will not lead to the extra hint of D_f.

Table 13: Ablation studies of GA ratio p for random forgetting on CIFAR-10. The forgetting set size is 10% training set. The method is UGradSL. We fix α as -0.4. The first row is the retraining results for reference.

p	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
-	8.07	17.41	100.00	91.61	-
0.80	$12.47_{\pm 1.01}$	$20.24_{\pm 2.12}$	$94.11_{\pm 0.71}$	$88.21_{\pm 0.57}$	4.13
0.81	$11.57_{\pm 1.69}$	$19.68_{\pm 3.31}$	$94.39_{\pm 1.41}$	$88.56_{\pm 1.04}$	3.61
0.82	$10.61_{\pm 0.08}$	$17.85_{\pm 0.97}$	$94.92_{\pm 0.18}$	$88.94_{\pm 0.37}$	2.68
0.83	$9.64_{\pm 0.52}$	$16.49_{\pm 0.96}$	$95.54_{\pm 0.71}$	$89.63_{\pm 0.71}$	2.23
0.84	$9.33_{\pm 1.04}$	$15.84_{\pm 1.17}$	$95.43_{\pm 0.95}$	$89.48_{\pm 0.71}$	2.38
0.85	$8.27_{\pm 0.82}$	$14.74_{\pm 1.36}$	$96.05_{\pm 0.41}$	$90.08_{\pm 0.40}$	2.09
0.86	$7.96_{\pm 0.42}$	$15.45_{\pm 2.00}$	$96.10_{\pm 0.44}$	$90.22_{\pm 0.18}$	**1.84**
0.87	$7.51_{\pm 0.26}$	$15.26_{\pm 3.47}$	$96.05_{\pm 0.18}$	$90.20_{\pm 0.51}$	2.02
0.88	$6.87_{\pm 0.37}$	$13.18_{\pm 1.26}$	$96.43_{\pm 0.35}$	$90.29_{\pm 0.64}$	2.58
0.89	$6.91_{\pm 0.56}$	$14.44_{\pm 2.46}$	$96.38_{\pm 0.73}$	$90.47_{\pm 0.36}$	2.22
0.90	$6.92_{\pm 1.08}$	$13.60_{\pm 3.42}$	$96.00_{\pm 0.50}$	$90.26_{\pm 0.14}$	2.58
0.91	$6.44_{\pm 1.30}$	$14.16_{\pm 2.27}$	$95.93_{\pm 1.18}$	$90.17_{\pm 0.72}$	2.60
0.92	$6.50_{\pm 0.69}$	$14.35_{\pm 0.72}$	$95.64_{\pm 0.50}$	$90.06_{\pm 0.12}$	2.64
0.93	$5.88_{\pm 0.82}$	$14.84_{\pm 1.26}$	$96.03_{\pm 0.84}$	$90.31_{\pm 0.54}$	2.51
0.94	$5.65_{\pm 0.30}$	$13.55_{\pm 0.78}$	$96.25_{\pm 0.44}$	$90.54_{\pm 0.10}$	2.77
0.95	$6.13_{\pm 1.29}$	$13.14_{\pm 2.43}$	$95.73_{\pm 1.03}$	$89.88_{\pm 0.75}$	3.05
0.96	$6.07_{\pm 0.91}$	$14.28_{\pm 2.15}$	$95.64_{\pm 0.79}$	$90.15_{\pm 0.36}$	2.74
0.97	$5.83_{\pm 1.25}$	$14.07_{\pm 1.98}$	$95.20_{\pm 0.98}$	$89.67_{\pm 0.59}$	3.08
0.98	$5.73_{\pm 0.84}$	$13.19_{\pm 1.99}$	$95.43_{\pm 0.98}$	$89.82_{\pm 0.38}$	3.23
0.99	$5.83_{\pm 1.05}$	$12.98_{\pm 1.37}$	$94.99_{\pm 0.79}$	$89.46_{\pm 0.59}$	3.46

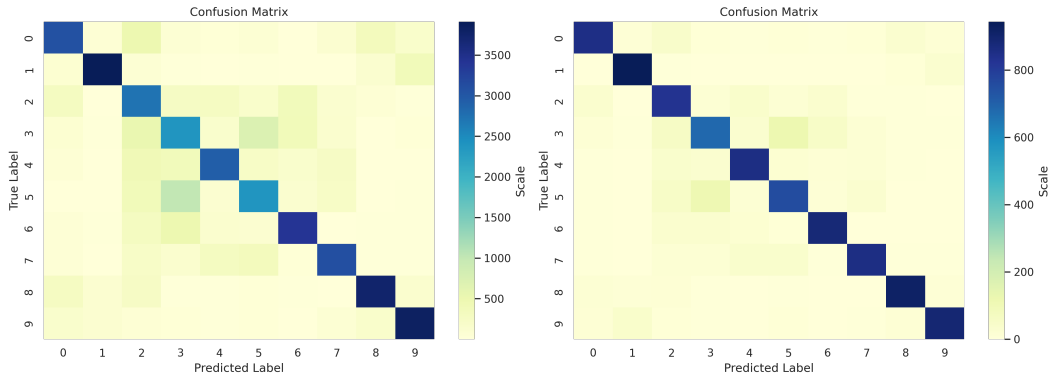


Figure 5: The confusion matrix of testing set and forgetting set D_f using our method on CIFAR-10 with random forgetting across all the classes. There is no big difference between the prediction distribution. Our method will not make D_f more distinguishable.

