

Supplementary Materials of Submission 12088

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1 A more complicated example of our forward chaining algorithm

2 We next illustrate the graph construction and the corresponding forward chaining algorithm by a
3 more complicated example. Consider the following rule set $\Gamma_{\mathcal{R}}$:

$$\{\neg A \wedge B \Rightarrow \neg M, A \wedge \neg B \Rightarrow \neg N, A \wedge \neg B \Rightarrow P, \neg M \wedge N \Rightarrow Q, \neg N \wedge P \Rightarrow K\}$$

4 We first obtain all the literals as $\text{Lit}_{\mathcal{R}} = \{A, \neg A, B, \neg B, \neg M, N, \neg N, P, Q, K\}$, construct the
5 literal nodes correspondingly, and construct the LHS nodes as $\{\neg A \wedge B, A \wedge \neg B, \neg M \wedge N, \neg N \wedge P\}$
6 according to $\Gamma_{\mathcal{R}}$. Then, directed edges are added to represent the relationships between LHS and
7 RHS, leading to the construction of the graph illustrated in Figure 1a.

8 Assume that the initially defined literals are $\text{Lit} = \{A, B, \neg M\}$ with truth value as $A = \neg M = \text{True}$
9 and $B = \text{False}$, then we have the initial node set with value True as $\text{Lit}^{\uparrow} = \{A, \neg B, \neg M\}$ (marked
10 in green) and the initial false node set with value False as $\text{Lit}^{\downarrow} = \{\neg A, B\}$ (marked in red) in
11 Figure 1b. Since $\{A, \neg B\} \subseteq \text{Lit}^{\uparrow}$, by applying one time of modus ponens inference rule on
12 $A \wedge \neg B \Rightarrow \neg N$ and $A \wedge \neg B \Rightarrow P$, we can update Lit^{\uparrow} as $\{A, \neg B, \neg M, \neg N, P\}$ and Lit^{\downarrow} as
13 $\{\neg A, B, N\}$, as shown in Figure 1c.

14 Similarly, based on $\{\neg N, P\} \subseteq \text{Lit}^{\uparrow}$, we update the nodes set Lit^{\uparrow} as $\text{Lit}^{\uparrow} \cup \{K\}$ and remain
15 Lit^{\downarrow} unchanged, as shown in Figure 1d. Finally, with $\{\neg M, K\} \subseteq \text{Lit}^{\uparrow}$ for the LHS node
16 $\neg M \wedge K$, we update the node set Lit^{\uparrow} as $\text{Lit}^{\uparrow} \cup \{N\} = \{A, \neg B, \neg M, \neg N, P, K, N\}$. Since
17 $\text{Lit}^{\uparrow} \cup \text{Lit}^{\downarrow} = \{N\}$, an inconsistency is identified. Note that if $\neg M$ is removed from the initially
18 defined literal set Lit , then there will be no inconsistency, and we will just obtain values for the
19 undefined literals $\{\neg N, N, P, K\}$ following this FC procedure.

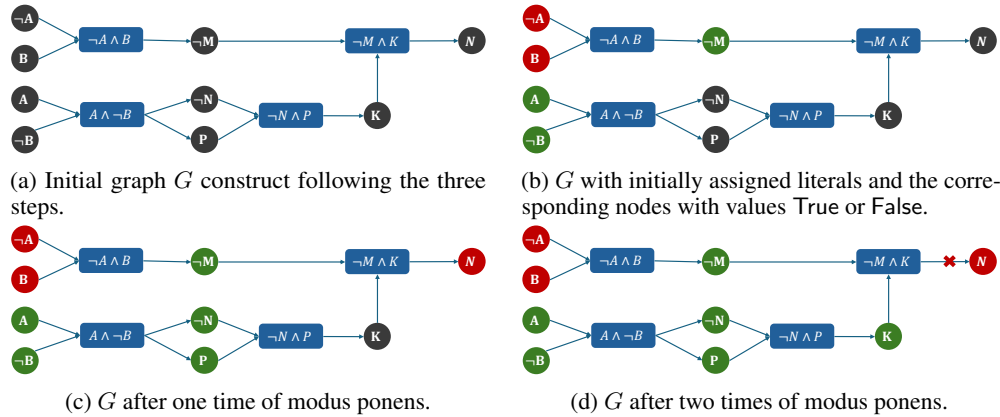


Figure 1: The initial graph G is constructed in (1a). The literal node sets Lit^{\uparrow} and Lit^{\downarrow} are updated step by step (from (1b) to (1d)), following the modus ponens inference rules, and an inconsistency is detected on the literal node N in (1d).

2 Dataset and experiment design

To evaluate the effectiveness of RvLLM, we consider three case studies: i) violation detection under the Singapore Rapid Transit Systems Act [1], ii) numerical comparison tasks, and iii) inequality solving problems. In the following, we present an in-depth overview of each case study and the corresponding datasets used.

Singapore Rapid Transit Systems Act [1]. This dataset comprises a collection of regulations governing passenger conduct on the railway. Among the full set of regulations, we identify and select 31 types that pertain specifically to passenger behaviors. These include: *Trespass, Smoking, Luggage, Foldable bicycle, Bringing animal, Dangerous goods, Offensive matter, Compliance with instructions, Overcrowding, Consumption, Spitting, Causing nuisance, Loitering, Throwing missiles, Wrongfully entering, Hawking, Meddling with plant, Misuse of an escalator, Misuse of an emergency safety device, Interference with doors, Improperly removing, Causing obstruction and danger, Transferring articles, Damaging a ticket, Hand in tickets, Failure to pay, Leaving motor vehicles, Failure of a vehicle driver, Dangerous driving, Driving vehicles on certain parts, Lost property*. For each of these regulation types, the work of [4] provides a set of scenarios—both compliant and non-compliant—that serve as our benchmark for conducting violation detection.

For example, the regulation in the category of *Consumption* corresponds to the relevant provision 14 **No consumption of food or drinks** in the Singapore Rapid Transit Systems Act [1], as follows:

“No person shall – (a) consume or attempt to consume any food or drinks while in or upon any part of the railway premises except in such places as are designated for this purpose by the Authority or its licensee; or (b) consume or attempt to consume any chewing gum or bubble gum while in or upon any part of the railway premises.”

In the experimental setup, we first query the target LLM to determine whether a given scenario is “Safe”—that is, free of any violations of the Singapore Rapid Transit Systems Act. This provides a baseline for violation detection using only the LLM. Subsequently, we employ RvLLM, which takes the scenario solely as the domain of discourse based on a Level-1 interpretation. The violation detection results are then collected from RvLLM. For scenarios that contain actual violations, if the target LLM classifies the scenario as “Safe” (i.e., fails to detect the violation), while RvLLM identifies a violation and returns “Inconsistent”, we consider this as evidence that RvLLM improves the detection accuracy of the target LLM on that particular example.

Numerical comparison. For this dataset, we randomly generate 100 numerical comparison problems to serve as our benchmark. This design is motivated by well-documented errors in LLMs when handling numerical comparisons, particularly in cases where the integral parts are identical but the fractional parts differ. A notorious example is the comparison between 9.11 and 9.9, which has been widely observed to cause incorrect behavior in LLMs [2]. In this case, models often erroneously compare the fractional part 0.11 with 0.9. Building on this insight, we construct 100 pairs of decimal numbers that share the same integer part but differ in their fractional components, specifically in the form $x.y$ vs. $x.z$, where $y < 10 < z$. Example pairs include “7.47 vs. 7.9” and “33.3 vs. 33.27”. These are designed to systematically evaluate how well RvLLM and baseline methods detect inconsistency in LLMs with respect to the comparison rule.

In the experimental setup, each numerical comparison question is presented to the target LLM with the instruction: “Please answer which one is greater, but nothing else.” The LLM’s response is then collected. Subsequently, RvLLM takes as the domain of discourse domain of both the original comparison question (as context) and the LLM output, and performs a verification procedure based on the predefined specification using a Level-2 interpretation. This enables RvLLM to assess the consistency underlying the LLM’s behavior.

Inequality solving. For this case study, we collect 40 inequality-solving problems from A-Level H2 Mathematics examination papers [3]. All of these problems involve a single free variable and are expressed as polynomial inequalities of degree two or higher, including examples such as fourth-degree polynomials. Here we give some example question used in the experiment: $\frac{3x-4x^2}{2x+1} \geq 0$, $\frac{x+5}{(x-3)^2} \geq \frac{x-7}{x(x-3)}$, and $\frac{4x^2-8x+2}{x-2} > \frac{1}{x^2}$.

In the experimental design, each inequality solving question is posed to the target LLM with the instruction: "Please solve the following inequality problem." The resulting step-by-step reasoning generated by the LLM is then provided as the domain of discourse for RvLLM. During this verification, RvLLM checks the logical consistency of the LLM's reasoning steps against three specification files encoding fundamental algebraic properties of inequalities taught at the junior college level. These specifications are specifically designed to detect three common categories of reasoning errors in LLM-generated solutions: incorrect factorization, flawed interval analysis, and the omission of endpoint or critical point evaluations.

3 Specification

Care should be taken in the design of specifications, as our specification language ESL supports three truth values for propositions: True, False, and Unknown, to support the open-world assumption. Given this, for a rule-like formula, if the left-hand side (LHS) evaluates to True, then the formula is considered valid if and only if the right-hand side (RHS) also evaluates to True—either a False value or an Unknown value will return as an inconsistency. Therefore, in certain cases, it becomes necessary to introduce “unknown”-type predicates due to the current limitations in the expressiveness of the ESL. For example, predicates such as $\text{UnkProperMeans}(x)$ and $\text{UnkAuthorDirect}(x)$ are defined in the Trespass (Type-1) specification within case study 1 to capture scenarios where the truth value cannot be definitively determined. A more detailed explanation of the usage of ESL is provided in the README.md file included in the supplementary code archive.

In the following, we provide the detailed specifications used across each case study.

