SUPPLEMENTARY MATERIAL FOR: EFFICIENT APPROXIMATION OF NEURAL POPULATION STRUCTURE AND CORRELATIONS WITH PROBABILISTIC CIRCUITS

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In the EIF network, each neuron's membrane voltage, $V_i (1 \leq i \leq N)$ evolves as follows:

$$\begin{aligned} \tau_m V_i' &= -V_i + \Delta_T e^{\frac{V_i - V_S}{\Delta_T}} + I_i(t), \\ I_i(t) &= \gamma + \sqrt{\sigma^2 \tau_m}\big[\sqrt{1-\lambda}\xi_i(t) + \sqrt{\lambda}\xi_c(t)\big]. \end{aligned} \tag{1}$$

When the membrane voltage V_i reaches the soft threshold $V_S = -53$ mV, it starts to diverge rapidly until it hits the hard threshold of $V_H = 20$ mV. At that point, the neuron spikes and the membrane voltage resets to $V_R = -60$ mV. The voltage stays at V_R for the refractory period of 3 ms. The input current $I_i(t)$ has a DC level of $\gamma = -60$ mv, and a white noise term with amplitude of $\sigma = 6.23$ mV. The white noise consists of a common input for all neurons, $\xi_c(t)$ and also an independent one, $\xi_i(t)$ for neuron i. Parameter λ controls the correlation between neurons by changing the weights of these two inputs. Other parameters are membrane time constant $\tau_m = 5$ ms, and $\Delta_T = 3$ mv which gives the spike initiation slope. We simulated this network with the backward Euler method, using time bins of 1 ms.

To simulate a heterogeneous network, we used various voltage membrane voltage V_R (to change firing rate) and correlation parameter λ (to change dependency/correlation with other neurons) for different subsets of the population. Specifically, neurons were divided into 4 groups with $V_R = [-58, -59, -60, -61]$ mv and $\lambda = [.25, .30, .45, .59]$, respectively. Neuron i and j had the same parameter if and only if $i = j \pmod 4$. Moreover, when $\lambda_2 > 0$, we paired neurons with $V_R = -58$ mv with neurons with $V_R = -59$ mv, and neurons with $V_R = -60$ mv with neurons with $V_R = -61$ mv.

For simulating networks with heterogeneous connectivity we used the well-known connected "balanced" network of excitatory and inhibitory spiking neurons from Brunel (2000). This network is composed of $N_E = 10000$ excitatory neurons and $N_I = 2500$ inhibitory neurons. The dynamics of the membrane voltage of each neuron is governed by:

$$\tau_m V_i' = -V_i + RI_i(t), \quad RI_i(t) = \tau_m \sum_j J_{ij} \sum_k \delta(t - t_j^k - D), \tag{2}$$

where $I_i(t)$ are the synaptic currents caused by spikes arriving at synapses of neuron i and R is the membrane resistance. $\tau_m = 20\text{ms}$ and the synaptic delay $D = 1.5\text{ms}$. J_{ij} gives the PSP amplitudes for each synapse. Excitatory synapses have $J_{ij} = J = 0.1\text{mV}$ and inhibitory synapses have $J_{ij} = -gJ$, where $J = 0.1\text{mV}$. Each neuron has synapses from a randomly chosen fraction $\epsilon = 0.1$ of neurons from the excitatory and inhibitory populations. External synapses are driven by a Poisson process with a given rate ν_{ext}. When the membrane potential reaches a given threshold $\theta = 20\text{mV}$, it is reset to a refractory potential $V_r = 10\text{mV}$ and the neuron enters a refractory period of fixed duration $\tau_{rp} = 2\text{ms}$ during which it ignores incoming stimulation. We specifically fit the models to two stable setups (figure 8, c and d) of the original paper with "stationary" and "slow oscillation" global activity Brunel (2000). These configurations correspond to $g = 5, \nu_{ext} = 20\text{Hz}$ and $g = 4.5, \nu_{ext} = 9\text{Hz}$, respectively.

We used the Bernoulli RBM implementaion of scikit-learn library (Pedregosa et al., 2011) with the default learning rate of .1.

REFERENCES

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