E.7 Ablation Study: Forgetting Set Size

Since the size of the forgetting set can affect unlearning performance, we further evaluate the robustness of our method under varying forgetting ratios. In addition to the 10% random forgetting results reported in Table 2 and Table 4, we consider forgetting set sizes of 20%, 30%, 40%, and 50% of the training data on CIFAR-10 and CIFAR-100. The results are summarized in Tables 17 and 18.

E.8 Ablation Study: GA Ratio p

In addition to an overview of the performance fluctuation in Figure 3. We provide the specific value of the ablation study regarding GA ratio p. We test the performance on random forgetting on CIFAR-10. The forgetting set size is 10% of the training set. The results of UGradSL and UGradSL+ are given in Tables 13 and 14.

Table 14: Ablation studies of GA ratio p for random forgetting on CIFAR-10. The forgetting set size is 10% training set. The method is UGradSL+. We fix α as -0.4. The first row is the retraining results for reference.

p	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
-	8.07	17.41	100.00	91.61	-
0.80	$18.84_{\pm 0.71}$	$26.78_{\pm 1.78}$	$91.95_{\pm 0.86}$	$86.05_{\pm 1.21}$	8.44
0.81	$17.00_{\pm 0.47}$	$24.55_{\pm 0.76}$	$93.21_{\pm 0.21}$	$87.50_{\pm 0.49}$	6.74
0.82	$16.45_{\pm 0.33}$	$24.22_{\pm 0.46}$	$93.60_{\pm 0.49}$	$87.64_{\pm 0.25}$	6.39
0.83	$14.45_{\pm 0.73}$	$21.73_{\pm 0.82}$	$94.66_{\pm 0.38}$	$88.59_{\pm 0.19}$	4.76
0.84	$13.44_{\pm 0.77}$	$20.92_{\pm 1.15}$	$94.67_{\pm 0.71}$	$88.67_{\pm 0.44}$	4.29
0.85	$12.57_{\pm 0.65}$	$19.18_{\pm 0.64}$	$95.25_{\pm 1.02}$	$89.33_{\pm 1.09}$	3.32
0.86	$11.42_{\pm 0.14}$	$18.34_{\pm 0.61}$	$95.56_{\pm 0.46}$	$89.49_{\pm 0.27}$	2.71
0.87	$10.90_{\pm 0.72}$	$17.22_{\pm 0.51}$	$95.79_{\pm 0.39}$	$89.77_{\pm 0.81}$	2.27
0.88	$10.13_{\pm 0.42}$	$17.85_{\pm 2.11}$	$95.97_{\pm 0.17}$	$90.03_{\pm 0.60}$	2.03
0.89	$8.98_{\pm 0.29}$	$14.94_{\pm 0.09}$	$96.20_{\pm 0.27}$	$90.23_{\pm 0.33}$	2.14
0.90	$8.41_{\pm 0.33}$	$16.87_{\pm 1.17}$	$96.53_{\pm 0.03}$	$90.64_{\pm 0.09}$	1.33
0.91	$8.01_{\pm 0.30}$	$17.33_{\pm 1.17}$	$96.50_{\pm 0.36}$	$90.68_{\pm 0.40}$	**1.14**
0.92	$7.74_{\pm 0.33}$	$15.62_{\pm 1.80}$	$96.28_{\pm 0.26}$	$90.48_{\pm 0.46}$	1.75
0.93	$6.67_{\pm 0.12}$	$15.93_{\pm 0.22}$	$96.86_{\pm 0.10}$	$90.96_{\pm 0.34}$	1.67
0.94	$6.79_{\pm 0.71}$	$16.47_{\pm 0.52}$	$96.42_{\pm 0.83}$	$90.74_{\pm 0.45}$	1.67
0.95	$6.03_{\pm 0.26}$	$14.82_{\pm 1.39}$	$96.76_{\pm 0.41}$	$90.94_{\pm 0.35}$	2.14
0.96	$5.78_{\pm 0.24}$	$14.79_{\pm 1.14}$	$96.90_{\pm 0.19}$	$91.30_{\pm 0.16}$	2.08
0.97	$5.98_{\pm 0.49}$	$14.96_{\pm 0.34}$	$96.56_{\pm 0.45}$	$90.81_{\pm 0.53}$	2.20
0.98	$6.46_{\pm 0.74}$	$15.15_{\pm 1.76}$	$95.52_{\pm 0.67}$	$90.15_{\pm 0.89}$	2.45
0.99	$5.67_{\pm 0.27}$	$14.40_{\pm 1.18}$	$96.17_{\pm 0.46}$	$90.61_{\pm 0.24}$	2.56

Table 15: The ablation study of smoothing rate α for random forgetting on CIFAR-10. The forgetting set size is 10% training set. The method we use is UGradSL. We fix p as 0.9.

α	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
-	8.07	17.41	100.00	91.61	-
−0.9	$8.17_{\pm 1.74}$	$14.96_{\pm 2.47}$	$95.81_{\pm 1.97}$	$89.97_{\pm 1.46}$	**2.10**
−0.8	$6.98_{\pm 0.47}$	$13.41_{\pm 0.99}$	$96.67_{\pm 0.88}$	$90.75_{\pm 0.22}$	2.32
−0.7	$7.23_{\pm 0.56}$	$14.33_{\pm 1.23}$	$96.28_{\pm 0.11}$	$90.47_{\pm 0.26}$	2.20
−0.6	$6.69_{\pm 0.22}$	$12.93_{\pm 0.67}$	$96.46_{\pm 0.39}$	$90.64_{\pm 0.04}$	2.59
−0.5	$6.56_{\pm 0.29}$	$13.00_{\pm 0.50}$	$96.58_{\pm 0.23}$	$90.66_{\pm 0.20}$	2.57
−0.4	$6.92_{\pm 1.08}$	$13.60_{\pm 3.42}$	$96.00_{\pm 0.50}$	$90.26_{\pm 0.14}$	2.58
−0.3	$6.32_{\pm 0.43}$	$13.63_{\pm 0.67}$	$96.18_{\pm 0.41}$	$90.52_{\pm 0.27}$	2.61
−0.2	$6.95_{\pm 0.54}$	$13.98_{\pm 1.99}$	$95.41_{\pm 0.68}$	$89.65_{\pm 0.56}$	2.77
−0.1	$7.13_{\pm 1.44}$	$14.47_{\pm 1.91}$	$95.08_{\pm 1.55}$	$89.57_{\pm 1.04}$	2.71