3.1 Specifications used in case study 1

We encode the following 31 regulations related to passengers' behavior into ESL-based specifications.

```
Type-1:      Trespass # (Regulation No.5) No person shall -- (a) enter into or upon any part of the
↳ railway other than those parts clearly defined by means of notices, signs and other directional
↳ indicators provided for persons using the railway premises; or (b) enter or leave such parts other
↳ than by the proper use of such means as ticket gates provided for entry or exit, or by such other
↳ means as may be directed by any authorised person.

"Variables":  ["x"],
"Predicates": ["IsPassenger(x) := a person x is a railway passenger.",
               "InRailwayPremises(x) := a person x is in some railway premises or on the railway
↳ station.",
               "InLimitedArea(x) := a person x is in some parts of the railway premises where usage
↳ notices are not clearly given.",
               "ProperMeans(x) := a person x uses ticket gates or other authorized means to entry or
↳ exit the railway premises.",
               "UnkProperMeans(x) := it is unknown that if a person x uses ticket gates or other
↳ authorized means to entry or exit the railway premises.",
               "AuthorDirect(x) := the action of a person x entries or exits the railway premises is
↳ directed by some authorized person.",
               "UnkAuthorDirect(x) := it is unknown that if the action of a person x entries or exits
↳ the railway premises is directed by some authorized person."],
"rules":     ["InRailwayPremises(x) => not InLimitedArea(x)",
               "InRailwayPremises(x) & not AuthorDirect(x) & not UnkProperMeans(x) => ProperMeans(x)",
               "InRailwayPremises(x) & not ProperMeans(x) & not UnkAuthorDirect(x) => AuthorDirect(x)"]
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Type-2:      Smoking # (Regulation No.6) No person shall -- (a) smoke; (b) carry a lighted pipe, cigar
↳ or cigarette; or (c) carry or use any item or object which has a naked flame, in any part of the
↳ railway premises where smoking is expressly prohibited by notice.

"Variables":  ["x"],
"Predicates": ["LightItem(x) := a person x lights a pipe, cigar, or cigarette.",
               "UnkLightItem(x) := it is unknown that if a person x lights a pipe, cigar, or
↳ cigarette.",
               "Smoke(x) := a person x smokes.",
               "UnkSmoke(x) := it is unknown that if a person x smokes.",
               "WithNakedFlame(x) := a person x carries or uses an item with a naked flame.",
               "UnkWithNakedFlame(x) := it is unknown that if a person x carries or uses an item with a
↳ naked flame.",
               "CarryLightedItem(x) := a person x carries a lighted pipe, cigar, or cigarette.",
               "UnkCarryLightedItem(x) := it is unknown that a person x carries a lighted pipe, cigar,
↳ or cigarette."],
```

```

    "InNoSmokingArea(x) := a person x is in some part of the railway premises where the
    ↪ smoking is prohibited."
"rules": ["InNoSmokingArea(x) & not UnkLightItem(x) => not LightItem(x)",
    "InNoSmokingArea(x) & not UnkSmoke(x) => not Smoke(x)",
    "InNoSmokingArea(x) & not UnkWithNakedFlame(x) => not CarryLightedItem(x)",
    "InNoSmokingArea(x) & not UnkCarryLightedItem(x) => not WithNakedFlame(x)"]

```

Type-3: **Luggage # (Regulation No.7.1) (1) No person shall bring into or upon any part of the**
 ↪ *railway premises -- (a) any luggage, article or thing which -- (i) exceeds the dimensions or weight*
 ↪ *restrictions specified on notices posted by the Authority or its licensee in the railway premises;*
 ↪ *(ii) cannot be carried or otherwise accommodated on the railway without risk of damage to railway*
 ↪ *property; or (iii) causes a nuisance or inconvenience to other persons using the railway premises,*
 ↪ *except in accordance with such conditions as may be specified by the Authority or its licensee on*
 ↪ *notices posted by the Authority or its licensee in the railway premises; or (b) any item which is*
 ↪ *prohibited by an officer or employee of the Authority or its licensee at the railway premises or by*
 ↪ *notices posted by such person in the railway premises.*

"Variables": ["x", "y"],
"Predicates": ["InRailwayPremises(x) := a person x is in some railway premises or on the railway
 ↪ station.",
 "Bring(x, y) := A person x brings or takes y onto railway premises or railway
 ↪ stations.",
 "ExceedsSize(y) := item y exceeds the specified dimensions or size or weights
 ↪ restrictions.",
 "RiskOfDamage(y) := item y cannot be carried without risk of damage to railway
 ↪ property.",
 "Nuisance(y) := item y causes nuisance or inconvenience to other persons.",
 "ProhibitedItem(y) := item y is prohibited by an officer, employee, or posted notices.",
 "SpecialConditions(y) := Taking the y mentioned in the context onto the railway premises
 ↪ have been explicitly allowed or authorized by the Authority or notices posted.",
 "UnknownSpecial(y) := It is unknown if taking the y mentioned in the context onto the
 ↪ railway premises is explicitly allowed or authorized by some Authority or notices."]
"rules": ["InRailwayPremises(x) & Bring(x, y) & ExceedsSize(y) => SpecialConditions(y)",
 "InRailwayPremises(x) & Bring(x, y) & RiskOfDamage(y) => SpecialConditions(y)",
 "InRailwayPremises(x) & Bring(x, y) & Nuisance(y) => SpecialConditions(y)",
 "InRailwayPremises(x) & Bring(x, y) & UnknownSpecial(y) => not ExceedsSize(y)",
 "InRailwayPremises(x) & Bring(x, y) & UnknownSpecial(y) => not RiskOfDamage(y)",
 "InRailwayPremises(x) & Bring(x, y) & UnknownSpecial(y) => not Nuisance(y)",
 "InRailwayPremises(x) & Bring(x, y) => not ProhibitedItem(y)"]

Type-4: **Foldable bicycle # (Regulation No.7.2) (2) Despite paragraph (1)(a) but without affecting**
 ↪ *any other provision in these Regulations, a person may, subject to the conditions specified in*
 ↪ *paragraph (3), bring into or upon any part of the railway premises a single foldable bicycle or*
 ↪ *personal mobility device, provided that -- (a) the foldable bicycle or the personal mobility device,*
 ↪ *if foldable, is folded and kept folded, with its longest side in a vertical position; (b) the*
 ↪ *dimensions of the folded bicycle, personal mobility device or folded personal mobility device (if*
 ↪ *foldable) do not exceed 120 cm by 70 cm by 40 cm; (c) if so required by an authorised person at the*
 ↪ *entrance to the railway premises, the wheels of the foldable bicycle or the personal mobility device*
 ↪ *must be covered so as to prevent any dirt on the wheels being deposited onto the railway premises or*
 ↪ *soiling the clothing of any passenger; (d) if the handlebars or pedals of the foldable bicycle or the*
 ↪ *personal mobility device extend beyond its frame, they are enclosed so as to minimise the risk of*
 ↪ *injury to any person or damage to any property; and (e) where the personal mobility device is*
 ↪ *propelled by an electric motor, the personal mobility device is switched off.*

"Variables": ["x", "y"],
"Predicates": ["IsBicycle(x) := x is a bicycle.",
 "IsPerson(x) := x is a person",
 "IsMobility(x) := x is a personal mobility device.",
 "CauseNuisance(x) := a person x conducts himself or his vehicle so as to cause a
 ↪ nuisance or annoyance or inconvenience to other passengers on train or railway
 ↪ premises.",
 "Foldable(x) := the vehicle x is foldable.",
 "Folded(x) := the vehicle x is folded and kept folded with its longest side in a
 ↪ vertical position.",
 "WithinSize(x) := x is within the carry-on size required by the railway.",
 "UnkWithinSize(x) := it is unknown that if x is within the carry-on size required by the
 ↪ railway.",
 "Bring(x, y) := A person x brings or takes y onto railway premises or railway
 ↪ stations.",
 "RequireCover(x) := some officer or staff requires that the wheels of vehicle x should
 ↪ be covered.",
 "WheelsCovered(x) := the wheels of vehicle x are finally covered.",
 "ExtendedFrame(x) := the handlebars or pedals of vehicle x extend beyond its frame.",
 "HandlebarsEnclosed(x) := the handlebars or pedals of vehicle x are finally enclosed.",
 "MotorPropelled(x) := x is propelled by an electric motor.",

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"rules":      "MotorOff(x) := the electric motor of vehicle x is switched off."
["IsPerson(x) & IsBicycle(y) & Bring(x, y) => not CauseNuisance(x)",
"IsPerson(x) & IsBicycle(y) & Bring(x, y) => Folded(y)",
"IsPerson(x) & IsBicycle(y) & Bring(x, y) & not UnkWithinSize(y) => WithinSize(y)",
"IsPerson(x) & IsMobility(y) & Bring(x, y) => not CauseNuisance(x)",
"IsPerson(x) & IsMobility(y) & Bring(x, y) => Folded(y)",
"IsPerson(x) & IsMobility(y) & Bring(x, y) & not UnkWithinSize(y) => WithinSize(y)",
"IsPerson(x) & IsBicycle(y) & Bring(x, y) & RequireCover(y) => WheelsCovered(y)",
"IsPerson(x) & IsMobility(y) & Bring(x, y) & RequireCover(y) => WheelsCovered(y)",
"IsPerson(x) & IsBicycle(y) & Bring(x, y) & ExtendedFrame(y) => HandlebarsEnclosed(y)",
"IsPerson(x) & IsMobility(y) & Bring(x, y) & ExtendedFrame(y) => HandlebarsEnclosed(y)",
"IsPerson(x) & IsBicycle(y) & Bring(x, y) & MotorPropelled(y) => MotorOff(y)",
"IsPerson(x) & IsMobility(y) & Bring(x, y) & MotorPropelled(y) => MotorOff(y)"]

