# A Complete Proof

## A.1 Maximum Independent Set

In maximum independent set, we use the energy function:

$$f(x) := -\sum_{i=1}^{n} w_i x_i + \sum_{(i,j)\in E} \beta_{ij} x_i x_j \tag{25}$$

We are going to prove the following proposition.

**Proposition A.1.** *If $\beta_{ij} \geq \min\{w_i, w_j\}$ for all $(i,j) \in E$, then for any $x \in \{0,1\}^n$, there exists a $x' \in \{0,1\}^n$ that satisfies the constraints in (16) and has lower energy: $f(x') \leq f(x)$.*

*Proof.* For arbitrary $x \in \{0,1\}^n$, if $x$ satisfies all constraints, we only need to let $x' = x$. Else, there must exist an edge $(i,j) \in E$, such that $x_i x_j = 1$. Denote $k = \arg\min\{w_i, w_j\}$, we define $x'_i = x_i$ if $i \neq k$ and $x'_k = 0$. In this case, we have:

$$f(x') - f(x) = w_k - \sum_{j\in N(k)} \beta_{k,j} x_j \leq w_k(1 - \sum_{j\in N(k)} x_j) \leq 0 \tag{26}$$

Thus we show $f(x') \leq f(x)$.

On the other side, consider a graph $G = (V = \{1,2\}, E = \{(1,2)\})$ and $\beta_{12} < w_1 < w_2$. Then the maximum independent set is $\{2\}$, which can be represented by $x = (0,1)$. However, in this case, let $x' = (1,1)$ is feasible while $f(x') \leq f(x)$. This means the condition we just derived is sharp. $\square$

## A.2 Maximum Clique

In maximum independent set, we use the energy function:

$$f(x) := -\sum_{i=1}^{n} w_i x_i + \sum_{(i,j)\in E^c} \beta_{ij} x_i x_j \tag{27}$$

We are going to prove the following proposition.

**Proposition A.2.** *If $\beta_{ij} \geq \min\{w_i, w_j\}$ for all $(i,j) \in E^c$, then for any $x \in \{0,1\}^n$, there exists a $x' \in \{0,1\}^n$ that satisfies the constraints in (18) and has lower energy: $f(x') \leq f(x)$.*

*Proof.* For arbitrary $x \in \{0,1\}^n$, if $x$ satisfies all constraints, we only need to let $x' = x$. Else, there must exist an edge $(i,j) \in E^c$, such that $x_i x_j = 1$. Denote $k = \arg\min\{w_i, w_j\}$, we define $x'_i = x_i$ if $i \neq k$ and $x'_k = 0$. In this case, we have:

$$f(x') - f(x) = w_k - \sum_{j:(k,j)\in E^c} \beta_{k,j} x_j \leq w_k(1 - \sum_{j:(k,j)\in E^c} x_j) \leq 0 \tag{28}$$

Thus we show $f(x') \leq f(x)$.

On the other side, consider a graph $G = (V = \{1,2\}, E = \{\})$ and $\beta_{12} < w_1 < w_2$. Then the maximum clique is $\{2\}$, which can be represented by $x = (0,1)$. However, in this case, let $x' = (1,1)$ is feasible while $f(x') \leq f(x)$. This means the condition we just derived is sharp. $\square$

## A.3 Minimum Dominate Set

In maximum independent set, we use the energy function:

$$f(x) := \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \beta_i (1 - x_i) \prod_{j\in N(i)} (1 - x_j) \tag{29}$$

We are going to prove the following proposition.

**Proposition A.3.** *If $\beta_i \geq \min_k\{w_k : k \in N(i)$ or $k = i\}$, then for any $x \in \{0,1\}^n$, there exists a $x' \in \{0,1\}^n$ that satisfies the constraints in (18) and has lower energy: $f(x') \leq f(x)$.*

*Proof.* For arbitrary $x \in \{0, 1\}^n$, if $x$ satisfies all constraints, we only need to let $x' = x$. Else, there must exist a node $t \in V$, such that $x_t = 0$ and $x_j = 0$ for all $j \in N(t)$. Let $k = \arg\min\{w_j : j \in N(t), \text{ or } j = t\}$, we define $x'_i = x_i$ if $i \neq k$ and $x'_k = 1$. In this case, we have:

$$f(x') - f(x) = w_k - \beta_t + \sum_{i \neq t} \beta_i \Big[(1 - x'_i) \prod_{j \in N(i)} (1 - x'_j) - (1 - x_i) \prod_{j \in N(i)} (1 - x_j)\Big] \leq 0 \quad (30)$$

Thus, we prove $f(x') \leq f(x)$.

On the other side, consider a graph $G = (V = \{1\}, E = \{\})$ and $\beta_1 < w_1$. Then the maximum clique is $\{1\}$, which can be represented by $x = (1)$. However, in this case, let $x' = (0)$ is feasible while $f(x') \leq f(x)$. This means the condition we just derived is sharp. □

### A.4 Minimum Cut

In maximum independent set, we use the energy function:

$$f(x) := \sum_{(i,j) \in E} x_i(1 - x_j)w_{ij} + \beta(\sum_{i=1}^{n} d_i x_i - D_1)_+ + \beta(D_0 - \sum_{i=1}^{n} d_i x_i)_+ \quad (31)$$

We are going to prove the following proposition.

**Proposition A.4.** *If $\beta \geq \max_i\{\sum_{j \in N(i)} |w_{i,j}|\}$, then any $x \in \{0, 1\}^n$, there exists a $x' \in \{0, 1\}^n$ that satisfies the constraints in (18) and has lower energy: $f(x') \leq f(x)$.*

## B Experiment Details

### B.1 Hardware

All methods were run on Intel(R) Xeon(R) Gold 5215 CPU @ 2.50GHz, with 377GB of available RAM. The neural networks were executed on a single RTX6000 25GB graphics card. The code was executed on version 1.9.0 of PyTorch and version 1.7.2 of PyTorch Geometric.