E.9 Smoothing Ratio α

Similar to p, we report the detailed results regarding the smoothing rate α. The results of UGradSL and UGradSL+ are given in Tables 15 and 16.

E.10 Gradient Analysis

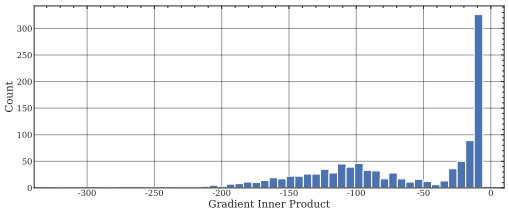
As mentioned in Section 3.3, $\langle \Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f, \Delta\boldsymbol{\theta}_n - \Delta\boldsymbol{\theta}_f \rangle \leq 0$ is always practically valid. We check the results on CelebA dataset (ResNet-18), ImageNet (ViT), CIFAR-100 (VGG-16) and CIFAR-10 (ResNet-18). The distribution of $\langle \Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f, \Delta\boldsymbol{\theta}_n - \Delta\boldsymbol{\theta}_f \rangle$ is shown in Figure 6, which aligns with our assumption.

E.11 The difference between UGradSL and UGradSL+

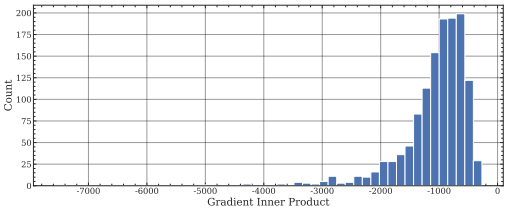
Although UGradSL and UGradSL+ look similar, the intuition of these two method is totally different because of the difference between FT and GA. We conducted experiments to illustrate the difference between GA and FT as well as UGradSL and UGradSL+. The results are given in Table 19. The dataset and forgetting paradigm is CIFAR-10 random forgetting. It can be found that the difference becomes much larger when the number of epochs is over 8. When the number of epochs is 10, the model becomes unusable because TA is less than 10%. We also report the performance of UGradSL

Table 16: The ablation study of smoothing rate α for random forgetting on CIFAR-10. The forgetting set size is 10% training set. The method we use is UGradSL+. We fix p as 0.9.

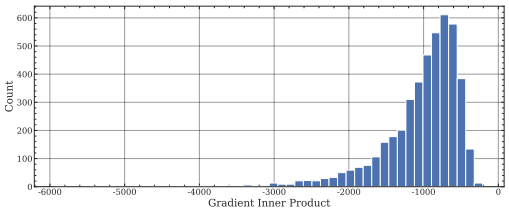
α	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
-	8.07	17.41	100.00	91.61	-
−0.9	$11.59_{\pm 0.40}$	$19.41_{\pm 0.59}$	$95.77_{\pm 0.58}$	$89.47_{\pm 0.56}$	2.97
−0.8	$10.68_{\pm 0.27}$	$18.41_{\pm 0.48}$	$95.94_{\pm 0.24}$	$89.91_{\pm 0.36}$	2.35
−0.7	$10.12_{\pm 1.01}$	$16.88_{\pm 0.69}$	$96.07_{\pm 0.89}$	$90.08_{\pm 0.78}$	2.01
−0.6	$8.98_{\pm 0.15}$	$16.29_{\pm 0.87}$	$96.64_{\pm 0.19}$	$90.75_{\pm 0.05}$	1.56
−0.5	$9.07_{\pm 0.21}$	$15.83_{\pm 0.25}$	$96.65_{\pm 0.34}$	$90.43_{\pm 0.50}$	1.78
−0.4	$8.41_{\pm 0.33}$	$16.87_{\pm 1.17}$	$96.53_{\pm 0.03}$	$90.64_{\pm 0.09}$	**1.33**
−0.3	$8.59_{\pm 0.07}$	$16.86_{\pm 2.15}$	$96.24_{\pm 0.38}$	$90.20_{\pm 0.15}$	1.56
−0.2	$7.55_{\pm 0.18}$	$16.68_{\pm 1.60}$	$96.43_{\pm 0.14}$	$90.86_{\pm 0.33}$	1.39
−0.1	$7.57_{\pm 0.18}$	$17.32_{\pm 0.23}$	$96.15_{\pm 0.38}$	$90.34_{\pm 0.27}$	1.43



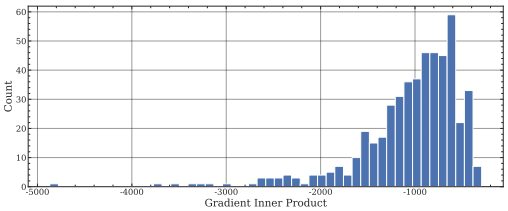
(a) The group forgetting on CelebA using ResNet-18



(b) The class-wise forgetting on ImageNet using ViT



(c) The random forgetting on CIFAR-100 using VGG-16



(d) The class-wise forgetting on CIFAR-10 using ResNet-18

Figure 6: The distribution of $\langle \Delta\boldsymbol{\theta}_r - \Delta\boldsymbol{\theta}_f, \Delta\boldsymbol{\theta}_n - \Delta\boldsymbol{\theta}_f \rangle$ using multiple models on multiple datasets.

and UGradSL+ in different epochs. For UGradSL, when the epochs are over 14, the model cannot be used at all. For UGradSL+, the algorithm is much more stable, showing the very good adaptive capability.