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Type-5: **Bring animal # (Regulation No.8)** (1) No person shall bring any animal into or upon, or
↳ allow any animal under his control to remain in or on, any part of the railway premises. (2) A person
↳ shall be responsible for any injury, loss or damage caused to the property or staff of the Authority
↳ or its licensee, or to any other person or property by such person or by any animal or article
↳ brought by him onto the railway premises and he shall indemnify the Authority and its licensee from
↳ and against any liability to any other person resulting therefrom. (3) The prohibition in paragraph
↳ (1) shall not apply to -- (a) a guide dog accompanying a person with a sight or hearing impairment;
↳ or (b) a dog under the effective control of any police officer on official duty or any member of an
↳ auxiliary police force in uniform or any security officer (within the meaning of the Private Security
↳ Industry Act (Cap. 250A)) engaged by the Authority or its licensee to provide security at the railway
↳ premises.

```

"Variables":  ["x", "y"],
"Predicates": ["IsAnimal(x) := x is an animal.",
"IsGuideDog(x) := x is a guide dog accompanying a person with a sight or hearing
↳ impairment.",
"IsOfficerDog(x) := x is a dog under the effective control of a police officer, auxiliary
↳ police officer, or security officer on duty.",
"CauseDamageOrInjury(x) := some person x or the animal of some person x brought with
↳ causes injury, loss, or damage to persons or property.",
"InRailwayPremises(x) := a person x or an animal x is in some railway premises or on the
↳ railway station."],
"Rules":      ["IsAnimal(x) & InRailwayPremises(x) & not IsOfficerDog(x) => IsGuideDog(x)",
"IsAnimal(x) & InRailwayPremises(x) & not IsGuideDog(x) => IsOfficerDog(x)"]

```

Type-6: **Dangerous goods # (Regulation No.9)** No person, not being an employee of the Authority or
↳ its licensee duly authorised in that behalf, shall bring into any part of the railway premises such
↳ dangerous or flammable substance or other thing as may be specified in the conditions of use.

```

"Variables":  ["x", "y"],
"Predicates": ["IsAllowed(x) := a person x is an employee of the Authority or certified or allowed by
↳ the Authority.",
"IsDangerOrFlammable(x) := x is a dangerous or flammable substance or other thing as may
↳ be specified as dangerous or issuing safety hazard.",
"Bring(x, y) := a person x brings y onto railway premises.",
"InRailwayPremises(x) := a person x is in some railway premises or on the railway
↳ station."],
"rules":      ["InRailwayPremises(x) & Bring(x, y) & IsDangerOrFlammable(y) => IsAllowed(x)"]

```

Type-7: **Offensive matter # (Regulation No.10)** No person shall cause any sewage, drainage waste or
↳ other offensive matter to flow onto or enter or be placed on any part of the railway premises.

```

"Variables":  ["x", "y"],
"Predicates": ["InRailwayPremises(x) := a person x is in some railway premises or on the railway
↳ station.",
"IsOffensive(y) := y is sewage, drainage waste, or other offensive matter.",
"CauseFlow(x, y) := a person x causes y to flow onto or enter or be placed on the
↳ railway premises.",
"Flow(y) := something y flows onto or enter or spilled on the railway premises.",
"Place(x, y) := a person x place y onto the railway premises."],
"rules":      ["InRailwayPremises(x) & CauseFlow(x, y) => not IsOffensive(y)",
"InRailwayPremises(x) & Place(x, y) => not IsOffensive(y)",
"Flow(x) => not IsOffensive(x)"]

```

Type-8: Compliance with instructions # (Regulation No.11) Every person while on the railway
 ↳ premises shall comply with all notices, signs and all reasonable directions and instructions of any
 ↳ authorised person.

"Variables": ["x", "y"],
 "Predicates": ["NoticeOrSign(x) := x is a notice or sign.",
 "Instruction(x) := x is a reasonable direction or instruction from an authorised person
 ↳ or a staff.",
 "ComplySign(x, y) := a person x complies with the notice or sign y.",
 "ComplyInstruct(x, y) := a person x complies with the instruction or direction y from an
 ↳ authorised person or a staff.",
 "InRailwayPremises(x) := a person x is in some railway premises or on the railway
 ↳ station."],
 "rules": ["InRailwayPremises(x) & NoticeOrSign(y) => ComplySign(x, y)",
 "InRailwayPremises(x) & Instruction(y) => ComplyInstruct(x, y)"]

Type-9: Overcrowding # (Regulation No.12) Without prejudice to regulation 11, where any
 ↳ authorised person determines that a train is full, no person shall enter or remain in the train if
 ↳ directed not to do so by him.

"Variables": ["x", "y"],
 "Predicates": ["IsFullTrain(y) := y is a train that is determined to be full or over-crowded by any
 ↳ authorised person or notification or signals.",
 "UnkFullTrain(y) := it is unknown that if y is a train that is determined to be full or
 ↳ over-crowded by any authorised person or notification or signals.",
 "Enter(x, y) := a person x enters the train y."],
 "rules": ["Enter(x, y) => not IsFullTrain(y)"]

Type-10: Consumption # (Regulation No.14) No person shall -- (a) consume or attempt to consume any
 ↳ food or drinks while in or upon any part of the railway premises except in such places as are
 ↳ designated for this purpose by the Authority or its licensee; or (b) consume or attempt to consume
 ↳ any chewing gum or bubble gum while in or upon any part of the railway premises.

"Variables": ["x", "y"],
 "Predicates": ["Consumption(x) := a person x consumes or attempts to consume any food or drinks.",
 "UnkConsumption(x) := it is unknown that if a person x consumes or attempts to consume
 ↳ any food or drinks.",
 "ChewGum(x) := a person x consumes or attempts to consume chewing gum or bubble gum.",
 "UnkChewGum(x) := it is unknown that if a person x consumes or attempts to consume
 ↳ chewing gum or bubble gum.",
 "InConsumptionArea(x) := a person x is in an area specifically designated by the
 ↳ Authority or its licensee for consumption.",
 "InRailwayPremises(x) := a person x is in some railway premises or on the railway
 ↳ station."],
 "rules": ["Consumption(x) & InRailwayPremises(x) => InConsumptionArea(x)",
 "InRailwayPremises(x) & not InConsumptionArea(x) & not UnkConsumption(x) => not
 ↳ Consumption(x)",
 "InRailwayPremises(x) & not UnkChewGum(x) => not ChewGum(x)"]

Type-11: Spitting # (Regulation No.15) (1) No person shall spit on any part of the railway
 ↳ premises. (2) No person shall -- (a) place or throw any litter; or (b) place or throw any chewing
 ↳ gum, upon any part of the railway premises, or into or on any fittings, equipment or property upon
 ↳ the railway premises, whether belonging to the Authority or its licensee or placed therein with the
 ↳ approval of the Authority or its licensee, except into receptacles expressly provided for that
 ↳ purpose. (3) No person shall soil any part of the railway premises, or any fittings, equipment or
 ↳ property upon the railway premises, whether belonging to the Authority or its licensee or placed
 ↳ therein with the approval of the Authority or its licensee.

"Variables": ["x", "y"],
 "Predicates": ["Spit(x) := a person or some persons x spits on some part of the railway premises or
 ↳ fittings, equipment, or property thereon.",
 "UnkSpit(x) := it is unknown that if some person x spits.",
 "Soil(x) := a person or some persons x soils some part of the railway premises, or any
 ↳ fittings, equipment or property upon the railway premises.",
 "UnkSoil(x) := it is unknown that if some person x soils something.",
 "IsGum(y) := y is a chewing gum.",
 "IsLitter(y) := y is some litter.",
 "ThrowOrPlace(x, y) := the person x drops or leaves or throws item y onto some part of
 ↳ the railway premises."],
 "rules": ["not UnkSpit(x) => not Spit(x)",
 "not UnkSoil(x) => not Soil(x)",
 "IsLitter(y) => not ThrowOrPlace(x, y)",
 "IsGum(y) => not ThrowOrPlace(x, y)"]

Type-12: **Causing nuisance # (Regulation No.17)** *No person shall conduct himself on any train or in any part of the railway premises so as to cause a nuisance or annoyance to other passengers.*

"Variables": ["x"],

"Predicates": ["CauseNuisance(x) := a person x causes a nuisance or annoyance to other passengers on train or railway premises.",
 "UnkCauseNuisance(x) := it is unknown that if a person x causes any nuisance or annoyance to other passengers on train or railway premises.",
 "InRailwayPremises(x) := a person x is in some railway premises or on the railway station."]

"rules": ["IsAnimal(x) & InRailwayPremises(x) & not IsOfficerDog(x) => IsGuideDog(x)",
 "IsAnimal(x) & InRailwayPremises(x) & not IsGuideDog(x) => IsOfficerDog(x)"]

Type-13: **Loitering # (Regulation No.18)** *No person, not being a passenger or having business in or in connection with the Authority or its licensee or its tenant, shall loiter or remain in or upon any part of the railway premises.*

"Variables": ["x"],

"Predicates": ["LoiterOrRemain(x) := the person x loiters or remains in some part of the railway premises.",
 "IsPassenger(x) := the person x is a passenger who intends to take or board the train.",
 "HasBusiness(x) := the person x has legitimate business in or in connection with the Authority or its licensee or its tenant."]

"rules": ["LoiterOrRemain(x) & not IsPassenger(x) => HasBusiness(x)",
 "LoiterOrRemain(x) & not HasBusiness(x) => IsPassenger(x)"]

Type-14: **Throwing missiles # (Regulation No.19)** *No person shall throw, drop or deposit, or cause to be thrown, dropped or deposited on, into or from the railway premises any missile or thing capable of injuring, damaging, endangering or inconveniencing any person or property.*

"Variables": ["x", "y"],

"Predicates": ["IsPerson(x) := x is a person.",
 "ThrowDropDeposit(x, y) := a person x throws, drops, or causes item y on, into, or near the railway premises.",
 "IsMissile(y) := y is a missile or other similar objects that could become a missile in the context.",
 "IsDanger(y) := y is something capable of damaging, endangering, or inconveniencing person or premise property."]

"rules": ["IsPerson(x) & ThrowDropDeposit(x, y) => not IsMissile(y)",
 "IsPerson(x) & ThrowDropDeposit(x, y) => not IsDanger(y)"]

Type-15: **Wrongfully entering # (Regulation No.20)** *No person, other than an authorised person, shall mount or enter or attempt to mount or enter any train or part thereof, except on such part as is provided for the carriage of passengers.*

"Variables": ["x", "y"],

"Predicates": ["Mount(x, y) := the passenger x mounts or attempts to mount some part y of train or railway premises.",
 "Enter(x, y) := the passenger x enters or attempts to enter part y of the train or railway premises.",
 "NotForPass(y) := y is some part of the train or railway premises that are not designated for passenger use."]