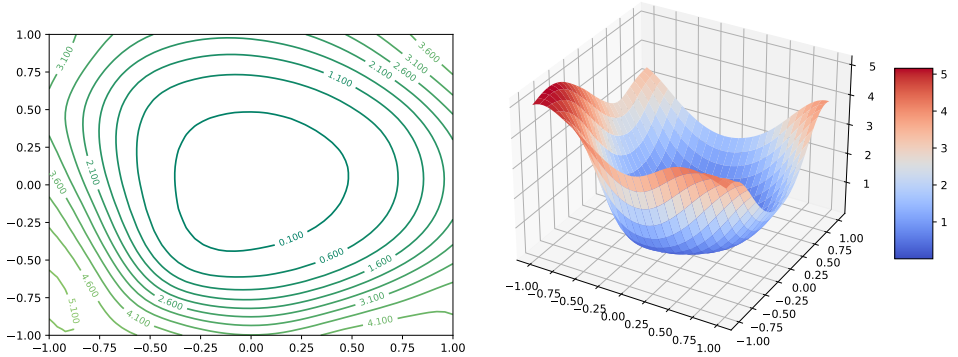


Figure 7: The loss landscape of $\boldsymbol{\theta}_r$ on CIFAR-10 and the model is ResNet-18.

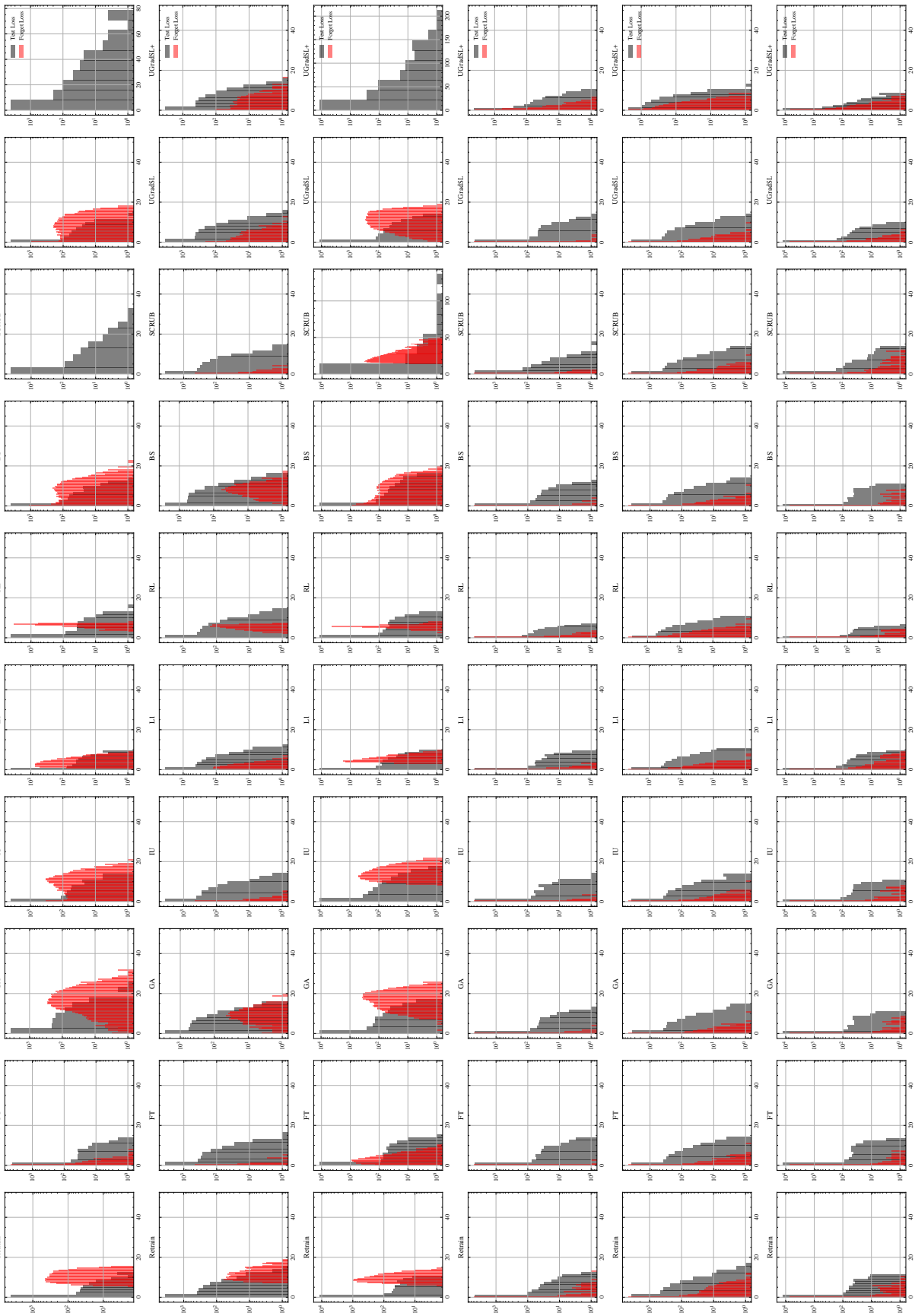


Figure 8: The distributions of the cross-entropy losses for the forget and test instances from the unlearned models. The y-axis is in log scale for better visualization. From the first to the last figure, they are random forgetting on CIFAR-10, CIFAR-100, SVHN and class-wise forgetting on CIFAR-10, CIFAR-100, SVHN.