"rules": ["Mount(x, y) => not NotForPass(y)",
 "Enter(x, y) => not NotForPass(y)"]

Type-16: **Hawking # (Regulation No.21)** *No person, while in or upon the railway premises shall, without the written permission of the Authority or its licensee -- (a) tout or solicit alms, rewards or employment of any description; (b) sell or offer for sale any article or goods, or carry on any business; or (c) display, exhibit or distribute any book or printed, written or pictorial matter or any such samples for the purpose of advertising or publicity.*

"Variables": ["x"],

"Predicates": ["InRailwayPremises(x) := a person x is in some railway premises or on the railway station.",
 "HasPermission(x) := the person x has got the official permission or certification.",
 "NoPermit(x) := the person x does not have any official permission or certification.",
 "ToutOrSolicit(x) := a person x touts or solicits alms, rewards, or employment.",
 "SellOrBusiness(x) := a person x sells or offers for sale any goods or carries on any business."]


```

"AdvOrPub(x) := a person x displays, exhibits, or distributes printed or pictorial
↳ matter for advertising or publicity."
"rules": ["InRailwayPremises(x) & ToutOrSolicit(x) => HasPermission(x)",
          "InRailwayPremises(x) & NoPermit(x) => not ToutOrSolicit(x)",
          "InRailwayPremises(x) & SellOrBusiness(x) => HasPermission(x)",
          "InRailwayPremises(x) & NoPermit(x) => not SellOrBusiness(x)",
          "InRailwayPremises(x) & AdvOrPub(x) => HasPermission(x)",
          "InRailwayPremises(x) & NoPermit(x) => not AdvOrPub(x)"]

```

Type-17: Meddling with plant # (Regulation No.22) No person shall improperly touch, use, meddle, damage or otherwise interfere with -- (a) any mechanical or electrical equipment or installation in or upon the railway premises, or any switch, lever or other device operating or controlling any such equipment; or (b) any locomotive, train, carriage, or any vehicle or equipment thereon used or employed in or upon, or in connection with the railway.

```

"Variables": ["x", "y"],
"Predicates": ["InRailwayPremises(x) := a person x is in some railway premises or on the railway
↳ station.",
               "IsRailEquip(y) := y is some equipment within a train or on the railway premises.",
               "Interfere(x, y) := the person x improperly touches, uses, meddles with, damages, or
↳ otherwise interferes with y."],
"rules": ["InRailwayPremises(x) & Interfere(x, y) => not IsRailEquip(y)"]

```

Type-18: Misuse of an escalator # (Regulation No.23) (1) No person shall -- (a) without reasonable excuse, sit on any step or pallet of any escalator or travelator in the railway premises; or (b) sit or ride on any handrail of any escalator or travelator in the railway premises. (2) No person shall use, or attempt to use any escalator or travelator in the railway premises except to travel from one end of the escalator or travelator to the other by means of the escalator stairway or the travelator pallet. (3) No person shall travel, or attempt to travel, upon any moving escalator or travelator in the railway premises in a direction other than the direction in which the escalator or travelator is moving.

```

"Variables": ["x"],
"Predicates": ["SitOnEscalator(x) := some person x sits on some step or handrail of an escalator or
↳ travelator in railway premises.",
               "UnkSitOnEscalator(x) := it is unknown that if the person x sits on some step or
↳ handrail of an escalator or travelator in railway premises.",
               "RideHandrail(x) := some person x rides on the handrail of an escalator or travelator in
↳ railway premises.",
               "UnkRideHandrail(x) := it is unknown that if the person x rides on the handrail of an
↳ escalator or travelator in railway premises.",
               "UseEscal(x) := a person x uses an escalator or travelator.",
               "ForTravel(x) := a person x uses an escalator or travelator in railway premises for
↳ travel.",
               "OppDirection(x) := a person x travels on an escalator or travelator in the opposite
↳ direction of its movement.",
               "UnkDirection(x) := it is unknown that which direction does the person x travels on an
↳ escalator or travelator."],
"rules": ["UseEscal(x) & not UnkSitOnEscalator(x) => not SitOnEscalator(x)",
          "UseEscal(x) & not UnkRideHandrail(x) => not RideHandrail(x)",
          "UseEscal(x) & not UnkDirection(x) => not OppDirection(x)"]

```

Type-19: Misuse of an emergency safety device # (Regulation No.25) No person, other than an authorised person, shall activate any emergency or safety device on the railway premises, except for the purpose for which the device is provided and in accordance with the instructions printed thereon.

```

"Variables": ["x", "y"],
"Predicates": ["IsEmergencyDevice(y) := y is an emergency or safety device on the railway premises.",
               "Activate(x, y) := some person x activates device y.",
               "CorrectPurpose(x, y) := a person x activates device y for a correct purpose that the
↳ device is provided for",
               "Comply(x, y) := a person x uses the device y in accordance with the instructions
↳ printed thereon",
               "IsPassenger(x) := the person x is a passenger."],
"rules": ["IsPassenger(x) & IsEmergencyDevice(y) & Activate(x, y) => CorrectPurpose(x, y)",
          "IsPassenger(x) & IsEmergencyDevice(y) & Activate(x, y) => Comply(x, y)"]

```


Type-20: **Interference with doors # (Regulation No.26)** *No person shall in any way interfere with*
↳ *any platform screen door or train door, or the operation of any such door, except -- (a) in an*
↳ *emergency and by means of any equipment on or near which is a notice indicating that it is intended*
↳ *to be used in an emergency; or (b) where he is an authorised person acting in the discharge of his*
↳ *duties.*

"Variables": ["x"],
"Predicates": ["Interfere(x) := a person x interferes with some platform screen door or train door, or
↳ the operation of any such door.",
 "IsPassenger(x) := a person x is a railway passenger or a commuter.",
 "ForEmergency(x) := a person x acts for solving an emergency.",
 "DesignatedEquip(x) := a person x is using some designated equipment.",
 "IsAuthority(x) := a person x is an authorised person acting in the discharge of
↳ duties.",
 "OnDuties(x) := a person x is performing some duties."]
"rules": ["Interfere(x) & IsPassenger(x) => ForEmergency(x)",
 "Interfere(x) & IsPassenger(x) => DesignatedEquip(x)",
 "Interfere(x) & not ForEmergency(x) => IsAuthority(x)",
 "Interfere(x) & not ForEmergency(x) => OnDuties(x)"]

Type-21: **Improperly removing # (Regulation No.28)** *No person shall deface, damage or improperly*
↳ *remove any part of the railway premises or fittings, equipment or property upon the railway premises,*
↳ *whether belonging to the Authority or its licensee or placed therein with the approval of the*
↳ *Authority or its licensee.*

"Variables": ["x"],
"Predicates": ["InRailwayPremises(x) := a person x is in some railway premises or on the railway
↳ station.",
 "Deface(x) := a person x defaces some part of the railway premises, equipment, or
↳ property.",
 "Damage(x) := a person x damages some part of the railway premises, equipment, or
↳ property.",
 "Remove(x) := a person x improperly removes some part of the railway premises,
↳ equipment, or property.",
 "HasPermission(x) := the person x has got the official permission or certification to do
↳ the deface, or damage or removal, or other things.",
 "Follow(x) := the person x follows the regulation to do the deface, or damage or
↳ removal, or other things."]
"rules": ["InRailwayPremises(x) & Deface(x) => HasPermission(x)",
 "InRailwayPremises(x) & Deface(x) => Follow(x)",
 "InRailwayPremises(x) & Damage(x) => HasPermission(x)",
 "InRailwayPremises(x) & Damage(x) => Follow(x)",
 "InRailwayPremises(x) & Remove(x) => HasPermission(x)",
 "InRailwayPremises(x) & Remove(x) => Follow(x)"]

Type-22: **Causing obstruction and danger # (Regulation No.29)** *No person shall place himself, or*
↳ *leave or place any animal or any object under his possession or charge, in or upon the railway*
↳ *premises so as to cause any obstruction, hindrance or danger to the Authority, its licensee or to any*
↳ *person using the railway.*

"Variables": ["x", "y"],
"Predicates": ["HindranceDanger(x) := x causes some obstruction, hindrance, or danger.",
 "InRailwayPremises(x) := a person or an object x is in some railway premises or on the
↳ railway station.",
 "NearRailwayPremises(x) := a person or an object x is near to the railway.",
 "PlaceOrLeave(x, y) := a person x places or leaves y under his possession or charge, in
↳ or upon some railway premises."]
"rules": ["InRailwayPremises(x) => not HindranceDanger(x)",
 "NearRailwayPremises(x) => not HindranceDanger(x)",
 "InRailwayPremises(x) & PlaceOrLeave(x, y) => not HindranceDanger(y)",
 "NearRailwayPremises(x) & PlaceOrLeave(x, y) => not HindranceDanger(y)"]

Type-23: **Transferring articles # (Regulation No.31)** *No person shall for the purpose of any trade*
↳ *or business transfer any article or goods between the paid area and unpaid area unless the article or*
↳ *goods is taken by a person through a ticket gate.*

"Variables": ["x", "y", "z"],
"Predicates": ["Trade(x, y, z) := a person x transfers y to another person z.",
 "TransferArea(x, y) := a person x transfers y from two different areas, one is paid area
↳ and another one is unpaid area.",
 "GoThruGate(x) := a person x goes through the designated gate.",
 "Moving(x) := a person x moves through stations on the railway.",
 "IsGoods(y) := y is an article or goods.",

```

"DifferentArea(x, y) := a person x is in paid area and another person y is in unpaid
↪ area, or vice versa.",
"ByPassGate(x, y, z) := a person x or a person z bypass the ticket gate to give the item
↪ y to the other."
"rules": ["DifferentArea(x, z) & IsGoods(y) & Trade(x, y, z) => ~ByPassGate(x, y, z)",
"TransferArea(x, y) & IsGoods(y) => GoThrGate(x)",
"Moving(x) => GoThrGate(x)"]