Table 17: MU Performance across different forgetting data amounts on ResNet-18, pre-trained on CIFAR-10 dataset, for random data forgetting.

Method	Random Set Size (10%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	8.07	17.41	100.00	91.61	-	24.66
FT	$1.10_{\pm 0.19}$	$4.06_{\pm 0.40}$	$98.83_{\pm 0.03}$	$93.70_{\pm 0.10}$	5.90	1.58
RL	$6.39_{\pm 1.09}$	$0.00_{\pm 0.00}$	$99.50_{\pm 0.10}$	$99.04_{\pm 0.08}$	6.76	1.92
GA	$0.56_{\pm 0.01}$	$1.19_{\pm 0.05}$	$99.48_{\pm 0.02}$	$94.55_{\pm 0.05}$	6.80	**0.31**
IU	$17.51_{\pm 2.19}$	$21.39_{\pm 1.70}$	$98.00_{\pm 0.38}$	$98.11_{\pm 0.38}$	5.48	1.18
BE	$0.00_{\pm 0.00}$	$0.26_{\pm 0.02}$	$100.00_{\pm 0.00}$	$95.35_{\pm 0.18}$	7.24	1.37
BS	$0.37_{\pm 0.10}$	$1.10_{\pm 0.43}$	$99.93_{\pm 0.01}$	$98.97_{\pm 0.02}$	7.86	1.21
ℓ_1-sparse	$2.80_{\pm 0.37}$	$19.59_{\pm 3.48}$	$99.07_{\pm 0.04}$	$98.00_{\pm 0.12}$	3.69	1.98
SalUn	$46.95_{\pm 0.15}$	$86.33_{\pm 2.58}$	$97.75_{\pm 0.42}$	$97.22_{\pm 0.77}$	28.92	2.42
UGradSL	$5.87_{\pm 0.50}$	$13.33_{\pm 0.20}$	$98.82_{\pm 0.28}$	$92.17_{\pm 0.20}$	**2.01**	0.45
UGradSL+	$6.03_{\pm 0.17}$	$10.65_{\pm 0.13}$	$99.79_{\pm 0.03}$	$93.64_{\pm 0.16}$	2.76	3.07

Method	Random Set Size (20%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	5.31	13.30	100.00	94.10	-	38.74
FT	$0.76_{\pm 4.55}$	$2.69_{\pm 10.61}$	$99.89_{\pm 0.11}$	$93.97_{\pm 0.13}$	3.85	2.17
RL	$6.47_{\pm 1.16}$	$28.62_{\pm 15.32}$	$99.60_{\pm 0.40}$	$92.39_{\pm 1.71}$	4.65	2.65
GA	$0.67_{\pm 4.64}$	$1.44_{\pm 11.86}$	$99.48_{\pm 0.52}$	$94.42_{\pm 0.32}$	4.33	0.26
IU	$2.91_{\pm 2.40}$	$5.53_{\pm 7.77}$	$97.30_{\pm 2.70}$	$90.64_{\pm 3.46}$	4.08	3.29
BE	$0.57_{\pm 4.74}$	$1.64_{\pm 11.66}$	$99.44_{\pm 0.56}$	$94.32_{\pm 0.22}$	4.29	0.53
BS	$0.62_{\pm 4.69}$	$1.62_{\pm 11.68}$	$99.46_{\pm 0.54}$	$94.20_{\pm 0.10}$	4.25	0.86
ℓ_1-sparse	$3.92_{\pm 1.39}$	$8.94_{\pm 4.36}$	$98.09_{\pm 1.91}$	$91.92_{\pm 2.18}$	2.46	2.20
SalUn	$3.73_{\pm 1.58}$	$13.18_{\pm 0.12}$	$98.61_{\pm 1.39}$	$92.75_{\pm 1.35}$	1.11	2.66
UGradSL	$6.07_{\pm 0.70}$	$13.82_{\pm 1.03}$	$95.71_{\pm 0.17}$	$90.19_{\pm 0.23}$	2.37	**0.24**
UGradSL+	$6.39_{\pm 0.19}$	$12.34_{\pm 1.79}$	$97.08_{\pm 0.44}$	$90.91_{\pm 0.95}$	**2.04**	0.31

Method	Random Set Size (30%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	6.64	14.60	100.00	92.78	-	33.65
FT	$0.56_{\pm 6.08}$	$1.66_{\pm 12.94}$	$99.83_{\pm 0.17}$	$94.22_{\pm 1.44}$	5.16	1.98
RL	$6.89_{\pm 0.25}$	$31.09_{\pm 16.49}$	$99.36_{\pm 0.64}$	$91.35_{\pm 1.43}$	4.70	2.63
GA	$0.65_{\pm 5.99}$	$1.50_{\pm 13.10}$	$99.46_{\pm 0.54}$	$94.44_{\pm 1.66}$	5.32	2.40
IU	$3.95_{\pm 2.69}$	$7.26_{\pm 7.34}$	$96.22_{\pm 3.78}$	$89.61_{\pm 3.17}$	4.24	3.32
BE	$0.63_{\pm 6.01}$	$3.35_{\pm 11.25}$	$99.39_{\pm 0.61}$	$94.19_{\pm 1.41}$	4.82	0.81
BS	$0.63_{\pm 6.01}$	$2.88_{\pm 11.72}$	$99.39_{\pm 0.61}$	$94.15_{\pm 1.37}$	4.93	1.28
ℓ_1-sparse	$4.70_{\pm 1.94}$	$9.97_{\pm 4.63}$	$97.63_{\pm 2.37}$	$91.19_{\pm 1.59}$	2.63	1.99
SalUn	$6.22_{\pm 0.42}$	$14.11_{\pm 0.49}$	$95.91_{\pm 4.09}$	$90.72_{\pm 2.06}$	1.76	2.64
UGradSL	$6.78_{\pm 0.66}$	$15.96_{\pm 0.12}$	$96.94_{\pm 0.56}$	$90.72_{\pm 0.80}$	1.66	**0.70**
UGradSL+	$6.36_{\pm 0.65}$	$14.99_{\pm 0.82}$	$97.35_{\pm 0.79}$	$91.10_{\pm 1.10}$	**1.25**	0.53