```

Type-24: **Damaging a ticket # (Regulation No.38) (1) No person shall improperly do anything to or**
↪ *with a ticket whereby -- (a) the coded data thereon are erased wholly or in part, or are otherwise*
↪ *altered or interfered with; or (b) the ticket is otherwise damaged. (2) No person shall use or*
↪ *attempt to use a ticket which has expired or has been improperly altered, damaged or interfered with*
↪ *for the purpose of entering or leaving the paid area or travelling upon the railway.*

```

"Variables": ["x", "y"],
"Predicates": ["IsPerson(x) := x is a person.",
"IsTicket(y) := y is a railway ticket.",
"DamageCodedData(x, y) := some person x erases or alters the coded data on ticket y.",
"UnkDamageCodedData(x, y) := it is unknown if some person x erases or alters the coded
↪ data on ticket y.",
"DamageTicket(x, y) := some person x damages ticket y on purpose.",
"UnkDamageTicket(x, y) := it is unknown if some person x damages ticket y on purpose.",
"IsDamagedTicket(x) := x is a damaged railway ticket",
"IsExpiredTicket(x) := x is an expired railway ticket.",
"IsValidTicket(x) := x is a valid railway ticket certified by some officer.",
"UseTicket(x, y) := a person x uses or attempts to use ticket y.",
"UnkUseTicket(x, y) := it is unknown that if a person x uses or attempts to use ticket
↪ y."
"rules": ["IsPerson(x) & IsTicket(y) & not UnkDamageCodedData(x, y) => not DamageCodedData(x, y)",
"IsPerson(x) & IsTicket(y) & not UnkDamageTicket(x, y) => not DamageTicket(x, y)",
"IsPerson(x) & IsDamagedTicket(y) & UseTicket(x,y) => IsValidTicket(y)",
"IsPerson(x) & IsExpiredTicket(y) & not UnkUseTicket(x,y) => not UseTicket(x, y)"]

```

Type-25: **Hand in tickets # (Regulation No.41) No person shall, without reasonable excuse, fail or**
↪ *refuse to deliver up his ticket to the Authority or its licensee under regulation 39 or upon the*
↪ *expiry of the validity of the ticket.*

```

"Variables": ["x"],
"Predicates": ["InRailwayPremises(x) := a person x is in some railway premises or on the railway
↪ station.",
"AskProduceOrHindIn(x) := a person x is asked to produce or hand in his railway ticket
↪ by some authorised person.",
"ProduceOrHandIn(x) := the passenger x produces or hand in a valid railway ticket on
↪ demand."
"rules": ["InRailwayPremises(x) & AskProduceOrHindIn(x) => ProduceOrHandIn(x)"]

```

Type-26: **Failure to pay # (Regulation No.42) No person shall, prior to leaving the paid area, fail**
↪ *or refuse to pay any administrative charge leviable in accordance with these Regulations.*

```

"Variables": ["x"],
"Predicates": ["PayInArea(x) := a person x pays the charge before leaving the paid area.",
"PayInTime(x) := a person x pays the charge before the deadline/due time.",
"RequireCharge(x) := a person x is required to or should pay the charge.",
"InRailwayPremises(x) := a person x is in some railway premises or on the railway
↪ station.",
"UnkSetting(x) := it is unknown if a person x is in some railway premises or on the
↪ railway station."
"rules": ["RequireCharge(x) & UnkSetting(x) => PayInTime(x)",
"RequireCharge(x) & InRailwayPremises(x) => PayInArea(x)"]

```

Type-27: **Leaving motor vehicles # (Regulation No.44) Except with the written permission of the**
↪ *Authority or its licensee, no person shall leave or cause to be left any motor car or other vehicle*
↪ *on any part of the railway premises or on any station approach road or entrance being under the*
↪ *control of the Authority or its licensee.*

```

"Variables": ["x", "y"],
"Predicates": ["IsPerson(x) := x is a person.",
"IsVehicle(x) := x is a motor car or other vehicle.",
"HasPermission(x, y) := a person x has permission or certification for leaving y on
↪ railway premises.",
"UnkPermitLeave(x, y) := it is unknown if a person x has permission or certification for
↪ leaving y on railway premises.",

```

```

"rules":
    "LeaveOnPremises(x, y) := a person x leaves or causes y to be left on the railway
    ↪ premises or station approach roads or entrances."
    ["IsPerson(x) & IsVehicle(y) & LeaveOnPremises(x, y) => HasPermission(x, y)",
     "IsPerson(x) & IsVehicle(y) & UnkPermitLeave(x, y) => not LeaveOnPremises(x, y)"]

```

Type-28: Failure of a vehicle driver # (Regulation No.46) Vehicle drivers shall, while in or upon
 ↪ any part of the railway premises, obey all traffic signs and signals and the reasonable instructions
 ↪ and directions of any authorised person.

```

"Variables": ["x", "y"],
"Predicates": ["IsDriver(x) := a person x is a driver or taking a vehicle on railway premises.",
               "IsTrafficSign(y) := y is a traffic sign or signal.",
               "ObeySign(x, y) := a person x obeys traffic sign or signal y.",
               "FollowDirection(x) := a person x follows the reasonable instructions or directions of
               ↪ any authorised person.",
               "InRailwayPremises(x) := a person x is in some railway premises or on the railway
               ↪ station."],
"rules": ["InRailwayPremises(x) & IsDriver(x) & IsTrafficSign(y) => ObeySign(x, y)",
          "InRailwayPremises(x) & IsDriver(x) => FollowDirection(x)"]

```

Type-29: Dangerous driving # (Regulation No.47) No person shall drive any motor car or other
 ↪ vehicle through, into or upon any part of the railway premises at a rate of speed or in a manner
 ↪ liable to involve danger to others.

```

"Variables": ["x"],
"Predicates": ["DangerSpeed(x) := a person x drives at a speed liable to involve danger to others.",
               "UnkDangerSpeed(x) := it is unknown that if a person x drives at a speed liable to
               ↪ involve danger to others.",
               "DangerManner(x) := a person x drives or operator the vehicle or train in a manner to
               ↪ involve danger to other people.",
               "UnkDangerManner(x) := it is unknown that if a person x drives or operator the vehicle
               ↪ or train in a manner to involve danger to other people.",
               "InRailwayPremises(x) := a person x is in some railway premises or on the railway
               ↪ station."],
"rules": ["InRailwayPremises(x) & not UnkDangerSpeed(x) => not DangerSpeed(x)",
          "InRailwayPremises(x) & not UnkDangerManner(x) => not DangerManner(x)"]

```

Type-30: Driving vehicles on certain parts # (Regulation No.48) No person shall drive any motor
 ↪ car or other vehicle upon or along any part of the railway premises set apart for the exclusive use
 ↪ of pedestrians.

```

"Variables": ["x"],
"Predicates": ["DrivesPedArea(x) := a person x drives in some area reserved for pedestrians.",
               "UnkDrivesPedArea(x) := it is unknown that if the person x drives in some area reserved
               ↪ for pedestrians.",
               "DrivesRestrictArea(x) := a person x drives in some restriction areas.",
               "UnkDrivesRestrictArea(x) := it is unknown if a person x drives in some restriction
               ↪ areas.",
               "InRailwayPremises(x) := a person x is in some railway premises or on the railway
               ↪ station."],
]
"rules": ["InRailwayPremises(x) & not UnkDrivesPedArea(x) => not DrivesPedArea(x)",
          "InRailwayPremises(x) & not UnkDrivesRestrictArea(x) => not DrivesRestrictArea(x)"]

```

Type-31: Lost property # (Regulation No.49) (1) Every person who finds any lost property in or
 ↪ upon any part of the railway premises shall hand over the property to any authorised person at the
 ↪ nearest station. (2) No person other than an authorised person shall remove from any part of the
 ↪ railway premises any property lost or left behind therein, except for the purpose of handing over the
 ↪ property to an authorised person.

```

"Variables": ["x", "y"],
"Predicates": ["IsPassenger(x) := the person x is a passenger.",
               "IsAuthority(x) := x is an authorised person on the railway premises.",
               "InRailwayPremises(x) := a person x is in some railway premises or on the railway
               ↪ station.",
               "FindsLost(x, y) := the person x finds lost property y in or upon the railway
               ↪ premises.",
               "HandOverLost(x, y) := a person x hands over the lost property y to an authorised person
               ↪ at the nearest station.",
               "RemoveLostProperty(x, y) := a person x removes lost property y from the railway
               ↪ premises."],

```

```

"rules":
  "PlanToHandOver(x, y) := a person x plans to hand over the property y to an authorised
  ↪ person."
  ["IsPassenger(x) & FindsLost(x, y) => HandOverLost(x, y)",
  "InRailwayPremises(x) & FindsLost(x, y) => HandOverLost(x, y)",
  "IsPassenger(x) & RemoveLostProperty(x, y) => PlanToHandOver(x, y)",
  "InRailwayPremises(x) & RemoveLostProperty(x, y) & not PlanToHandOver(x, y) =>
  ↪ IsAuthority(x)"]

```

95 3.2 Specifications used in case study 2

96 For the second case study, we use one specification rule to encode a simple algebraic property, i.e.,
 97 multiplying both sides of a comparison by a positive (resp. negative) number will maintain (resp.
 98 alter) the truth of the comparison.

```

Type-1: Numerical comparison # multiplying both sides of a comparison (greater than, less than, or
↪ equal to) by a positive (resp. negative) number will maintain (resp. alter) the truth of the
↪ comparison.

"Variables": ["x", "y", "z"],
"Predicates": ["IsLarger(x, y) := x is larger than y.",
  "Pos(x) := x is a positive number, i.e., x > 0.",
  "Neg(x) := x is a negative number, i.e., x < 0."],
"rules": ["IsLarger(x, y) & Pos(z) => IsLarger(x*z, y*z)",
  "IsLarger(x, y) & Neg(z) => IsLarger(y*z, x*z)"]

```

99 3.3 Specifications used in case study 3

100 In the third case study, we observe that it is not feasible to encode all algebraic properties, rules, or
 101 reasoning strategies into specifications using ESL, or even into natural language. In light of this
 102 limitation, we instead identify several common reasoning errors exhibited by LLMs and formalize
 103 these into three distinct specifications, each designed to capture a specific type of error.