Method	Random Set Size (40%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	7.01	18.37	100.00	92.52	-	28.47
FT	$0.77_{\pm 6.24}$	$2.88_{\pm 15.49}$	$99.96_{\pm 0.04}$	$94.27_{\pm 1.75}$	5.88	1.62
RL	$5.02_{\pm 1.99}$	$37.76_{\pm 19.39}$	$99.61_{\pm 0.39}$	$92.14_{\pm 0.38}$	5.54	2.68
GA	$0.67_{\pm 6.34}$	$1.57_{\pm 16.80}$	$99.47_{\pm 0.53}$	$94.38_{\pm 1.86}$	6.38	**0.53**
IU	$7.89_{\pm 0.88}$	$10.99_{\pm 7.38}$	$92.21_{\pm 7.79}$	$86.15_{\pm 6.37}$	5.60	3.27
BE	$0.86_{\pm 6.15}$	$15.72_{\pm 2.65}$	$99.27_{\pm 0.73}$	$93.46_{\pm 0.94}$	2.62	1.04
BS	$1.18_{\pm 5.83}$	$13.97_{\pm 4.40}$	$98.94_{\pm 1.06}$	$93.01_{\pm 0.49}$	2.95	1.72
ℓ_1-sparse	$2.84_{\pm 4.17}$	$7.09_{\pm 11.28}$	$98.75_{\pm 1.25}$	$92.20_{\pm 0.32}$	4.26	1.63
SalUn	$6.86_{\pm 0.15}$	$15.15_{\pm 3.22}$	$95.01_{\pm 4.99}$	$89.76_{\pm 2.76}$	2.78	2.67
UGradSL	$5.81_{\pm 0.11}$	$14.98_{\pm 2.65}$	$97.31_{\pm 1.06}$	$90.73_{\pm 0.48}$	**2.27**	0.62
UGradSL+	$5.82_{\pm 0.37}$	$14.53_{\pm 1.83}$	$97.11_{\pm 0.40}$	$90.74_{\pm 0.38}$	2.42	0.63

Method	Random Set Size (50%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	7.91	19.29	100.00	91.72	-	23.90
FT	$0.44_{\pm 7.47}$	$2.15_{\pm 17.14}$	$99.96_{\pm 0.04}$	$94.23_{\pm 2.51}$	6.79	1.31
RL	$7.61_{\pm 0.30}$	$37.36_{\pm 18.07}$	$99.67_{\pm 0.33}$	$92.83_{\pm 1.11}$	4.95	2.65
GA	$0.40_{\pm 7.51}$	$1.22_{\pm 18.07}$	$99.61_{\pm 0.39}$	$94.34_{\pm 2.62}$	7.15	**0.66**
IU	$3.97_{\pm 3.94}$	$7.29_{\pm 12.00}$	$96.21_{\pm 3.79}$	$90.00_{\pm 1.72}$	5.36	3.25
BE	$3.08_{\pm 4.83}$	$24.87_{\pm 5.58}$	$96.84_{\pm 3.16}$	$90.41_{\pm 1.31}$	3.72	1.31
BS	$9.76_{\pm 1.85}$	$32.15_{\pm 12.86}$	$90.19_{\pm 9.81}$	$83.71_{\pm 8.01}$	8.13	2.12
ℓ_1-sparse	$1.44_{\pm 6.47}$	$4.76_{\pm 14.53}$	$99.52_{\pm 0.48}$	$93.13_{\pm 1.41}$	5.72	1.31
SalUn	$7.75_{\pm 0.16}$	$16.99_{\pm 2.30}$	$94.28_{\pm 5.72}$	$89.29_{\pm 2.43}$	2.65	2.68
UGradSL	$6.83_{\pm 0.23}$	$12.73_{\pm 1.66}$	$97.62_{\pm 0.71}$	$90.27_{\pm 0.55}$	2.87	0.77
UGradSL+	$6.13_{\pm 1.35}$	$16.49_{\pm 2.73}$	$97.84_{\pm 0.34}$	$90.84_{\pm 0.69}$	**1.91**	0.77

Table 18: MU Performance across different forgetting data amounts on ResNet-18, pre-trained on CIFAR-100 dataset, for random data forgetting.

Method	Random Set Size (10%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	29.47	53.50	99.98	70.51	-	25.01
FT	$2.55_{\pm 0.03}$	$10.59_{\pm 0.27}$	$99.95_{\pm 0.01}$	$75.95_{\pm 0.05}$	18.83	1.95
RL	$4.06_{\pm 0.37}$	$50.12_{\pm 3.48}$	$99.92_{\pm 0.02}$	$71.30_{\pm 0.36}$	7.41	1.20
GA	$2.58_{\pm 0.06}$	$5.95_{\pm 0.17}$	$97.45_{\pm 0.02}$	$76.09_{\pm 0.01}$	20.64	0.29
IU	$15.71_{\pm 5.19}$	$18.69_{\pm 4.12}$	$84.65_{\pm 5.19}$	$62.20_{\pm 4.17}$	18.05	1.20
BE	$0.01_{\pm 0.00}$	$1.45_{\pm 0.02}$	$98.22_{\pm 1.26}$	$78.26_{\pm 0.00}$	22.76	**0.24**
BS	$2.20_{\pm 2.11}$	$10.73_{\pm 9.37}$	$98.22_{\pm 1.26}$	$70.23_{\pm 1.67}$	18.02	0.34
ℓ_1-sparse	$8.19_{\pm 0.38}$	$19.11_{\pm 0.52}$	$88.39_{\pm 0.31}$	$80.26_{\pm 0.16}$	19.25	1.00
SalUn	$35.23_{\pm 0.32}$	$89.39_{\pm 0.46}$	$99.53_{\pm 0.04}$	$64.26_{\pm 0.58}$	12.10	3.33
UGradSL	$18.36_{\pm 0.17}$	$40.71_{\pm 0.13}$	$98.38_{\pm 0.03}$	$68.23_{\pm 0.16}$	6.95	0.55
UGradSL+	$21.69_{\pm 0.59}$	$49.47_{\pm 1.25}$	$99.87_{\pm 0.34}$	$73.60_{\pm 0.26}$	**3.75**	3.52

Method	Random Set Size (20%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	26.84	52.41	99.99	73.88	-	36.88
FT	$2.70_{\pm 24.14}$	$11.63_{\pm 40.78}$	$99.95_{\pm 0.04}$	$75.51_{\pm 1.63}$	16.65	2.05
RL	$54.74_{\pm 27.90}$	$97.32_{\pm 44.91}$	$99.47_{\pm 0.52}$	$65.59_{\pm 8.29}$	20.41	2.11
GA	$6.79_{\pm 20.05}$	$13.22_{\pm 39.19}$	$94.11_{\pm 5.88}$	$71.39_{\pm 2.49}$	16.90	**0.26**
IU	$5.34_{\pm 21.50}$	$11.79_{\pm 40.62}$	$95.54_{\pm 4.45}$	$70.89_{\pm 2.99}$	17.39	3.77
BE	$2.51_{\pm 24.33}$	$6.70_{\pm 45.71}$	$97.38_{\pm 2.61}$	$75.07_{\pm 1.19}$	18.46	0.49
BS	$2.53_{\pm 24.31}$	$6.57_{\pm 45.84}$	$97.38_{\pm 2.61}$	$75.05_{\pm 1.17}$	18.48	0.82
ℓ_1-sparse	$37.83_{\pm 10.99}$	$38.90_{\pm 13.51}$	$76.63_{\pm 23.36}$	$58.79_{\pm 15.09}$	15.74	2.05
SalUn	$25.83_{\pm 1.01}$	$64.69_{\pm 12.28}$	$96.01_{\pm 3.98}$	$65.87_{\pm 8.01}$	6.32	2.12
UGradSL	$30.10_{\pm 1.03}$	$47.39_{\pm 1.17}$	$93.49_{\pm 0.24}$	$64.99_{\pm 0.04}$	**5.92**	0.83
UGradSL+	$27.29_{\pm 0.99}$	$35.92_{\pm 0.94}$	$93.36_{\pm 0.03}$	$66.59_{\pm 0.37}$	7.71	0.59

Method	Random Set Size (30%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	28.52	52.24	99.98	70.91	-	32.92
FT	$2.65_{\pm 25.87}$	$11.18_{\pm 41.06}$	$99.94_{\pm 0.04}$	$75.17_{\pm 4.26}$	17.81	1.44
RL	$51.46_{\pm 22.94}$	$96.34_{\pm 44.10}$	$99.32_{\pm 0.66}$	$62.77_{\pm 8.14}$	18.96	2.14
GA	$2.40_{\pm 26.12}$	$5.70_{\pm 46.54}$	$97.39_{\pm 2.59}$	$75.33_{\pm 4.42}$	19.92	**0.40**
IU	$5.96_{\pm 22.56}$	$12.63_{\pm 39.61}$	$94.59_{\pm 5.39}$	$69.74_{\pm 1.17}$	17.18	3.76
BE	$2.44_{\pm 26.08}$	$6.53_{\pm 45.71}$	$97.37_{\pm 2.61}$	$74.77_{\pm 3.86}$	19.56	0.76
BS	$2.49_{\pm 26.03}$	$6.40_{\pm 45.84}$	$97.33_{\pm 2.65}$	$74.65_{\pm 3.74}$	19.56	1.24
ℓ_1-sparse	$38.45_{\pm 9.93}$	$38.52_{\pm 13.72}$	$76.36_{\pm 23.62}$	$58.09_{\pm 12.82}$	15.02	1.47
SalUn	$27.34_{\pm 1.18}$	$62.99_{\pm 10.75}$	$94.50_{\pm 5.48}$	$63.10_{\pm 7.81}$	6.31	2.16
UGradSL	$30.10_{\pm 0.12}$	$47.39_{\pm 2.08}$	$93.49_{\pm 0.74}$	$64.99_{\pm 1.53}$	**4.71**	0.83
UGradSL+	$24.89_{\pm 0.24}$	$44.60_{\pm 0.94}$	$94.90_{\pm 0.88}$	$66.16_{\pm 0.78}$	5.28	0.79