104 **Factorization checking.** This specification encodes the properties that must hold for a correct
 105 factorization process, e.g., $a_0x^2 + b_0x + c_0 = (a_1x + b_1)(a_2x + b_2) \Rightarrow (a_0 = a_1a_2) \& (b_0 =$
 106 $a_1b_2 + b_1a_2) \& (c_0 = b_1b_2).$

```

Type-1: Factorization # factorization properties.

"Variables": ["a_0", "a_1", "a_2", "b_0", "b_1", "b_2", "c_0", "e_0", "e_1", "e_2", "x"],
"Predicates": ["Equal(a_0, a_1) := a_0 is equal to a_1, i.e., a_0 = a_1.",
  "IsPolyOne(a_0, a_1, a_2, b_0) := there is an expression a_0 in the form of a_1 * b_0 +
  ↪ a_2, where b_0 is the variable.",
  "IsPolyTwo(a_0, a_1, a_2, b_0, b_1) := there is an expression a_0 in the form of a_1 *
  ↪ b_1^2 + a_2 * b_1 + b_0, where b_1 is the variable.",
  "PolyFactorizeNeg(a_0, a_1, a_2) := there is an expression in the form of a_0 =
  ↪ -(a_1)(a_2).",
  "PolyFactorizePos(a_0, a_1, a_2) := there is an expression in the form of a_0 =
  ↪ (a_1)(a_2) or -a_0 = -(a_1)(a_2).",
  "PolyFactorizeSquareNeg(a_0, a_1) := there is an expression in the form of a_0 =
  ↪ -(a_1)^2.",
  "PolyFactorizeSquarePos(a_0, a_1) := there is an expression in the form of a_0 = (a_1)^2
  ↪ or -a_0 = -(a_1)^2."],
"rules": ["IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & IsPolyOne(e2, a2, b2, x) &
  ↪ PolyFactorizePos(e, e1, e2) => Equal(a, a1 * a2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & IsPolyOne(e2, a2, b2, x) &
  ↪ PolyFactorizePos(e, e1, e2) => Equal(b, a1 * b2 + b1 * a2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & IsPolyOne(e2, a2, b2, x) &
  ↪ PolyFactorizePos(e, e1, e2) => Equal(c, b1 * b2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & IsPolyOne(e2, a2, b2, x) &
  ↪ PolyFactorizeNeg(e, e1, e2) => Equal(a, -a1 * a2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & IsPolyOne(e2, a2, b2, x) &
  ↪ PolyFactorizeNeg(e, e1, e2) => Equal(b, -a1 * b2 - b1 * a2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & IsPolyOne(e2, a2, b2, x) &
  ↪ PolyFactorizeNeg(e, e1, e2) => Equal(c, -b1 * b2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & PolyFactorizeSquarePos(e, e1) =>
  ↪ Equal(a, a1 * a1)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & PolyFactorizeSquarePos(e, e1) =>
  ↪ Equal(b, a1 * b1 * 2)",
  "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & PolyFactorizeSquarePos(e, e1) =>
  ↪ Equal(c, b1 * b1)",

```

```

    "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & PolyFactorizeSquareNeg(e, e1) =>
    ↪ Equal(a, -a1 * a1)",
    "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & PolyFactorizeSquareNeg(e, e1) =>
    ↪ Equal(b, -a1 * b1 * 2)",
    "IsPolyTwo(e, a, b, c, x) & IsPolyOne(e1, a1, b1, x) & PolyFactorizeSquareNeg(e, e1) =>
    ↪ Equal(c, -b1 * b1)"]

```

107 **Interval analysis.** The interval analysis specification encodes the property that, for any given concrete
108 closed interval $[a, b]$ or open interval $(a, b]$, it must always hold that $a \leq b$. Furthermore, when the
109 target LLM attempts to select a point c within the interval between a and b to test the validity of
110 the interval as a solution, it must satisfy $a \leq c \leq b$. Note that the deduced properties $a \leq b$ and
111 $a \leq c \leq b$ represent necessary, but not sufficient, conditions for a correct interval analysis, and they
112 always hold in all valid cases.

```

Type-1:      Interval analysis

"Variables":  ["a_0", "b_0", "c_0"],
"Predicates": ["IsInterval(a_0, b_0) := a_0 and b_0 are endpoints for an interval, where a_0 is the
↪ left endpoint and b_0 is the right endpoint.",
    "InLessOrder(a_0, b_0, c_0) := there exist point order in the form of a_0 < b_0 < c_0,
    ↪ or a_0 ≤ b_0 ≤ c_0, or a_0 < b_0 ≤ c_0, or a_0 ≤ b_0 < c_0, where b_0 is
    ↪ the variable, a_0 and c_0 are concrete numbers.",
    "TestInterval(a_0, b_0, c_0) := choose a concrete number c_0 to test the concrete
    ↪ interval between the concrete number a_0 and the concrete number b_0",
    "NoGreaterThanOrEqual(a_0, b_0) := the value a_0 is no greater than the value b_0",
    "IsInfty(a_0) := a_0 is +\infty or -\infty.",
    "IsNegInfty(a_0) := a_0 is -\infty."],
"rules":     ["IsInterval(a_0, b_0) => NoGreaterThanOrEqual(a_0, b_0)",
    "InLessOrder(a_0, b_0, c_0) => NoGreaterThanOrEqual(a_0, c_0)",
    "TestInterval(a_0, b_0, c_0) => NoGreaterThanOrEqual(a_0, c_0)",
    "TestInterval(a_0, b_0, c_0) => NoGreaterThanOrEqual(c_0, b_0)"]

```

113 **Endpoints checking.** This specification encodes the requirement that if certain endpoints are
114 determined to be undefined, for example, due to division by zero, then such points must be excluded
115 from the final result. Conversely, points that are determined to be well-defined must be included in
116 the final answer.

```

Type-1:      Interval analysis

"Variables":  ["x", "y", "z"],
"Predicates": ["Defined(x, y) := No expression is undefined when x = y, where y is a number and x is
↪ the variable.",
    "Undefined(x, y) := some expression is undefined (e.g., make the denominator as zero)
    ↪ when x = y, where y is a number and x is the variable.",
    "Exclude(x, y) := according to the context, the variable x ≠ y, i.e., the variable x
    ↪ cannot be equal to value y.",
    "IsAns(x, y) := the variable x can take value y in the final answer, and y is neither
    ↪ -\infty nor \infty.",
    "IsCriticalPoint(x, y) := the number y is a critical point for the variable x according
    ↪ to the context.",
    "IsVar(x) := x is the variable of the raw inequality question."],
"rules":     ["Undefined(x, y) => ~IsAns(x, y)",
    "IsCriticalPoint(x, y) & IsAns(x, y) => not Undefined(x, y)",
    "Exclude(x, y) => ~IsAns(x, y)",
    "IsCriticalPoint(x, y) & Defined(x, y) => IsAns(x, y)",
    "IsAns(x, y) => Defined(x, y)",
    "IsAns(x, y) => not Undefined(x, y)"]

```

117 4 Prompt design for the perception agents

118 In this section, we present the prompt used across all experiments. It is worth noting that, for
 119 more effective perception, it is recommended to design distinct prompt templates tailored to specific
 120 downstream, domain-specific tasks.

121 4.1 Prompt design for case study 1

122 **Query prompt.** The prompt shown in Figure 2 serves as the original query template provided to the
 123 target LLM, with the testing scenario serving as the context part, i.e., `[[Context]]`.

```

Task Description:
  (1) Please evaluate the severity for the following context: True (There is no misconduct behavior in the context according to the railway
  laws in Singapore) or False (There exists some misconduct in the context according to the railway laws in Singapore).
  (2) Please only output the single word 'True' or 'False' but nothing else.
Context:
[[Context]]
  
```

Figure 2: The prompt template of the query to the target LLM used in case study 1, where the `[[Context]]` placeholder will be substituted with the corresponding testing scenario.

124 **Interpretation prompt.** The prompt template provided in Figure 3 is designed for the perception
 125 agent to perform interpretation, that is, to abstract all feasible facts, or instantiations of the given
 126 predicates, based on the specification file. Given a specific specification file, the corresponding
 127 interpretation prompt can be generated by substituting the placeholders: i) for `[[Variables]]` and
 128 `[[Predicates]]` with the corresponding elements from the specification file, and ii) for `[[Context]]` with
 129 the relevant testing scenario.

```

Task Description: You are a perception agent with NO reasoning abilities.
  (1) Get the "Objects" and "Interpretation" only based on the "Context" within the "Domain" following the way used in the "Template".
  (2) There are three values of each predicate: True, False, and Unknown.
  (3) Set "Unknown" to the predicate when there is no "explicit evidence".
  (4) The **arguments** in each interpretation must be **from Objects**.
  (5) Only output two lists but nothing else: "Objects" and "Interpretations".
----- Template Start -----
Variables: x, y
Predicates:
  inMRT(x) := x is on MRT.
  consuming(x) := x is consuming something.
Context:
  Alice and Bob are on the MRT. Alice is eating bread.
Objects:
  {Alice, Bob, bread}
Interpretations:
  {inMRT(Alice) = True; inMRT(Bob) = True; consuming(Alice, bread) = True; consuming(Bob, bread) = Unknown;}
----- Template End -----
----- Domain Start -----
Variables:
  [[Variables]]
Predicates:
  [[Predicates]]
Context:
  [[Context]]
----- Domain End -----
  
```

Figure 3: The prompt template of interpretation abstraction used in case study 1. The `[[Variables]]` and `[[Predicates]]` placeholders are substituted with the ones from the specification file. The `[[Context]]` placeholder is then substituted with the testing scenario.

130 **Inferred knowledge prompt.** The prompt template provided in Figure 4 is designed to issue a new
 131 query to the target LLM in order to question the inferred knowledge in the first case study. Note that
 132 the second round of queries related to the knowledge inferred must refer to its original context, i.e.,
 133 the testing scenario.

```

Task Description:
(1) Given the Context and Predicates, please check the Boolean value of the Proposition.
(2) Return True or False only.
(3) Only output True or False.
Context:
[[Context]]
Predicates:
[[Predicates]]
Proposition:
[[Proposition]]

```

Figure 4: The prompt template of query for the inferred knowledge used in case study 1. The `[[Predicates]]` are substituted with the ones from the specification file, the `[[Context]]` is replaced with the testing scenario, and the `[[Proposition]]` will be replaced with the inferred knowledge, which is represented as instantiated predicates.