Method	Random Set Size (40%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	30.07	58.06	99.99	69.87	-	28.29
FT	$2.66_{\pm 27.41}$	$11.05_{\pm 47.01}$	$99.95_{\pm 0.04}$	$75.35_{\pm 5.48}$	19.99	1.51
RL	$51.75_{\pm 21.68}$	$95.78_{\pm 37.72}$	$99.27_{\pm 0.72}$	$59.41_{\pm 10.46}$	17.64	2.12
GA	$2.46_{\pm 27.61}$	$5.91_{\pm 52.15}$	$97.39_{\pm 2.60}$	$75.40_{\pm 5.53}$	21.97	**0.51**
IU	$4.58_{\pm 25.49}$	$10.32_{\pm 47.74}$	$96.29_{\pm 3.70}$	$70.92_{\pm 1.05}$	19.49	3.78
BE	$2.54_{\pm 27.53}$	$7.44_{\pm 50.62}$	$97.35_{\pm 2.64}$	$74.56_{\pm 4.69}$	21.37	1.00
BS	$2.70_{\pm 27.37}$	$7.63_{\pm 50.43}$	$97.26_{\pm 2.73}$	$74.10_{\pm 4.23}$	21.19	1.66
ℓ_1-sparse	$38.49_{\pm 8.42}$	$40.21_{\pm 17.85}$	$78.43_{\pm 21.56}$	$57.66_{\pm 12.21}$	15.01	1.52
SalUn	$25.54_{\pm 4.53}$	$60.08_{\pm 2.02}$	$94.64_{\pm 5.35}$	$62.52_{\pm 7.35}$	4.81	2.14
UGradSL	$30.07_{\pm 1.58}$	$49.23_{\pm 1.07}$	$95.30_{\pm 0.34}$	$64.52_{\pm 0.28}$	**4.72**	1.08
UGradSL+	$30.42_{\pm 0.77}$	$45.94_{\pm 1.41}$	$93.98_{\pm 0.50}$	$63.21_{\pm 0.35}$	6.29	0.77

Method	Random Set Size (50%)					
	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	RTE (↓, min)
Retrain	32.69	61.15	99.99	67.22	-	25.01
FT	$2.71_{\pm 29.98}$	$10.71_{\pm 50.44}$	$99.96_{\pm 0.03}$	$75.11_{\pm 7.89}$	22.08	1.25
RL	$50.52_{\pm 17.83}$	$95.91_{\pm 34.76}$	$99.47_{\pm 0.52}$	$56.75_{\pm 10.47}$	15.90	2.13
GA	$2.61_{\pm 30.08}$	$5.92_{\pm 55.23}$	$97.49_{\pm 2.50}$	$75.27_{\pm 8.05}$	23.97	**0.66**
IU	$12.64_{\pm 20.05}$	$17.54_{\pm 43.61}$	$87.96_{\pm 12.03}$	$62.76_{\pm 4.46}$	20.04	3.80
BE	$2.76_{\pm 29.93}$	$8.85_{\pm 52.30}$	$97.39_{\pm 2.60}$	$74.05_{\pm 6.83}$	22.92	1.26
BS	$2.99_{\pm 29.70}$	$8.76_{\pm 52.39}$	$97.24_{\pm 2.75}$	$73.38_{\pm 6.16}$	22.75	2.08
ℓ_1-sparse	$39.86_{\pm 7.17}$	$40.43_{\pm 20.72}$	$78.17_{\pm 21.82}$	$55.65_{\pm 11.57}$	15.32	1.26
SalUn	$26.17_{\pm 6.52}$	$59.47_{\pm 1.68}$	$94.04_{\pm 5.95}$	$61.39_{\pm 5.83}$	5.00	2.13
UGradSL	$33.80_{\pm 1.61}$	$57.38_{\pm 2.31}$	$95.29_{\pm 0.11}$	$56.88_{\pm 0.80}$	**4.98**	0.95
UGradSL+	$32.20_{\pm 0.49}$	$49.20_{\pm 1.44}$	$94.47_{\pm 0.69}$	$61.53_{\pm 0.97}$	5.91	0.75

Table 19: The difference between GA and FT as well as UGradSL and UGradSL+ on CIFAR-10 regarding the number of epochs. The forgetting paradigm is random forgetting.

	Gradient Ascent					Fine-tuning				
Epoch	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
5	0	0.32	95.31	100	9.56	0.04	0.34	95.13	99.99	9.59
6	0	0.40	95.34	100	9.53	-	-	-	-	-
7	0.82	2.22	93.24	99.26	9.21	-	-	-	-	-
8	3.44	4.78	90.80	96.18	7.76	-	-	-	-	-
9	10.34	12.76	83.42	89.00	6.53	-	-	-	-	-
10	76.26	72.22	6.49	24.24	70.97	0.04	0.24	94.97	99.99	9.65
15	-	-	-	-	-	0.02	0.80	94.68	99.96	9.58
	UGradSL					UGradSL+				
Epoch	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)	UA	$\text{MIA}_{\text{Score}}$	RA	TA	Avg. Gap (↓)
10	14.98	33.22	77.18	84.07	13.27	6.26	14.10	93.39	99.62	4.94
11	24.26	34.38	68.22	75.06	20.37	6.52	11.66	93.04	99.37	5.51
12	28.70	24.62	68.17	74.39	19.22	21.46	27.38	89.41	97.07	9.85
13	38.46	72.90	61.78	64.72	37.75	29.48	31.92	87.74	94.93	12.88
14	99.86	86.74	0.45	0.20	88.02	31.62	32.68	86.53	93.36	13.51
Retrain	8.07	17.41	100.00	91.61	-	8.07	17.41	100.00	91.61	-