134 4.2 Prompt design for case study 2

135 **Interpretation prompt.** The prompt template given in Figure 5 is designed for the perception agent
 136 to do the interpretation for case study 2. The substitution of the placeholders `[[Variables]]` and
 137 `[[Predicates]]` follows the same procedure as illustrated in Figure 3. The `[[Context]]` in this case
 138 consists of the comparison query provided to the target LLM and its corresponding response.

```

Task Description: You are a perception agent with NO reasoning abilities.
(1) Get the "Objects" and "Interpretation" only based on the "Context" within the "Domain" following the way used in the "Template".
(2) There are three values of each predicate: True, False, and Unknown.
(3) Set "Unknown" to the predicate when there is no "explicit evidence".
(4) The **arguments** in each interpretation must be **from Objects**.
(5) Only output two lists but nothing else: "Objects" and "Interpretations".
----- Template Start -----
Variables: x, y
Predicates:
  IsLarger(x, y) := x is larger than y.
  Pos(x) := x is a positive number, i.e., x > 0.
  Neg(x) := x is a negative number, i.e., x < 0.
Context:
  Question: 1 and 10, which one is greater? Answer: 10
Objects:
  {1, 10}
Interpretations:
  {IsLarger(10, 1) = True; IsLarger(1, 10) = False; Pos(1) = Unknown; Neg(1) = Unknown; Pos(10) = Unknown; Neg(10) = Unknown;}
----- Template End -----
----- Domain Start -----
Variables:
  [[Variables]]
Predicates:
  [[Predicates]]
Context:
  [[Context]]
----- Domain End -----

```

Figure 5: The prompt template of interpretation abstraction used in case study 2. The `[[Variables]]` and `[[Predicates]]` placeholders are substituted with the ones from the specification file. The `[[Context]]` placeholder is replaced with the comparison question together with the corresponding response generated by the target LLM.

139 **Level-2 interpretation prompt (new instance generation).** The prompt template provided in
 140 Figure 6 is designed to generate an instance for a predicate that failed to be fully instantiated, resulting
 141 in a partially interpreted rule. Notably, we reuse the template part from Case Study 1 directly, as this
 142 allows us to avoid introducing a standard or expected answer. Instead, using a non-referential template
 143 ensures greater randomness in the instance generation process, which is essential for this comparison
 144 task. The placeholders `[[Variables]]` and `[[Predicates]]` are replaced with the corresponding elements
 145 from the specification file, while `[[Values]]` represent the predicate along with its assigned truth
 146 value—guiding the perception agent in generating the desired instance.

147 **Inferred knowledge prompt.** The prompt template provided in Figure 7 is designed to issue a
 148 new query to the target LLM in order to question the inferred knowledge in the second case study.
 149 Notably, the original context is not required for this new query as the inferred knowledge is all about


```

Task Description:
(1) Generate an instance for each variable in the predicate and satisfy the predicate value. You can follow the way used in "Template".
(2) The predicate values should be satisfied at the same time.
(3) Only output the variable instance value with nothing else.
----- Template Start -----
Variables: x, y
Predicates: InRailwayPremises(x, y) := a person x is in some railway premises y.
Values: {InRailwayPremises(x, y) = True}
Instance: {x = Alice; y = Railway station}
----- Template End -----
----- Domain Start -----
Variables:
[[Variables]]
Predicates:
[[Predicates]]
Values:
[[Values]]
----- Domain End -----

```

Figure 6: The prompt template of new instance generation for the partially interpreted rule. The `[[Variables]]` and `[[Predicates]]` are replaced with the ones from the specification file. The `[[Values]]` is replaced with the predicate along with its assigned truth value—guiding the perception agent in generating the desired instance.

150 concrete numerical comparison questions; only the relevant predicate definitions `[[Predicates]]` and
151 the inferred propositional knowledge `[[Propositions]]` are needed.

```

Task Description: You are a perception agent with NO reasoning abilities.
(1) Given the Predicates, please check the Boolean value of the Proposition.
(2) Return True or False only.
(3) Only output True or False.
Predicates:
[[Predicates]]
Proposition:
[[Proposition]]

```

Figure 7: The prompt template of query for the inferred knowledge used in case study 2. The `[[Predicates]]` is replaced with the ones from the specification file. The `[[Proposition]]` is replaced with the inferred knowledge, which is represented as instantiated predicates.

152 4.3 Prompt design for case study 3

153 **Interpretation prompt.** Unlike the prompt templates used in the previous two case studies, due to
154 the increased complexity of this case study, we reuse the same predicates within the template and
155 explicitly provide the corresponding context, objects, and interpretations. Within this setup, only the
156 `[[Context]]` placeholder needs to be replaced with the step-by-step reasoning process generated by
157 the target LLM, as illustrated in Figures 8, 9, and 10.

158 **Inferred knowledge prompt.** For the factorization and interval specifications, we employ the
159 same prompt template shown in Figure 7. However, for the endpoints specification, the target
160 LLM must refer to its original step-by-step reasoning process when answering inferred knowledge
161 related to endpoint information. Accordingly, when the inferred knowledge requires access to the
162 original context for an accurate response, the prompt template is designed to include such contextual
163 information. Hence, the corresponding prompt template is the same as the one given in Figure 4.

164 5 Experiment statistical significance

165 The only source of variability in RvLLM arises from the perception results generated by the perception
166 agent. To minimize this effect, we set the temperature parameter of the perception agent to a relatively
167 low value (currently 0.5) and perform multiple runs during preliminary experiments until the outputs
168 stabilize and remain consistent across iterations. This ensures that the experimental results reported
169 in the paper exhibit negligible error margins.

```

Task Description: You are a perception agent with NO reasoning abilities.
(1) Get the "Objects" and "Interpretation" ONLY based on the "Context" within the "Domain" without any reasoning procedure.
(2) There are three values of each predicate: True, False, and Unknown.
(3) The **arguments** in each interpretation must be **from Objects**.
(4) Only output two lists: Objects and Interpretation.
----- Template Start -----
Variables: [[Variables]]
Predicates:
[[Predicates]]
Context:
### Factor the quadratic expression
 $-x^2 - ax + b = -(x^2 + cx + d) = -(x+e)^2$ 
Simplify  $(x+a)(x-b)$  and get  $(x^2 + cx - d)$ .
Factor  $y^2 - cy + d = (y-a)(y+b)$ 
Objects:
{minus_x_squared_minus_2_x_minus_1, x_squared_plus_c_x_minus_d, y_squared_minus_c_y_plus_d,
x_plus_e, x_plus_a, x_minus_b, y_minus_a, y_plus_b, -1, -a, b, 1, c, d, e, a, -b, -d, -c}
Interpretations:
{PolyFactorizeSquareNeg(minus_x_squared_minus_a_x_plus_b, x_plus_e) = True;
PolyFactorizeSquarePos(x_squared_plus_c_x_plus_d, x_plus_e) = True;
PolyFactorizePos(x_squared_minus_c_x_minus_d, x_plus_a, x_minus_b) = True;
PolyFactorizePos(y_squared_minus_c_y_plus_d, y_minus_a, y_plus_b) = True;
IsPolyTwo(minus_x_squared_minus_a_x_plus_b, -1, -a, b, x) = True; IsPolyTwo(x_squared_plus_c_x_plus_d, 1, c, d, x) = True;
IsPolyTwo(x_squared_minus_c_x_minus_d, 1, c, -d, x) = True; IsPolyTwo(y_squared_minus_c_y_plus_d, 1, -c, d, x) = True;
IsPolyOne(x_plus_e, 1, e, x) = True; IsPolyOne(x_plus_a, 1, a, x) = True; IsPolyOne(x_minus_b, 1, -b, x) = True;
IsPolyOne(y_minus_a, 1, -a, y) = True; IsPolyOne(y_plus_b, 1, b, y) = True;}
----- Template End -----
----- Domain Start -----
Context:
[[Context]]
----- Domain End -----

```

Figure 8: The prompt template of interpretation abstraction used in case study 3: Factorization Error. The `[[Variables]]` and `[[Predicates]]` are substituted with the ones from the specification file, the `[[Context]]` is replaced with the target LLM’s step-by-step reasoning process.

6 Broader impacts

The integration of domain-specific knowledge into LLM verification, as proposed in this work, offers significant benefits by enhancing the reliability and trustworthiness of LLMs in high-stakes applications. By introducing a general specification language ESL and a runtime verification framework RvLLM, our approach empowers domain experts to encode structured knowledge that can systematically detect and mitigate inconsistencies in LLM outputs. This not only improves the accuracy of model responses in specialized domains—such as legal compliance or mathematical reasoning—but also paves the way for deploying LLMs in safety-critical systems where formal guarantees are essential. However, potential risks remain. The effectiveness of RvLLM depends on both the performance of the perception agent and the quality and correctness of the specifications provided by human experts. If these components are not carefully designed and validated, they may introduce new sources of bias or error. Therefore, while our framework represents a promising step toward improving the robustness of AI systems, it should be regarded as a complementary mechanism rather than a complete solution to the broader challenges of ensuring reliability and consistency in LLM-based reasoning at the current stage of investigation.

```

Task Description: You are a perception agent with NO reasoning abilities.
(1) Get the "Objects" and "Interpretation" ONLY based on the "Context" within the "Domain" without any reasoning procedure.
(2) There are three values of each predicate: True, False, and Unknown.
(3) The arguments in each interpretation must be from Objects.
(4) Only output two lists: Objects and Interpretation.
----- Template Start -----
Variables: [[Variables]]
Predicates:
[[Predicates]]
Context:
- For  $a < x < b$ 
- For  $b < x < c$ 
 $[c, d], [d, e], [1-\sqrt{3}, 1+\sqrt{3}]$ 
For  $\forall (x \in (Val\_0, Val\_1))$ : Choose  $\forall (x = Val\_2)$ .
#### Interval  $(Val\_0, Val\_1)$ : Choose  $\$x = Val\_3$ .
Objects:
{a, x, b, c, d, e, Val_0, Val_1, Val_2, -a, -1,  $1-\sqrt{3}$ ,  $1+\sqrt{3}$ }
Interpretations:
{InLessOrder(a, x, b) = True; InLessOrder(b, x, c) = True;
IsInterval(c, d) = True; IsInterval(d, e) = True; IsInterval( $1-\sqrt{3}$ ,  $1+\sqrt{3}$ ) = True;
TestInterval(Val_0, Val_1, Val_2) = True; TestInterval(Val_0, Val_1, Val_3) = True;}
----- Template End -----
----- Domain Start -----
Context:
[[Context]]
----- Domain End -----

```

Figure 9: The prompt template of interpretation abstraction used in case study 3: Interval Error. The [[Variables]] and [[Predicates]] are substituted with the ones from the specification file, the [[Context]] is replaced with the target LLM’s step-by-step reasoning process.

```

Task Description: You are a perception agent with NO reasoning abilities.
(1) Get the "Objects" and "Interpretation" ONLY based on the "Context" within the "Domain" without any reasoning procedure.
(2) There are three values of each predicate: True, False, and Unknown.
(3) The arguments in each interpretation must be from Objects.
(4) Only output two lists: Objects and Interpretation.
----- Template Start -----
Variables: [[Variables]]
Predicates:
[[Predicates]]
Context:
Determine the critical points:
 $\forall \{a, b, Val\_0, Val\_1, Val\_2\}$ 
Note:
-  $\forall (x \neq Val\_0)$ 
-  $x = Val\_1$ : some expression is undefined.
- At  $\forall (x = Val\_2)$ : the denominator is zero
Final Answer: The final answer is  $\boxed{(Val\_2, Val\_3) \cup [Val\_4, \infty)}$ .
Objects:
{a, b, Val_0, Val_1, Val_2, Val_3, Val_4, x,  $1-\sqrt{3}$ ,  $1+\sqrt{3}$ }
Interpretations:
{IsCriticalPoint(x, a) = True; IsCriticalPoint(x, b) = True; IsCriticalPoint(x, Val_0) = True;
IsCriticalPoint(x, Val_1) = True; IsCriticalPoint(x, Val_2) = True; Defined(x, a) = True; Defined(x, b) = True;
Exclude(x, Val_0) = True; Undefined(x, Val_1) = True; Undefined(x, Val_2) = True;
IsAns(x, Val_2) = False; IsAns(x, Val_3) = False; IsAns(x, Val_4) = True;}
----- Template End -----
----- Domain Start -----
Context:
[[Context]]
----- Domain End -----

```

Figure 10: The prompt template of interpretation abstraction used in case study 3: Endpoints Error. The [[Variables]] and [[Predicates]] are substituted with the ones from the specification file, the [[Context]] is replaced with the target LLM’s step-by-step reasoning process.

7 An illustration example

In this section, we illustrate the usage of RvLLM through a concrete example drawn from Case Study 1. Now consider the following scenario which return “Safe” by the target LLM qwen-plus:

“A commuter on the MRT carefully transported their pet snake in a secure, locked container designed to ensure the safety of other passengers. The container was opaque to minimize any potential disturbance or alarm among fellow commuters. The owner calmly explained to curious onlookers that the snake was being moved to a new home and assured them of the safety measures taken. Throughout the journey, the snake remained quiet and undetectable, causing no disruption to the train’s operation or its passengers.”

Given the input scenario, RvLLM first invokes the perception agent, which, in this setting, is also qwen-plus, to extract the objects and their associated interpretations, all of which are listed in Table 1.

Table 1: The observations obtained by the perception agent

objects_all	“commuter”, “snake”, “container”
interPre_all	“IsAnimal(snake) = True”, “IsGuideDog(snake) = False”, “IsOfficerDog(snake) = False” “CauseDamageOrInjury(snake) = False”, “InRailwayPremises(commuter) = True” “InRailwayPremises(snake) = True”

Recall that the specification rules provided in the “Bring Animal” regulation are as follows:

1. $\text{IsAnimal}(x) \ \& \ \text{InRailPremises}(x) \ \& \ \neg \text{IsOfficerDog}(x) \Rightarrow \text{IsGuideDog}(x)$
2. $\text{IsAnimal}(x) \ \& \ \text{InRailPremises}(x) \ \& \ \neg \text{IsGuideDog}(x) \Rightarrow \text{IsOfficerDog}(x)$

Following the interpretation process, we obtain the propositional, rule-like formulas listed in Table 2, which are instantiated based on the defined predicates and the corresponding set of objects.

Table 2: The interpreted propositional rule-like formula derived from the specification rules, where formulae (i)-(iii) are obtained from the interpretation of the first rule, and formulae (iv)-(vi) are derived from the second rule.

i	$\text{IsAnimal}(\text{commuter}) \ \& \ \text{InRailPremises}(\text{commuter}) \ \& \ \neg \text{IsOfficerDog}(\text{commuter}) \Rightarrow \text{IsGuideDog}(\text{commuter})$
ii	$\text{IsAnimal}(\text{snake}) \ \& \ \text{InRailPremises}(\text{snake}) \ \& \ \neg \text{IsOfficerDog}(\text{snake}) \Rightarrow \text{IsGuideDog}(\text{snake})$
iii	$\text{IsAnimal}(\text{container}) \ \& \ \text{InRailPremises}(\text{container}) \ \& \ \neg \text{IsOfficerDog}(\text{container}) \Rightarrow \text{IsGuideDog}(\text{container})$
iv	$\text{IsAnimal}(\text{commuter}) \ \& \ \text{InRailPremises}(\text{commuter}) \ \& \ \neg \text{IsGuideDog}(\text{commuter}) \Rightarrow \text{IsOfficerDog}(\text{commuter})$
v	$\text{IsAnimal}(\text{snake}) \ \& \ \text{InRailPremises}(\text{snake}) \ \& \ \neg \text{IsGuideDog}(\text{snake}) \Rightarrow \text{IsOfficerDog}(\text{snake})$
vi	$\text{IsAnimal}(\text{container}) \ \& \ \text{InRailPremises}(\text{container}) \ \& \ \neg \text{IsGuideDog}(\text{container}) \Rightarrow \text{IsOfficerDog}(\text{container})$

For clarity and ease of representation, we use the notation summarized in Table 3 to represent the propositions appearing in the interpreted rule-like formulas. The table also includes the corresponding truth values, which are determined by the perception results from the perception agent. If a truth value cannot be obtained through perception, it is assigned the value Unknown directly (under the open-world assumption).

Table 3: The propositional representations of the interpreted predicates, where \top , \perp , and Unk. denote the truth value as True, False, and Unknown, respectively.

$p_0: \text{IsAnimal}(\text{commuter}) = \text{Unk.}$	$p_1: \text{IsAnimal}(\text{snake}) = \top$	$p_2: \text{IsAnimal}(\text{container}) = \text{Unk.}$
$p_3: \text{InRailPremises}(\text{commuter}) = \top$	$p_4: \text{InRailPremises}(\text{snake}) = \top$	$p_5: \text{InRailPremises}(\text{container}) = \text{Unk.}$
$p_6: \text{IsOfficerDog}(\text{commuter}) = \text{Unk.}$	$p_7: \text{IsOfficerDog}(\text{snake}) = \perp$	$p_8: \text{IsOfficerDog}(\text{container}) = \text{Unk.}$
$p_9: \text{IsGuideDog}(\text{commuter}) = \text{Unk.}$	$p_{10}: \text{IsGuideDog}(\text{snake}) = \perp$	$p_{11}: \text{IsGuideDog}(\text{container}) = \text{Unk.}$

We then obtain the following rephrased propositional, rule-like formulas, as presented in Table 4.

After conducting the forward chaining procedure, we identify an inconsistency arising from the following propositional rules: $p_1 \ \& \ p_4 \ \& \ \neg p_7 \Rightarrow p_{10}$ and $p_1 \ \& \ p_4 \ \& \ \neg p_{10} \Rightarrow p_7$. The corresponding

Table 4: The rephrased propositional rule-like formula as well as the truth value of the propositions.

i	$ p_0 \& p_3 \& \neg p_6 \Rightarrow p_9 $	$p_0 = p_6 = p_9 = \text{Unk.}, p_3 = \top$
ii	$ p_1 \& p_4 \& \neg p_7 \Rightarrow p_{10} $	$p_1 = p_4 = \top, p_7 = p_{10} = \perp$
iii	$ p_2 \& p_5 \& \neg p_8 \Rightarrow p_{11} $	$p_2 = p_5 = p_8 = p_{11} = \text{Unk.}$
iv	$ p_0 \& p_3 \& \neg p_9 \Rightarrow p_6 $	$p_0 = p_9 = p_6 = \text{Unk.}, p_3 = \top$
v	$ p_1 \& p_4 \& \neg p_{10} \Rightarrow p_7 $	$p_1 = p_4 = \top, p_{10} = p_7 = \perp$
vi	$ p_2 \& p_5 \& \neg p_{11} \Rightarrow p_8 $	$p_2 = p_5 = p_{11} = p_8 = \text{Unk.}$

inconsistent knowledge are:

$\text{IsAnimal}(\text{snake}) \& \text{InRailwayPremises}(\text{snake}) \& \neg \text{IsOfficerDog}(\text{snake}) \Rightarrow \text{IsGuideDog}(\text{snake})$

$\text{IsAnimal}(\text{snake}) \& \text{InRailwayPremises}(\text{snake}) \& \neg \text{IsGuideDog}(\text{snake}) \Rightarrow \text{IsOfficerDog}(\text{snake})$

Analysis. It is worth noting that the target LLM does not identify any violation, as the context appears to indicate no disruption to the train’s operation or its passengers. However, according to the strict regulations on bringing animals aboard, only guide dogs and official dogs are permitted on the premises. By leveraging domain-specific knowledge encoded in the ESL-based specification, RvLLM successfully detects the violation that the LLM overlooks.

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